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Abstract

The aim of this research paper is to evaluate hedge fund returns Value-at-Risk by using GARCH models. To perform the empirical analysis, one uses the HFRX daily performance hedge fund strategy subindexes and spans the period March 2003 – March 2008. I found that skewness and kurtosis are substantial in the hedge fund returns distribution and the clustering phenomenon is pointed out. These features suggest the use of GARCH models to model the volatility of hedge fund return indexes. Hedge fund return conditional variances are estimated by using linear models (GARCH) and non-linear asymmetric models (EGARCH and TGARCH). Performance of several Value at Risk models is compared; the Gaussian VaR, the student VaR, the cornish fisher VaR, the normal GARCH-type VaR, the student GARCH-type VaR and the cornish fisher GARCH-type VaR. Our results demonstrate that the normal VaR underestimates accurate hedge fund risks while the student and the cornish fisher GARCH-type VaR are more reliable to estimate the potential maximum loss of hedge funds.

JEL Classification: G11, G12, G23
Keywords: Hedge Fund, Value at Risk, GARCH models

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Since the 1990's, a substantial growth of hedge funds has been observed. According to HedgeFund Intelligence, assets under management accounted for $2, 65 trillion in 2007. Since 2000, investors have been looking for new investments. Fuelled by the prospect of double and triple-digit returns and the losses caused by the 2000 "dot-com bubble", a wide range of them have sought exposure to hedge funds. They have been getting more and more importance within the financial community since the beginning of this decade. It is thus important to keep an eye on hedge funds particularly because they have attracted a growing share of institutional investors capital. This share has been constantly progressing as a result of the growth of incoming liquidity under their management, then delegate to hedge funds. The institutionalization of hedge funds raises the issue of a good risk management because public savings are concerned.

In the same way, as hedge funds seek positive absolute returns they therefore engage in aggressive strategies leading to extreme losses. Their potential impact on systemic risk is real. This is particularly true in regard of the leverage hedge funds undertake. It may raise financial stability issues. Its use has been drawn to the attention since the beginning of the nineties. In 1993/94, several highly-leveraged funds were said to have amplified volatility in the US bond market because of the forced liquidation of their positions. When the Federal Reserve unexpectedly raised interest rates, funds were forced to deleverage causing bond prices to fall. Additionally, to meet margin calls, these funds had to sell off their long positions in European securities markets, transmitting the disturbance to European markets. In the same way, in 1997/98, hedge funds had been able to take substantial short positions in Asian markets as a result of the leverage provided by their counterparties. Those episodes fuelled complaints that hedge funds using leverage destabilize the markets. Concerns about hedge fund leverage reach its highest point with the collapse of Long-Term Capital Management in 1998. In order to deliver attractive returns, the fund had highly leveraged positions as price discrepancies in US securities market were small. In September 1998, LTCM's leverage was very high. It had 600 million in capital to offset positions worth $1 billion. With a high probability of failure, fears arose that the distressed sales of its positions in US securities markets might destabilize financial markets. These fears led to the rescue of LTCM by the FED. In 2007, hedge funds own most of the risky CDO tranches (around 46%, OECD). As they are very big players, the counterparty credit exposures are very large. Problems arose when hedge funds started failing. Hedge funds have been experiencing extreme funding liquidity risk since banks were short on capital, especially because they relied on leverage. They then faced higher margins. The interaction between the liquidity risk and leverage lead to extreme losses.

Thus, the use of leverage by hedge funds to a considerable extent raises the debate about the implications of hedge fund operations for systemic stability and increases their exposure to market risk. The objective of absolute performance conducts them to such a fragile situation. And the fact that they implicitly manage public savings justifies this need of a good risk management that has never been more pressing.

All these worries highlight the importance of a reliable way to evaluate hedge fund risks. However, the traditional measures of risk are not suitable for hedge funds as they not deal with some of hedge funds specificities. Indeed, under the mean-variance approach, hedge funds are very attractive as they generate high returns with low risk. But, the expansive literature about hedge fund returns analysis concludes that the mean-variance approach is not adequate to investigate hedge funds risk. Fung and Hsieh [1997], Brook and Kat [2001] or Amenc, Curtis and Martellini [2003] demonstrated that the monthly returns distribution of hedge fund strategies present negative skewness and excess kurtosis. Thus, according to Kat [2003], volatility underestimates hedge fund risks since volatility does not give any information about asymmetric distribution of returns and extreme losses. But, as noticed by Scott and Horvath [1980], investors are interested in higher moments of the distribution function. These drawbacks underline the importance of an adequate measure of hedge funds risk that accounts for extreme losses in order to capture as much as possible their characteristics. This paper is focused on this concern. One aims to go further the traditional measures of risk and introduce a more appropriate method to hedge funds.

While we point out that positions undertake by directional hedge funds strategies[3] increase the market risk and extreme losses have to be considered, we decided to focus on Value at Risk (VaR). VaR is widely used in the practice of risk management within the financial industry. Its popularity is mainly due to the expression of the market risk in only one figure. For a methodology to be considered sound, the probability of loss should be reflected accurately. Different types of VaR have been introduced. Riskmetrics assumes that the continuously compounded daily return of a portfolio follows a conditional normal distribution. But in the case of skewed and fat-tails returns, estimates result in an underestimation of the true risk. Favre and Galeano [2002] introduced the modified VaR that is based on the cornish fisher expansion quantile in order to take into account the asymmetry and the kurtosis of the distribution. The historical estimation methodology in turn provides a non-parametric estimate of VaR. It does not make any assumption about the distribution of the portfolio

[3] Fund positions are based on the evolution of the market as a whole.

return. It assumes that the distribution of returns will remain the same in the past and in the future. While these methods have a real contribution, they are not forward-locking and not adapted to capture hedge fund specificities. And as we seek to predict VaR, GARCH-type VaR appears to be more appropriate. In the same way, hedge fund returns exhibit nonlinear dependencies (Fung and Hsieh [1997]). These nonlinearities are caused by the use of derivatives, leverage and illiquid assets. It is thus important to capture time-variation in hedge fund returns. GARCH-type VaR allows the inclusion of time-varying conditional volatility in the VaR and deals with asymmetric and leptokurtosism phenomenon. Nonlinearities also often arise from the presence of dynamic strategies as hedge fund managers are able to shift their exposures rapidly. To account for the nonlinearity aspect and to analyze the dynamics of hedge fund risk, one also considers asymmetric GARCH models to trace hedge fund returns volatility process more effectively.

On the other hand, because of their private nature, hedge funds do not have to disclose their results. Most of researches on hedge funds are based on monthly data. The "democratisation" of hedge funds has encouraged them to provide data with higher frequency. Another contribution of this paper is the use of daily data to measure all hedge fund strategies risks.

This article is organised as follows: Firstly, I present hedge fund strategies according to Hedge Fund Research data provider. Secondly, one introduces the Value at Risk concept and the conventional variance models considered. While the normal distribution is widespread, it however cannot describe fat-tails returns. Hence, the student-t distribution of returns is also applied dealing with leptokurtosis. GARCH, EGARCH and TGARCH process take into account the observed volatility clustering of returns. Thirdly, the GARCH models are applied to forecast a 1 day-ahead Value at Risk for various thresholds (5%, 2.5% and 1%). The forecasts are compared with the VaR based on the standard deviation. Finally, one backtests the different VaR approaches to test the relevance of the VaR models considered.

Hedge fund Strategies Description

Hedge fund strategies may be grouped in three different areas; directional, arbitrage and specific situations. Directional strategies entail a bet on the direction of the overall market. They involve taking positions on forward and option markets, and in global markets. Arbitrage strategy managers seek to exploit price discrepancies. Finally specific situations strategy managers tend to benefit from events affecting companies such a merger arbitrage and a restructuration.

Directional strategies	Arbitrage strategies	Specific situation strategies
Equity hedge *Macro*	*Convertible arbitrage* *Equity market neutral* *Relative value arbitrage*	*Distressed securities* *Event driven* *Merger arbitrage*

To perform the empirical analysis, one uses the HFRX hedge fund indexes. The description of hedge fund strategies is directly taken from HFR documentation.

Global Index

The HFRX Global Hedge Fund Index is designed to be representative of the overall composition of the hedge fund universe. It is comprised of eight strategies; convertible arbitrage, distressed securities, equity hedge, equity market neutral, event driven, macro, merger arbitrage, and relative value arbitrage. The strategies are asset weighted based on the distribution of assets in the hedge fund industry.

Convertible arbitrage

Convertible Arbitrage involves taking long positions in convertible securities and hedging those positions by selling short the underlying common stock. A manager will, in an effort to capitalize on relative pricing inefficiencies, purchase long positions in convertible securities, generally convertible bonds, convertible preferred stock or warrants, and hedge a portion of the equity risk by selling short the underlying common stock.

Distressed Securities

Distressed Securities managers invest in, and may sell short, the securities of companies where the security's price has been, or is expected to be, affected by a distressed situation. Managers will seek profit opportunities arising from inefficiencies in the market for such securities and other obligations.

Equity Hedge

Equity Hedge, also known as long/short equity, combines core long holdings of equities with short sales of stock or stock index options. Equity hedge portfolios may be anywhere from net long to net short depending on market conditions. Equity hedge managers generally increase net long exposure in bull markets and decrease net long exposure or even are net short in a bear market

Equity Market Neutral

Equity Market Neutral strategies strive to generate consistent returns in both up and down markets by selecting positions with a total net exposure of zero. Trading managers will hold a large number of long equity positions and an equal, or close to equal, dollar amount of offsetting short positions for a total net exposure close to zero. By taking long and short positions in equal amounts, the equity market neutral manager seeks to neutralize the effect that a systematic change will have on values of the stock market as a whole.

Event driven

Event Driven investment strategies or "corporate life cycle investing" involves investments in opportunities created by significant transactional events, such as spin-offs, mergers and acquisitions, industry consolidations, liquidations, reorganizations, bankruptcies, recapitalizations and share buybacks and other extraordinary corporate transactions. Event driven trading involves attempting to predict the outcome of a particular transaction as well as the optimal time at which to commit capital to it.

Macro

Macro strategies attempt to identify extreme price valuations in stock markets, interest rates, foreign exchange rates and physical commodities, and make leveraged bets on the anticipated price movements in these markets. Profits are made by correctly anticipating price movements in global markets and having the flexibility to use any suitable investment approach to take advantage of extreme price valuations

Merger Arbitrage

Merger Arbitrage, also known as risk arbitrage, involves investing in securities of companies that are the subject of some form of extraordinary corporate transaction, including acquisition or merger proposals, exchange offers, cash tender offers and leveraged buy-outs. Typically, a manager purchases the stock of a company being acquired or merging with another company, and sells short the stock of the acquiring company. A manager engaged in merger arbitrage transactions will derive profit (or loss) by realizing the price differential between the price of the securities purchased and the value ultimately realized when the deal is consummated.

Relative Value Arbitrage

Relative Value Arbitrage is a multiple investment strategy approach. The overall emphasis is on making "spread trades" which derive returns from the relationship between two related securities rather than from the direction of the market. Generally, trading managers will take offsetting long and short positions in similar or related securities when their values, which are mathematically or historically interrelated, are temporarily distorted. Profits are derived when the skewed relationship between the securities returns to normal.

Methodology

VaR is defined by the following relationship:

$$P_r \left\lfloor r_t \prec VaR_{t|t-1}(\alpha) \right\rfloor = \alpha \qquad (1)$$

r_t = *return on asset at time t*
$1-\alpha$ = *confidence level*

It describes the estimated maximum potential loss of an asset not exceeded with a given probability defined as the confidence level, over a given period of time.

It is computed as follows:

$$VaR_{t|t-1}(\alpha) = T \times E(r) + z(\alpha) \times \sqrt{T \times} h_t(r) \qquad (2)$$

T = *time horizon*
$E(r)$ = *average return*
$z(\alpha)$ = *quantile depending on the level of confidence* α
$h_t(r)$ = *volatility* $\left(\sqrt{h_t}\right.$ *in case of GARCH – type estimate of volatility*$\left.)\right)$

The VaR is the result of a product of the scalar denoted $z(\alpha)$ representing a quantile depending on the level of confidence with the volatility in $t-1$ at horizon t.
According to Wilmott [1998], the assumption of zero mean is valid over short-term horizons. This assumption is based on the conjecture that the magnitude of mean is substantially smaller than the magnitude of the standard deviation and therefore can be ignored.

Therefore, the VaR formula considered is:

$$VaR_{t|t-1}(\alpha) = \underbrace{z(\alpha)}_{1.} \times \sqrt{T} \times \underbrace{h_t(r)}_{2.} \tag{3}$$

To evaluate hedge fund risks, one considers different types of VaR. The difference is made upon the quantile and the volatility model considered.

