http://economix.u-paris10.fr/

Document de Travail

Working Paper

2007-20

Maximizing minimal rights for sustainability: a viability approach

Vincent MARTINET

UMR 7166 CNRS

Université Paris X-Nanterre
Maison Max Weber (bâtiments K et G)
200, Avenue de la République
92001 NANTERRE CEDEX

Tél et Fax : 33.(0)1.40.97.59.07
Email : secretariat-economix@u-paris10.fr

Université Paris X Nanterre

Maximizing minimal rights for sustainability: a viability approach

Vincent MARTINET
EconomiX,
Université Paris X Nanterre,
200 av. de la République, 92001 Nanterre Cedex.
Tel: +33 1 40 97 59 36;
Email: vincent.martinet@u-paris10.fr

March 21, 2007

Abstract

This paper examines how the viability approach can be used to define sustainability goals. In an economic model with a non renewable natural resource, we define minimal rights to be guaranteed for all generations. These rights can include a minimal consumption (economic goal) and the preservation of natural resources (environmental goal). From a given economic state, it is possible to define the set of minimal rights that can be provided for all generation. To address the intergenerational equity issue, we propose to use a criterion that define the set of minimal rights that provide the maximal utility, in a Rawlsian perspective (Rawls, 1971). We describe how this criterion can be applied and computed, and discuss it with respect to usual criteria, including the maximin criterion, the *Green Golden Rule*, the Chichilnisky approach and the Mixed Bentham-Rawls criterion.

Key-words: sustainability, intergenerational equity, minimal rights, viability.
JEL Classification: Q01, Q32, O13, C61.

1 Introduction

Solow (1993, p.167-168) claimed that

> *"If the sustainability means anything more than a vague emotional commitment, it must require that something be conserved for the very long run. It is very important to understand what that thing is: I think it has to be a generalized capacity to produce economic well-being."*

Asheim et al. (2001) argue that sustainability requires the utility to be non decreasing along time, for equity concerns. Stavins et al. (2003) interpret sustainability as 'intergenerational equity' plus 'economic efficiency'. Intergenerational equity is defined by requiring a non decreasing utility through time, and economic efficiency is defined with respect to the neo-classical discounted utilitarian criterion. In these approaches, the social objective (sustainability of the utility) is not considered in the objective function (in the criterion to optimize) but as an added constraint to an economic criterion. This approach is criticized by Krautkraemer (1998), Pezzey and Toman (2002) and Cairns and Long (2006) who argue that the objective function has to be defined in order to consider the sustainability issue, and especially intergenerational equity. From that point of view, the debate on sustainability becomes a debate on the social criterion to be optimized.

Heal (1998) examines the various criteria proposed to cope with the sustainability issue. Each criterion characterizes the optimal (sustainable) path and defines what is preserved for sustainability (if anything is preserved).

The most commonly used criterion is the intertemporal sum of discounted utilities

$$\max \int_0^\infty \Delta(t) U_t dt. \tag{1}$$

This criterion, which has a solution only if the discount factor $\Delta(t)$ decreases towards zero at the infinite time[1], is criticized because it does not take long term

[1] Usually, the discount factor is decreasing at a constant rate, i.e. $\Delta(t) = e^{-\delta t}$. Other alternatives have been proposed, including hyperbolic discounting which lead to a decreasing discount rate. A general form of hyperbolic discounting is $\Delta(t) = (1 + \alpha t)^{-\gamma/\alpha}$. Nevertheless, the discount factor is still decreasing and the utility of the far future is not taken into account, which raises the equity issue.

utility into account. According to Chichilnisky (1996) this criterion is a dictatorship of the present.

Another proposed criterion (Beltratti et al., 1995) is inspired by the economic Golden Rule, and defines the highest indefinitely maintainable level of instantaneous utility. The criterion reads

$$\max \lim_{t \to \infty} U_t \tag{2}$$

This approach, called the *green golden rule*, does not take into account the present, and is called a "dictatorship of the future" by Chichilnisky (1996).

Solow (1974) and Cairns and Long (2006) address the sustainability issue by using the maximin criterion. This criterion defines the maximal sustainable level of utility, in the sense that it maximizes the utility of the poorest generation:

$$\max \left(\min_t U_t \right) \tag{3}$$

This criterion has been criticized as it may lead to maintain the initial poverty by restricting the investment if the first generation is the poorest.

As mentioned by Beltratti et al. (1995, p.179), "an important task ... lies in the analysis of criteria that combine maximization of discounted utility with elements related to the long run". In other words, sustainability issue relies on the definition of criteria that take into account both short and long run. Chichilnisky (1996) develops such a study and provides the general form that the criterion must have:

$$\max \left(\theta \int_0^\infty \Delta(t) U_t dt + (1-\theta) \lim_{t \to \infty} U_t \right) \tag{4}$$

where $\Delta(t)$ is the discount factor. Nevertheless, the solution of criteria of this form is not easy to compute (See Heal, 1998). Moreover, the criterion is not unique and depends on the choice of the parameter θ and of the discount factor $\Delta(t)$.[2]

In a recent contribution, Long (2006) proposes, in a complementary way of Chichilnisky (1996), to mix a maximin criterion and the utilitarian discounted

[2]The term lim can also be replaced by any function that depends only on the limiting behavior of the utility over time, such as long-run average for example.

criterion. The criterion he proposes, called a "Mixed Bentham-Rawls criterion" reads[3]

$$\max \left(\theta \int_0^{\infty} \Delta(t) U_t dt + (1-\theta) \min_t U_t \right). \tag{5}$$

This criterion is sensitive to the present, to the future and to the least advantaged generation.

As mentioned in the begining of this introduction, sustainability can be interpreted as the requirement to have something preserved in the long-run, in an intergenerational equity perspective. To discuss various criteria with respect to their implications[4], the debate mainly emerges in the intertemporal resource allocation framework and aims at defining an equitable share of consumption and access to natural resources among generations. Criteria are more or less preservative with respect to the resource use and the level of consumption. The maximin criterion is the most explicit in the definition of what is preserved, as it targets the preservation of the utility along time.

In particular, the equity issue is mainly addressed with respect to the consumption of each generation, and the utility function depends on the instantaneous consumption c_t, namely $U(c_t)$. Each criterion leads to a infinite stream of consumption. The emergence of environmental issue has lead to to into account natural resource amenities, which emphasizes the preservation issue. In that perspective, the utilitarian approach can be completed: Natural resources x_t can be a component of the utility function, $U(c_t, x_t)$. Heal (1998) examines the implications of such an approach in the sustainability debate. Depending on the criteria, there is more or less consumption and thus "less or more" preservation of resource stocks.

