http://economix.u-paris10.fr/

Document de Travail
Working Paper
2010-17

On Legal Cooperation and the Dynamics of Legal Convergence

Bertrand Crettez
Bruno Deffains
Olivier Musy

FRE 3257

Université de Paris Ouest Nanterre La Défense
(bâtiments T et G)
200, Avenue de la République
92001 NANTERRE CEDEX

Tél et Fax : 33.(0)1.40.97.59.07
Email : secretariat-economix@u-paris10.fr

On Legal Cooperation and the Dynamics of Legal Convergence*

Bertrand Crettez[†]
ERMES, University Panthéon-Assas Paris II,

Bruno Deffains[‡]
ERMES, University Panthéon-Assas Paris II,

Olivier Musy[§]
EconomiX, University Paris-Ouest Nanterre La Défense

May 27, 2010

Abstract

In this paper, we study the dynamics of legal convergence and the comparison between the different instruments of legal convergence based on cooperative strategies (*i.e.*, harmonization and unification) or not. To study these questions we use a model with two nation-states which is inspired in part by that used in Carbonara and Parisi (2008) where preferences of each nation-state are such that it is costly to change the law, but it is also costly to have a different legal system from the other nation-state. We show that legal unification could be achieved in the long-run through small step by step changes despite the existence of huge harmonization costs in the short run. We also show that legal cooperation is not always necessary to achieve legal convergence.

JEL Classification : C72-K00

Key words : Law-and-Economics, Legal Convergence, Legal harmonization, Legal Uniformization.

*We thank Susan Crettez and Régis Deloche for helpful comments on a previous version of this work.

[†]Université Panthéon-Assas, Paris II, ERMES EAC 4441 CNRS, 26 rue des Fossés Saint-Jacques, 75005, Paris. Phone: (33) 01 44 41 89 61. Email: bertrand.crettez@me.com.

[‡]Université Panthéon-Assas, Paris II, ERMES EAC 4441 CNRS, 26 rue des Fossés Saint-Jacques, 75005, Paris. Phone: (33) 01 44 41 89 61. Email: bruno.deffains@free.fr.

[§]EconomiX, University Paris Ouest-Nanterre la Défense, 200 Avenue de la République, Bt. K - 92001 Nanterre Cedex Phone: 01 40 97 59 07, Fax: 01 40 97 70 57. Email : olivier.musy@wanadoo.fr.

1 Introduction

1.1 Motivations

Law is a fundamental instrument of international economic integration. Different legal systems increase transaction costs in cross-border business, because, on the one hand, costs occur through the provision of information about, and adapting to, the respective national regulations, and, on the other hand, the great number of legal provisions and processes increases the uncertainty which surrounds cross-border transactions procedures where different legal systems are in place.

Rodrik (2004) argues thatthe diversity of national institutional arrangements is the most important source of transaction costs in international exchanges. According to this author, these costs broadly represent nearly 35% in ad-valorem terms. In the same vein, the European Commission noticed in the recent period that legal uncertainty is regarded as the main reason for the fact that the economic dynamics triggered by the process of European integration develop more slowly than expected and desired entrepreneurs (see its Communication to the Council and the European Parliament on European contract law of September (2001)).

Therefore, increasing globalization naturally seems to call for increasing standardization of the law. Standardization in the legal field aims to render uniform the legal responses to the same facts or situations, irrespective of the place they occur or of the national elements involved. In a certain measure, the convergence of rules, even though still formally belonging to different legal systems, is more or less imposed by the reality of increasing transnational interaction. The result of this evolution is also called *legal convergence*. However, there are different techniques to achieve legal convergence. One possibility is to create a set of rules that is applicable regardless of national borders. For instance, there are numerous sets of universal rules on contracts, covering different aspects of international commerce and created by different institutions (see *e.g.*, CISG, Unidroit, ICC Incoterms).

Other possibilities imply a more cooperative solution between countries. They refer mainly to *harmonization* and *unification*. Harmonization obliges different national legislations not to be contradictory with regard to a certain common aim. Unification occurs at an international or supranational level. It refers to the substitution of multiple rules by a new legislation (or the substitution of one legal rule for others). As noted by de Cruz (1999), unlike unification which contemplates the substitution of two or more legal systems with one single system, harmonization of law seeks to promote coordination of different legal provisions or systems by eliminating major differences and creating minimum requirements or standards. Harmonization can be seen as a step towards unification.

1.2 Related literature

Globalization calls for more legal standardization but is it really desirable and possible to achieve legal convergence and if so, what kind of cooperation does it require ?

As for the first question the very possibility of achieving legal harmonization or unification has been discussed in the comparative law literature (see *e.g.*, Legrand (1996), (1997)). The major reason is that legal harmonization and unification are seen as a threat to the legal culture and history of a country. Indeed, legal traditions may be so distant from each other that society would simply resist the proposed legal change. Therefore, huge adaptation costs may make harmonization and unification impossible. Even Law and Finance scholars who compare the quality of legal institutions and argue that common law systems perform better than their civil law counterparts, explicitly recognize that it would be economically impossible for France or Germany to change their legal system in favor of common law mechanisms (La Porta *et al.* (2001)). A more optimistic viewpoint is developed in Merryman *et al.* (1994), building on an idea of "natural convergence". For them, similar nations have a tendency to have similar problems, and to arrive at similar legal ways of dealing with them. Hence, legal convergence is intrinsically inevitable in the sense that it is the logical response to common problems.

As for the second question, while some scholars have observed some remarkable efforts of cooperation (see, *e.g.*, Kramer (2008) in the European Union context), the issue of cooperation in the process of legal convergence itself has been addressed only recently in the literature. In fact it is not evident that legal convergence requires legal cooperation. Loeper (2008)-(2009) compares decentralization with unification and highlights the importance of patterns of jurisdictional interdependence. He considers a large class of situations in which external costs are driven by the differences between local policies. He finds that if external costs are sufficiently salient then no uniform policy Pareto dominates decentralization: salient enough externalities have a self-disciplining effect which induces local jurisdictions to choose similar policies. He also finds that if external effects are symmetrical, decentralization is again socially preferred to any uniform policy. Importantly, he finds that cost of decentralization is not monotonic in the magnitude of externalities and vanishes as they become arbitrarily large. Another argument against centralization is put forward by Baniak and Grajzl (2009). Like Loper (2008)- (2009), they study the advantages of cooperation in a general framework, that is with more than two nation-states, where each nation-state has imperfect information about the preferences of the other states. They find notably that interventionist harmonization is not justified unless there are structural asymmetries in the patterns of interjurisdictional linkages (for instance when there exists a jurisdiction that the rest of the world strives to synchronize with, but whose local conditions are unknown). Even if cooperation is apparently better than non-cooperation, we must take care of the context in which it is done. In particular, it may be that before cooperating, countries increase their legal switching costs so as to shift the burden of legal adaptation to their partners. Carbonara and Parisi (1988) show that two outcomes are possible in this perspective. First, it may be that there is less legal convergence with endogenous switching costs than without (whether or not nation-states cooperate). This is what they call the paradox of harmonization. Second, it may be that with endogenous switching costs, non-cooperation is better than legal cooperation, and yields more legal convergence than without endogenous switching costs (whether or not nation-states cooperate in this later case).

A common feature of the literature quoted above is that the viewpoint is static (or limited to two periods) and theoretical. While to the best of our knowledge there is no literature on the dynamics of legal convergence, there is a burgeoning empirical literature on legal convergence. Much of this literature is based on cross-sectional data, due to non-availability of robust comparative time series data. However, some authors have recently tried to construct such data, in order to measure and compare the evolution of laws in different countries. Building such a set of longitudinal data, Armour *et al.* (2008) show that the gap between countries concerning protection of shareholders has been narrowing during the period 1995-2005, even if differences still exist (the degree of shareholder protection is still higher in common law systems, but civil law systems are catching up). Siems (2008) displays some evidence that one of the sources of the convergence of shareholder law between countries is the actions of multinational companies and law and accounting firms, which result in the propagation of similar standards around other countries. Armour *et al.* (2009) extend the analysis to creditor protection and labour regulation (worker protection), and show that there is at least a slight process of convergence for these fields during the recent period. They show that different legal origins do not prevent convergence, and highlight the role played by transnational setting processes, and the emergence of some international consensus about ideas of what constitutes the best practices.

The apparent legal convergence phenomenon that seems to be at work according to the previous studies leads us to ask a few questions. How can we explain this phenomenon ? Does it have a limit, as suggested in the work of Legrand ? What part of this phenomenon is explained by legal cooperation ?

1.3 Objectives

In this paper, we try to give theoretical answers to the above questions. Indeed, we are particularly interested in the dynamics of legal convergence and in the comparison between the different instruments of legal convergence based on cooperative strategies (*i.e.*, harmonization and unification).

We show that Legrand's viewpoint may be correct but only in the short-run. Indeed, legal unification could be achieved in the long-run through small step by step changes despite the existence of huge harmonization costs in the short-run. We also show that legal cooperation is not always necessary to achieve legal convergence.

To study these questions we use a model with two nation-states which is inspired in part by that used in Carbonara and Parisi (2008). The preferences of each nation-state are such that it is costly to change the law, but it is also costly to have a legal system which is different to the other nation-state. At each time, both nation-states must decide to adapt or not their legal systems. They balance the advantage of adapting their legal system to the foreign nation-state with the cost of changing. In the model, we do not assume that there are intrinsic preferences for legal systems. We believe indeed that most political reasons supposed to explain why there cannot be legal convergence have little to do with moral, political or philosophical values and more to do with the difficulties of compensating the agents economically hurt by the decrease in the legal distance. We use the model to study three kinds of interactions.