1. Types of quantile.

- Normal quantile
- Student quantile. The Student's *t* distribution deals with the phenomenon of excess kurtosis by modelling tail thickness by a parameter called "degree of freedom".
- Cornish fisher quantile. Favre and Galeano [2002] introduced the modified VaR that is based on the cornish fisher expansion quantile. It is an expansion around the normal distribution in order to take into account the asymmetry and the fat tails of the distribution.

$$z_{CF}(\alpha) = z_{normal}(\alpha) + \frac{1}{6}(z_{normal}(\alpha)^2 - 1)S + \frac{1}{24}(z_{normal}(\alpha)^3 - 3z_{normal}(\alpha))(K-3) - \frac{1}{36}(2z_{normal}(\alpha)^3 - 5z_{normal}(\alpha))S^2$$

$$\begin{aligned} z_{normal}(\alpha) &= quantile\ \ of\ \ the\ \ normal\ \ distribution \\ S &= skewness\ \ coefficient \\ K &= kurtosis\ \ coefficient \end{aligned}$$

2. Types of volatility model.

- Volatility estimate is based on standard deviation
- Volatility estimate is based on GARCH models. In forecasting a GARCH model of a time series of returns R_t, three distinct specifications have to be provided; one for the conditional mean equation, one for the conditional variance, and one for the conditional error distribution, conditional on Ω_{t-1}, the information set available at time t-1.

GARCH models assume that the conditional mean equation is modelled as follows:

$$\begin{aligned} R_t &= c + \varepsilon_t \\ \varepsilon_t &= \eta_t \sigma_t \\ \eta_t\ \ &iid\ \ N(0,1) \end{aligned} \tag{4}$$

However, when returns exhibit serial dependence structure, the mean equation is modelled as an AR, MA or ARMA process.

We first focus on GARCH model introduced by Bollerslev [1986]:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 \tag{5}$$
$$\alpha_i \succ 0 \quad \forall \quad i = 1 \cdots\cdots q \quad and \quad \beta_j \succ 0 \quad \forall \quad j = 1 \cdots\cdots p$$

We also considered Nelson [1991] exponential GARCH model (EGARCH) in which the logarithm of conditional variance is specified as:

$$\log \sigma^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{k=1}^{r} \beta_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} + \sum_{j=1}^{p} \gamma_j \log \sigma_{t-j}^2 \tag{6}$$

Finally we deal with Zakoian [1990] threshold GARCH model (TGARCH) in which the conditional variance is specified as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \beta_i \varepsilon_{t-1}^2 d_{t-1} + \sum_{j=1}^{p} \gamma_j \sigma_{t-j}^2 \tag{7}$$
$$d_{t-1} = \begin{cases} 1 & if \quad \varepsilon_{t-1} \prec 0 \\ 0 & if \quad \varepsilon_{t-1} \geq 0 \end{cases}$$

An assumption about the conditional distribution of the error term ε_t is required. While the normal (Gaussian) distribution is widespread; it however cannot describe fat-tails returns. Hence the student's *t* distribution is also applied, modelling tail thickness by a parameter called "degree of freedom".

To sum up, 21 VaRs are computed; those based on standard deviation and computed with normal, student and cornish fisher quantile and those based on conditional volatility models (GARCH, TGARCH and EGARCH under the assumption of both normal and student assumption) computed with the same quantiles.

Empirical Analysis of Hedge Fund Return Indexes Volatility

Data

Hedge fund providers licence their indexes to partners who can then create investable products. These products track the index by investing in a weighted portfolio of its constituents. To this end, only a limited numbers of liquid hedge funds are selected which leads to a sub-representativity bias.

Investable indexes providers have very strict hedge fund selection criterion in order to end up with a product easy to manage. However, very few hedge funds fulfil them. Consequently, the whole universe of hedge funds is not represented. Investable indexes are less representative than non investable indexes.

To perform the empirical analysis, one uses the HFRX (investable Hedge Fund Research indices) daily performance subindexes split by an investment style and an aggregate index which encompasses all hedge fund strategies and spans the period March 31st 2003 - March 3rd 2008 (1241 observations).

The HFRX indexes are based on the Hedge Fund Research (HFR) database. The indexes measure the net of fee returns denominated in US dollar. Funds included must be currently open to new transparent investment, have at least $50 Million under management and meet 24 month track record. The HFRX indexes consist of eight single strategies presented above; convertible arbitrage, distressed securities, equity hedge, equity market neutral, event driven, macro, merger arbitrage and relative value arbitrage. The HFRX global hedge fund index encompasses over 55 funds.

Summary Statistics

Globally it can be noticed that hedge fund index returns[4] are quite favourably compared to stocks and bonds (appendix A). HFRX global index has an average mean return of 5.73 %. Amongst investment styles, event driven exhibits the highest average mean return with 7.78 % followed by macro (7.58 %). Convertible arbitrage and equity market neutral strategies exhibit the lowest returns.

Appendix A shows that standard deviations of stock market indexes and JP Morgan EMU Bond Index are much higher than those of hedge funds. Russell 2000 has an average standard deviation of 18.50%, followed by NASDAQ (16.59%) and Dow Jones EURO STOXX 50 (16.21%), while HFRX global index exhibits a weak average standard deviation of 3.74%.

[4] Returns of hedge fund indexes at time t are computed as follows:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \qquad (8)$$

$P_t = Value\ \ of\ \ the\ \ hedge\ \ fund\ \ strategy\ \ index\ \ at\ \ the\ \ time\ \ t$

$P_{t-1} = Value\ \ of\ \ the\ \ hedge\ \ fund\ \ strategy\ \ index\ \ at\ \ the\ \ time\ \ t-1$

The most attractive strategies in term of volatility are distressed securities (2.36%), merger arbitrage (3.55%) and convertible arbitrage (3.61%).

None of the hedge fund returns distributions seem to be symmetric. As a matter of fact, their skewness coefficients are different from zero and negative. The same conclusion is reached when we look at the stock and bond market indexes.
Globally, hedge funds, stocks and bonds show evidence of fat tails, since the kurtosis exceeds 3, which is the normal value. It means that extreme returns (either losses or gains) are more likely than they are with the normal distribution. The returns distribution is leptokurtic. The combination of negative skewness and excess kurtosis denote a high probability of negative returns. Equity market neutral strategy exhibits the highest kurtosis coefficient with 38.03 followed by relative value arbitrage with 14.62 and merger arbitrage with 13.96.

According to those findings, hedge fund returns distribution seems to be far from normally distributed. According to the Jarcque-Bera test of normality, the null hypothesis of normally distributed returns is not accepted for none of the indexes. Indeed, the results show that the statistic is higher than theoretical value read in Chi-Square table with two degrees of freedom at the significant value of 5% (5.99) for all hedge fund return indexes and stock and bond market indexes. These results demonstrate that measures based on mean-variance approach are not reliable. On the other hand, table 1 below shows the existence of ARCH effects for most hedge fund strategies. This might be linked to the non-normality of the returns but also to the conditional distribution. That's why, one decided to go further the normal distribution and consider the Skewness one and the Cornish-Fisher expansion.

Table 1: ARCH Test of Hedge Fund Return Indexes

	$T \times R^2\,(prob)$	$Q(5)\,(prob)$	$Q(10)\,(prob)$
Convertible Arbitrage	28.25 (0.00)	19.28 (0.00)	33.76 (0.00)
Distressed Securities	5.36 (0.25)	84.75 (0.00)	115.70 (0.00)
Event Driven	122.75 (0.00)	30.00 (0.00)	38.65 (0.00)
Equity Hedge	202.79 (0.00)	47.18 (0.00)	58.46 (0.00)
Equity Market Neutral	429.76 (0.00)	43.86 (0.00)	49.68 (0.00)
Macro	191.88 (0.00)	42.57 (0.00)	51.25 (0.00)
Merger Arbitrage	269.26 (0.00)	14.42 (0.01)	28.85 (0.00)
Relative Value Arbitrage	282.26 (0.00)	11.49 (0.04)	24.21 (0.01)
Global	240.90 (0.00)	79.74 (0.00)	94.30 (0.00)

Volatility clustering is exhibited in appendix B. We can see that periods of high and low volatility are grouped together. These stylized facts are typical features of financial time series and demonstrate that the volatility of hedge funds varies through time. This suggests the use of non-linear time series structures to model the volatility of hedge fund returns.

Volatility Forecasting Using GARCH Models

As hedge fund strategies exhibit serial correlation induced by discontinuous trading (table 1), we represent the return generating process in the mean equation of the GARCH-type model with an ARMA-like component[5] in order to desmooth hedge fund returns. A lag structure of p=1 and q=1 seems to be the most suitable (for equity market neutral, merger arbitrage and relative value arbitrage, the appropriate lag structure is p=2 and q=2). The autocorrelation and partial autocorrelation functions are used to identify the form of the mean equation. The choice of the adequate mean equation is based on selection criterion[6].

Then, one models the hedge fund strategies conditional variance as a GARCH, TGARCH and EGARCH process as they deal with nonlinearities and asymmetries. The conditional variances were estimated under the assumption that residuals ε_t follow the normal and the student law. A lag structure of p=1 and q=1 is also considered.

Appendixes D and E present the hedge fund strategies parameters estimates. Whatever the distribution followed by residuals ε_t, the conditional variance parameters are significant in all cases. The Ljung-Box and the ARCH-LM tests confirm that models considered take the heteroscedasticity into account for almost all hedge fund strategies.

GARCH(1;1) process exhibit $\hat{\alpha}_1 + \hat{\beta}_1 \prec 1$ which means that hedge fund strategies show a mean-reverting behaviour.

TGARCH(1;1) model shows a value of $\hat{\beta}_1 \succ 0$ (excepted for macro strategy), which indicates that bad news have larger impact on the volatility of the returns.

We observe that EGARCH(1;1) process display a value of $\hat{\beta}_1 \prec 0$ and $\hat{\gamma}_1 \succ 0$. Bad news increase hedge fund strategies volatility at time $t+1$ and the extent of bad news on the volatility is important.

[5] Either an AR, a MA or an ARMA.

[6] R^2, AIC, SIC and Log Likelihood.

According to these findings, all hedge fund strategies present asymmetric effects. This feature is in line with the negative skewness, nonlinearities and dynamic trading strategies that characterise hedge fund returns.

Table 2: Value at Risk Forecasts Results

Normal quantiles

	Standard Deviation	GARCH	TGARCH	EGARCH
	VaR	*VaR*	*VaR*	*VaR*
Convertible Arbitrage	-0,529%	-0,656%	-0,678%	-0,703%
Distressed Securities	-0,346%	-0,350%	-0,351%	-0,366%
Event Driven	-0,647%	-0,646%	-0,619%	-0,676%
Equity Hedge	-0,842%	-0,949%	-1,008%	-0,910%
Equity Market Neutral	-0,604%	-0,739%	-0,796%	-0,738%
Macro	-1,094%	-1,574%	-1,897%	-1,856%
Merger Arbitrage	-0,521%	-0,334%	-0,343%	-0,349%
Relative Value Arbitrage	-0,512%	-0,816%	-0,876%	-0,816%
Global Index	-0,548%	-0,653%	-0,637%	-0,599%

Student quantiles

	Standard Deviation	1% GARCH	TGARCH	EGARCH
	VaR	*VaR*	*VaR*	*VaR*
Convertible Arbitrage	-0,562%	-0,702%	-0,723%	-0,753%
Distressed Securities	-0,394%	-0,429%	-0,424%	-0,430%
Event Driven	-0,705%	-0,704%	-0,688%	-0,733%
Equity Hedge	-0,886%	-1,014%	-1,080%	-0,978%
Equity Market Neutral	-0,646%	-0,808%	-0,854%	-0,802%
Macro	-1,207%	-1,743%	-2,131%	-2,098%
Merger Arbitrage	-0,583%	-0,378%	-0,397%	-0,400%
Relative Value Arbitrage	-0,573%	-0,898%	-0,971%	-0,919%
Global Index	-0,591%	-0,701%	-0,718%	-0,639%

Cornish fisher quantiles[7]

	1%			
	Standard Deviation	GARCH	TGARCH	EGARCH
	VaR	*VaR*	*VaR*	*VaR*
Convertible Arbitrage	-0,690%	-0,857%	-0,885%	-0,918%
Distressed Securities	-0,404%	-0,408%	-0,409%	-0,427%
Event Driven	-0,990%	-0,988%	-0,947%	-1,034%
Equity Hedge	-1,125%	-1,268%	-1,346%	-1,216%
Equity Market Neutral	-2,774%	-3,395%	-3,660%	-3,392%
Macro	-2,207%	-3,173%	-3,825%	-3,743%
Merger Arbitrage	-1,171%	-0,750%	-0,771%	-0,786%
Relative Value Arbitrage	-1,136%	-1,811%	-1,945%	-1,812%
Global Index	-0,862%	-1,027%	-1,001%	-0,942%

Table 2 reports results of 1 day-ahead VaR forecasts computed for 1 % threshold using normal, student and cornish fisher quantiles.