The Brundtland report, *Our Common Future* (WCED, 1987), defined the *sustainable development* as a "development that meets the needs of the present without compromising the ability of future generations to meet their own needs". This explicit reference to the "needs" of the various generations can be a way to address the issue. An important challenging question is then the definition of the basic

[3]Long (2006) develops the criterion in a discreet time framework and over a finite time horizon. He also consider a constant discount rate. To be consistent within the presentation of the various criteria, we present the criterion a continuous time over an infinite horizon.

[4]According to Dasgupta and Heal (1979, p.311), "*it is legitimate to revise or criticize ethical norms in the light of their implications*".

needs to be guaranteed for a sustainable development.

In the late 70's, the economic development issues where much more concerned with the intragenerational equity issue, and the development of poor countries. It has been argued (Chichilnisky, 1977) that aggregated economic indicators like GDP were not able to encompass the development issue, as it neglects an important dimension of development: the satisfaction of basic needs. In fact, the scarcity of some basic goods can jeopardize economic development, or at least, modify the priority of the social planner: the maximization of Present Net Value of the economy may not be the primary goal of development.

From a more general point of view, one way to address the sustainability issue is to consider the satisfaction of basic needs, or equivalently, of minimal rights. Among these basics needs, one can consider a minimal consumption and a minimal environmental quality. The issue is then to determine the way we define these minimal rights, in an equity concern. According to John Rawls' conception of justice (Rawls, 1971), the first requirement for equity is to choose the allocation of resources that provides the maximal number of minimal rights every one can enjoy.[5] Even if the Rawls' conception of justice was not built for intergenerational equity issues[6], we can examine what minimal rights can be provide to all generations. In our sustainable development issue, it is equivalent to examine what level of consumption, and what level of environmental quality can be provided to all generations.

[5]This result comes from the allocation of rights one would made under the "veil of ignorance". Rawls argues that justice should be based on two principles, with a priority order. The first principle is the definition of fundamental rights every one can enjoy ("*each person is to have an equal right to the most extensive scheme of equal basic liberties compatible with a similar scheme of liberties for others*"). The second principle is based on (with here again a priority order) "fair equality of opportunity" to a social position and on the "difference principle" that stipulates that inequality in the wealth distribution is justified if it is beneficial for the poorest individual, i.e. if the poorest individual in this configuration in richer than the poorest individual in all other possible allocations. This last statement leads to the maximin criterion, which is thus the less important point in Rawls' theory of justice.

[6]According to Rawls, his theory of justice should not be applied to intergenerational problems. Nevertheless, one of the propositions of Rawls (1971) has already been applied to intertemporal equity problems: the maximin criteria (Solow, 1974; Cairns and Long, 2006).

In a way similar to Chichilnisky (1977) that argues that economic development must be consistent with the attainment of adequate levels of *per capita* consumption of basic goods, a primary goal to achieve intergenerational equity can be define as a minimal *per generation* access to basic goods. The satisfaction of basic needs criterion aims at guaranteeing that each generation gets a minimal level of primary goods, while allowing some generation to have more than what is considered sufficient.

In this paper, we propose a methodological approach to determine minimal rights representing sustainability. Minimal rights are defined as a set of constraints (sustainability objectives) that must be respected forever, in an intergenerational equity perspective. Such constraints refer to basic needs of each generation. Chatterjee and Ravikumar (1999) study the implications of minimum consumption requirements on the rate of growth and the evolution of wealth distribution. We focus here on the implications of such a constraint on intertemporal consumption and investment paths, and on intergenerational equity, when the economy relies on the extraction of a non-renewable resource. At the same time, we consider a minimal resource constraint ensuring the preservation of a part of the natural resource.

Using the viability framework, we exhibit intertemporal paths that make it possible to satisfy a set of constraints representing minimal rights for all generations, including a minimal consumption level and a guaranteed resource stock to be preserved. It is a much more general way to address the minimal consumption and viability issue than found in Dawid and Day (2006).

Our approach is however quite different from the sufficientarianism. We do not take a given level of minimal consumption and resource preservation, and then determine the intertemporal path that make it possible to satisfy these particular constraints in the future, like in Chichilnisky (1977). On the contrary, we aim at defining the set of all the achievable goals from the initial state, i.e. the set of minimal consumption levels and preserved resource stock that can be guaranteed forever given the initial state. We then propose a criterion to choose within the set of all possible sustainability goals.

It leads to define a possible mathematical formalization to define minimal rights to be provided for each generation. In that framework, we do not optimize intertemporal paths but level of constraints to be guaranteed forever.

This approach is applied to an intertemporal resource allocation model with a manufactured capital stock and a non renewable natural resource. It is a canonical model severally used in the literature on sustainability (Dasgupta and Heal, 1974; Solow, 1974; Heal, 1998), which allow a comparison of results with the existing criteria, including the maximin, the green golden rule, the Chichilnisky approach and the mixed Bentham-Rawls criterion.

The paper is organized as follows. In section 2, we define the set of minimal rights that can be guaranteed to all generations in a standard economic model with an exhaustible resource. We present the framework of analysis (2.1), the model (2.2), and the results (2.3). A criterion based on the maximization of minimal rights is proposed in 3, and applied to our analytical model. The results are interpreted in section 4. We discuss the implications of this criterion with respect to the results of other criteria (maximin, green golden rule, Chichilnisky's criterion, mixed Bentham-Rawls criterion). Section 5 concludes.

2 On the possibility to achieve some minimal rights

In this section, we develop a framework to define the set of sustainability goals that can be achieved by an economy, given initial endowments. This framework is based on an extension of the viable control framework (Aubin, 1991).

2.1 Viability framework and minimal rights

If we consider that sustainability encompasses several objectives from different nature (ecological, economic and social) it may be difficult to adopt an optimization approach and define efficiency in a multicriteria context. Another possibility is to adopt the viability approach, which studies the effectiveness of dynamic intertemporal paths, with respect to a set of objectives defined by constraints. In the viability framework, the constraint must be respected forever, inducing an intergenerational equity perspective. The constraints can thus be interpreted as minimal rights to be guaranteed to any generation.

The purpose of the viability approach (Aubin, 1991) is to study the behavior of dynamic systems subject to constraints. The desired characteristics for the system are defined by a set of constraints, and the viability approach examines the conditions for these constraints to be satisfied forever, in a dynamic perspective. The first step of the analysis is to define the states of the system from which at least one intertemporal path that satisfies the constraints forever starts. The set of such states is called the *viability kernel* of the problem. By definition, from any state in the viability kernel, there is at least one admissible decision that keeps the trajectory in the kernel. Thus, from any state outside the viability kernel, there are no decisions that make it possible to respect the constraints forever. Whatever the decisions, the dynamics of the system will violate the constraint in a finite time.[7] Viable paths are paths that stay in the kernel. The second step of the analysis is the definition of the viable decisions associated with a viable state. Not all decisions may be viable and some of the admissible decisions may lead the system outside the viability kernel.

