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## The Dynamics of Intensive Cultivation

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# THE DYNAMICS OF INTENSIVE CULTIVATION

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Abstract. An increase in the demand for agricultural goods leads to the use of more intensive cultivation methods. Though Ricardo sees no difficulties in the intensification process, their existence is revealed by the possible occurrence of multiple equilibria. A general theory of intensive rent is based on a formal parallel with single-product systems without land.

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## 1. INTRODUCTION

The<sup>2</sup> recent rise in the prices of agricultural products and raw materials is generally thought to be a long-term phenomenon, which puts again to the fore the old Ricardian theory of rent: an increase in the demand for agricultural goods leads to a rise in their prices, which makes the exploitation of lands of lower quality profitable or allows for the introduction of more intensive cultivation methods. Some doubts on the validity of that classical piece of economic analysis arose when Montani (1975) and D'Agata (1983) provided examples showing that different ways to meet a given demand basket may exist, each corresponding to different agricultural methods and being associated with different prices, wages and rents. Though established in a static framework, the phenomenon sets a challenge to Ricardo's dynamic analysis which suggests that a progressive increase in demand is met by introducing more productive methods which are land saving but were too costly to be used when corn was cheap. The sequence of substitutions determines a unique path and a unique solution is expected for each level of demand. The multiplicity puzzle has been analyzed in specific cases, assuming one agricultural good and no or restricted substitutions in the industrial sector, without convincing results. Let us isolate the intensification process by assuming that land is of uniform quality. A general theory of intensive rent proper, with one land of uniform quality, several agricultural and industrial goods and any number of methods in each sector, is deemed to be out of reach (Kurz and Salvadori, 1995).

We show on the contrary that the general theory of intensive rent is simple. Under the hypothesis, faithful to Ricardo, that the given distribution variable is the real wage basket, the key argument is that the formalization of intensive cultivation with a uniform rent per acre is very close to that of single-product systems with a uniform wage per worker. This feature is used to transfer well known results relative to single production without scarce resources to the study of intensive cultivation.

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Section 2 introduces the intensive cultivation model. Section 3 describes the Ricardian dynamics and analyzes the value side, then the physical side of the problem. Section 4 identifies the conditions of validity of those dynamics.

## 2. COUNTING EQUATIONS

The data of the general problem of intensive rent proper are the  $n$  reproducible commodities (their decomposition as industrial goods and agricultural goods does not matter, as soon as there is at least one agricultural good), the set of industrial and agricultural methods of production, which are of the single-product type with constant returns (land is not treated here as an output of agriculture), the total area  $\bar{l}$  of the uniform grade land and the semipositive final demand basket  $d$ . After incorporation of the real wage basket into the physical inputs of production, labor does not appear explicitly: the  $j$ th method of production is described by the column-vector  $a_j$  of its  $n$  input coefficients and the land area  $l_j$  per unit produced (the land input of an industrial method is zero). On the physical side, the unknowns are the semipositive activity levels, represented by a column-vector  $y$  which satisfies  $n$  constraints stemming from the adjustment of the net product to demand

$$y - Ay = d \tag{1}$$

and one constraint relative to the full cultivation of land

$$ly = \bar{l} \tag{2}$$

(taking into account the fallow method, with land as only input and no product, allows us to replace the inequality  $ly \leq \bar{l}$  by an equality). Flukes apart, these conditions imply that at least  $n+1$  methods are operated. On the value side, the unknowns of a long-term equilibrium are the uniform rate of profit  $r$ , the price row-vector  $p$  and the uniform rent  $\rho$  per acre (fallowing requires a zero rent). For a given numéraire, their number is reduced to  $n+1$ . Each method  $j$  producing commodity  $i$  satisfies an inequality with complementarity relationship

$$(1+r)pa_j + \rho l_j \geq p_i \quad [y_j] \quad (3)$$

Equalities hold for the operated methods ( $y_j > 0$ ). Flukes apart, at most  $n+1$  equalities are met by the  $n+1$  unknowns. On the whole, it turns out that the number of operated processes exceeds that of commodities by one, therefore two processes produce the same commodity. That coexistence may well occur in industry: since land enters in the production of iron, the use of a land-saving iron method has the same effects as that of more intensive corn method (Saucier, 1981).