Among investment styles, equity hedge, merger arbitrage and relative value exhibit the highest VaR for almost all methods. These strategies seek to benefit from a spread which leads to a significant use of leverage increasing the level of risk. This is particularly true for relative value arbitrage (cf LTCM). Managers experience small price discrepancies and use a high level of leverage in order to generate returns. In the same way, merger arbitrage managers are exposed to market risk. Indeed when the market is down, merger arbitrage activity suffers.

Results show that VaR based on the normal quantile underestimate market risk. Those based on student and cornish fisher quantiles seem to be more relevant methods as their values are higher.

Among VaR methods, whatever the quantile may be, TGARCH and EGARCH-type VaR exhibit the highest values.

If one has a glance to the appendixes G, H and I, we can see that the GARCH/TGARCH and EGARCH-type VaR do react immediately to small or large price changes.

[7] GARCH, TGARCH and EGARCH parameters are estimated under the assumption that residuals follow the normal law

Backtesting VaR Models

To support the conclusions found above, it is necessary to test these VaR models. The quality of the VaR forecasts depends on the quality of the VaR method. One has to judge whether our VaR forecasts are consistent with subsequently realized returns given the confidence level. When the number of realized observations falling outside VaR predictions is in line with the confidence level, the VaR model is adequate.

The tests start from a hit sequence function. It describes whether or not a loss in excess of the reported VaR has been realized. The function is defined as follows:

$$I_{t+1}(\alpha) = \begin{cases} 1 & if \quad R_t \prec VaR_{t|t-1}(\alpha) \\ 0 & if \quad R_t \succ VaR_{t|t-1}(\alpha) \end{cases} \tag{19}$$

$R_t = daily\ ex - post\ returns$
$VaR_t(\alpha) = ex - ante\ Value - at - Risk\ \ forecats$

A VaR model will be accurate if and only if the hit sequence function satisfies both the unconditional coverage property and the independence property[8] (Christoffersen [1998]).

Backtesting Results

The number of observations for each hedge fund index is 1241. One uses the last 250 observations for out-of-sample forecasting. For each model, the first 991 daily returns are used to form a VaR forecast for day 992. Then, data from day 2 to day 992 are used to form a VaR forecast for day 993 and so on. 250 out-of-sample forecasts are generated recursively by moving the estimation-window forward through time.

Table 3: Number and Proportion of Hedge Funds VaR Failures

Normal quantiles

	1%			
	Standard Deviation	GARCH	TGARCH	EGARCH
	VaR	*VaR*	*VaR*	*VaR*
Convertible Arbitrage	-0,53%	-0,66%	-0,68%	-0,70%
Distressed Securities	-0,35%	-0,35%	-0,35%	-0,37%
Event Driven	-0,65%	-0,65%	-0,62%	-0,68%

[8] These tests are presented in appendix G.

Equity Hedge	-0,84%	-0,95%	-1,01%	-0,91%
Equity Market Neutral	-0,60%	-0,74%	-0,80%	-0,74%
Macro	-1,09%	-1,57%	-1,90%	-1,86%
Merger Arbitrage	-0,52%	-0,33%	-0,34%	-0,35%
Relative Value Arbitrage	-0,51%	-0,82%	-0,88%	-0,82%
Global Index	-0,55%	-0,65%	-0,64%	-0,60%

Student quantiles[9]

	Standard Deviation	1% GARCH	TGARCH	EGARCH
	VaR	*VaR*	*VaR*	*VaR*
Convertible Arbitrage	-0,56%	-0,70%	-0,72%	-0,75%
Distressed Securities	-0,39%	-0,43%	-0,42%	-0,43%
Event Driven	-0,71%	-0,70%	-0,69%	-0,73%
Equity Hedge	-0,89%	-1,01%	-1,08%	-0,98%
Equity Market Neutral	-0,65%	-0,81%	-0,85%	-0,80%
Macro	-1,21%	-1,74%	-2,13%	-2,10%
Merger Arbitrage	-0,58%	-0,38%	-0,40%	-0,40%
Relative Value Arbitrage	-0,57%	-0,90%	-0,97%	-0,92%
Global Index	-0,59%	-0,70%	-0,72%	-0,64%

Cornish fisher quantiles

	Standard Deviation	1% GARCH	TGARCH	EGARCH
	VaR	*VaR*	*VaR*	*VaR*
Convertible Arbitrage	-0,69%	-0,86%	-0,89%	-0,92%
Distressed Securities	-0,40%	-0,41%	-0,41%	-0,43%
Event Driven	-0,99%	-0,99%	-0,95%	-1,03%
Equity Hedge	-1,13%	-1,27%	-1,35%	-1,22%
Equity Market Neutral	-2,77%	-3,40%	-3,66%	-3,39%
Macro	-2,21%	-3,17%	-3,83%	-3,74%
Merger Arbitrage	-1,17%	-0,75%	-0,77%	-0,79%
Relative Value Arbitrage	-1,14%	-1,81%	-1,95%	-1,81%
Global Index	-0,86%	-1,03%	-1,00%	-0,94%

A glance to table 3 shows that the ex-post violations rate is larger than the initial coverage rate. For most of the models, losses in excess of the reported VaR occurred more frequently, no matter the threshold. This suggests that VaR understates the actual level of risk.

Under the cornish fisher GARCH-type (2.5% and 1%), losses in excess occurred less frequently for several hedge fund strategies. This means that VaR is too conservative.

[9] No reported conditional variance for several GARCH and TGARCH estimations

Table 4: Unconditional Coverage, Independence and Conditional Coverage Test

Normal quantiles

	1%							
	Standard Deviation		GARCH		TGARCH		EGARCH	
	Nb of F	*POF*	*Nb of F*	*POF*	*Nb of F*	*POF*	*Nb of F*	*POF*
Convertible Arbitrage	11	4.40%	6	2.40%	6	2.40%	7	2.80%
Distressed Securities	4	1.60%	3	1.20%	3	1.20%	3	1.20%
Event Driven	17	6.80%	7	2.80%	8	3.20%	7	2.80%
Equity Hedge	23	9.20%	5	2.00%	7	2.80%	11	4.40%
Equity Market Neutral	15	6.00%	11	4.40%	10	4.00%	10	4.00%
Macro	16	6.40%	5	2.00%	4	1.60%	4	1.60%
Merger Arbitrage	16	6.40%	12	4.80%	12	4.80%	12	4.80%
Relative Value Arbitrage	22	8.80%	8	3.20%	9	3.60%	9	3.60%
Global Index	24	9.60%	8	3.20%	11	4.40%	10	4.00%

Student quantiles[10]

	1%							
	Standard Deviation		GARCH		TGARCH		EGARCH	
	Nb of F	*POF*	*Nb of F*	*POF*	*Nb of F*	*POF*	*Nb of F*	*POF*
Convertible Arbitrage	9	3,60%	5	2,00%	6	2,40%	5	2,00%
Distressed Securities	3	1,20%	-	-	-	-	2	0,80%
Event Driven	11	4,40%	6	2,40%	4	1,60%	4	1,60%
Equity Hedge	21	8,40%	4	1,60%	6	2,40%	6	2,40%
Equity Market Neutral	13	5,20%	10	4,00%	9	3,60%	10	4,00%
Macro	15	6,00%	4	1,60%	3	1,20%	4	1,60%
Merger Arbitrage	15	6,00%	-	-	-	-	11	4,40%
Relative Value Arbitrage	19	7,60%	-	-	4	1,60%	6	2,40%
Global Index	21	8,40%	2	0,80%	4	1,60%	5	2,00%

Cornish fisher quantiles[11]

	1%							
	Standard Deviation		GARCH		TGARCH		EGARCH	
	Nb of F	*POF*	*Nb of F*	*POF*	*Nb of F*	*POF*	*Nb of F*	*POF*
Convertible Arbitrage	6	2,40%	3	1,20%	2	0,80%	2	0,80%
Distressed Securities	2	0,80%	3	1,20%	2	0,80%	3	1,20%
Event Driven	4	1,60%	2	0,80%	1	0,40%	2	0,80%
Equity Hedge	8	3,20%	1	0,40%	1	0,40%	0	0,00%
Equity Market Neutral	1	0,40%	0	0,00%	0	0,00%	0	0,00%
Macro	4	1,60%	0	0,00%	0	0,00%	0	0,00%
Merger Arbitrage	4	1,60%	0	0	0	0	1	0,40%

[10] No reported conditional variances for several GARCH and TGARCH estimations
[11] GARCH, TGARCH and EGARCH parameters are estimated under the assumption that residuals follow the normal law

Relative Value Arbitrage	2	0,80%	0	0	0	0,00%	0	0,00%
Global Index	7	2,80%	1	0,40%	1	0,40%	1	0,40%

The first table exhibits results when VaRs are computed using the normal quantile. The worst VaR model is performed by the one based on the standard deviation. It fails the unconditional and conditional coverage tests for all the hedge fund strategies (except distressed securities). However, it passes the independence test for most of the strategies (except equity market neutral, macro and global index).

On the opposite side, all GARCH-type VaR models pass successfully all the tests for most of the hedge fund strategies. GARCH and TGARCH-type VaR do not pass the unconditional and conditional coverage for equity market neutral and merger arbitrage. EGARCH-type VaR fails these tests for equity hedge, equity market neutral, merger arbitrage and global index.

When VaRs are computed using the student quantile, the best performers models are still GARCH-type VaR. Again the VaR model based on standard deviation fails almost all tests (except for distressed securities and independence test for several hedge fund strategies). VaRs models pass all the tests except the GARCH and TGARCH type VaR (it fails the unconditional coverage for equity market neutral and merger arbitrage. The EGARCH-type VaR does not pass the conditional coverage for merger arbitrage).

When VaRs are computed using the cornish fisher quantile, the worst performer model is once more the one based on the standard deviation. The GARCH, TGARCH and EGARCH -type VaR pass all tests.

Conclusion

This research paper aimed to investigate hedge funds market risk. One demonstrates that daily hedge fund return distributions are asymmetric and leptokurtic. Furthermore, volatility clustering phenomenon and the existence of ARCH effects demonstrate that hedge funds volatility varies through time. These features suggest the modelisation of their volatility using symmetric (GARCH) and asymmetric models (EGARCH and TGARCH).

The conditional variances were estimated under the assumption that residuals ε_t follow the normal and the student law. The conditional variance of hedge fund strategies exhibits asymmetric effects and mean reversion among investment styles.

The knowledge of the conditional variance was used to forecast 1-day-ahead ahead VaR. The estimations are compared with the Gaussian, the student and the modified VaR. The results demonstrate that VaR models based on normal quantile underestimate risk while those based on student and cornish fisher quantiles seem to be more relevant measurements. GARCH-type VaR are very sensitive to changes in the return process.

Backtesting results show that the choice of the model used to forecast volatility is important. Indeed, the VaR based on standard deviation is not relevant to measure hedge funds risks as it fails the appropriate tests. On the opposite side, GARCH, TGARCH and EGARCH-type VaR are accurate as they pass successfully the backtesting tests. The quantile used has also an impact on the relevance of the VaR models considered. GARCH-type VaR computed with the student and especially cornish fisher quantiles lead to better results.