It means that if the initial state of the economy is not within the viability kernel, the initial endowments of the economy do not allow the sustainability goals to be achieved. The objectives (constraints level) should be diminished to be fulfilled. In that context, it is possible to extend the viability analysis by defining, for a given initial state, the set of reachable goals.

In our context, the constraints include a minimal consumption $\underline{c}$ and a minimal resource stock $\underline{x}$ to be preserved. We will examine the set of goals $(\underline{c}, \underline{x})$ that are reachable from the initial capital stocks of the economy. Doing that, we define the set of minimal rights that can be guaranteed to any generation.

As mentioned in the introduction, a particular problem one can face when trying to achieve some particular goals is that the defined goals may not be reachable from initial conditions through feasible paths of the system. In that case, the goals can be reached in some future date. This issue is discussed in section 4.3.

[7]This is related to the inertia of the system. There are states that satisfy the viability constraints but from which no viable path exists. The crisis is unavoidable.

2.2 Model

To make comparisons with other approaches easier, we develop the proposed approach for a standard model. We consider an economy with infinitely many generations, and make the simplifying assumption that each generation is composed by an unique representative agent.[8] A non renewable resource x_t is extracted (r_t) and used with capital k_t to produce capital. The production function is denoted $f(k_t, r_t)$. The capital can be ever consumed or invested. Such a model has been studied in Dasgupta and Heal (1974); Solow (1974), and is an useful stylized model for addressing the sustainability issue: the intertemporal allocation of the exhaustible resource, and the stream of consumption through time make intertemporal comparisons possible, in an intergenerational equity perspective.

The dynamics are

$$\dot{k} = k_t^\alpha r_t^\beta - c_t, \tag{6}$$

$$\dot{x} = -r_t. \tag{7}$$

We take a Cobb-Douglas production function which is argued to be the most interesting case for studying sustainability, as mentioned by Dasgupta and Heal (1979).[9]

In that model, we wonder what are the level of guaranteed consumption $\underline{c}$ and preserved resource stock $\underline{x}$ that are compatible with the initial state (k_0, x_0). We are thus considering the following viability constraints

$$c_t \geq \underline{c}, \tag{8}$$

$$x_t \geq \underline{x}. \tag{9}$$

[8] The intragenerational equity issue is not addressed. However, the perspective of growing population is discussed in section 4.3, in a discussion on the possibility to increase minimal rights along time.

[9] If we consider Constant Elasticity of Substitution (CES) production functions, if the elasticity of substitution between the capital and the resource is greater than one, there is no sustainability issue as it is possible to produce without using the natural resource. The resource is not essential to produce. On the contrary, an elasticity of substitution lower than one implies that the resource is essential to produce, and the intertemporal production is bounded. No consumption can be sustained. The intermediate case with elasticity equals to one, i.e. the Cobb-Douglas case, makes it possible to substitute capital to the resource in the production even if the resource is essential to produce. According to Dasgupta and Heal (1979), this case is the most interesting.

2.3 Admissible sustainability objectives for a given initial state

In order to define the minimal rights to be guaranteed for a sustainable development, we need to define the set of all possible rights, i.e. the rights that are compatible with the initial state of the economy and with the studied dynamics. We want to define the set of

$$\mathcal{S} = \left\{ (\underline{c}, \underline{x}) \;\middle|\; \begin{array}{l} \text{there exist paths } (k(.), x(.)) \text{ starting from } (k_0, x_0) \\ \text{that satisfy the constraints (8) and (9)} \end{array} \right\}. \tag{10}$$

Martinet and Doyen (2007) describe the relationship between consumption and preservation goals in the model we consider. They examine the conditions for a minimal consumption $\underline{c}$ to be guaranteed when there is also a constraint on the preservation of the resource $\underline{x}$. It means that they determine if there are intertemporal decisions $(c(.), r(.))$ that make it possible to respect the constraints $c_t \geq \underline{c}$ and $x_t \geq \underline{x}$ for all $t \geq 0$, from a given initial state (k_0, x_0).

As a consequences it is possible to determine which levels of guaranteed consumption $\underline{c}$ and preserved resource stock $\underline{x}$ are reachable from the initial state. An easy way to define levels of consumption that can be guaranteed along with the preservation of a part of the resource stock is to consider the maximal level of consumption that can be sustained, under a resource preservation constraint.

$$\begin{aligned} c^+(k_0, x_0, \underline{x}) = \max\Big(c^\sharp \;\Big|\; & \text{given } (k_0, x_0), \text{there exists } (c(.), r(.)) \\ & \text{such that } \forall t \geq 0, c_t \geq c^\sharp \text{ and } x_t \geq \underline{x} \Big). \end{aligned} \tag{11}$$

This approach is linked with the maximin approach (Solow, 1974; Cairns and Long, 2006). In the classical model we study, the maximal sustainable level of consumption, given the preservation goal $\underline{x}$, is defined as follows.

Result 1 *Consider a Cobb-Douglas function with $\alpha > \beta$. The maximal sustainable consumption from initial state (k_0, x_0), associated with the guaranteed stock $\underline{x}$ is*

$$c^+(k_0, x_0, \underline{x}) = (1 - \beta)\big((x_0 - \underline{x})(\alpha - \beta)\big)^{\frac{\beta}{1-\beta}} k_0^{\frac{\alpha-\beta}{1-\beta}}. \tag{12}$$

This result is proven in Martinet and Doyen (2007, Proposition 3), but an easier way to insure this result is to note that equation (12) leads exactly to the maximin result of Solow (1974, p.39) for $\underline{x} = 0$. Solow's result is thus extended to take into account the resource constraint.

Fig. 1 represents equation (12). It represents all consumption and resource preservation levels that can be guaranteed to all generation. Eq. (12) is the upper bound of the set of all reachable goals.

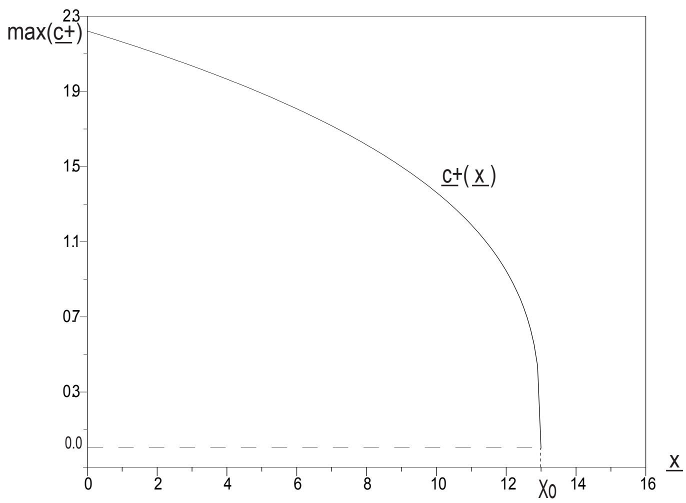

Figure 1: Substitution between guaranteed consumption and resource conservation for a Cobb-Douglas technology.