First of all, nation-states may act non-cooperatively, each of them adapting to the other without coordination. In this case, we show that, though there is no cooperation, legal unification may be achieved in the long-run. Two lessons can be drawn from this result. On one hand, the absence of legal unification in the short-run does not preclude it in the long-run. On the other hand, legal unification does not require coordination. Second, following Carbonara and Parisi (2008), we assume that the two nation-states choose cooperatively to harmonize their legal systems. This solution implies coordination costs so that, as in Carbonara and Parisi (2008), legal unification is an exception. Third, we assume that the two nation- states cooperate to achieve legal unification (but without coordination costs: legal unification entails less coordination costs than legal harmonization). We introduce this third kind of interaction to explain why legal unification is sometimes chosen (as, *e.g.*, in the European Union). This is necessary since with the first two kinds of interactions (as in Carbonara and Parisi (2008)) legal unification is never chosen[1].

Next, we let the two nation-states choose across time to harmonize or not, to unify or not. We use several examples to study these choices[2]. These examples illustrate two kinds of results with regard to legal convergence. First, it is possible that nation-states never choose to cooperate. This case is like the situation described by Legrand, but legal unification is achieved in the long-run. Second, it is possible that nation-states choose not to cooperate during a finite time (however long it may be) and then decide after a while to cooperate. Thus, the absence of legal unification during several periods does not mean that this may never happen. These results indicate that legal cooperation should not be a matter of principle but a matter of choice.

1.4 Organization of the paper

The paper unfolds as follows. Section 2 presents the basic setup. Section 3 discusses non-cooperative processes of legal changes. Sections 4 and 5 study

[1]We show however that legal unification may be chosen with non smooth payoffs.

[2]With a little work, it can be shown that the comparison results at one point of time between centralization and non-cooperation are special cases of the more general results of Loeper's (2008). But we prove them in different and much simpler way, avoiding the clever but long arguments used by Loeper in his general setting. We came across Loeper's paper only when our paper was nearly completed, after reading Baniak and Grajzl (2009)'s paper.

the cooperation regimes (respectively harmonization and unification). Section 6 presents the comparison between the different modes of legal interactions. Section 7 discusses the results and section 8 concludes the paper. All the proofs are listed in the appendixes to this paper.

2 The Model

We use a model with two nation-states which is inspired in part by that used in Carbonara and Parisi (2008). At each time, two nation-states must decide whether or not to adapt their legal systems (we imagine that there are non-overlapping generations of decision-makers with a decision-making horizon of one period ahead of them). They balance the advantage of adapting their legal system to the foreign system with the cost of changing.

According to Wagner (2002), "there are different types of costs when we face the question of legal convergence. First are the costs for one country to change from period to period its legal rules and/or Institutions. Second, are the costs for the same country to adapt its legal system to its international environment. Third, under certain circumstances, the country has also to support coordination costs, for instance when it decides to cooperate with another country to harmonize certain legal standards". Finally, another obstacle to complete legal harmonization is the lobbying cost (Casella (2001)). We will detail the coordination costs in section 4. For the time being, we shall concentrate on the other costs which we model as follows [3].

We assume that legal systems can be described by the set of real numbers. We let x_t^i denote the legal system of nation-state i, $i = 1, 2$ at date t. To have an interesting problem, we assume that initially the two legal systems are different: $x_0^1 \neq x_0^2$. For simplicity, we assume that these numbers are always common knowledge[4].

We assume that law makers' preferences in nation-state i are represented by the real valued utility function $U^i(x_t^i, x_t^j) + V^i(x_t^i, x_{t-1}^i)$ where the subutility functions $U^i(x_t^i, x_t^j)$, $V^i(x_t^i, x_{t-1}^i)$, $i, j = 1, 2$, $i \neq j$, are defined in $\mathbb{R}^2$. The subutility function $U^i(x_t^i, x_t^j)$ gives the utility of having a new legal system x_t^i when the other nation-state chooses legal system x_t^j. The subutility function $V^i(x_t^i, x_{t-1}^i)$ gives the utility of having a new legal system x_t^i when the prevailing legal system is x_{t-1}^i. Thus the different costs identified by Wagner enter in a separable way in the utility function. This assumption is made to simplify the analysis.
utility In that

We shall need the the next definition, which was introduced by Hamada in international economics (see, *e.g.*, McMillan (1986)). A function $f : \mathbb{R}^2 \to \mathbb{R}$ and $y \in \mathbb{R}$ is *single peaked* in its second (resp. first) argument y (resp. x)

[3]Contrary to Carbonara and Parisi, we do not assume that nation-states can modify their switching costs. This possibility is discussed in section 7.

[4]See, *e.g.*, Baniak and Grajzl (2009) for an analysis which takes into account incomplete information about the local conditions prevailing in other nation-states.

when $f(x, y)$ is increasing with respect to x (resp. y) if $x < y$ (resp. $y < x$), decreasing with respect to x (resp. y) if $x > y$ (resp. $x < y$) and thus reaches a maximum at $x = y$.

Assumption 1. The functions U^i and V^i are single-peaked in their second argument.

Therefore the maximal values of each of these functions are realized when their respective arguments are equal (*e.g.*, when $x_t^1 = x_t^2$ for U^i and $x_t^i = x_{t-1}^i$ for V^i). As for function U^i, it means that it is costly for each nation-state to have a new law which is too different from that of the other nation-state. As for function V^i, it means that it is costly for each nation-state to modify the law which was chosen in the past, *i.e.*, x_{t-1}^i.

It is easy to check that the next functions satisfy assumption 1:

- The absolute value function $- \mid x_t^1 - x_t^2 \mid$ and its generalization, *i.e.*[5],

$$-\sqrt{a + (x_t^1 - x_t^2)^2},\ a \geq 0, \tag{1}$$

- The quadratic function $-(x_t^1 - x_t^2)^2$.

One can observe that we focus on a process of legal convergence in which there are no fundamental preferences such as constitutional values to be taken into account. Therefore we assume that there are no superior reasons that would make a nation-state always prefer a particular legal system over another one. As a consequence, the most important obstacles to legal change are the *costs of legal changes*. We believe indeed that most political reasons supposed to explain why there cannot be legal convergence have little to do with moral, political or philosophical values. More probably, the lack of legal convergence is due to the difficulty to compensate some agents for the loss that they could incur if there were progress towards more legal standardization. For instance, while it may be difficult to unify social laws, or abolish the death penalty everywhere, it is clear that in the first case, the difficulty lies more in the reluctance of economic agents to bear the costs of harmonization (this would be the case of workers if unification yields less regulation), whereas in the second, it lies more in differing moral values. We disregard the case where legal convergence is prevented because of diverging moral values.

Our model generalizes that of Carbonara and Parisi (2008) which was used in a two-period setup. Specifically, these authors use the following specification for the overall utility of a nation-state: $f_i - d_i(1 - x^i - x^j) - s_i(x^i)$, where f_i is the maximal trade gain that arises when there is legal unification, $d_i(1 - x^i - x^j)$ is the cost of legal diversity in terms of trade gains, and $s_i(x^i)$ is the cost of changing one's law - again in terms of trade gains. Here x^i and x^j are in $[0, 1]$. They are interpreted as the fraction of the foreign law which is transplanted into the legal system of nations i and j respectively. Function $d_i(.)$ reaches a minimum at 0 and both $d_i(.)$ and $s_i(.)$ are convex. In a dynamic framework

[5] One obtains the absolute value function when $a = 0$.

with more than two periods, the objective of nation-state i, with no intrinsic preferences for a legal system, would be: $f_i - d_i(1 - x_t^i - x_t^j) - s_i(x_t^i - x_{t-1}^i)$.

Having laid out our model, we can now consider starting the analysis of non-cooperative processes of legal change.

3 Non-cooperative Processes of Legal Change

Does legal standardization necessarily imply legal cooperation between nations ? Many examples plead for a negative answer. A well-known argument explaining the convergence of national legal rules is found in the law-and-economics literature where it is argued that national legal rules will converge spontaneously in order to implement the efficient allocation of scarce resources (see, *e.g.* the papers in Marciano and Josselin (2002) and notably Smits (2002), Mattei (1994), Ogus (1999), Garoupa and Ogus (2003)). To put it in a nutshell, convergence will be achieved through the works of legislators, judges and arbitrators, who will choose the same *efficient* legal rules. Therefore, legal convergence is the result of decentralized decisions, sometimes made by private agents, which are not necessarily coordinated[6].

Some decisions are also made by nation-states in a unilateral way and refer to what is called "legal transplantation" in the comparative law literature. Carbonara and Parisi (2008) define legal transplantation as "the introduction, in national legal systems, of statutes and principles belonging to other systems, be they legal rules of other countries or customs whose acceptance is widespread"[7]. For instance, several nation-states decided unilaterally to ratify the United Nations Convention on Contracts for the International Sale of Goods (CISG) after 1980, the year when a diplomatic conference representing 62 states finalized the text in Vienna [8]. By doing so, the nation-states who ratified after 1980, have acted in a non-cooperative way as they only had to decide to ratify or not an existing text.

In the remainder of this section we shall study the dynamics of legal change under the assumptions that there is no coordination at all. We shall study the relevance of the argument mentioned above according to which legal convergence can be achieved without resorting to formal legal cooperation.

Let us recall that we assume an international context where nation-state i, $i = 1, 2$, adapts its legal system by maximizing $U^i(x_t^i, x_t^j) + V^i(x_t^i, x_{t-1}^i)$, $i = 1, 2$ with respect to x_t^i only $(i = 1, 2)$[9].