### 3. RICARDO'S ANALYSIS

With two methods producing the same good, the extension of the more productive method to the expense of the other allows for an increase in production. During this phase, the value equations are not affected and the prices remain constant. This continuous expansion comes to an end when the activity level of the less productive method vanishes. At this stage, a spasmodic rise in the price of the product in short supply (say, corn) occurs, the surge being sufficient for the introduction of a more productive method which was not used before because of its cost. The level of the uniform rent rises, the rate of profit falls and, since corn enters in the production of other commodities (and indirectly of all others, if the economy is basic), all prices change. To an observer, the new value conditions let appear a new method of production in some branch when another disappears: the stage is thus set for another phase of expansion followed by another jump in prices and distribution. This scheme is the core of the Ricardian dynamics of intensive cultivation.

The value side of these dynamics has a noticeable property: the value relationships (3) bear a striking formal similarity with those prevailing for the choice of methods in a purely industrial framework without land (Sraffa, 1960, chapter XII) if the land vector is identified with the labor vector and the rent per acre with the wage per worker. The value problem can

therefore be solved by transposing the standard tools and results: the numéraire being chosen, consider an arbitrary  $n$ -set of methods, one for each commodity, and draw the  $\rho-r$  curve obtained by eliminating the prices from the corresponding value equations. Repeat the procedure for all possible  $n$ -sets and consider the upper envelope of these curves. Each curve being decreasing, the same for their envelope, a property which expresses the trade-off between profits and rents. On the factor price frontier, each farmer or entrepreneur maximizes profits for the given prices and rent. The only significant difference with the industrial framework is that the number of equalities now exceeds that of commodities by one, but one more equality is met at a switch point between two consecutive  $n$ -sets along the frontier. That is, the switch points  $(\rho, r)$ , at which two methods producing the same good are equally costly and can operate simultaneously, are the noteworthy points for the theory of intensive cultivation. The Ricardian dynamics are made of two phases: (i) at each node, there are  $n+1$  operated methods, with one 'old' method and one 'new' method producing the same commodity, and a progressive substitution of the new method for the old; and, (ii) when the old method disappears, the value system jumps from a node to the next on the factor price frontier, with one new method appearing in the same or another industry.

Consider now the quantity side. At a switch point, the coexistence of  $n+1$  methods for  $n$  goods can be viewed as the coexistence of two neighboring square  $n$ -sets, with  $n-1$  common methods and an alternative for the production of some commodity. Consider first a given  $n$ -set. The input matrix  $A$  has dimension  $n \times n$  and the net baskets  $d$  which can be produced under the scarcity constraint (2) are those belonging to the simplex  $S = \{d; d > 0 \text{ and } Vd = \bar{l}\}$ , where  $V = l(I - A)^{-1}$  is the row-vector representing the land values of the commodities. These land values are similar to the more usual labor values and the condition  $Vd = \bar{l}$  means that the whole land enters directly or indirectly into the production of basket  $d$ . Consider now two  $n$ -sets  $A$  and  $A'$  which differ by one method. At any given rate of profit, all prices in terms

of wage are smaller in the selected system (Sraffa, 1960, section 94), therefore all labor values (which coincide formally with prices for zero profits) are also smaller in one system. Transposed into the agricultural framework, the property means that one of the vectors  $V$  or  $V'$  of land values is smaller than the other, therefore one of the simplices  $S$  or  $S'$  is entirely above the other: without ambiguity, the corresponding  $n$ -set is more productive. Finally, consider a Ricardian node: two neighboring  $n$ -sets work together and, depending on relative activity levels, can produce any basket which is a barycenter of baskets in  $S$  and  $S'$ , therefore any basket belonging to the 'slice' between  $S$  and  $S'$ .

#### **4. THE RICARDIAN DREAM**

It is worth noting that, in the above analysis, the dynamics result from the examination of the value conditions and that the posterior study of quantities has shown that the land value constitutes the relevant index to measure physical productivity. Ricardo's views on the dynamics of intensive cultivation can be criticized because they proceed by identifying profitability and productivity conditions. Let us rather quote Sraffa (1960, section 88): "The existence side by side of two methods can be regarded as a phase in the course of a progressive increase of production on the land. The increase takes place through the gradual extension of the method that produces more corn at a higher unit cost, at the expense of the method that produces less. As soon as the former method has extended to the whole area, the rent rises to the point where a third method which produces still more corn at a still higher cost can be introduced to take the place of the method that has just been superseded." The reference to a change in *one* method corresponds to the jump from a switch point to the next. The implicit hypothesis is that the new method then introduced is indeed more productive, but the property is not ensured.