We thus know the set of all reachable goals for sustainability. Any inner pair $(\underline{c}, \underline{x})$ such that $\underline{c} \leq c^+(\underline{x})$ is feasible.[10] Note that on the border, a rise of resource preservation implies a fall of sustainable consumption.

A question that arises now is to define sustainability goals in order to satisfy some intergenerational equity.

[10] For the sake of simplicity, we will omit the initial state in the notation of function $c^+(k_0, x_0, \underline{x})$.

3 Choosing among sustainability goals

In this section we introduce an objective function to define which sustainability rights should be chosen among the possible set $\mathcal{S}$.

3.1 An objective function related to minimal rights

Consider that minimal rights are based on a minimal consumption $\underline{c}$ and a minimal resource stock $\underline{x}$. We define social preferences among the minimal rights to be guaranteed.

Definition 1 *A minimal right utility function $U(\underline{c}, \underline{x})$ represents the preferences on minimal rights. We postulate*

- $\underline{c} \in \mathbb{R}_0^+$; $\underline{x} \in \mathbb{R}_0^+$
- $U(\underline{c}, \underline{x}) : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \mapsto \mathbb{R}_0^+$
- $U_c \geq 0$; $U_x \geq 0$
- $U_{c,x} \leq 0$.

This utility function represents the preferences of a virtual representative agent placed under the veil of ignorance, and that should choose minimal rights to be guaranteed to any generation in a sustainability concern.

We propose to choose the pair of minimal rights $(\underline{c}, \underline{x})$ that maximizes this utility function. Hence, our approach is not to maximize some intertemporal welfare but to maximize the utility associated with some minimal rights to be guaranteed for all generations.

Definition 2 *We define the Minimal Rights criterion as*

$$\max_{\underline{c}, \underline{x}} U(\underline{c}, \underline{x}) \tag{13}$$

subject to

$$\begin{aligned} (k_0, x_0) & \quad given & (14) \\ c_t & \geq \underline{c} & (15) \\ x_t & \geq \underline{x} & (16) \\ \dot{k} & = k_t^\alpha r_t^\beta - c_t & (17) \\ \dot{x} & = -r_t & (18) \end{aligned}$$

3.2 Results

Using the result 1, this problem is equivalent to the maximization of this criterion among the possible pairs $(\underline{c}, \underline{x})$, i.e.

$$\max_{\underline{c}, \underline{x}} U(\underline{c}, \underline{x})$$

$$s.t.$$

$$\begin{aligned} 0 & \leq \underline{x} & (19) \\ 0 & \leq \underline{c} & (20) \\ 0 & \leq x_0 - \underline{x} & (21) \\ 0 & \leq c^+(\underline{x}) - \underline{c} & (22) \end{aligned}$$

This problem is a classical static optimization problem under inequality constraints (Léonard and Long, 1992).[11]

To solve this problem, we define the following functional form

$$\phi(\mu_1, \mu_2, \mu_3, \mu_4, \underline{c}, \underline{x}) = U(\underline{c}, \underline{x}) + \mu_1 \underline{x} + \mu_2 \underline{c} + \mu_3 (x_0 - \underline{x}) + \mu_4 (c^+(\underline{x}) - \underline{c}) \quad (23)$$

where the μ_i are the dual variables of the problem.

According to the Khun-Tucker theorem, the optimality conditions of the prob-

[11] Such problems are generally easier to solve than complex dynamic optimization programs.

lem are[12]

$$\phi_{\mu_1} = \underline{x} \geq 0, \qquad \mu_1 \geq 0, \qquad \mu_1 \underline{x} = 0 \tag{24}$$

$$\phi_{\mu_2} = \underline{c} \geq 0, \qquad \mu_2 \geq 0, \qquad \mu_2 \underline{c} = 0 \tag{25}$$

$$\phi_{\mu_3} = x_0 - \underline{x} \geq 0, \qquad \mu_3 \geq 0, \qquad \mu_3(x_0 - \underline{x}) = 0 \tag{26}$$

$$\phi_{\mu_4} = c^+(\underline{x}) - \underline{c} \geq 0, \qquad \mu_4 \geq 0, \qquad \mu_4(c^+(\underline{x}) - \underline{c}) = 0 \tag{27}$$

$$\phi_{\underline{x}} = U_{\underline{x}} + \mu_1 + \mu_4 \frac{dc^+(\underline{x})}{d\underline{x}} \leq 0, \qquad \underline{x} \geq 0, \qquad \underline{x}\left(U_{\underline{x}} + \mu_1 + \mu_4 \frac{dc^+(\underline{x})}{d\underline{x}}\right) = 0 \tag{28}$$

$$\phi_{\underline{c}} = U_{\underline{c}} + \mu_2 - \mu_4 \leq 0, \qquad \underline{c} \geq 0, \qquad \underline{c}(U_{\underline{c}} + \mu_2 - \mu_4) = 0 \tag{29}$$

Strictly positive solutions

First assume that, at the optimum, both the optimization variables $\underline{x}$ and $\underline{c}$ are strictly positives. From eq. (24) and (25), we get $\mu_1 = \mu_2 = 0$. Moreover, if consumption is positive, the preserved resource stock will be lower than the initial stock x_0. Eq. (26) then leads to $\mu_3 = 0$. We thus get a system from equations (27), (28) and (29), in which there are three equations and three variables.

$$\mu_4(c^+(\underline{x}) - \underline{c}) = 0 \tag{30}$$

$$\underline{x}\left(U_{\underline{x}} + \mu_4 \frac{dc^+(\underline{x})}{d\underline{x}}\right) = 0 \tag{31}$$

$$\underline{c}(U_{\underline{c}} - \mu_4) = 0 \tag{32}$$

As we have assumed that $\underline{c} \neq 0$, eq. (32) leads to $\mu_4 = U_{\underline{c}}$. We thus get from eq. (30) and (31) the conditions

$$\underline{c} = c^+(\underline{x}) \tag{33}$$

$$\frac{dc^+(\underline{x})}{d\underline{x}} = -\frac{U_{\underline{x}}}{U_{\underline{c}}} \tag{34}$$

It leads to the following result

$$\frac{d\underline{c}^+(\underline{x})}{d\underline{x}} = -\frac{U_{\underline{x}}}{U_{\underline{c}^+(\underline{x})}} \tag{35}$$

Fig. 2 illustrates this result.