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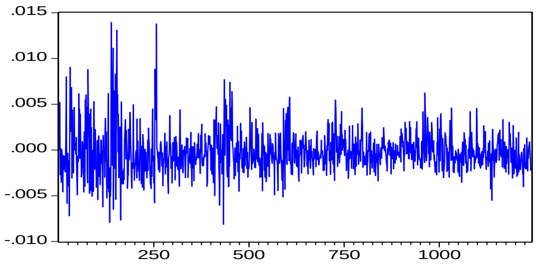
Appendix A: Summary statistics for Hedge Fund Daily Return Indexes from March 2003 to March 2008

	Mean [12]	*Volatility* [13]	*Skewness*	*Kurtosis*	*Jarque Bera Test*	*Probability*	*Normality*
HF Strategies							
HFRX Convertible Arbitrage	0.44%	3.61%	-0.44	4.97	241.01	0.00	No
HFRX Equity Hedge	5.53%	5.74%	-0.62	5.02	289.12	0.00	No
HFRX Equity Market Neutral	0.70%	4.12%	-0.28	38.03	63454.58	0.00	No
HFRX Merger Arbitrage	5.16%	3.55%	-1.17	13.96	6489.18	0.00	No
HFRX Relative Value Arbitrage	3.62%	3.49%	-0.18	14.62	6987.81	0.00	No
HFRX Event Driven	7.78%	4.42%	-0.71	6.84	867.56	0.00	No
HFRX Distressed Securities	7.52%	2.36%	0.36	5.99	488.82	0.00	No
HFRX Macro	7.58%	7.47%	-1.35	11.80	4385.33	0.00	No
HFRX Gobal Index	5.73%	3.74%	-1.09	7.18	1147.09	0.00	No
Stock Market Indexes							
Dow Jones	8.25%	12.28%	-0.28	4.26	98.31	0.00	No
Russel 2000	11.69%	18.50%	-0.21	3.26	13.14	0.00	No
Nasdaq	9.78%	16.59%	-0.16	3.51	18.67	0.00	No
S&P 500	8.72%	12.88%	-0.32	4.50	137.21	0.00	No
DJ EUROSTOXX 50	13.82%	16.21%	-0.41	7.94	1298.84	0.00	No
Bond Market Indexes							
JP Morgan EMU Bond Index	2.92%	1.06%	-0.29	5.12	250.68	0.00	No
Lehman Bond Composite US Index	4.46%	3.93%	-0.11	9.98	2525.08	0.00	No

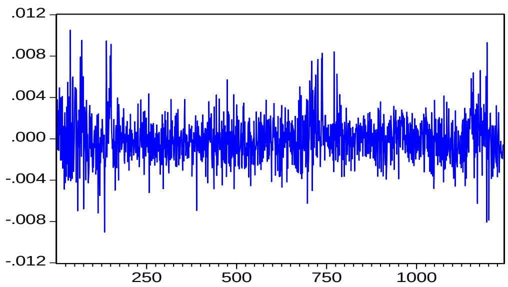
[12] Annualized returns are computed as follows: $prod\,(1+R_{\alpha})^{\frac{scale}{n}} - 1$

[13] Annualized standard deviation is computed as follows: $\sigma \times \sqrt{periods}$

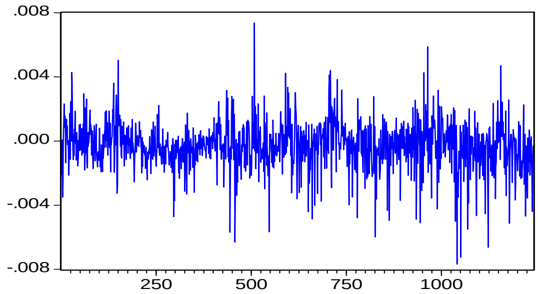
Appendix B: Continuously Compounded Hedge Fund Daily Return Indexes



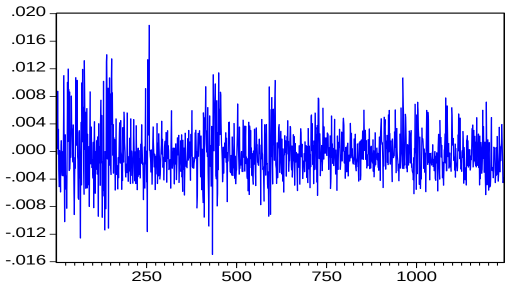
Global Index



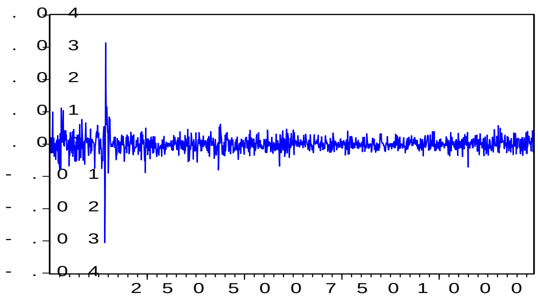
Convertible Arbitrage



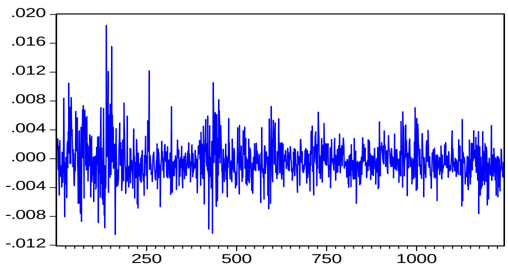
Distressed Securities



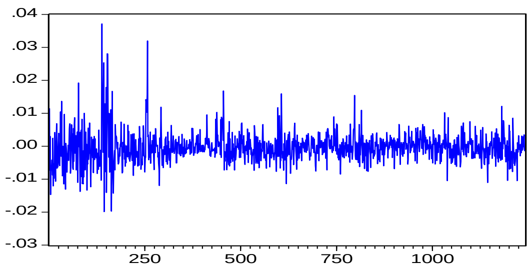
Equity Hedge



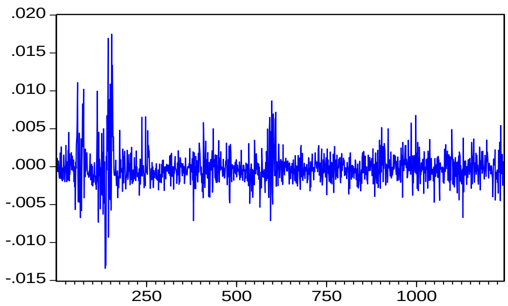
Equity Market Neutral



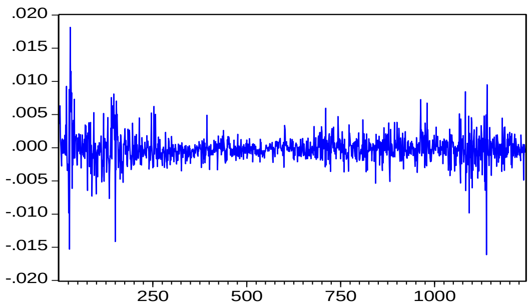
Event Driven



Macro



Merger Arbitrage



Relative Value Arbitrage

Appendix C: Hedge Fund Strategies Mean Equation Modelling

	Global	CA	DS	ED	EH	EMN	M	MA	RVA
Mean Equation	**ARMA(1;1)**	**AR(1)**	**ARMA(1;2)**	**ARMA(1;1)**	**AR(1)**	**ARMA(2;2)**	**ARMA(1;1)**	**ARMA(2;2)**	**ARMA(2;2)**
$\hat{c}$	2.20E-04	1.99E-05	2.60E-04	2.95E-04	2.16E-04	3.91E-05	2.99E-04	2.03E-04	1.44E-04
	(9.09E-05)	(6.08E-05)	(1.03E-04)	(1.06E-04)	(1.24E-04)	(5.86E-05)	(1.76E-04)	(6.68E-05)	(6.59E-05)
$\hat{\varpi}_1$	0.470	-0.061	0.960	0.750	0.187	0.645	0.616	0.498	0.432
	(0.105)	(0.028)	(0.019)	(0.105)	(0.028)	(0.127)	(0.123)	(0.059)	(0.108)
$\hat{\theta}_1$	-0.261		-0.838	-0.661		-0.480		-0.438	-0.440
	(0.114)		(0.034)	(0.119)		(0.120)		(0.055)	(0.095)
$\hat{\varpi}_2$						0.166	-0.488	-0.859	-0.689
						(0.125)	(0.137)	(0.053)	(0.108)
$\hat{\theta}_2$			-0.064			-0.368		0.879	0.773
			(0.030)			(0.118)		(0.048)	(0.095)
R^2	0.053	0.004	0.058	0.018	0.035	0.041	0.026	0.019	0.016
AIC	-9.314	-9.336	-10.236	-8.944	-8.439	-9.106	-7.902	-9.376	-9.408
SIC	-9.301	-9.328	-10.220	-8.932	-8.431	-9.086	-7.889	-9.356	-9.388
Log Likelihood	5777.519	5790.334	6350.500	5548.312	5234.034	5646.316	4902.126	5813.718	5833.495
Q(5)	1.2311	17.029**	2.077	2.279	0.951	2.526	4.151	4.7959**	0.249
Q(10)	21.026**	33.379**	14.546**	12.159	12.806	9.769	13.883	10.370	13.016**
ARCH LM-Test	240.904**	28.253**	5.358	122.754**	202.789**	429.757**	191.881**	269.260**	282.266**

Appendix D: Hedge Fund Strategies Parameters Estimates (errors follow the normal law)

ARMA - GARCH(1;1) modelling[14]

	Global	CA	DS	ED	EH	EMN	M	MA	RVA
Mean Equation	**ARMA(1;1)**	**AR(1)**	**ARMA(1;2)**	**ARMA(1;1)**	**AR(1)**	**ARMA(2;2)**	**ARMA(1;1)**	**ARMA(2;2)**	**ARMA(2;2)**
$\hat{c}$	2.75E-04	1.36E-04	3.15E-04	4.02E-04	3.23E-04	8.10E-05	2.81E-04	2.74E-04	2.67E-04
	(7.65E-05)	(5.32E-05)	(1.15E-04)	(8.59E-05)	(1.15E-04)	(4.67E-05)	(1.30E-04)	(4.85E-05)	(6.64E-05)
$\hat{\varpi}_1$	0.405	-0.107	0.962	0.706	0.190	1.144	0.382	0.201	0.113
	(0.111)	(0.031)	(0.019)	(0.126)	(0.031)	(0.101)	(0.202)	(0.007)	(0.156)
$\hat{\theta}_1$	-0.185		-0.825	-0.619		-1.094	-0.254	-0.207	-0.122
	(0.122)		(0.037)	(0.143)		(0.097)	(0.212)	(0.004)	(0.166)
$\hat{\varpi}_2$						-0.257		-0.976	0.837
						(0.147)		(0.006)	(0.155)
$\hat{\theta}_2$			-0.074			0.193		0.995	-0.799
			(0.033)			(0.144)		(0.003)	(0.163)
Variance Equation									
$\hat{\alpha}_0$	1.13E-07	9.52E-08	2.86E-08	2.25E-07	3.97E-07	2.77E-07	7.82E-07	2.34E-07	7.19E-08
	(3.32E-08)	(4.10E-08)	(9.51E-09)	(6.36E-08)	(1.20E-07)	(5.49E-08)	(1.77E-07)	(3.97E-08)	(1.70E-08)
$\hat{\alpha}_1$	0.068	*0.057*	0.031	0.077	0.079	0.144	0.118	0.164	0.147
	(0.010)	(0.010)	(0.006)	(0.014)	(0.013)	(0.017)	(0.018)	(0.015)	(0.014)
$\hat{\beta}_1$	0.909	0.925	0.956	0.892	0.888	0.801	0.847	0.783	0.849
	(0.015)	(0.016)	(0.008)	(0.021)	(0.020)	(0.024)	(0.023)	(0.019)	(0.013)
R^2	0.053	-0.002	0.057	0.017	0.034	0.017	0.024	0.009	-0.004
AIC	-9.563	-9.448	-10.266	-9.120	-8.610	-9.550	-8.195	-9.764	-9.845
SIC	-9.538	-9.427	-10.237	-9.095	-8.589	-9.517	-8.170	-9.731	-9.812
Log Likelihood	5935.133	5862.712	6371.816	5660.325	5343.113	5924.464	5086.696	6056.671	6107.113
Q(5)	0.896	9.125	2.7275	2.321	2.0362	3.1429	0.419	6.547**	1.521
Q(10)	11.800	22.743**	10.959	6.607	7.1936	10.872	7.192	13.895**	8.574
ARCH LM-Test	12.676	7.047	15.167	8.746	5.609	6.451	16.183	15.361	9.841**

[14] ** no significant at 5% confidence level

ARMA - TGARCH(1;1) modelling[15]