A non-cooperative equilibrium process of *legal change* at a given date t is a Nash equilibrium where the strategies are $(\overline{x}_t^1, \overline{x}_t^2)$.

[6]This conclusion is discussed and criticized by Carbonara and Parisi (2009), who show that agents can coordinate their choices on an inefficient legal rule.

[7]Carbonara and Parisi (1988) give several examples of legal transplants.

[8]For a more general discussion, see Herings and Kanning (2008) who argue that the CISG itself is the result of a non-cooperative process.

[9]The case with n countries and its specificity is discussed in section 7.

Let us assume the existence of a non-cooperative equilibrium process of legal changes across time. How does legal convergence evolve along a dynamic path ? Are legal systems more and more alike or do they differ more and more ? The following result ensures that legal systems cannot differ across times.

Proposition 1. *Let a non-cooperative equilibrium process of legal changes $(\overline{x}_t^1, \overline{x}_t^2)_{t\geq 0}$ be given. Let us assume that legal systems are different at date $t-1$, i.e., $\overline{x}_{t-1}^1 < \overline{x}_{t-1}^2$. Then:*

1. *If $\max\{\overline{x}_t^1, \overline{x}_t^2\} < \overline{x}_{t-1}^1$, or $\overline{x}_{t-1}^2 < \max\{\overline{x}_t^1, \overline{x}_t^2\}$, there is complete convergence from date t on : $\overline{x}_s^1 = \overline{x}_s^2$, $\forall s \geq t$.*
2. *If not, we necessarily have :*
$$\overline{x}_{t-1}^1 \leq \overline{x}_t^1 \leq \overline{x}_t^2 \leq \overline{x}_{t-1}^2. \tag{2}$$
3. *Moreover, either there is complete convergence in finite time, or the equilibrium sequence of legal changes $(\overline{x}_t^1, \overline{x}_t^2)_t$ converges to a limit $(\overline{x}^1, \overline{x}^2)$, satisfying $\overline{x}^1 \leq \overline{x}^2$.*

The last part of the above Proposition does not ensure that legal unification may be achieved in the long-run, either because the status quo is chosen in each period, or because $\overline{x}^1$ is different from $\overline{x}^2$. However, due to assumption 1, if the objective functions are differentiable everywhere, choosing the status quo is never an equilibrium choice (unless $\overline{x}_{t-1}^1 = \overline{x}_{t-1}^2$). Indeed, at a Nash equilibrium, we have:

$$U_1^1(\overline{x}_t^1, \overline{x}_t^2) + V_1^1(\overline{x}_t^1, \overline{x}_{t-1}^1) = 0 \tag{3}$$
$$U_1^2(\overline{x}_t^2, \overline{x}_t^1) + V_1^2(\overline{x}_t^2, \overline{x}_{t-1}^2) = 0 \tag{4}$$

The meaning of these equations is that each nation-state balances the benefit of having a legal system more like that of the other nation-state ($U_1^i(\overline{x}_t^i, \overline{x}_t^j)$) and the cost of adaptation ($V_1^i(\overline{x}_t^i, x_{t-1}^i)$).

Now, if the status quo is chosen at date t, *i.e.* if $\overline{x}_t^1 = \overline{x}_{t-1}^1$ and $\overline{x}_t^2 = \overline{x}_{t-1}^2$, by assumption 1 we have $V_1^i(\overline{x}_t^i, x_{t-1}^i) = 0$ (the marginal cost of changing the system is nil) so that the above conditions can now be written as follows:

$$U_1^1(\overline{x}_t^1, \overline{x}_t^2) = 0 \tag{5}$$
$$U_1^2(\overline{x}_t^2, \overline{x}_t^1) = 0 \tag{6}$$

Thus the marginal cost of having a different legal system from the other nation-state is nil. But by assumption 1 again, this is only possible if $\overline{x}_t^1 = \overline{x}_t^2$, which is then impossible unless $\overline{x}_{t-1}^1 = \overline{x}_{t-1}^2$.

For similar reasons, when the objective functions are differentiable everywhere, legal unification is *never* a Nash equilibrium in the short-run (unless $\overline{x}_{t-1}^1 = \overline{x}_{t-1}^2$). Indeed, when legal unification is realized, *i.e* $\overline{x}_t^1 = \overline{x}_t^2$, by assumption 1 and (3)-(4) we have:

$$V_1^1(\overline{x}_t^1, \overline{x}_{t-1}^1) = 0 \tag{7}$$
$$V_1^2(\overline{x}_t^2, \overline{x}_{t-1}^2) = 0 \tag{8}$$

This means that the marginal cost of legal change is nil in both nation-states. But by assumption 1 again this implies that $\overline{x}_t^1 = \overline{x}_{t-1}^1$ and $\overline{x}_t^2 = \overline{x}_{t-1}^2$, which implies that $\overline{x}_t^1 = \overline{x}_{t-1}^1 = \overline{x}_t^2 = \overline{x}_{t-1}^2$.

The reason why, neither the status quo nor legal unification are Nash equilibria in the short-run is as follows. In both cases, the marginal value of L^i, the cost of having a different legal system, or V^i, the cost of changing the legal system, is nil. But in every equilibrium, the marginal value of $L^i + V^i$ is also nil. Hence, if, say, the marginal value of L^i is nil, so must be that of V^i. However, assumption 1 implies that this is impossible (unless $\overline{x}_{t-1}^1 = \overline{x}_{t-1}^2$). When the differentiability of the utility function is not satisfied everywhere (as in example 2 below), the above conclusions do not hold anymore.

The next Proposition, which summarizes the preceding discussion, and whose first part generalizes Lemma 1 and Proposition 1 of Carbonara and Parisi (2008), explains what happens to the legal distance between the two nation-states in the long-run.

Proposition 2. *Let us assume that the utility functions U^i, V^i, $i = 1, 2$, are continuously differentiable everywhere.*

1. *Then, if the legal systems are different at date 0, at each date t there is neither status quo nor legal unification.*
2. *However, legal systems are more and more alike, $\overline{x}_{t-1}^1 < \overline{x}_t^1 < \overline{x}_t^2 < \overline{x}_{t-1}^2$ and the sequence of legal systems $(\overline{x}_t^1, \overline{x}_t^2)_t$ converges to a limit $(\overline{x}^1, \overline{x}^2)$ satisfying $\overline{x}^1 = \overline{x}^2$ so that legal unification is achieved in the long-run.*

The last part of the Proposition shows that Legrand's viewpoint may be correct but only in the short-run. Indeed, legal unification can be achieved in the long-run through small step by step changes despite the existence of harmonization costs in the short-run. The key reason for the continuous decrease in legal distance between the two nation-states relies on assumption 1. Indeed, if $\overline{x}_{t-1}^1 < \overline{x}_t^2$ and $x_t^1 < \overline{x}_{t-1}^1$, $U^1(x_t^1, \overline{x}_t^2) + V^1(x_t^1, \overline{x}_{t-1}^1)$ increases whereas if $\overline{x}_t^2 < x_t^1$, $U^1(x_t^1, \overline{x}_t^2) + V^1(x_t^1, \overline{x}_{t-1}^1)$ decreases. In both cases, it pays for nation-state 1 to have a legal system more like that of nation-state 2 and not to change too much. The same argument applies for nation-state 2.

The steady-state $(\overline{x}^1, \overline{x}^2)$, whose existence is asserted in the above Proposition is certainly not globally stable. This is so as it is located in the interval $[x_0^1, x_0^2]$ and starting with different initial conditions would change the interval and would lead to a different steady-state. We will show by way of an example that the steady-state may not be locally stable. Thus, there is a considerable path dependency at work. History matters.

One interesting point however is that legal convergence may be achieved without cooperation, at least in the long-run[10]. Let us now illustrate the preceding discussion by way of two examples.

Example 1. The quadratic case

[10]We will discuss this result for the case when there are more than two nation-states in section 7.

Let us consider the case where for $i = 1, 2$:

$$U^i(x_t^i, x_t^j) + V^i(x_t^i, x_{t-1}^i) = -\frac{1}{2}\left(x_t^i - x_t^j\right)^2 - \frac{\theta}{2}\left(x_t^i - x_{t-1}^i\right)^2 \tag{9}$$

The next quadratic function has been used in a different context by Alonso *et al.* (2008).
It is easy to see that the non-cooperative equilibrium at date t is given by the following expressions:

$$\overline{x}_t^1 = \frac{1+\theta}{2+\theta}\overline{x}_{t-1}^1 + \frac{1}{2+\theta}\overline{x}_{t-1}^2 \tag{10}$$

$$\overline{x}_t^2 = \frac{1}{2+\theta}x_{t-1}^1 + \frac{1+\theta}{2+\theta}x_{t-1}^2 \tag{11}$$

We notice that when θ – the relative weight of V^i relative to U^i in law-makers' preferences – goes to infinity, $\overline{x}_t^1$ goes to x_{t-1}^1. When changing one legal system is very costly, the status quo is indeed the best decision for both nation-states.