[12]The variables are at optimal values. In order to get simple notations, we do not note them $\underline{x}^*$ and $\underline{c}^*$ but simply $\underline{x}$ and $\underline{c}$

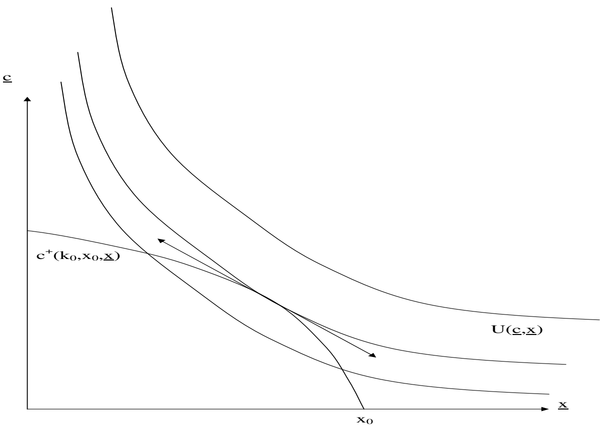

Figure 2: Optimal minimal rights when $\underline{c} > 0$ and $\underline{x} > 0$.

Corner solution $\underline{x} = 0$

Assume now that $\underline{x} = 0$. It implies that $\mu_3 = 0$ (from eq. 26).

If $\underline{c} = 0$, eq. (27) would require $\mu_4 = 0$. But it is in contradiction with relation (29) which requires $U_{\underline{c}} + \mu_2 - \mu_4 \leq 0$. Thus, we have $\mu_4 > 0$ and $\underline{c} = c^+(0)$, from eq. (27).

As $\underline{c} \neq 0$, we get $\mu_2 = 0$ from eq. (25). We then get $\mu_4 = U_{\underline{c}}$ from eq.(29). Finally, the inequality condition (28) requires

$$U_{\underline{x}|(\underline{x}=0)} + \mu_1 + U_{\underline{c}} \frac{dc^+(\underline{x})}{d\underline{x}}_{|(\underline{x}=0)} \leq 0 \tag{36}$$

This equation can be expressed with respect to μ_1

$$\mu_1 \leq -U_{\underline{x}|(\underline{x}=0)} - U_{\underline{c}} \frac{dc^+(\underline{x})}{d\underline{x}}_{|(\underline{x}=0)} \tag{37}$$

As $\mu_1 \geq 0$, it is possible only if $-U_{\underline{x}|(\underline{x}=0)} - U_{\underline{c}} \frac{dc^+(\underline{x})}{d\underline{x}}_{|(\underline{x}=0)} \geq 0$, or equivalently if

$$0 \leq \frac{U_{\underline{x}}}{U_{\underline{c}}} \leq -\frac{dc^+(\underline{x})}{d\underline{x}}_{|(\underline{x}=0)} \tag{38}$$

It means that such a corner solution is possible if the slope of the utility function is smaller than that of the function $c^+(\underline{x})$. Such a solution is then possible only if the marginal utility of preservation for a nil resource stock is small with respect to the marginal utility of consumption. It implies that $U_{\underline{x}|(\underline{x}=0)} < \infty$.

Fig. 3 illustrates this result.

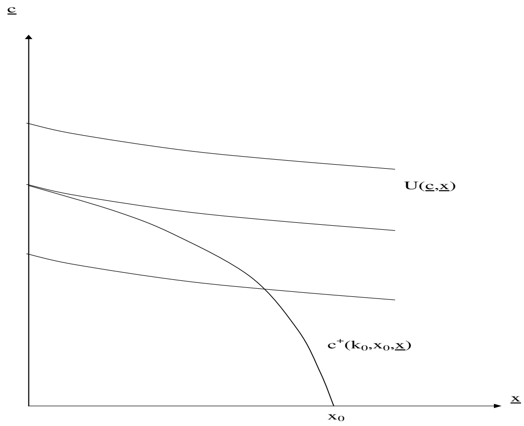

Figure 3: Optimal minimal rights when $\underline{x} = 0$.

Corner solution $\underline{c} = 0$

We now turn toward the other case : $\underline{c} = 0$. The inequality from eq. (29) implies

$$U_c + \mu_2 - \mu_4 \leq 0 \quad \Longrightarrow \quad \mu_4 \geq U_{\underline{c}|(\underline{c}=0)} + \mu_2 > 0 \tag{39}$$

It is only possible if $U_{\underline{c}|(\underline{c}=0)} < \infty$.

As $\mu_4 > 0$, we know from eq. (27) that $\underline{c} = c^+(\underline{x})$ which means, as $\underline{c} = 0$, that $\underline{x} = x_0$. We have $\mu_1 = 0$ from eq. (24). Thus, $\underline{x} > 0$ requires from eq. (28) that

$$\mu_4 = -\frac{U_{\underline{x}|(\underline{x}=x_0)}}{\frac{dc^+(\underline{x})}{d\underline{x}}_{|(\underline{x}=x_0)}} \tag{40}$$

Combining this condition with eq. (39), we get

$$0 \leq \mu_2 \leq -U_{\underline{c}|(\underline{c}=0)} - \frac{U_{\underline{x}|(\underline{x}=x_0)}}{\frac{dc^+(\underline{x})}{d\underline{x}}_{|(\underline{x}=x_0)}} \tag{41}$$

We thus have a condition on the marginal utilities in $(\underline{c} = 0, \underline{x} = x_0)$:

$$0 \leq -\frac{dc^+(\underline{x})}{d\underline{x}} \leq \frac{U_{\underline{x}}}{U_{\underline{c}}} \tag{42}$$

The slope of the utility function must be greater than that of the function $c^+(\underline{x})$. In particular, the marginal utility of consumption, when consumption is zero, must be finite (and small with respect to the marginal utility of preservation).

Fig. 4 illustrates this result.

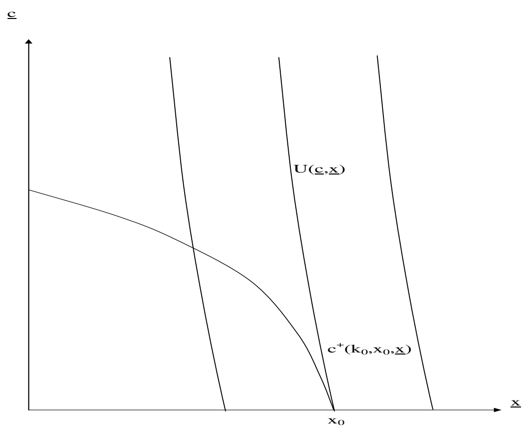

Figure 4: Optimal minimal rights when $\underline{c} = 0$.