	Global	CA	DS	ED	EH	EMN	M	MA	RVA
Mean Equation	**ARMA(1;1)**	**AR(1)**	**ARMA(1;2)**	**ARMA(1;1)**	**AR(1)**	**ARMA(2;2)**	**ARMA(1;1)**	**ARMA(2;2)**	**ARMA(2;2)**
$\hat{c}$	2.55E-04	1.23E-04	3.09E-04	2.88E-04	2.15E-04	7.17E-06	3.02E-04	2.50E-04	2.25E-04
	(8.12E-05)	(5.37E-05)	(1.15E-04)	(1.44E-04)	(1.15E-04)	(5.74E-05)	(1.23E-04)	(4.84E-05)	(9.13E-05)
$\hat{\varpi}_1$	0.446	-0.109	0.962	0.971	0.201	0.418	0.255	-0.655	0.129
	(0.107)	(0.031)	(0.019)	(0.017)	(0.032)	(0.671)	(0.227)	(0.034)	(0.145)
$\hat{\theta}_1$	-0.221		-0.826	-0.945		-0.358	-0.143	0.644	-0.139
	(0.119)		(0.037)	(0.023)		(0.671)	(0.234)	(0.036)	(0.154)
$\hat{\varpi}_2$						0.035		-0.901	0.838
						(0.434)		(0.034)	(0.143)
$\hat{\theta}_2$			-0.073			-0.064		0.898	-0.795
			(0.033)			(0.413)		(0.034)	(0.150)
Variance Equation									
$\hat{\alpha}_0$	1.98E-07	1.41E-07	2.72E-08	3.05E-07	7.10E-07	3.14E-07	2.24E-07	1.97E-07	7.82E-08
	(4.12E-08)	(4.94E-08)	(9.38E-09)	(6.26E-08)	(1.34E-07)	(5.87E-08)	(6.33E-08)	(3.11E-08)	(1.58E-08)
$\hat{\alpha}_1$	0.038	0.037	0.027	0.014	-0.018	0.051	0.109	0.052	0.089
	(0.025)	(0.010)	(0.007)	(0.014)	(0.022)	(0.021)	(0.017)	(0.016)	(0.020)
$\hat{\beta}_1$	0.066	0.038	0.007	0.097	0.158	0.176	-0.086	0.142	0.097
	(0.028)	(0.015)	(0.009)	(0.021)	(0.031)	(0.031)	(0.018)	(0.026)	(0.023)
$\hat{\gamma}_1$	0.883	0.915	0.957	0.891	0.869	0.798	0.930	0.825	0.853
	(0.021)	(0.019)	(0.008)	(0.020)	(0.026)	(0.027)	(0.009)	(0.016)	(0.014)
R^2	0.053	-0.001	0.057	0.012	0.035	0.016	0.022	0.004	-0.004
AIC	-9.564	-9.450	-10.265	-9.128	-8.628	-9.569	-8.207	-9.770	-9.851
SIC	-9.535	-9.425	-10.231	-9.099	-8.604	-9.532	-8.178	-9.732	-9.814
Log Likelihood	5936.803	5864.869	6372.012	5666.274	5355.654	5936.974	5095.600	6061.265	6111.762
Q(5)	0.709	8.125	2.7426	9.123**	2.2849	2.5914	2.750	6.530**	1.505
Q(10)	12.012	20.878**	11.066	12.499	7.4772	10.354	9.813	14.028**	7.958
ARCH LM-Test	11.375	9.331	15.552	7.270	2.361	5.764	12.570	17.593	5.240

[15] ** no significant at 5% confidence level

ARMA - EGARCH(1;1) modelling

	Global	CA	DS	ED	EH	EMN	M	MA	RVA
Mean Equation	**ARMA(1;1)**	**AR(1)**	**ARMA(1;2)**	**ARMA(1;1)**	**AR(1)**	**ARMA(2;2)**	**ARMA(1;1)**	**ARMA(2;2)**	**ARMA(2;2)**
$\hat{c}$	2.90E-04	1.05E-04	3.32E-04	3.40E-04	2.52E-04	8.25E-06	2.73E-04	2.38E-04	2.18E-04
	(7.70E-05)	(5.39E-05)	(1.01E-04)	(9.33E-05)	(1.16E-04)	(5.35E-05)	(1.21E-04)	(4.66E-05)	(4.47E-05)
$\hat{\varpi}_1$	0.463	-0.105	0.955	0.780	0.197	1.108	0.322	-1.676	-0.813
	(0.104)	(0.031)	(0.021)	(0.089)	(0.032)	(0.418)	(0.216)	(0.219)	(0.243)
$\hat{\theta}_1$	-0.260		-0.830	-0.699		-1.058		1.685	0.812
	(0.114)		(0.037)	(0.104)		(0.419)		(0.217)	(0.245)
$\hat{\varpi}_2$						-0.472	-0.202	-0.703	-0.735
						(0.228)	(0.224)	(0.212)	(0.258)
$\hat{\theta}_2$			-0.060			0.432		0.713	0.737
			(0.033)			(0.227)		(0.210)	(0.257)
Variance Equation									
$\hat{\alpha}_0$	-0.423	-0.477	-0.331	-0.530	-0.705	-0.890	-0.384	-0.718	-0.595
	(0.094)	(0.129)	(0.088)	(0.127)	(0.138)	(0.166)	(0.081)	(0.087)	(0.078)
$\hat{\alpha}_1$	0.144	0.132	0.097	0.144	0.126	0.244	0.186	0.239	0.281
	(0.023)	(0.023)	(0.017)	(0.025)	(0.030)	(0.028)	(0.023)	(0.019)	(0.023)
$\hat{\beta}_1$	-0.021	-0.039	-0.007	-0.040	-0.100	-0.108	0.038	-0.096	-0.048
	(0.016)	(0.010)	(0.009)	(0.016)	(0.020)	(0.017)	(0.016)	(0.015)	(0.014)
$\hat{\gamma}_1$	0.975	0.969	0.980	0.965	0.947	0.943	0.978	0.957	0.969
	(0.007)	(0.010)	(0.006)	(0.010)	(0.011)	(0.012)	(0.006)	(0.007)	(0.006)
R^2	0.053	0.000	0.057	0.018	0.035	0.010	0.023	0.004	0.001
AIC	-9.561	-9.439	-10.275	-9.120	-8.615	-9.566	-8.205	-9.761	-9.857
SIC	-9.532	-9.415	-10.242	-9.091	-8.590	-9.528	-8.177	-9.724	-9.820
Log Likelihood	5934.727	5858.412	6378.497	5661.190	5347.394	5934.838	5094.360	6055.782	6115.459
Q(5)	1.719	8.994	2.6167	1.583	2.3516	3.1758	1.492	6.963**	2.728
Q(10)	12.489	22.524**	10.855	5.882	8.4889	9.5340	7.824	13.396**	7.800
ARCH LM-Test	2.992	11.469**	17.044	6.459	5.677	2.750	9.769	15.310	7.177**

Appendix E: Hedge Fund Strategies Parameters Estimates (errors follow the student law)

ARMA - GARCH(1;1) modelling[16]

	Global	CA	DS	ED	EH	EMN	M	MA	RVA
Mean Equation	**ARMA(1;1)**	**AR(1)**	**ARMA(1;2)**	**ARMA(1;1)**	**AR(1)**	**ARMA(2;2)**	**ARMA(1;1)**	**ARMA(2;2)**	**ARMA(2;2)**
$\hat{c}$	3.86E-04	1.32E-04	2.06E-04	4.76E-04	4.40E-04	8.29E-05	3.58E-04	3.25E-04	3.72E-04
	(6.70E-05)	(5.03E-05)	(7.52E-05)	(7.91E-05)	(1.05E-04)	(4.55E-05)	(1.10E-04)	(4.29E-05)	(1.83E-04)
$\hat{\varpi}_1$	0.397	-0.111	0.952	0.603	0.179	1.109	0.373	-0.665	0.314
	(0.113)	(0.029)	(0.019)	(0.150)	(0.031)	(0.129)	(0.266)	(0.040)	(0.611)
$\hat{\theta}_1$	-0.187		-0.843	-0.504		-1.062	-0.283	0.656	-0.315
	(0.122)		(0.034)	(0.164)		(0.127)	(0.276)	(0.037)	(0.618)
$\hat{\varpi}_2$						-0.230		-0.876	0.676
						(0.144)		(0.032)	(0.607)
$\hat{\theta}_2$			-0.052			0.168		0.893	-0.660
			(0.028)			(0.142)		(0.029)	(0.608)
Variance Equation									
$\hat{\alpha}_0$	1.18E-07	8.31E-08	5.98E-08	2.01E-07	3.70E-07	2.50E-07	6.88E-07	2.62E-07	8.65E-08
	(4.19E-08)	(4.38E-08)	(3.05E-08)	(7.69E-08)	(1.36E-07)	(7.38E-08)	(2.34E-07)	(7.70E-08)	(2.92E-08)
$\hat{\alpha}_1$	0.097	0.049	0.064	0.090	0.091	0.144	0.115	0.140	0.135
	(0.020)	(0.012)	(0.019)	(0.021)	(0.020)	(0.026)	(0.025)	(0.030)	(0.025)
$\hat{\beta}_1$	0.880	0.935	0.913	0.885	0.878	0.809	0.853	0.799	0.856
	(0.023)	(0.018)	(0.025)	(0.025)	(0.025)	(0.032)	(0.029)	(0.038)	(0.022)
DOF	8	10	4	7	11	8	6	5	5
R^2	0.050	-0.002	0.057	0.015	0.032	0.017	0.021	0.002	-0.001
AIC	-9.620	-9.471	-10.380	-9.167	-8.634	-9.583	-8.270	-9.835	-9.927
SIC	-9.591	-9.446	-10.347	-9.138	-8.609	-9.546	-8.241	-9.797	-9.890
Log Likelihood	5971.602	5878.178	6443.856	5690.284	5358.774	5945.494	5134.469	6101.563	6158.984
Q(5)	1.382	9.867**	3.811	2.096	2.607	3.227	1.870	4.240**	2.015
Q(10)	11.536	23.826**	10.207	6.555	7.727	10.911	8.116	13.069**	8.134
ARCH LM-Test	9.255	7.938	13.770	6.903	4.236	5.955	16.090	3.650	11.613**

16 ** no significant at 5% confidence level

ARMA - TGARCH(1;1) modelling[17]

	Global	CA	DS	ED	EH	EMN	M	MA	RVA
Mean Equation	**ARMA(1;1)**	**AR(1)**	**ARMA(1;2)**	**ARMA(1;1)**	**AR(1)**	**ARMA(2;2)**	**ARMA(1;1)**	**ARMA(2;2)**	**ARMA(2;2)**
$\hat{c}$	3.54E-04	1.26E-04	1.94E-04	4.32E-04	3.41E-04	3.39E-05	3.88E-04	3.10E-04	4.05E-04
	(7.04E-05)	(5.06E-05)	(7.61E-05)	(8.15E-05)	(1.06E-04)	(5.39E-05)	(1.06E-04)	(4.31E-05)	(3.02E-04)
$\hat{\varpi}_1$	0.460	-0.111	0.952	0.642	0.192	0.453	0.080	-0.670	0.373
	(0.103)	(0.029)	(0.019)	(0.137)	(0.031)	(0.453)	(0.313)	(0.042)	(0.662)
$\hat{\theta}_1$	-0.241		-0.844	-0.542		-0.396	0.008	0.661	-0.376
	(0.113)		(0.033)	(0.151)		(0.451)	(0.315)	(0.040)	(0.671)
$\hat{\varpi}_2$						0.080		-0.873	0.621
						(0.381)		(0.032)	(0.659)
$\hat{\theta}_2$			-0.051			-0.124		0.888	-0.602
			(0.028)			(0.371)		(0.029)	(0.661)
Variance Equation									
$\hat{\alpha}_0$	1.98E-07	1.16E-07	6.32E-08	2.95E-07	6.26E-07	3.04E-07	3.35E-07	2.32E-07	9.47E-08
	(5.11E-08)	(5.11E-08)	(3.13E-08)	(8.89E-08)	(1.44E-07)	(7.94E-08)	(1.42E-07)	(5.98E-08)	(2.89E-08)
$\hat{\alpha}_1$	0.037	0.031	0.052	0.036	-0.015	0.064	0.135	0.040	0.084
	(0.030)	(0.016)	(0.019)	(0.023)	(0.027)	(0.029)	(0.033)	(0.030)	(0.029)
$\hat{\beta}_1$	0.120	0.031	0.040	0.103	0.171	0.168	-0.098	0.132	0.096
	(0.039)	(0.018)	(0.029)	(0.035)	(0.037)	(0.043)	(0.031)	(0.041)	(0.040)
$\hat{\gamma}_1$	0.854	0.929	0.908	0.869	0.865	0.792	0.906	0.828	0.853
	(0.026)	(0.019)	(0.026)	(0.027)	(0.028)	(0.035)	(0.021)	(0.030)	(0.022)
DOF	8	10	4	7	13	9	6	5	5
R^2	0.052	-0.002	0.057	0.016	0.034	0.019	0.018	0.002	-0.001
AIC	-9.626	-9.472	-10.381	-9.174	-8.651	-9.593	-8.279	-9.841	-9.930
SIC	-9.593	-9.443	-10.344	-9.141	-8.622	-9.551	-8.246	-9.800	-9.888
Log Likelihood	5975.947	5879.471	6445.054	5695.612	5370.694	5952.649	5141.284	6106.575	6161.533
Q(5)	1.156	8.687	3.736	3.894	2.633	3.430	5.613	3.802	1.720
Q(10)	11.661	21.638**	10.548	6.803	7.763	11.141	12.461	11.322	7.125
ARCH LM-Test	8.151	10.076	15.265	49.782**	2.184	5.468	9.712	20.387	5.384

[17] ** no significant at 5% confidence level

ARMA - EGARCH(1;1) modelling[18]