It can be shown that the dynamics of the non-cooperative equilibria may be rewritten as:

$$\overline{x}_t^1 = \frac{x_0^1 + x_0^2}{2} - (\frac{\theta}{2+\theta})^t \frac{(x_0^2 - x_0^1)}{2} \tag{12}$$

$$\overline{x}_t^2 = \frac{x_0^1 + x_0^2}{2} + (\frac{\theta}{2+\theta})^t \frac{(x_0^2 - x_0^1)}{2} \tag{13}$$

Several comments are in order. First, legal systems tend to be alike and legal unification is achieved in the long-run at the level $(x_0^1 + x_0^2)/2$. Second, when $x_0^1 < x_0^2$, the sequence $(\overline{x}_t^1)_t$ is increasing, whereas $(\overline{x}_t^2)_t$ is decreasing. The first sequence goes to the steady-state from below, whereas the second goes to it from above. Third, we can see that the steady-state depends on the values of x_0^1 and x_0^2 which illustrates the fact that legal convergence is history dependent. Fourth, the steady-state is not locally stable (changing the initial conditions changes the value of the steady-state). Finally, along the equilibrium path, the values of the utility functions are similar and equal to:

$$\mathcal{U}_t^{nc} = -\left[\frac{\theta\,(1+\theta)}{2\,(2+\theta)^2}\right]\left(\overline{x}_{t-1}^1 - \overline{x}_{t-1}^2\right)^2 \tag{14}$$

While this example shows that legal convergence is realized in the long-run, it does not follow that it always takes an infinite amount of time to achieve it. Legal unification could be the issue of non-cooperative interactions even in the short-run. Thus one does not need to resort to cooperation to achieve perfect legal convergence, even in the short-run. This is shown in the next example:

Example 2. The absolute value-quadratic case

Let us consider the case where, for $i = 1, 2$:

$$U^i(x_t^1, x_t^2) + V^i(x_t^1, x_{t-1}^1) = - \mid x_t^1 - x_t^2 \mid - \frac{\theta}{2}(x_t^1 - x_{t-1}^1)^2 \tag{15}$$

It is obvious that the objectives are not differentiable at $x_t^1 = x_t^2$. This example is studied in detail in the appendix B to this paper. There it is shown that the best response function of nation-state 1 is given as follows (by symmetry, the best response function of nation-state 2 has the same features):

$$x_t^1(x_t^2) = \begin{cases} -\frac{1}{\theta} + x_{t-1}^1 \text{ if } x_t^2 \leq -\frac{1}{\theta} + x_{t-1}^1 \\ x_t^2 \text{ if } \frac{1}{\theta} + x_{t-1}^1 \leq x_t^2 \\ \frac{1}{\theta} + x_{t-1}^1 \text{ if } x_t^2 \geq \frac{1}{\theta} + x_{t-1}^1 \end{cases} \tag{16}$$

One can show that there are two kinds of equilibria at date t (we assume without loss of generality that $\overline{x}_{t-1}^1 < \overline{x}_{t-1}^2$):

- If $\overline{x}_{t-1}^2 - \overline{x}_{t-1}^1 < \frac{2}{\theta}$, there are multiple Nash equilibria satisfying:

$$-\frac{1}{\theta} + x_{t-1}^2 \leq \overline{x}_t^1 = \overline{x}_t^2 \leq \frac{1}{\theta} + x_{t-1}^1. \tag{17}$$

- If $\overline{x}_{t-1}^2 - \overline{x}_{t-1}^1 \geq \frac{2}{\theta}$, there is a unique Nash equilibrium where:

$$\overline{x}_t^1 = \frac{1}{\theta} + \overline{x}_{t-1}^1 \tag{18}$$

$$\overline{x}_t^2 = -\frac{1}{\theta} + \overline{x}_{t-1}^2 \tag{19}$$

These results are a special case of Proposition 10 (and footnote 32) of Loeper (2008)[11].

One may also show that the dynamics of legal convergence are as follows.

- If $\overline{x}_{t-1}^2 - \overline{x}_{t-1}^1 \leq \frac{2}{\theta}$, legal unification is achieved from date t on.
- If not, there is a finite date $t' > t$ at which legal unification is realized from this date on.

The two cases are illustrated in figures 1 and 2. In figure 1, the distance between legal systems at date t is relatively small and both law-makers choose non-cooperatively to unify their laws. However, there are multiple ways of doing that, so that there is a degree of inderterminacy with regard to the legal unification outcome. In figure 2, the distance between legal systems at date t is relatively large, and legal unification is not a Nash equilibrium (we observe that there is a unique equilibrium).

This example is clearly special in that the function U^i is not differentiable at $x_t^1 = x_t^2$. To understand again the importance of non-differentiability, we may see that when $\overline{x}_t^1$ goes to $\overline{x}_t^2$ from above, $\frac{\partial}{\partial x^1} U^1(x_t^1, \overline{x}_t^2)$ does not necessarily vanish. Thus, it is possible that the marginal gain of having $\overline{x}_t^1 = \overline{x}_t^2$ is higher than the marginal loss ($\frac{\partial}{\partial x^1} V^1(x_t^1, x_{t-1}^1)$) of having $\overline{x}_t^1 \neq \overline{x}_{t-1}^1$. This ensures that legal unification may be a best-response choice.

The preceding example has shown that legal unification may be achieved in the short-run without resorting to cooperation. This does not however imply that cooperation should not be preferred over non-cooperation. Before addressing this issue, a study of legal cooperation is in order.

[11] Indeed Loeper assumes that $U^i(x_t^1, x_t^2) + V^i(x_t^1, x_{t-1}^1) = - \mid x_t^1 - x_t^2 \mid^\alpha - \frac{\theta}{2}(x_t^1 - x_{t-1}^1)^2$, $\alpha \in]0, 1]$. While our results are a special case of Loeper's (2009).

4 Cooperative Legal Harmonization Equilibria With Coordination Costs

International public law gives many examples of legal cooperation at a globalized level. The most important institutional arrangement of the post-World War II trading system is the GATT/WTO. The GATT/WTO regime is a multilateral regime that originated with the General Agreement on Tariffs and Trade, evolved through many trade rounds, and culminated with the creation of the WTO. Concerning the “trade liberalization game”, economic theory tends to evoke images of a Harmony game where each State has a dominant strategy to cooperate by liberalizing trade with every other State. In practice, cooperation is based on substantive obligations, principles, commitments, and compliance institutions (see *e.g.*, Trachtman, 2008).

The central argument is that the WTO has emerged as a major player in the field of global harmonization of national laws. Of course, the WTO is usually perceived as an organization whose mandate is mainly to lower artificial trade barriers between nations such as custom tariffs, quotas and other border measures. However, the GATT/WTO regime has gradually moved towards a system whose features are not only the elimination of artificial trade barriers, but also the harmonization of domestic policies. At first, the harmonization of certain rules and procedures was simply a by-product of the need to eliminate straightforward trade barriers. Gradually, however, the harmonizing measures were deliberately expanded so as to affect pure domestic policies and rules even if they remain significantly determined at national level.

As an example of the harmonization function of the GATT, let us consider the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPs). This agreement, concluded as part of the Uruguay Round, could not be considered as dealing with the elimination of international trade barriers – at least not in the traditional sense. Its subject matter is not international trade measures, but rather domestic laws and policies, most of which apply equally to both domestic and foreign players. Those policies in the field of intellectual property must, under the TRIPs Agreement, comply with certain minimum standards of legal protection which are prescribed by it, and all WTO members whose domestic legal systems do not yet provide such protection are required to amend their laws accordingly. Those “minimum” standards are in fact more than minimal, and tend to reflect the level of intellectual property protection prevailing in industrialized countries and mandated by leading international treaties.

In the same way, the WTO recently contributed to the harmonization of certain domestic policies in the future. For instance, the connection between competition law and international trade has always been quite obvious. Indeed, as traditional barriers to trade have been reduced, there have been increasing concerns that the gains from such liberalization may be thwarted by private anti-competitive practices. It is probably with this concern in mind that the Treaty of Rome from the outset included rules on competition in order to ensure that competition in the European Common Market is not distorted.

But not only the Treaty of Rome, which of course goes far beyond what any conventional trade agreement would seek to achieve, included such provisions. They have also been included in all the free trade agreements concluded by the EC with its associated trading partners, as well as in other international trade agreements.

A final example concerns the issue of environmental standards. It is sufficient to say that the many problems that have surfaced in the last two decades as a result of the differences in the level of environmental regulation prevailing in different countries could also be alleviated if agreement could be reached on common minimum standards. As in the cases of competition policy and TRIPs, the WTO would hardly be in a position to set those standards by itself, but would have to refer to existing multilateral environmental agreements or standards. Of course, these evolutions have to be distinguished from legal unification because for all the considered fields, the national authorities have a large autonomy in the enforcement of standards.

In this section we analyze the mechanisms of cooperative legal harmonization in the framework introduced in the preceding sections. In so doing, we follow the approach of legal coordination introduced by Carbonara and Parisi (2008)[12]. These authors model coordination by assuming that the two nation-states solve the following problem at each date t:

$$\max_{x_t^1, x_t^2} U^1(x_t^1, x_t^2) + U^2(x_t^2, x_t^1) + V^1(x_t^1, x_{t-1}^1) + V^2(x_t^2, x_{t-1}^2) - M(x_t^1, x_t^2)$$

where $M(x_t^1, x_t^2) = M > 0$ if $x_t^1 \neq x_{t-1}^1$ and/or $x_t^2 \neq x_{t-1}^2$, and $M(x_t^1, x_t^2) = 0$ whenever $x_t^1 = x_{t-1}^1$ and $x_{t-1}^2 = x_{t-1}^2$.

Thus, nation-states only bear coordination costs when at least one of them changes its legal system. These coordination costs result from the existence of different legal systems across nation-states who decide to cooperate. They are supported by legal authorities to harmonize national legal rules (for instance a new international treaty). Costs of coordination certainly depend on whether countries trust each other, whether international agreements are enforced and whether governments maintain stable and effective law. As noted by Goldsmith and Posner (2005), such problem solving is difficult because the costs of coordination rise exponentially with the number of states.

We define a cooperative harmonization equilibrium as a solution to the above problem.