3.3 Interpretation

This approach has methodological intersections with the maximin approach (Solow, 1974; Cairns and Long, 2006), in that the efficient growth paths are solutions of a constrained optimization problem: maximin under constraint

Whatever the case, the optimal solution always satisfies $\underline{c} = c^+(\underline{x})$. The only intertemporal path that satisfies the optimal minimal rights is then a maximin under constraints and is efficient from an economic point of view. As a consequence, optimal minimal rights with respect to our criterion entail efficient resource use. The only intertemporal path that satisfies the optimal constraint is the same path as a maximin path (on consumption) under a natural resource constraint fixed at the level $\underline{x}^*$.

Note that on the boundary of the set $\mathcal{S}$, there is only one intertemporal path that satisfies the constraints. As this path is the solution of a maximin problem under constraint, it satisfies both Hartwick's and Hotelling's rules: the intertemporal use of the natural resource and the investment are optimal.

Our approach can then be interpreted as the definition of optimal constraint levels for maximin under constraint problems.

The level of the resource preservation will depend on the marginal utilities of consumption and resource stock. If we assume that the consumption marginal utility for a nil consumption $U_{c|(c=0)}$ is infinite, and that that of the resource for a nil resource stock $U_{x|(x=0)}$ is also infinite, the result will satisfy $\underline{c} > 0$ and $\underline{x} > 0$.

As the solution is in any case on the boundary of the set $\mathcal{S}$, the solution is "pareto efficient" as it is not possible to increase one of the minimal rights without decreasing the other.

4 Discussion

In this section, we discuss our result with respect to existing criteria. We emphasize the links between our approach and the maximin framework. We then discuss the interest of our approach with respect to the "green golden rule" (Beltratti et al., 1995) and the Chichilnisky's criterion (Chichilnisky, 1996). We also refer to the Mixed Bentham-Rawls criterion recently proposed by Long (2006).

4.1 Minimal rights and sustainability criteria

As mentioned in the introduction, various criteria have been proposed to deal with the sustainability issue. In order to take into account both intertemporal equity issue and preservation issue, Heal (1998) discusses the results of these criteria using an utility function that takes into account both the consumption c_t and the resource stock x_t; i.e. $U(c_t, x_t)$. The natural resource thus has an amenity value. Each criterion leads to the preservation of a part of the natural resource, and an intertemporal consumption path.

According to Heal (1998), the criteria are more or less conservative with respect to the resource. From the less to the more conservative, we have the discounted utilitarian criterion, the Chichilnisky criterion and the green golden rule.

For the maximin criterion, it depends on the way the problem is defined and on the initial configuration.

We propose here a short comparison of the results of these criterion and ours.

Discounted utilitarian criterion The usual economic criterion is the neoclassical sum of discounted utilities.

$$\max_{c(.),r(.)} \int_0^{\infty} \Delta(t) U(c_t, x_t) dt \tag{43}$$

Heal (1998) analyses the solution of this criterion in a production-consumption model with a non renewable resource, when utility depends both on the consumption and the resource stock. This analysis was started by Krautkraemer (1985). Unfortunately, usual mathematical tools don't make it possible to find out an explicit solution. These kind of problems are not easy to solve.

The maximin The maximin criterion maximizes the utility of the poorest generation. Within the sustainability debate, Cairns and Long (2006) propose two ways to consider the environment in a maximin problem. The first one is to introduce the resource stock in the utility function.[13] The criterion reads

$$\max_{c(.),r(.)} \left(\min_t U(c_t, x_t) \right) \tag{44}$$

Such a criterion implies an increasing consumption like the resource stock decreases. As far as we can say, the optimal solution has not been computed in our model.

The second possible approach is to consider a maximin under natural resource constraint.[14] A question then arises: at which level should we set the constraint?

Our approach proposes a framework to set the level of such sustainability constraints.

[13] This utilitarian approach is related to the "weak sustainability".

[14] This conservative approach is more related to the "strong sustainability".

The Green Golden Rule The "green golden rule" criterion consists in maximizing the utility attained by the policy at an infinite time, when utility depends both on the consumption and the resource stock, i.e. $U(c_t, x_t)$. The criterion is of the form

$$\max \lim_{t\to\infty} U(c_t, x_t).$$

The interpretation of this criterion from a sustainability point of view is studied in Heal (1998). The solution of the criterion is given for two particular cases of a production-consumption model, depending on the characteristics of the production function $f(k, r)$:

- with a non-renewable resource necessary for production, in the sense that $f(k, 0) = 0$, "the maximum sustainable utility level is attained by conserving the entire stock and consuming none of the produced good" (Heal, 1998, proposition 35, p.135).
- with a non-renewable resource that is not necessary to production, i.e. $f(k, 0) > 0$, the maximal sustainable utility level is attained without extracting any part of the stock (Heal, 1998, proposition 36, p.136).

The Cobb-Douglas case is not studied, and thus, the substitutability issue is not addressed.[15]

It thus seems that this criterion implies the preservation of the initial resource stock, whatever is the technological configuration. We will show that it is not always true by studying the particular case of a Cobb-Douglas production function. In that case, we will show that it is not optimal to preserve the whole resource stock unless the marginal utility of consumption for nil consumption is small with respect to the marginal utility of the resource preservation.

Proposition 1 *If* $U_{c|(c=0)} \geq -\frac{U_{x|(x=x_0)}}{\frac{dc^+(x)}{dx}_{|(x=x_0)}}$, *then*

$$argmax\Big(\lim_{t\to\infty} U(c_t, x_t)\Big) \neq (c_t = 0, x_t = x_0) \tag{45}$$

[15] Being precise, the Cobb-Douglas case satisfies $f(k, 0) = 0$. Nevertheless, the solution is not the one described by Heal (1998) as we will prove it. In fact, the "green golden rule" criterion has no solution in the Cobb-Douglas case.

Proof – To prove proposition 1, we demonstrate that the stationary state (no resource extraction and no consumption of the capital good) is dominated, with respect to the GGR criterion, by the solution of our criterion.

It is straightforward, using the result of eq. (42) and the solution of the optimization problem 13 in the case $\underline{c} = 0$. $\diamond$

Our criterion can be proposed to maximize the utility that is reached at an infinite time when the "green golden rule" does not lead to a solution. As the resource is non-renewable and its harvest irreversible, the resource stock is decreasing along time. Along the optimal path of the "minimal rights" criterion, the consumption is constant and the resource stock decreases towards the minimal value. The utility of the minimal rights (the one that has been maximized) is the utility at an infinite time. In fact, we applied some "infinite utility criterion" on a reduced set of paths: maximin under constraints. In our particular model, the criterion can be expressed as $\lim_{t\to\infty} U(c_t, x_t) = U(\underline{c}, \underline{x})$. In fact, as the non renewable resource stock is necessarily decreasing, and if the consumption can be smoothed along time in order to increase the level of the minimal consumption, the utility level that is sustained in the long-run is the utility associated with the minimal rights.