	Global	CA	DS	ED	EH	EMN	M	MA	RVA
Mean Equation	**ARMA(1;1)**	**AR(1)**	**ARMA(1;2)**	**ARMA(1;1)**	**AR(1)**	**ARMA(2;2)**	**ARMA(1;1)**	**ARMA(2;2)**	**ARMA(2;2)**
$\hat{c}$	3.60E-04	1.12E-04	1.93E-04	4.20E-04	3.61E-04	3.00E-05	3.93E-04	2.95E-04	2.40E-04
	(6.74E-05)	(5.09E-05)	(7.45E-05)	(8.07E-05)	(1.05E-04)	(5.24E-05)	(1.06E-04)	(5.81E-05)	(3.53E-05)
$\hat{\varpi}_1$	0.445	-0.107	0.951	0.637	0.191	0.487	0.330	0.036	-0.454
	(0.106)	(0.029)	(0.019)	(0.125)	(0.031)	(0.355)	(0.278)	(0.072)	(0.796)
$\hat{\theta}_1$	-0.233		-0.847	-0.530		-0.430	-0.244	-0.022	0.473
	(0.116)		(0.033)	(0.139)		(0.354)	(0.286)	(0.078)	(0.797)
$\hat{\varpi}_2$						0.076		0.882	0.061
						(0.281)		(0.062)	(0.567)
$\hat{\theta}_2$			-0.049			-0.124		-0.870	-0.029
			(0.028)			(0.276)		(0.067)	(0.569)
Variance Equation									
$\hat{\alpha}_0$	-0.576	-0.381	-0.576	-0.549	-0.643	-0.897	-0.396	-0.747	-0.717
	(0.133)	(0.140)	(0.227)	(0.149)	(0.142)	(0.216)	(0.117)	(0.166)	(0.025)
$\hat{\alpha}_1$	0.196	0.116	0.160	0.169	0.149	0.261	0.198	0.226	0.363
	(0.036)	(0.028)	(0.041)	(0.036)	(0.037)	(0.042)	(0.035)	(0.041)	(0.044)
$\hat{\beta}_1$	-0.059	-0.030	-0.029	-0.066	-0.108	-0.095	0.045	-0.093	-0.025
	(0.023)	(0.014)	(0.022)	(0.023)	(0.023)	(0.023)	(0.020)	(0.026)	(0.025)
$\hat{\gamma}_1$	0.966	0.976	0.965	0.965	0.954	0.944	0.977	0.954	0.966
	(0.010)	(0.011)	(0.016)	(0.011)	(0.011)	(0.016)	(0.009)	(0.012)	(0.003)
DOF	8	9	4	7	12	9	6	5	5
R^2	0.051	0.000	0.057	0.017	0.034	0.019	0.020	0.005	-0.001
AIC	-9.624	-9.464	-10.385	-9.172	-8.643	-9.590	-8.276	-9.834	-9.921
SIC	-9.591	-9.435	-10.348	-9.139	-8.614	-9.549	-8.243	-9.793	-9.880
Log Likelihood	5974.683	5874.854	6447.829	5694.718	5365.801	5950.979	5139.411	6102.300	6156.288
Q(5)	1.087	9.638**	3.361	0.722	2.641	3.593	4.156	4.954**	1.152
Q(10)	11.782	23.406**	10.683	5.009	8.604	10.252	10.174	9.578	5.704
ARCH LM-Test	7.503	12.299**	18.020	42.953	1.581	6.815	9.756	3.004	4.016

[18] ** no significant at 5% confidence level

Appendix F: Backtesting VaR model tests

Test of Unconditional Coverage

This test examines how many times an estimated VaR is violated in a given time period. When the number of violations differs from $\alpha \times 100\,\%$, the estimated VaR method either understates or overstates the risk. It states that the probability of realizing a loss in excess of the reported VaR, $VaR_t(\alpha)$ must be precisely $\alpha\,\%$:

$$\Pr\left(I_{t+1}(\alpha)=1\right)=\alpha \tag{20}$$

Kupiec's Proportion of Failures Test [1995]

The Kupiec's POF statistic is computed as follows:

$$POF = 2\log\left(\left(\frac{1-\hat{\alpha}}{1-\alpha}\right)^{T-I(\alpha)}\left(\frac{\hat{\alpha}}{\alpha}\right)^{I(\alpha)}\right) \tag{21}$$

$$I(\alpha)=\sum_{t=1}^{T} I_t(\alpha)$$

$$\hat{\alpha}=\frac{I(\alpha)}{T}$$

$$T = \textit{the number of observations}$$

Under the null of the unconditional coverage test, the POF statistic is distributed as a χ^2 with one degree of freedom.

The LR test of unconditional coverage [1998]

Christoffersen developed an equivalent test; the likelihood ratio test of unconditional coverage. The test is:

$$LR_{uc} = -2\log\left(\frac{(1-p)^{n_0}\, p^{n_1}}{(1-\hat{\pi})^{n_0}\, \hat{\pi}^{n_1}}\right) \tag{22}$$

$$n_0 = T - n_1$$

$$n_1 = \textit{number of ones in the sample}$$

$$\hat{\pi} = \frac{n_1}{n_0 + n_1}$$

The likelihood ratio LR_{uc} has an asymptotic $\chi^2(1)$ distribution.

Test of Independence

VaR violations at various periods must be independent over time. Christoffersen [1998] introduced the likelihood ratio of independence which examines the serial independence of the hit sequence function given the confidence level. It is expressed as follows:

$$LR_{ind} = -2\log\left(\frac{(1-\hat{\pi}_2)^{n_{00}+n_{10}}\,\hat{\pi}_2^{n_{01}+n_{11}}}{(1-\pi_{01})^{n_{00}}\,\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\,\pi_{11}^{n_{11}}}\right) \qquad (23)$$

$n_{ij} = number\ of\ observations\ with\ value\ i\ followed\ by\ j$

$$\hat{\pi}_2 = \frac{(n_{01}+n_{11})}{(n_{00}+n_{10}+n_{01}+n_{11})}$$

$$\hat{\pi}_{01} = \frac{n_{01}}{n_{00}+n_{01}}$$

$$\hat{\pi}_{11} = \frac{n_{11}}{n_{10}+n_{11}}$$

The LR test of independence is asymptotically distributed as a $\chi^2(1)$.

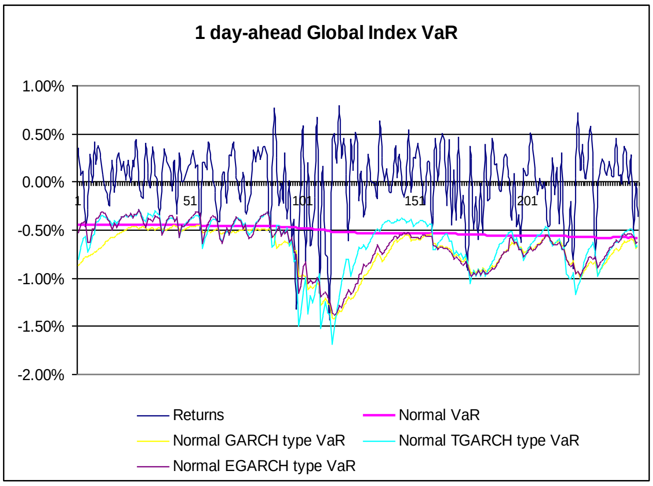
Test of Conditional Coverage

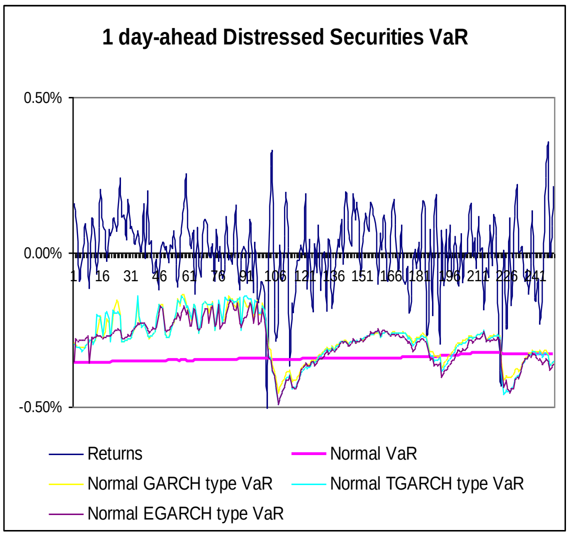
Christoffersen [1998] combined the unconditional coverage test and the independence test to form a test of conditional coverage. It is written as follows:

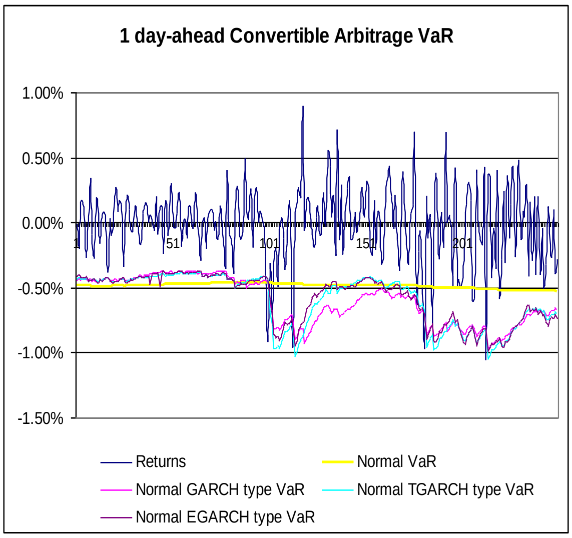
$$LR_{cc} = LR_{uc} + LR_{ind} \qquad (24)$$

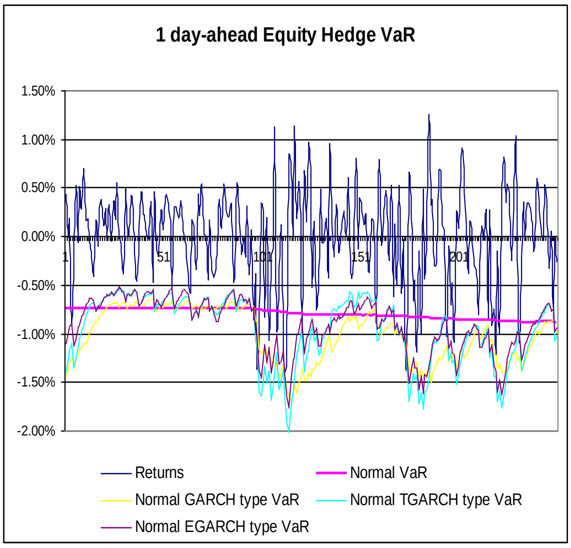
The distribution of the conditional coverage test is asymptotically χ^2 with two degrees of freedom.

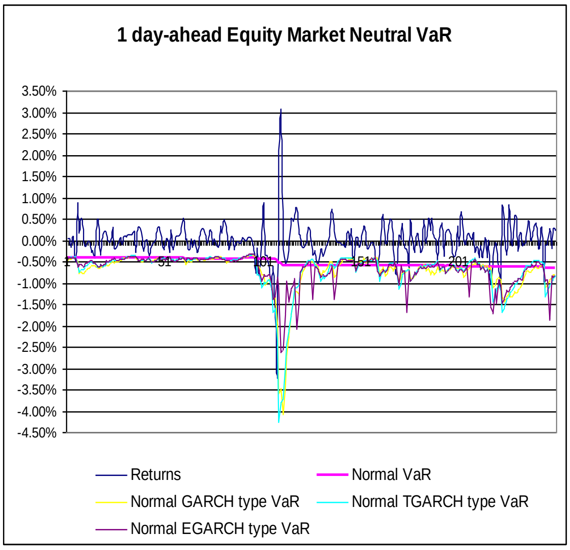
Appendix G: Hedge Fund Historical Returns and VaR Forecasts (normal quantile, 1% confidence level)