Notice that both nations have equal weight in the problem (this is assumed for simplicity). We do not assume as Carbonara and Parisi (2008) that nation-states can increase ex-ante the cost of harmonization in order to make it more difficult ex-post. We do this to simplify the analysis and because we are interested in the long-run of the dynamics of legal standardization (this assumption is discussed in section 7).

To ease the analysis, we introduce the next assumption.

[12] Another example of this approach can be found in a different framework in Monheim Helstroffer and Obidzinski (2010).

Assumption 2. The utility functions U^i are single peaked in their first argument.

Applied to the function U^i, this assumption means that the greater the difference between the foreign legal system and the national one, the lower the utility (the national legal system is considered as fixed: what changes is the value of the foreign legal system).

We remark that assumption 1 does not imply assumption 2. Indeed, we assume for instance that:

$$U^i(x,y) = \begin{cases} x+y \text{ if } \ x \leq y \\ -x+2y \text{ if } x > y \end{cases} \tag{20}$$

This function satisfies assumption 1 and assumption 2. We also remark that all the examples given in section 2 satisfy the symmetry property: $U^i(x,y) = U^i(y,x)$, for all x, y. In this case, assumption 1 implies assumption 2[13].

We now study the properties of the cooperative legal harmonization equilibria. As for legal convergence, we have:

Proposition 3. *Let us suppose that the functions U^i satisfy assumption 2. Let a cooperative legal harmonization equilibrium $(\tilde{x}_t^1, \tilde{x}_t^1)$ be given at date t. Then if $x_{t-1}^1 < x_{t-1}^2$, we have:*

$$x_{t-1}^1 \leq \tilde{x}_t^1 \leq \tilde{x}_t^2 \leq x_{t-1}^2 \tag{21}$$

The preceding result shows that as long as the status quo is not chosen, legal systems tend to be more alike. This is because if, for instance, $\tilde{x}_t^2$ is in $[x_{t-1}^1, x_{t-1}^2]$, assumptions 1 and 2 ensure that $U^1(x_t^1, \tilde{x}_t^2) + U^2(\tilde{x}_t^2, x_t^1) + V^1(x_t^1, x_{t-1}^1) + V^2(\tilde{x}_t^2, x_{t-1}^2)$ is increasing with respect to x_t^1 when it is lower than x_{t-1}^1 and decreasing when it is higher than x_{t-1}^2. Our assumptions thus ensure that it never pays to coordinate on laws which are too different from those inherited by the nation-states. A compromise is therefore always chosen, the new laws being inbetween the initial levels of both nation-states. In general, the new laws will never be identical: the compromise requires the new laws should be less distant from each other, but they also should not be too different from their historically given levels. As a consequence, legal unification may never be chosen, nor reached in the long-run. To address this issue we now introduce the next assumption:

Assumption 3. The functions $U^i(x,x)$ are constant.

This assumption means that the gain resulting from unifying the legal systems does not depend on its nature (*i.e.*, on the value of x). This assumption is satisfied by all the examples used in this paper. This is not surprising as we do not assume that nation-states have intrinsic preferences for a given legal system.

[13]Indeed, if $x < y < y'$, one has: $U^i(x,y') = U^i(y',x) < U^i(y,x) = U^i(x,y)$ (and conversely if $x > y$).

Proposition 4. *Let us assume that the functions U^i satisfy assumptions 2 and 3 and that the utility functions are differentiable everywhere. Consider a sequence of cooperative legal harmonization equilibria with $x_0^1 \neq x_0^2$. Then legal unification is not chosen and nation-states choose the status quo in a finite period.*

Two important assumptions yield the above results, which generalizes Proposition 2 and Lemma 2 of Carbonara and Parisi (2008). First, differentiability of the objective function must be satisfied everywhere. Under this assumption, the reasons why legal unification is never chosen in the short-run are exactly the same as those seen in the previous section. Second, the cost M must not depend upon the difference in the legal systems. Without this cost, we would have a result similar to that of the preceding section: legal systems will be more and more alike and converge to a unique value. But this would imply that the values of the utility functions would tend to a limit. At a certain moment, the increase in utility resulting from legal unification would be lower than M. Therefore, when there are coordination costs, it never pays to choose legal unification from this date on.

Of course, the assumption that M does not depend on the distance between legal systems is strong. It is indeed likely that the coordination cost M decreases as legal systems are more alike. It would not be difficult to take this property into account, but we refrain from doing this because it simplifies the analysis and especially the comparisons with the other kinds of interactions. Indeed what matters, is that there are some coordination costs when legal systems are at a certain distance and that these costs are lower with other kinds of interactions. This seems to be plausible since nation-states are sometimes reluctant to cooperate or, quite the contrary, choose to unify their laws. If legal harmonization were not generating higher coordination costs sometimes, it would be difficult to explain why cooperation is not always chosen since one can always duplicate the decisions made with other kinds of interactions[14].

We now illustrate the notion of a cooperative legal harmonization equilibrium with the two examples introduced in the previous section.

Example 1 (Continued)

Assuming that there are no coordination costs, the cooperative equilibrium is:

$$\tilde{x}_t^1 = x_{t-1}^1 - \frac{2}{4+\theta}(x_{t-1}^1 - x_{t-1}^2) \tag{22}$$

$$\tilde{x}_t^2 = x_{t-1}^2 - \frac{2}{4+\theta}(x_{t-1}^2 - x_{t-1}^1) \tag{23}$$

[14]Of course, cooperation could not be realized if nation-states cannot trust each other, or if the resulting surplus could not be shared in a balanced way. While these difficulties may exist, it is hard to believe that they could not be worked out.

It can be shown that the dynamics are given by :

$$\tilde{x}_t^1 = \frac{x_0^1 + x_0^2}{2} - (\frac{\theta}{4+\theta})^t \frac{(x_0^2 - x_0^1)}{2} \quad (24)$$

$$\tilde{x}_t^2 = \frac{x_0^1 + x_0^2}{2} + (\frac{\theta}{4+\theta})^t \frac{(x_0^2 - x_0^1)}{2} \quad (25)$$

We see again that $(\tilde{x}_t^1)_t$ is increasing while $(\tilde{x}_t^2)_t$ is decreasing. Also, convergence to the steady-state is more rapid than when there is non-cooperation (this is because $\theta/(4+\theta)$ is lower than $\theta/(2+\theta)$).

Denoting $\mathcal{U}_t^h$ the value of the objective of a nation-state at a cooperative legal harmonization equilibrium with coordination costs, we have:

$$\mathcal{U}_t^h = -\frac{\theta(x_{t-1}^1 - x_{t-1}^2)^2}{2(4+\theta)} - \frac{M}{2} \quad (26)$$

If the status quo is chosen, this value is:

$$\mathcal{U}_t^h = -\frac{1}{2}(x_{t-1}^1 - x_{t-1}^2)^2 \quad (27)$$

The status quo is then chosen when:

$$\frac{4-\theta}{2(4+\theta)}(x_{t-1}^1 - x_{t-1}^2)^2 < M \quad (28)$$

This solution is always chosen whenever $\theta \geq 4$: it costs a lot for each nation-state to move from the status quo. When $\theta < 4$, the status quo is chosen only if the distance between legal systems is not too high. Otherwise, it is better to harmonize.

Example 2 (Continued)

Assuming $x_{t-1}^1 < x_{t-1}^2$, and that there are no coordination costs, one can show that the cooperative equilibrium is[15]:

$$\tilde{x}_t^i = \begin{cases} \frac{x_{t-1}^1 + x_{t-1}^2}{2}, \text{ if } x_{t-1}^2 - x_{t-1}^1 \leq \frac{4}{\theta} \\ \frac{(-1)^{1+i}}{2}\theta + x_{t-1}^i \text{ if } x_{t-1}^2 - x_{t-1}^1 \geq \frac{4}{\theta} \end{cases} \quad i = 1, 2. \quad (29)$$

The value of the cooperative objective is:

$$\mathcal{U}_t^h = \begin{cases} -\frac{\theta}{4}(x_{t-1}^1 - x_{t-1}^2)^2, \text{ if } x_{t-1}^2 - x_{t-1}^1 \leq \frac{4}{\theta} \\ x_{t-1}^1 - x_{t-1}^2 + \frac{2}{\theta} \text{ if } x_{t-1}^2 - x_{t-1}^1 \geq \frac{4}{\theta} \end{cases} \quad (30)$$

Now, let us assume that there are coordination costs, *i.e.*, M is positive. Then, one may show that it is optimal to choose the status quo if $M > \frac{4}{\theta}$, that is, if M is relatively high. If, on the other hand, $M \leq \frac{4}{\theta}$, it is optimal to choose the status quo only if $x_{t-1}^2 - x_{t-1}^1 \leq \frac{4}{\theta}$. Otherwise, the difference between both legal systems is too high, and nation-states must make their legal systems more alike. Thus, if $M \leq 4/\theta$, the values of the utility functions are:

$$\mathcal{U}_t^h = \begin{cases} -\frac{\theta}{4}(x_{t-1}^1 - x_{t-1}^2)^2, \text{ if } x_{t-1}^2 - x_{t-1}^1 \leq \frac{4}{\theta} \\ x_{t-1}^1 - x_{t-1}^2 + \frac{2}{\theta} - \frac{M}{2} \text{ if } x_{t-1}^2 - x_{t-1}^1 \geq \frac{4}{\theta} \end{cases} \quad (31)$$

[15]See appendix B for details.

5 Cooperative Legal Unification Equilibria

In the past, legal unification has been frequently produced in a cooperative way by federal systems. There are "vertical" models like Germany in which executive, legislative, or judicial powers are vertically integrated, as well as "horizontal" models like the United States in which each level of government makes, executes, and adjudicates its own laws separately (see *e.g.*, Halberstam et Reinmann (2010)).