Doing so, one takes into account a constant consumption path, the resource preservation, and the utility of the very-long run. Thus, we combine a *maximin* approach under constraint and determine the constraint level with a criterion closed to the *green golden rule*.

The Chichilnisky's approach Chichilnisky (1996) has developed a criterion that makes it possible to avoid both the "dictatorship of the present" and the "dictatorship of the future". The criterion is

$$W = \theta \int_0^\infty \Delta(t) U(c_t, x_t) dt + (1-\theta) \lim_{t\to\infty} U(c_t, x_t) \tag{46}$$

where $\Delta(t)$ is the discount factor that can be exponentially decreasing or hyperbolic. According to Heal (1998, p.69), *"Intuitively,* [the] *second term* [of Chichilnisky's criterion] *reflects the* sustainable utility level *attained by a policy".*

The Chichilnisky criterion is a convex combination of the utilitarian discounted criterion and the green golden rule criterion. However, as noted by Heal (1998) and Long (2006), the Chichilnisky criterion suffers a major limitation: for many growth models (including a standard two-sector growth model with a renewable resource) there is no optimal path for the criterion.

Any path that leads to the preservation of a part of the stock will be beaten by another path that burns just an extra piece of the stock to increase the capital stock. So, an optimal path does not exist. There will be no consumption until an infinite time.

According to Gerlagh and Keyser (2003), conservationist policies can be Pareto efficient, and strict resource conservation is equivalent to non-dictatorship of the present, as defined by Chichilnisky (1996). They argue that *"strict conservationist policies that impose explicit exploitation constraints ensure sustainability, and are far simpler to implement, compared to the more complex resource management rules that aim at a careful balancing of costs and benefits"* (Gerlagh and Keyser, 2003, p.312).

Our approach includes such a resource conservation constraint. We thus avoid the dictatorship of the present.

The Mixed Bentham-Rawls criterion This criterion is proposed by Long (2006). It is a convex combination of the maximin criterion and the utilitarian discounted criterion.

$$\max \left(\theta \int_0^\infty \Delta(t) U(c_t, x_t) dt + (1-\theta) \min_t U(c_t, x_t) \right). \tag{47}$$

Comparing this approach to Chichilnisky's one, we have three criteria (discounted utilitarian, maximin and green golden rule), and two convex combination have been proposed. In the following section, we argue that our criterion is a particular limit case of the third combination: maximin and green golden rule.

Is our criterion a particular case of the Chichilnisky's one ? The optimal path of Chichilnisky's criterion is not easy to compute. Heal (1998) develops a particular approach combining two steps that we will detail below, in a particular case of the criterion.

Consider a maximin program under constraint

$$\max_{c(.),r(.)} \quad \left(\min_t U(c_t) \right) \tag{48}$$

$$s.t. \qquad x_t \geq \underline{x} \tag{49}$$

and consider that this criterion is the first part of a "virtual" Chichilnisky's criterion.[16] It leads to the following special case of Chichilnisky's criterion:

$$\max \left(\theta \int_0^\infty \pi(t) U(c) dt + (1-\theta) \lim_{t \to \infty} U(c_t, x_t) \right) \tag{50}$$

where $\pi(t)$ is the discount factor associated with the maximin path for this model.

If one applies the method developed by Heal (1998) to exhibit the solution of such a criterion, it leads to the following: First, consider that a part x^* of the resource is preserved forever, and then that the first part of the criterion is optimized. We obtain a maximin path under constraint, which is a point $c^+(x^*)$, defined by eq. (12). The second step consists in maximizing the level of x^* in order to maximize criterion (50). If we take the limit of this criterion when $\theta \to 0$, we maximize the utility at infinite time or, equivalently, the "minimal rights" criterion we defined.

The method we propose and that consists in two steps (first defining the maximal guaranteed consumption for a level of resource preservation and then defining the level of the resource constraint) is closed from the method used to determine the optimum of the Chichilnisky criterion in Heal (1998). First, a part of the stock is put away and the discounted criterion is optimized. Then, the level of the resource preserved is optimized. Our approach can then be interpreted as way to avoid the difficulty induced by the resolution of Chichilnisky's criterion.

[16] A maximin program can be expressed as the discounted sum of intertemporal utility (see Cairns and Long, 2006) for some particular discount factor $\pi(t)$ such that a path $[\bar{U^*}, S^*, c^*]$ (where S represents the states of the system and c the controls) solves the maximin problem if and only if the path $[S^*, c^*]$ solves the following control problem

$$\max_c \int_0^\infty \pi(t) U(S, c, t) dt,$$

where $\pi(t)$ is the virtual discount factor of the maximin program with constant utility $\bar{U}^*$. Thus, a maximin program could be the first part of the chichilnisky's criterion. However, note that the virtual discount factor of a maximin problem is endogenous to that problem. Thus, all that is done in this section is a stylized fact for the following discussion.

4.2 Defining minimal objectives to implement sustainability

Until now, we have presented an approach to define minimal rights for sustainable development. We have compared the result of this approach in a canonical model with usual criteria, and argued that it allows avoiding some difficulties in defining optimal paths. However, any reader would argue that the use of our criterion relies on the definition of the set of all reachable goals, and then, on the viability approach. Defining such minimal rights is not an easy task, and one can argue that the approach is no more applicable than any discussed criterion.

However, there is an increasing body of literature using viability approach in complex bioeconomic systems, emphasizing its applicability is one accepts to use numerical methods to solve the problem.[17] The criterion we propose, more than a stylized criterion used to interpret the sustainability in a normative debate, can be applied to real bioeconomic problems, and help choosing between reachable sustainability objectives.

If sustainability encompasses several objectives, an optimization approach can be hard to apply. Our approach combining viability analysis and the optimization of constraints (goals) levels is an alternative solution.

In fact, when usual criteria do not have solutions, a possibility is to reduce the set of admissible path (by selecting paths that satisfy a set of constraints) and then to apply a criterion. For example, the green golden rule criterion will always postpone consumption. We propose to examine the problem when a minimal consumption is required. The problem is then to define the maximal utility level sustained by a path with a positive consumption.

The criterion is not used to define optimal paths under (already defined) constraint, but to define the set of reachable goals and to choose the sustainability target (minimal rights for sustainability). Once such goals are defined, there is no utility comparisons between generations. Everything is set with respect to the state of the system (cloosed-loop control problem) and not along a time dependent optimal path.

[17] See Martinet et al. (2007) for example.

4.3 Reaching minimal right and minimal time of crisis

It is possible that none of the reachable constraints are relevant for sustainability. For example, the level of sustainable consumption may be lower that a subsistence level. It is the purpose of the sufficientirianism approach to determine how to reach economic development paths that satisfy basic needs.