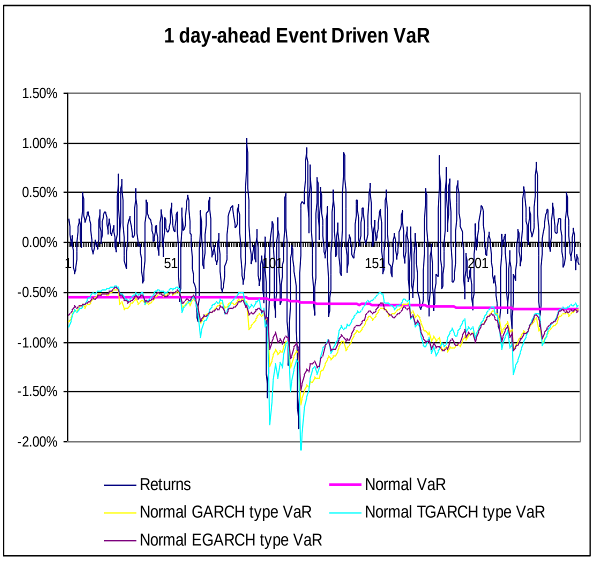


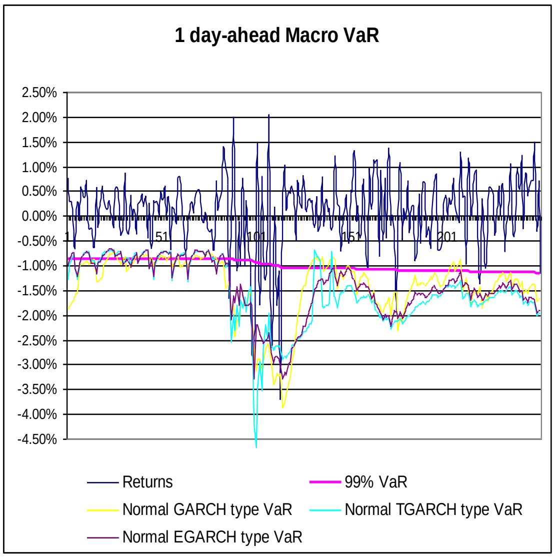


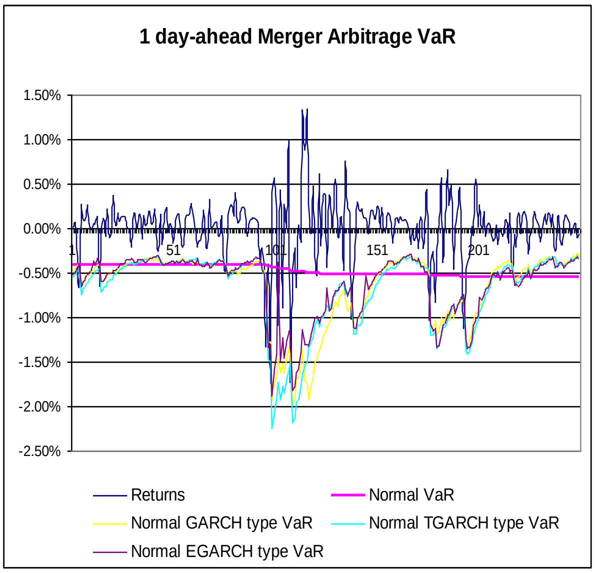


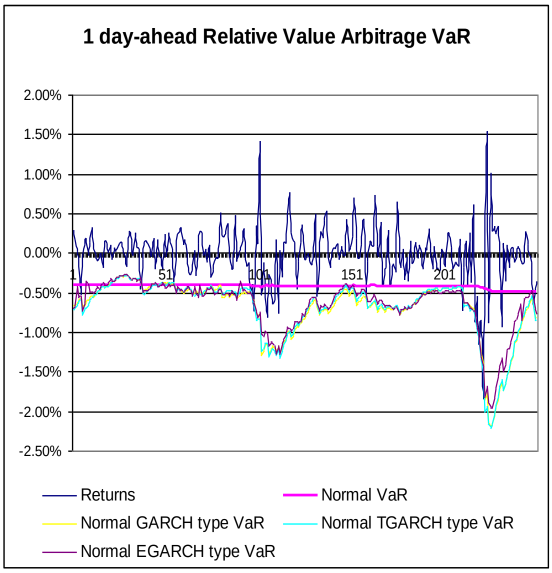




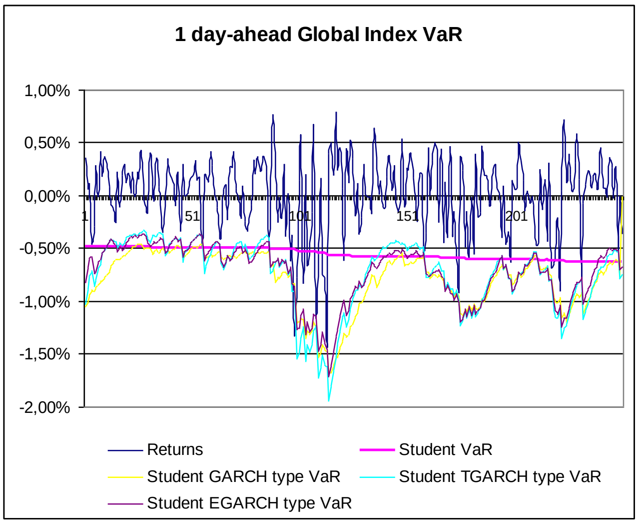

1 day-ahead Event Driven VaR
Returns
Normal VaR
Normal GARCH type VaR
Normal TGARCH type VaR
Normal EGARCH type VaR

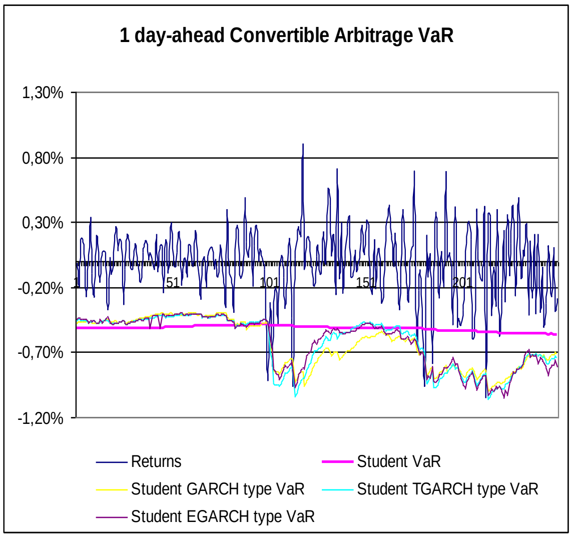

1 day-ahead Macro VaR
Returns
99% VaR
Normal GARCH type VaR
Normal TGARCH type VaR
Normal EGARCH type VaR

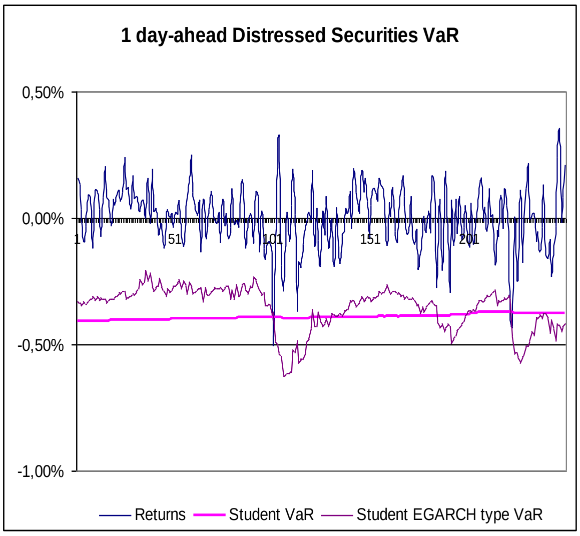

1 day-ahead Merger Arbitrage VaR
Returns
Normal VaR
Normal GARCH type VaR
Normal TGARCH type VaR
Normal EGARCH type VaR

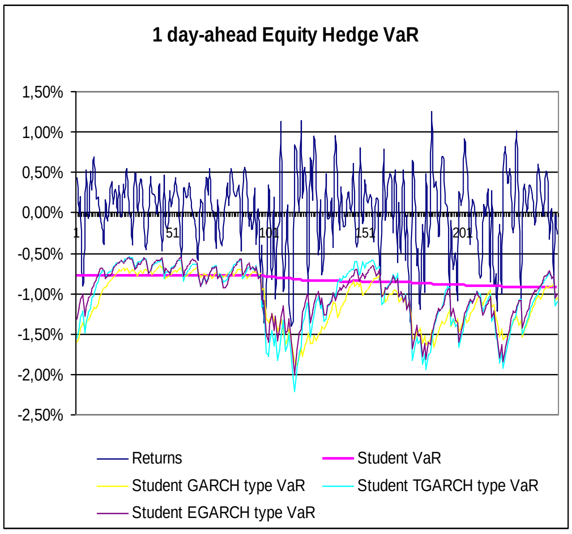

1 day-ahead Relative Value Arbitrage VaR
Returns
Normal VaR
Normal GARCH type VaR
Normal TGARCH type VaR
Normal EGARCH type VaR

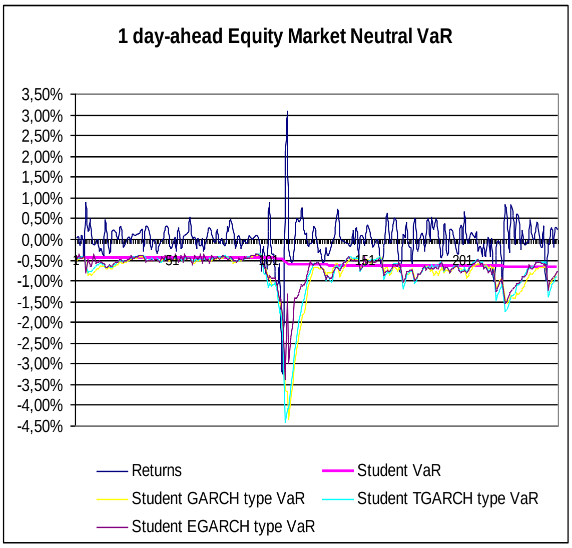
Appendix H: Hedge Fund Historical Returns and VaR Forecasts (student quantile, 1% confidence level)