An interesting modern example of legal unification concerns the experience of the European Union (EU) in the field of competition law or contract law. The European Union has wide experience with making law for diverging jurisdictions. The Treaty of Rome (1958) was the point of departure for organizing cooperation between European countries. Under the regime of the Treaty, many directives or regulations had led to a uniform law in Europe (see *e.g.*, competition policy). The European Commission, which is responsible for issuing these directives and regulations, can fine individuals, firms, organizations and States under certain circumstances and can also refer cases to the European Court of Justice, which is the final arbiter of European Law[16].

However, an important question is whether convergence of private law is at all possible in the European Union. Some have argued that the differences among the 28 private law systems in Europe (27 national systems and Scottish law) are too large to come to any real convergence. According to scholars, various other methods to reach (further) convergence of private law in Europe could be considered. Should the European Union continue with the present harmonization process by issuing European directives or should other methods (also) be used to reach more convergence of law ? For instance, wide-ranging pleas have been made for promoting a European legal science and education and for convergence of law through competition of legal systems.

These difficulties motivate an analysis of legal unification and its comparison with other methods of legal standardization. In the following we concentrate on the case where two nation-states cooperate to reach legal unification. Formally, two nation-states are assumed to choose a *common* value x_t^u for their legal system (that is $x_t^1 = x_t^2 = x_t^u$). In a way, legal harmonization is a particular case of the cooperative equilibrium studied in the previous section (since one could always choose $x_t^1 = x_t^2$). However, here, for simplicity, it is assumed that there are no coordination costs. We think indeed that it is easier to coordinate on a single legal system rather than to try to make different systems more compatible. In our view, the advantage of legal unification is to eliminate the costs of interacting with different legal systems *and* to economize on the coordination costs[17]. For sure, this viewpoint is disputable. It is probably more relevant when there are more than two nation-states. We nevertheless restrict the analysis to the case of two nation-states to simplify the analysis.

We now suppose that assumption 3 holds and we define a cooperative legal

[16] See Crettez and Deloche (2006) for an analysis of the role of the Commission in the convergence of legal rules in the European Union.

[17] We will discuss this assumption in section 7.

unification equilibrium at date t as legal system x_t^u satisfying:

$$V^1(x_t^u, x_{t-1}^1) + V^2(x_t^u, x_{t-1}^2) \geq V^1(x_t, x_{t-1}^1) + V^2(x_t, x_{t-2}^2),\ \forall x_t.$$

This definition does not take into account the terms U^1 and U^2 since $x_t^1 = x_t^2 = x_t^u$, and by assumption 3, the values $U^1(x,x)$ and $U^2(x,x)$ do not depend on x. Thus a legal unification equilibrium is a law which minimizes the losses incurred by the nation-states in choosing a common level which is not too different from the historically given ones[18]. As the next proposition shows, under our assumptions, the unified legal system is chosen in between the initial levels.

Proposition 5. *Let us assume that assumption 3 holds. If $x_{t-1}^1 < x_{t-1}^2$, then*

$$x_{t-1}^1 \leq x_t^u \leq x_{t-1}^2. \tag{32}$$

If both functions V^i, $i = 1,2$ are differentiable with respect to x, then one necessarily has $x_{t-1}^1 < x_t^c < x_{t-1}^1$

The intuition for this result is exactly the same as for Proposition 3. We remark that the value x_t^u where convergence is achieved is history dependent (since it is located in the historically given levels of the laws). We also remark that at a cooperative unification equilibrium, we have complete legal convergence from date t on whether nation-states cooperate later or not. Indeed, at date $t+1$, the laws of these nation-states are similar, so even if they do not cooperate they will always choose the status quo which, here, means choosing the unified level x_t^u.

If one drops assumption 3, the legal system x chosen by the law-makers would solves: $\max_x U^1(x,x)+U^2(x,x)+V^1(x,x_{t-1}^1)+V^2(x,x_{t-1}^2)$. Without further assumptions on $L^i(x,x)$, it is difficult to say if x will be in $[x_{t-1}^1, x_{t-1}^2]$. It could also be that while legal convergence would be achieved from date t on, nation-states may find it interesting to change the law at each period. Indeed, one cannot exclude that the solution of $\max_x U^1(x,x)+U^2(x,x)+V^1(x,x_t^c)+V^2(x,x_t^c)$ be x_t^c.

Examples 1 and 2 (Continued)

When unification is chosen, by assumption, $x_t^1 = x_t^2$, and the terms $U(x_t^1, x_t^2)$ vanish (there are nil for both examples). We are hence led to study the following problem:

$$\max_x \left[-\frac{\theta}{2}\left(x - x_{t-1}^1\right)^2 - \frac{\theta}{2}\left(x - x_{t-1}^2\right)^2 \right]$$

It is easy to find that the optimal value of x_t^u is given by:

$$x_t^u = \frac{1}{2}\left(x_{t-1}^1 + x_{t-1}^2\right)$$

The values of the utilities are again similar and equal to

$$\mathcal{U}_t^u = -\frac{\theta}{8}\left(x_{t-1}^1 - x_{t-1}^2\right)^2 \tag{33}$$

[18] As an alternative, one could use the assumption that harmonization is realized by delegation, *i.e.*, a nation-state imposing its preferred legal rule as in Baniak and Grajzl (2009).

6 Choosing between Non-cooperative and Cooperative Processes of Legal Change

In this section we study the choice of the different means used to achieved legal convergence. We will mainly use the examples studied in the previous sections.

As for the choice between cooperative harmonization and the alternatives (non-cooperative legal processes of legal change and legal unification), what matters is the size of the cooperation cost (see Carbonara and Parisi (2008)). Indeed, as all legal decisions may be chosen in cooperative equilibrium, the latter case would always dominate the alternatives with no coordination costs. Therefore, to save place we shall concentrate on the more interesting choice between the non-cooperative solution and the cooperative legal unification equilibrium.

As for the choice between the non-cooperative solution and cooperative legal unification, what matters is the trade-off between eliminating the costs stemming from different legal systems and keeping control of the evolution of national legal systems. The next Proposition gives a necessary condition for preferring the non-cooperative solution over legal unification.

Proposition 6. *Let us assume that assumption 3 holds. A necessary condition for the non-cooperative solution to be preferred over legal unification is:*

$$V^i(\overline{x}_t^i, x_{t-1}^i) \geq V^i(x_t^u, x_{t-1}^i), \ i = 1, 2. \tag{34}$$

If the above inequalities hold, it is easy to see that assumption 1 and Proposition 1 imply:

$$x_{t-1}^1 \leq \overline{x}_t^1 \leq x_t^u \leq \overline{x}_t^2 \leq x_{t-1}^2. \tag{35}$$

Beyond this Proposition, it is difficult to present general results. As a consequence, we shall only consider examples that illustrate interesting possibilities of legal standardization. In the first example (the quadratic case), non-cooperative legal standardization is always preferred over legal standardization. In the second example (the absolute value-quadratic case), legal unification is preferred over non-cooperative legal standardization when the difference in the initial legal systems is low. In the third example, legal unification is always preferred to non-cooperative legal standardization.

6.1 The quadratic case

In this example, each nation-state has the same objective function:

$$U^i(x_t^i, x_t^j) + V^i(x_t^i, x_{t-1}^i) = -\frac{1}{2}\left(x_t^i - x_t^j\right)^2 - \frac{\theta}{2}\left(x_t^i - x_{t-1}^i\right)^2, \ i \neq j. \tag{36}$$

We recall that:

$$\mathcal{U}_t^{nc} = -\left(x_{t-1}^1 - x_{t-1}^2\right)^2 \left[\frac{\theta\,(1+\theta)}{2\,(2+\theta)^2}\right] \tag{37}$$

$$\mathcal{U}_t^h = \max\{-\frac{1}{2}\frac{\theta(x_{t-1}^1 - x_{t-1}^2)^2}{(4+\theta)} - \frac{M}{2}, -\frac{1}{2}(x_{t-1}^1 - x_{t-2}^2)^2\} \tag{38}$$

$$\mathcal{U}_t^u = -\frac{\theta}{8}\left(x_{t-1}^1 - x_{t-1}^2\right)^2 \tag{39}$$

We recall also that that:

$$\overline{x}_t^1 = \frac{1+\theta}{2+\theta}\overline{x}_{t-1}^1 + \frac{1}{2+\theta}\overline{x}_{t-1}^2$$
$$\overline{x}_t^2 = \frac{1}{2+\theta}x_{t-1}^1 + \frac{1+\theta}{2+\theta}x_{t-1}^2$$
$$x_t^u = \frac{1}{2}\left(x_{t-1}^1 + x_{t-1}^2\right)$$

It is easy to check that:

$$\overline{x}_t^1 < x_t^u < \overline{x}_t^2$$

We observe that:

$$\mathcal{U}_t^u > \mathcal{U}_t^{nc} \tag{40}$$

$$\Longleftrightarrow -\frac{\theta}{8}\left(x_{t-1}^1 - x_{t-1}^2\right)^2 > -\left(x_{t-1}^1 - x_{t-1}^2\right)^2 \left[\frac{\theta\,(1+\theta)}{2\,(2+\theta)^2}\right] \tag{41}$$

$$\Longleftrightarrow 2\theta^2 < 0. \tag{42}$$

This is impossible. Legal unification is *never* preferred over non-cooperation[19].

In this example, eliminating the costs of having different legal systems never compensates the loss of the freedom to ajust one's legal system. It is noteworthy that this result does not depend on the weight given to the cost of changing one's legal system (*i.e.*, θ).