Our approach is somewhat different from the sufficientarianism, or the satisfaction of basic needs criterion, as it is presented in Chichilnisky (1977) and Long (2006). In their perpectives, the criterion consists in the minimization of the time needed to reach an economic path that satisfies the basic needs. Long (2006) defines a minimal level of utility $\hat{u}$ to be reached, and Chichilnisky (1977) considers a multiobjective approach in which various basics needs have to be fulfilled, without considering an associated utility. Chichilnisky (1977) gives a mathematical formalization of economic efficiency in an economy concerned with the satisfaction of the consumption of basic goods in some near future. Efficiency is defined with respect to the minimization of the time horizon at which the minimal consumption level is reached. The criterion can be written as

$$\min_{u(.)} T \;\Big|\; u_\tau \geq \hat{u} \quad \forall \tau > T \tag{51}$$

Our approach is different as the "basic needs" are not defined *a priori* with respect to some real level of consumption or environment quality. On the contrary, our approach aims at defining the possible sustainable development goals, interpreted as "minimal rights" to be provided to all generations. Doing this, we avoid the issue of the (normative) definition of the level at which "basic needs" are defined.

However, the viability framework can be extended by using the concept of "time of crisis" (Doyen and Saint-Pierre, 1997) which is the time during which the constraints are not met. The *minimal time of crisis* is thus a relevant approach to address the basic needs issue. This approach has been developed in Martinet et al. (2007) in a bioeconomic model.

5 Conclusion

Sustainability can be defined by several objectives/goals of different nature. An optimization approach is then hard to adopt. In this paper, we proposed to apply a

Rawlsian approach of justice to choose among the possible sustainability goals. We describe how to apply this approach in the classical Dasgupta-Heal-Solow model.

Our approach consists in maximizing the utility associated with minimal rights to be guaranteed to all generations. It is thus a static optimization problem among all possible sustainability goals. The set of possible sustainability goals, from initial economic state, are defined using the viability approach, like in Martinet and Doyen (2007).

We discussed the interpretation of the "minimal rights criterion" with respect to existing criteria, and show that it could provide some alternative to classical criteria (Maximin, Green Golden Rule, Chichilnisky) when the solution does not exist or is not easy to compute.

We then discussed the benefits and limits of such an approach. We showed that it can be applied to real cases sustainability problem, and that adequate numerical and algorithmic methods exist for application studies. This framework can even be extended to address the issue of reaching sustainability goals that are not achievable with the initial endowment of the economy.

Ackwnoledgements I am grateful to participants of the CIREQ seminar (June 3, 2005, Montreal) for their comments and remarks on the first draft of the paper, especially to Bob Cairns and Hassan Benchekroun. I am also indebted to the members of the French research program "Economic Modeling of Sustainable Development", especially Mabel Tidball. The research program on viability and sustainability relies on collaboration with Luc Doyen. All errors remain my own.

References

Asheim G., Buchholz W. and Tungodden D. (2001) Justifying sustainability. *Journal of Environmental Economics and Management*, 41, pp. 252-268.

Aubin J.P. (1991) *Viability Theory.* Birkhäuser, Springer Verlag.

Beltratti A., Chichilnisky G. and Heal G. (1995) The Green Golden Rule. *Economics Letters*, 49(2), pp. 175-179.

Cairns R. and Long N.V. (2006) Maximin, a direct approach to sustainability. *Environmental and Development Economics*, 11, pp. 275-300.

Chatterjee S. and Ravikumar B. (1999) Minimum consumption requirements: theoretical and quantitative implications for growth and distribution. *Macroeconomic Dynamics*, 3, pp.482-505.

Chichilnisky G. (1977) Economic Development and efficiency criteria in the satisfaction of basic needs. *Applied Mathematical Modelling*, 1, p.290-298.

Chichilnisky G. (1996) An axiomatic approach to sustainable development. *Social Choice and Welfare*, 13(2), pp. 219-248.

Dasgupta P. and Heal G. (1974) The Optimal Depletion of Exhaustible Resources. *Review of Economic Studies. Symposium on the Economics of Exhaustible Resources*, 41, pp. 1-28.

Dasgupta P. and Heal G. (1979) *Economic Theory and Exhaustible Resources.* Cambridge University Press.

Dawid H. and Day R. (2006) On sustainable growth and collapse: Optimal and adaptative paths. *Journal of Economic Dynamics and Control*, in press.

Doyen L. and Saint-Pierre P. (1997) Scale of viability and minimal time of crisis. *Set-Valued Analysis*, 5, p.227-246.

Gerlagh R. and Keyser M.A (2003) Efficiency of conservationist measures: an optimistic viewpoint. *Journal of Environmental Economics and Management*, 46, pp. 310-333.

Hartwick J. (1977) Intergenerational Equity and the Investing of Rents from Exhaustible Resources. *American Economic Review*, 67, pp. 972-974.

Heal G. (1998) *Valuing the Future, Economic Theory and Sustainability.* Columbia University Press, New York.

Hotelling H. (1931) The Economics of Exhaustible Resources. *Journal of Political Economy*, 39, pp. 137-175.

Krautkraemer J. (1985) Optimal Growth, Resource Amenities and the Preservation of Natural Environments. *Review of Economic Studies*, 52, pp.153-170.

Krautkraemer J. (1998) Nonrenewable resource scarcity, *Journal of Economic Literature*, XXXVI, pp. 2065-2107.

Léonard D. and Long N.V. (1992) *Optimal control theory and static optimization in economics*, Cambridge University Press.

Long N. (2006) A mixed Bentham-Rawls Criterion for Intergenerational Equity. *working paper*. McGill University.

Martinet V. and Doyen L. (2007) Sustainability of an economy with an exhaustible resource: a viable control approach. *Resource and Energy Economics* **29**, pp.17-39.

Martinet V., Thébaud O. and Doyen L. (2007) Defining viable recovery paths toward sustainable fisheries. *Ecological Economics*, to appear.

Pezzey J. and Toman M. (2002) Progress and Problems in Economics of Sustainability. In *International Yearbook of Environmental and Resource Economics 2002-3*, p. 165-232, T. Tietenberg and H. Folmer (eds), Edward Algar, UK.

Rawls J. (1971) *A theory of Justice*. Oxford, England: Clarendon.

Solow R.M.(1974) Intergenerational Equity and Exhaustible Resources. *Review of Economic Studies. Symposium on the Economics of Exhaustible Resources*, 41, pp. 29-45.

Solow, R. (1993) An Almost Practical Step Towards Sustainability, Resources Policy 19, pp.162-172.

Stavins R., Wagner A. and Wagner, G. (2003) Interpreting sustainability in economic terms: dynamic efficiency plus intergenerational equity, *Economics Letters*, 79, pp. 339-343.

WCED (1987). "Our common Future", World Commission on Environment and Development. *Oxford University Press.*