A 'Ricardian reversal' occurs when the newly introduced method is less productive than the one with which it is jointly operated. The phenomenon explains the occurrence of

multiple solutions: if, after a series of jumps which allow to produce up to a certain level  $d$ , the new method reduces it to a lesser level  $d'$ , there are two possibilities to produce an intermediate basket. Suppose moreover that the next switch is of the normal type and allows to increase production up to some high level. In a static framework, one can find values and activity levels sustaining the production of a basket greater than  $d$  but, when demand is continuously increasing beyond  $d$ , a jump appears with the sudden introduction of new methods of cultivation at positive activity levels (on the factor price frontier, it corresponds to a jump over the next switch point). These discontinuities do not fit with smooth substitution on the physical side. From a theoretical point of view, the occurrence of a Ricardian reversal is linked with the so-called paradoxes of capital theory: 'capital reversal' occurs when more capital intensive methods are less productive.

The Ricardian story holds if the substitution along the frontier leads to the introduction of more productive methods. This condition can be characterized by an algebraic criterion which is some resemblance with Erreygers's (1995) uniqueness criterion:

*Definition.* Let us associate with a method of production and an arbitrary rate of profit  $r$  the  $(n+1)$ -column vector made of its output vector minus  $(1+r)$  times its material inputs, with the scalar  $-l_i$  in last position.  $\Delta(r)$  is the determinant obtained by stacking  $n+1$  vectors, with two methods competing to produce the last good.

We adopt the temporary convention that the coefficients of the old method are in the last but one column whereas those of the new method are in the last column. Since the next criterion derives from a property of single-product systems, we momentarily replace 'land' by 'labor' and 'rent' by 'wage'. When the rate of profit varies continuously, the determinant  $\Delta(r)$  vanishes at switch points because the value equalities (3) show that the product of price-and-

wage vector by the corresponding matrix is zero. The determinant preserves a constant sign between switch points and this sign indicates which of the alternative methods is then cheaper. If it is positive, a fictitious increase in the labor coefficient of the method described in the last column lets decrease the determinant (by the Hawkins-Simon criterion) down to zero, the fictitious method is then as costly as the old, therefore the method before the fictitious change was cheaper. The argument shows that the determinant  $\Delta(r)$  is positive when the method ranked in last position is cheaper at the rate of profit  $r$  (Bidard, 2004). On the value side, the determinant vanishes at a switch point  $r$  and takes the sign of  $-\Delta'(r)$  after it ('minus' because the rate of profit is decreasing), and since the introduction of the new method is justified by its lower costs,  $-\Delta'(r)$  is positive. On the physical side, the change introduces a method with smaller labor values if  $\Delta(0)$  is positive. In the agricultural framework, and after abandoning the convention on the ranking of the columns of the determinant, the conclusion is:

*Proposition.* The substitution of methods along the frontier for decreasing rates of profit (or increasing rents) lets come in at each switch point a new method which operates jointly with an old method producing the same commodity. The Ricardian dynamics hold if the new method is more productive, which is the case if the derivative  $\Delta'(r)$  at the switch point and  $\Delta(0)$  have opposite signs.

Under the alternative hypothesis of a given rate of profit (Sraffa's hypothesis), similar lessons are obtained by showing that only the upper envelope of the wage-rent curves matters and that a jump from a switch to the next on the frontier correspond to the change in one method of production. Again, the question is whether the new method, which is selected for profitability reasons, is indeed more productive. Some differences regarding the conclusions

are illustrated by the behavior of a simple corn-labor-land model with corn as the only produced commodity: for a given real wage, the model behaves like a neoclassical corn-land model with smooth substitution and the Ricardian dynamics hold; on the contrary, multiple solutions may coexist for a given rate of profit (Freni, 1991).

## **5. CONCLUSION**

Following an increase in the demand for agricultural goods, a rise in prices provides an incentive to use more costly methods on the already cultivated lands. The progressive substitution of a new method for an old one presumes that the new method, which has become profitable at the new prices and distribution levels, is indeed more productive, a coincidence which is not automatically ensured. The conditions for smooth Ricardian dynamics in a general intensive cultivation model have been identified. If they are not met, the occurrence of a reversal implies severe discontinuities on the physical side which make the dynamics of intensification obscure. From a theoretical standpoint, another implication of this study follows from a return to the traditional framework with homogeneous labor and no land. The construction then leads to an endogenous determination of the wage rate under a full employment hypothesis, for a given labor supply. The stage is thus set for a solid comparison between the classical approach and the conventional theory.

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