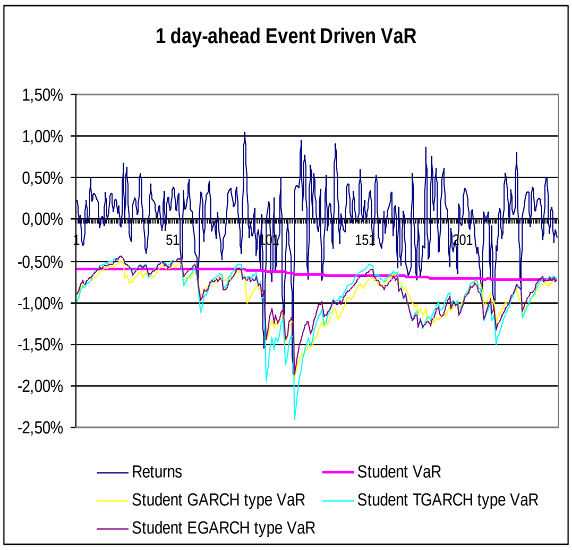


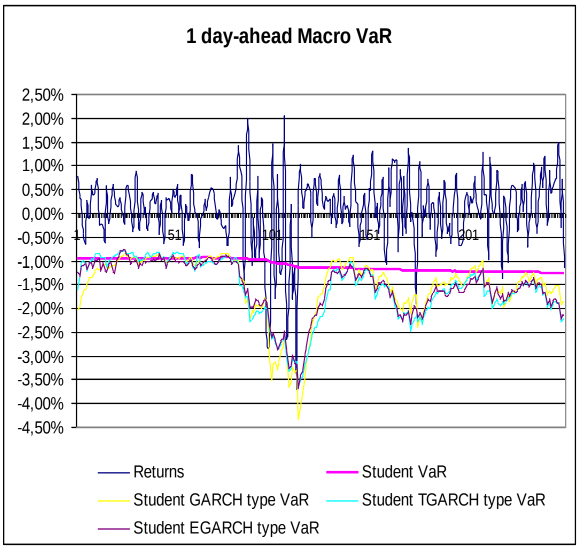


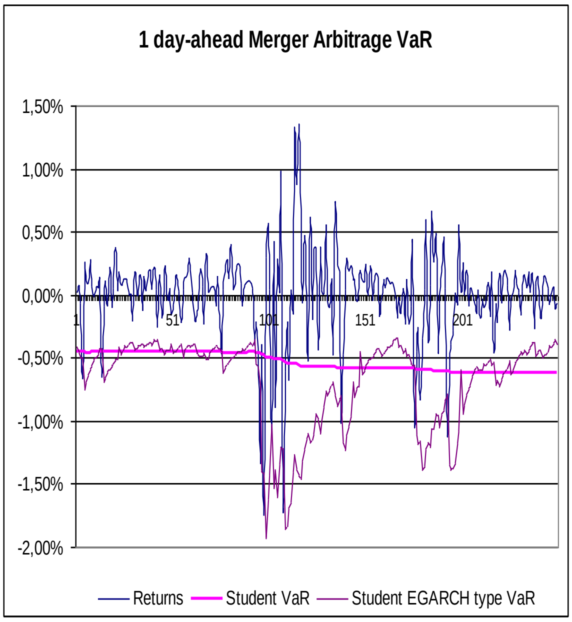


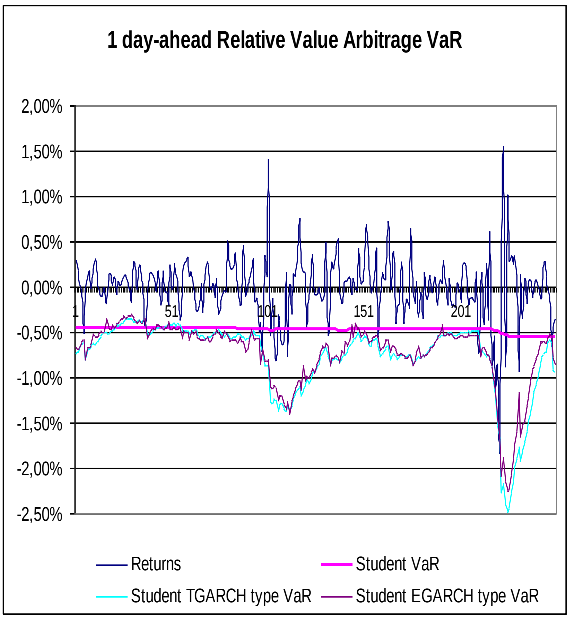




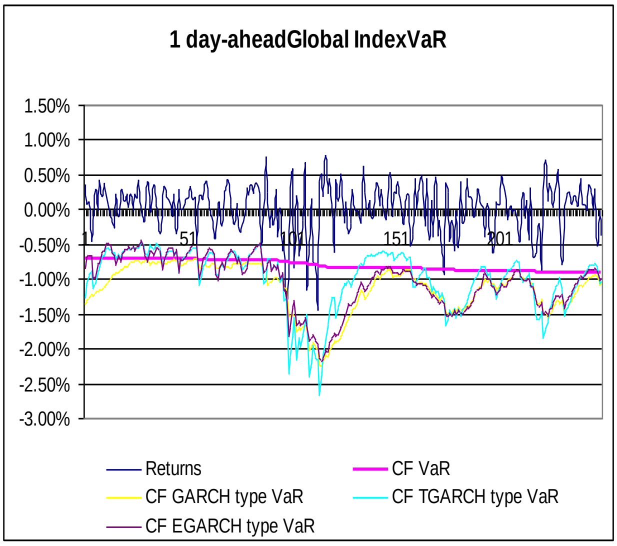

1 day-ahead Event Driven VaR
1,50%
1,00%
0,50%
0,00%
-0,50%
-1,00%
-1,50%
-2,00%
-2,50%
1
51
101
151
201
Returns
Student VaR
Student GARCH type VaR
Student TGARCH type VaR
Student EGARCH type VaR

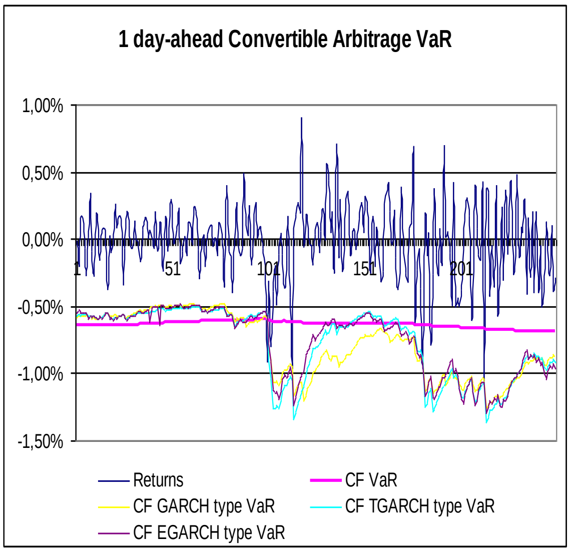

1 day-ahead Macro VaR
2,50%
2,00%
1,50%
1,00%
0,50%
0,00%
-0,50%
-1,00%
-1,50%
-2,00%
-2,50%
-3,00%
-3,50%
-4,00%
-4,50%
1
51
101
151
201
Returns
Student VaR
Student GARCH type VaR
Student TGARCH type VaR
Student EGARCH type VaR

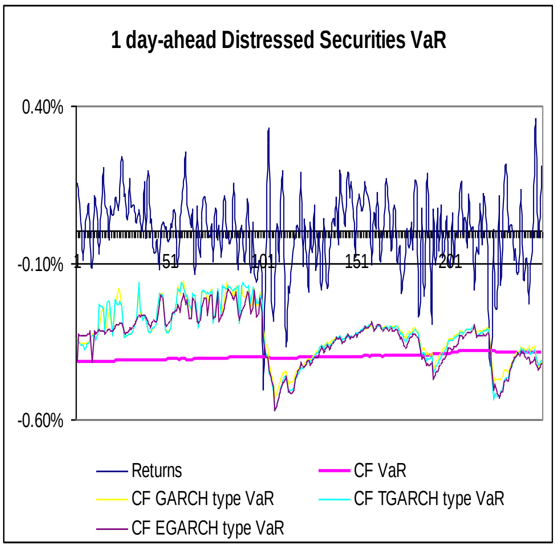

1 day-ahead Merger Arbitrage VaR
1,50%
1,00%
0,50%
0,00%
-0,50%
-1,00%
-1,50%
-2,00%
1
51
101
151
201
Returns
Student VaR
Student EGARCH type VaR

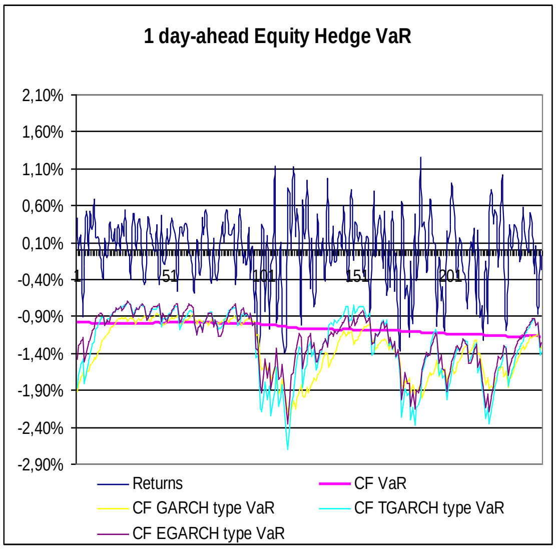

1 day-ahead Relative Value Arbitrage VaR
2,00%
1,50%
1,00%
0,50%
0,00%
-0,50%
-1,00%
-1,50%
-2,00%
-2,50%
1
51
101
151
201
Returns
Student VaR
Student TGARCH type VaR
Student EGARCH type VaR

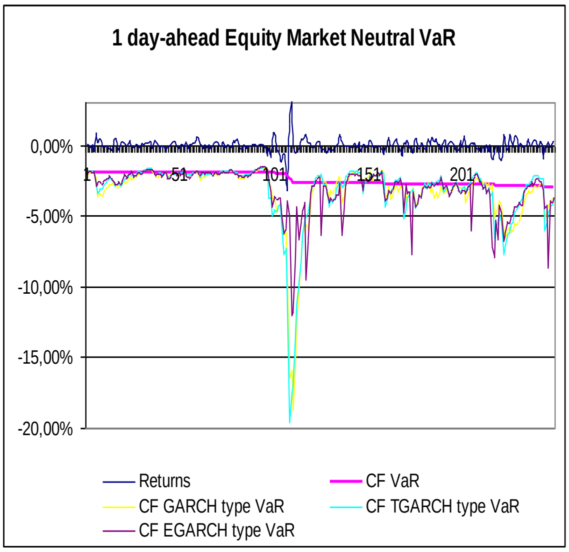
Appendix I: Hedge Fund Historical Returns and VaR Forecasts (cornish fisher quantile, 1% confidence level)

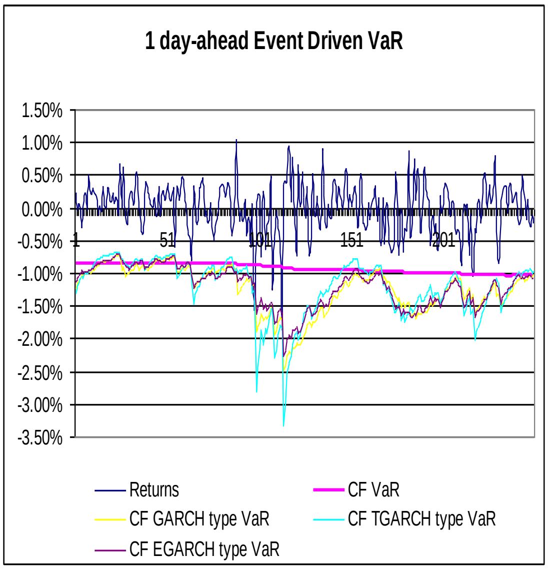


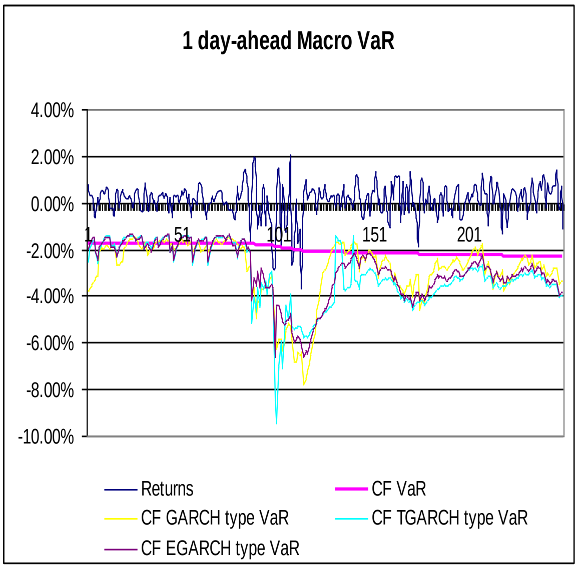


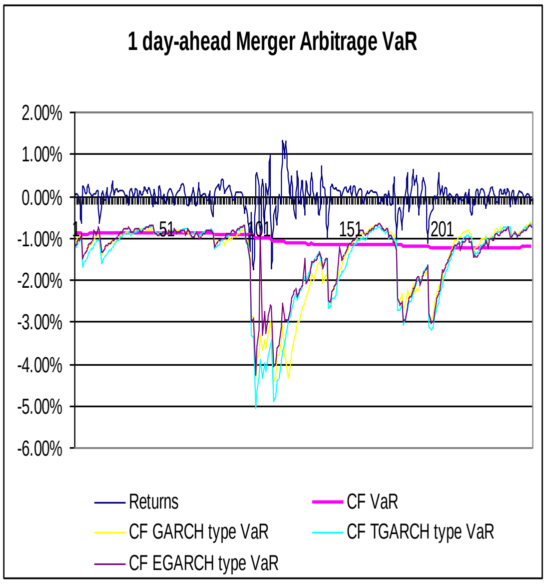


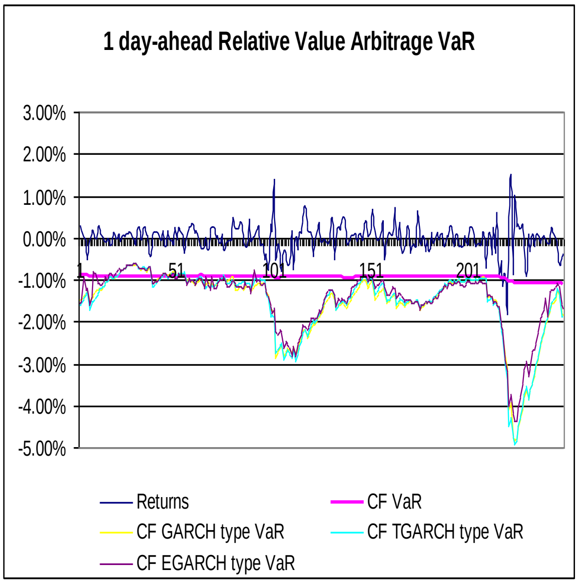





1 day-ahead Event Driven VaR
1.50%
1.00%
0.50%
0.00%
-0.50%
-1.00%
-1.50%
-2.00%
-2.50%
-3.00%
-3.50%
1
51
101
151
201
Returns
CF VaR
CF GARCH type VaR
CF TGARCH type VaR
CF EGARCH type VaR


1 day-ahead Macro VaR
4.00%
2.00%
0.00%
-2.00%
-4.00%
-6.00%
-8.00%
-10.00%
1
51
101
151
201
Returns
CF VaR
CF GARCH type VaR
CF TGARCH type VaR
CF EGARCH type VaR


1 day-ahead Merger Arbitrage VaR
2.00%
1.00%
0.00%
-1.00%
-2.00%
-3.00%
-4.00%
-5.00%
-6.00%
1
51
101
151
201
Returns
CF VaR
CF GARCH type VaR
CF TGARCH type VaR
CF EGARCH type VaR


1 day-ahead Relative Value Arbitrage VaR
3.00%
2.00%
1.00%
0.00%
-1.00%
-2.00%
-3.00%
-4.00%
-5.00%
1
51
101
151
201
Returns
CF VaR
CF GARCH type VaR
CF TGARCH type VaR
CF EGARCH type VaR