6.2 The absolute value-quadratic case

Again, we assume that the preferences of the law-makers are identical and equal to:

$$U(x_t^1, x_t^2) + V(x_t^1, x_{t-1}^1) = - \mid x_t^1 - x_t^2 \mid -\frac{\theta}{2}(x_t^1 - x_{t-1}^1)^2 \tag{43}$$

[19]This result is implied by Proposition 3 of Loeper (2008).

We recall that:

$$\mathcal{U}_t^{nc} = \begin{cases} -\frac{\theta}{2}(x - x_{t-1}^i)^2 & \text{if } x_{t-1}^2 - x_{t-1}^1 \le \frac{2}{\theta} \\ \text{where } -\frac{1}{\theta} + x_{t-1}^2 \le x \le \frac{1}{\theta} + x_{t-1}^1, & \\ x_{t-1}^1 - x_{t-1}^2 + \frac{3}{2\theta} & \text{if } x_{t-1}^2 - x_{t-1}^1 > \frac{2}{\theta}. \end{cases} \tag{44}$$

$$\mathcal{U}_t^h = \begin{cases} -\frac{\theta}{4}(x_{t-1}^1 - x_{t-1}^2)^2, \text{if } M \le \frac{4}{\theta} \text{ and } x_{t-1}^2 - x_{t-1}^1 \le \frac{4}{\theta} \\ x_{t-1}^1 - x_{t-1}^2 + \frac{2}{\theta} - \frac{M}{2}, \text{ if } M \le \frac{4}{\theta} \text{ and } x_{t-1}^2 - x_{t-1}^1 \ge \frac{4}{\theta}, \\ x_{t-1}^1 - x_{t-1}^2, \text{ if } M \ge \frac{4}{\theta}. \end{cases} \tag{45}$$

$$\mathcal{U}_t^u = -\theta/8(x_{t-1}^2 - x_{t-1}^1)^2 \tag{46}$$

Let us concentrate on the comparison between $\mathcal{U}_t^{nc}$ and $\mathcal{U}_t^u$. First of all, it is easy to see that when the difference in legal systems satisfies $x_{t-1}^2 - x_{t-1}^1 \le 2/\theta$, and $x = (x_{t-1}^1 + x_{t-1}^2)/2$, the non-cooperative solution delivers the same welfare as legal unification (we have $\overline{x}_t^1 = x_t^u = \overline{x}_t^2$). This is because at the non-cooperative equilibrium nation-states choose the same legal system. When x takes a different value, the non-cooperative solution is strictly preferred over legal unification by a nation-state (the cooperative unified legal system being less close to the previous legal systems than the non-cooperative unified one). The other nation-state, of course, prefers legal unification for the opposite reason.

Next, when the difference in legal systems is moderately low, *i.e.*, $x_{t-1}^2 - x_{t-1}^1 \in [2/\theta, 6/\theta]$, legal unification is preferred over the non-cooperative solution. Indeed, in this case, nation-states use different legal systems and the cost of this difference does not compensate the freedom of choosing one's legal system. When the initial difference between legal systems is high enough (*i.e.*, higher than $6/\theta$), it pays no more to have a unified system. That would lead to a legal system too different from the initial national systems[20]. In any case, one can check that : $\overline{x}_t^1 \le x_t^u \le \overline{x}_t^2$.

Finally a comment is in order with regard to the choice between cooperative harmonization and cooperative legal unification. When coordination costs are not too high, *i.e.*, when $x_{t-1}^2 - x_{t-1}^1 \le 8/\theta$, it is better to have legal unification. The argument is that having different legal systems is costly, even if they are well conceived, whereas having identical systems maximizes the gain U^i.

6.3 The case where $U^i = V^i$

We now assume that the preferences of the decision makers are described using a single continuously differentiable strictly concave even function $Z(.)$ defined in $\mathbb{R}$, *i.e.*: $Z(x_t^1 - x_t^2) + Z(x_t^1 - x_{t-1}^1)$. From the even property : $Z(x) = Z(-x)$, it follows that: $Z'(x-y) = -Z'(y-x)$, so that $Z(.)$ reaches its maximum at 0. Thus assumption 1 is satisfied.

The quadratic functions is an example of the function $Z(.)$ as is the function: $-\sqrt{a + (x_t^1 - x_t^2)^2}$.

[20]These results are a special case of Proposition 10 (and footnote 32) of Loeper (2008).

Leaving $Z(.)$ unspecified for a while, we may derive the values of the legal systems chosen in the non-cooperative solution and in the legal unification equilibrium. These values are given in the next Proposition:

Proposition 7. *When $U = V = Z$ and Z is a strictly concave continuously differentiable even function, we have:*

$$\overline{x}_t^1 = \frac{2x_{t-1}^1 + x_{t-1}^2}{3} \tag{47}$$

$$\overline{x}_t^2 = \frac{x_{t-1}^1 + 2x_{t-1}^2}{3} \tag{48}$$

$$x_t^u = \frac{1}{2}(x_{t-1}^1 + x_{t-1}^2) \tag{49}$$

Moreover, we have $\mathcal{U}_t^{nc} = Z(\frac{x_{t-1}^1 - x_{t-1}^2}{3}) + Z(\frac{x_{t-1}^2 - x_{t-1}^1}{3}) = 2Z(\frac{x_{t-1}^1 - x_{t-1}^2}{3})$ *and* $\mathcal{U}_t^u = Z(\frac{x_{t-1}^2 - x_{t-1}^1}{2})$.

We observe that:

$$\overline{x}_t^1 \leq x_t^u \leq \overline{x}_t^2 \tag{50}$$

Below, we give an example of a function $Z(.)$ for which the legal unification equilibrium is *always* preferred over the non-cooperative solution[21] .

Proposition 8. *Let $Z(x) = -\sqrt{a + x^2}$, $a > 0$. Then we always have $\mathcal{U}_t^u > \mathcal{U}_t^{nc}$.*

What this example shows is that the elimination of the costs stemming from the existence of different legal systems brings about an increase in welfare that more than compensates the loss of the choice of one's legal system.

It is interesting to notice that when $Z(x) = -\frac{1}{2}x^2$, we always have $\mathcal{U}_t^{nc} > \mathcal{U}_t^u$ (this is hardly surprising since this case corresponds to the quadratic example with $\theta = 1$ and we have seen that in this case, non-cooperation is always preferred over legal unification).

7 Discussion

In this section, we discuss some assumptions used of our model. We focus on two major points of the analysis. The first point relates to the modelization of the costs of legal changes. The second concerns the generalization of the model to the case where there are more than two players[22].

[21] This result is a special case of Proposition 9 of Loeper (2008), who does not use the specification $Z(x) = -\sqrt{a + x^2}$.

[22] A minor remark concerns the temporal horizon of the agents. Our propositions do not depend on the fact that governments have a one period-ahead horizon. If the forecast horizons were more distant, the consequence would be simply to reinforce the convergence effect even if the speed of convergence would be different.

7.1 The costs of legal changes

We will address two kinds of concerns. First we shall explain why we did not retain the assumption made by Carbonara and Parisi (2008) that nation-states may change ex ante their switching costs. Second, we will discuss the assumption that legal harmonization is the only case where there is a fixed cost to change the law.

7.1.1 Absence of endogenous switching costs

While we build upon the analysis of Carbonara and Parisi (2008) we have amended their framework. We do not assume that there are endogenous switching costs. That is, the possibility for a nation-state to increase or decrease ex ante the value taken by the functions U^i and V^i. This assumption is used by these authors to explain what they call the paradox of harmonization (that is, when there are endogenous switching costs there may be more legal convergence with non-cooperation than with cooperation). In our view, this assumption is perfectly relevant in a static framework, but in a dynamic framework, with repeated interactions, it is plausible that cooperation would also extend to the choice of switching costs. This is likely to be the case since some switching costs are of a legal nature (*e.g.*, the introduction of referendums as an additional barrer to adopt laws decided at an international level). Thus cooperation would concern both switching costs and the law. To simplify the analysis, *i.e.*, to keep it one- dimensional, we leave out the possibility of endogenous switching costs. In a way, since switching costs are mainly of legal nature, this amounts to aggregating further the legal system considered in the analysis.

7.1.2 Harmonization and fixed costs

As we have noticed, there are only fixed costs when there is legal harmonization. We have assumed that this is due to the existence of coordination costs. But there are several other reasons explaining the existence of fixed costs (see *e.g.*, Gomez Pomar (2008)), and these apply to the other alternative ways to standardize the law:

1. to collect reliable evidence on the actual state of the events one desires to regulate;
2. to consider a range of alternatives for each issue under consideration;
3. to estimate the likely impact of the regulatory alternatives on the position of the relevant individuals and groups;
4. to draft the law, and legal drafting may be costly depending on the kind and length of the exercise;
5. to invest political capital to convince the relevant public of the virtues of the new legislation, and to overcome opposition from the interest groups who may be harmed by the legal reform, even when overall it enhances social welfare.

It follows that many preparatory activities are costly and that a significant fraction of those costs are invariant to the number of individuals and firms that will benefit from a new legal regime.

Building on these motivations, we can assume that choosing a new legal system implies a fixed cost L. In addition, if nation-states make their decisions in a cooperative fashion, they bear an additionnal fixed cost M representing the costs of cooperation. This cost is shared equally among nation-states. If both nation-states choose to unify their legal system, the overall cost faced by each of them is simply: $(L+M)/2$. Of course, if a nation-state chooses the status quo, there is no fixed cost.

With this new set of assumptions, some of our results may be modified. In order to save space, we concentrate on the quadratic case. The following table summarizes the fixed costs per nation-state, the choices of legal moves, and the corresponding utility in each case (we set $\lambda = (x^1_{t-1} - x^2_{t-2})$).

Case	Fixed costs	Choices	Utilities
Harm.	$L + \frac{M}{2}$	$\tilde{x}^i = x^i_0 - \frac{2(x^i_0 - x^j_0)}{4+\theta},\ i \neq j$	$\mathcal{U}^h_t = -\frac{\theta\lambda^2}{2(4+\theta)} - \left(L + \frac{M}{2}\right)$
Non coop.	L	$\overline{x}^i = x^i_0 - \frac{(x^i_0 - x^j_0)}{2+\theta},\ i \neq j$	$\mathcal{U}^{nc}_t = -\left[\frac{\theta(1+\theta)\lambda^2}{2(2+\theta)^2}\right] - M$
Unification	$\frac{L+M}{2}$	$x^{c,i} = \frac{\left(x^i_0 + x^j_0\right)}{2},\ i \neq j$	$\mathcal{U}^c_t = -\frac{\theta\lambda^2}{8} - \left(\frac{L+M}{2}\right)$
Status Quo	0	$x^i = x^i_0$	$\mathcal{U}^{sq}_t \equiv -\frac{1}{2}\lambda^2$

Table 1: Fixed costs, choices and payoffs

Here, we focus on symmetric equilibria. Indeed, in some configurations, there could be some interesting equilibria with one nation moving, the other choosing the status quo.

Assuming as an example that $x^1_0 = 0$ and $x^2_0 = 1$, figure 3 displays the moves of each nation-state for each configuration.

Nation-states move more in the harmonized (H) case than in the non cooperative (NC) case because they take into account the effect of their individual move on the other nation's utility. In the NC case, this externality is not taken into account, which results in smaller changes. When nation-states unify their legal systems, all the variable costs are put in the term V of legal change, but this cost is shared equally between the two countries. In the case of the Status Quo, the whole cost is put on the distance between nation-states and this distance is maximal because nobody is moving. In return, no one has to pay for fixed costs.

As it is never optimal to have all the distance in one of the two terms, the only advantage of unification and status quo is to reduce fixed costs. They are then preferred to NC and H only when the initial distance between the states (λ) is small. Indeed, we have the following conditions:

$$\mathcal{U}_t^u > \mathcal{U}_t^h \text{ if } \lambda < L\left[\frac{4(4+\theta)}{\theta^2}\right] \quad (51)$$

$$\mathcal{U}_t^u > \mathcal{U}_t^{nc} \text{ if } \lambda < (L-M)\left[\frac{4(2+\theta)^2}{\theta^3}\right] \quad (52)$$

$$\mathcal{U}_t^h > \mathcal{U}_t^{nc} \text{ if } \lambda > M\left[\frac{(4+\theta)(2+\theta)^2}{\theta^2}\right] \quad (53)$$

$$\mathcal{U}_t^u > \mathcal{U}_t^{sq} \text{ if } \lambda > (L+M)\left(\frac{4}{4-\theta}\right) \quad (54)$$

When λ is sufficiently high, variable costs are relatively more important than other costs, and then harmonization is always preferred. When λ is sufficiently small, the inverse is true and the status quo is always preferred. Between these two situations, unification and non-cooperation can be optimal for intermediate values of λ, and for specific values of θ, L and M.

As an example, assuming $\theta = 1$, and $L > M$, any decision may be optimal for some subsets of λ, as shown in figure 4. The areas corresponding to unification and non-cooperation can disappear for other values of parameters.

When λ is such that nation-states find it optimal to choose either unification or the status quo solution, there are no further legal changes. To be located in one area or the other is only a matter of initial conditions. In a dynamic framework, the unification equilibrium is preferable to the status quo because all further costs will be nil. The myopic behavior of deciders can trap them in an inefficient situation.

7.2 Generalizations when there are more than two nation-states

Here we only consider the possibility that legal unification can be achieved in the long-run when there are more than 2 nation-states. Specifically, we shall only consider the dynamics for the a priori less favorable case for legal convergence, that is the non cooperative solution.

Let us make the following assumptions:

1. There are N nations-states, $i = 1, \cdots, N$, $N > 2$.
2. Each nation-state i is endowed with a utility function:

   $$L^i(x^i, x^{-i}) + V^i(x^i, x^i_{-1}) \quad (55)$$

   where $L^i : \mathbb{R}^N \to \mathbb{R}$ and $V^i : \mathbb{R}^2 \to \mathbb{R}$ are smooth functions, x^{-i} is a $N-1$ vector with coordinates x^j, $j = 1, \cdots, N$, $j \neq i$.
3. For each i, the function L^i is increasing with respect to x^i when $x^i < \min_{j\neq i} x^j$, decreasing when $x^i > \max_{j\neq i} x^j$. More precisely, we assume

that:

$$x^i < \min_{j \neq i} x^j \Rightarrow \frac{\partial L^i}{\partial x^i} > 0 \text{ and when } x^i = \min_{j \neq i} x^j, \frac{\partial L^i}{\partial x^i} \geq 0 \tag{56}$$

$$x^i > \max_{j \neq i} x^j \Rightarrow \frac{\partial L^i}{\partial x^i} < 0 \text{ and when } x^i = \max_{j \neq i} x^j, \frac{\partial L^i}{\partial x^i} \leq 0 \tag{57}$$

4. For each i, the function $V^i(.)$ satisfies assumption 1.

The third assumption generalizes assumption 1 used in the two nation-states case: it never pays to increase the legal distance with all the nation-states.

A non-cooperative equilibrium process of legal change at date t is a N-vector $\overline{x}_t = (\overline{x}_t^1, \cdots, \overline{x}_t^N)$, which satisfies the following inequalities:

$$L^i(\overline{x}_t^i, \overline{x}^{-i})_t + V^i(\overline{x}_t^i, \overline{x}_{t-1}^i) \geq L^i(x_t^i, \overline{x}_t^{-i}) + V^i(x_t^i, x_{t-1}^i), \ \forall i = 1, \cdots, N. \tag{58}$$

At an equilibrium, we necessarily must have:

$$\frac{\partial L^i}{\partial x^i}(\overline{x}_t^i, \overline{x}_t^{-i}) + \frac{\partial V^i}{\partial x^i}(\overline{x}_t^i, x_{t-1}^i) = 0, \ i = 1, \cdots, N. \tag{59}$$

We now study the convergence property satisfied in equilibrium.

Proposition 9. *Let us assume that the x_{t-1}^i are all different. Let a non-cooperative equilibrium process of legal change at date t be given. Then we have:*

$$\min_i x_{t-1}^i < \min_i \overline{x}_t^i < \max_i \overline{x}_t^i < \max_i x_{t-1}^i. \tag{60}$$

As shown in the above Proposition, legal distance is still decreasing in a non-cooperative setting with more than two nation-states. However, if some but not all of x_{t-1}^j are equal, the above statement might not be true anymore. In particular, it might be false at date $t+1$ if some $\overline{x}_t^i$ are equal. Nevertheless, one can show that the decrease in legal distance can still be realized at each date if we further assume that: if $x^j = \min_i x^i$ (resp. $x^j = \max_i x^i$), and there is an integer k such that $x^j \neq x^k$, then :

$$\frac{\partial L^j}{\partial x^j}(x^j, x^{-j}) > 0 \text{ (resp. } \frac{\partial L^j}{\partial x^j}(x^j, x^{-j}) < 0)$$

With this assumption, nation-state j would always choose to reduce its legal distance with the other nation-states.

8 Conclusion

Since the emergence of the Nation-state, law-making has primarily been a task for national legislatures and courts. They "make" law for relatively homogeneous societies that are usually characterized by a common language and culture. As a result of increasing globalization, this is now rapidly changing.

An important problem of law-making in a globalizing world is how to deal with the diversity national legal cultures.

In this paper, we have studied the dynamics of legal convergence and the the comparison between different instruments of legal convergence based on cooperative or non-cooperatives strategies (*i.e.*, harmonization and unification).

We have first shown that Legrand's viewpoint may be correct but only in the short-run. Indeed, legal unification may be achieved in the long-run through small step by step changes despite the existence of huge harmonization costs in the short-run.

While our results generally suggest that legal convergence can be achieved, there are some empirical cases where this is not true. Indeed, Balas *et al.* (2009), evaluating the degree of "procedural formalism" in civil procedures, show that differences between common and civil law have widened during the period from 1950 to 2000. The degree of procedural formalism is measured for simple types of disputes: the eviction of a nonpaying tenant and the collection of a bounced check. However, as these procedures mainly concern disputes internal to nation-states, it is not clear that these parts of the law are concerned by our conclusions which focus more on parts of the law used by actors implicated in international transactions.

We have also shown that for many configurations, convergence between legal systems does not require any form of cooperation between nation-states. The creation of more cooperative frameworks of decisions essentially has an impact on the speed of convergence, but not on the fact that the convergence will occur. The forces that prevent convergence, such as the presence of fixed costs, have the same impact in cooperative and non cooperative frameworks. Moreover, a faster convergence is not necessarily a "good thing", and in many configurations, there is no interest for nation-states to be boost the process of convergence. The more brutal form of convergence, which in our model takes the form of an immediate unification of legal systems, is an optimal choice for nation-states only in some special cases. To take its time remains a valuable option for nation-states, and this could maybe explain why there are some discontents in political unions when many choices are taken in harmonized or unified fashion.

In our framework, no legal system is inherently "weaker" or "better" than others. Indeed we have argued that the costs associated to legal standardization are more of an economical rather than of a political or philosophical nature. There are however some topics on which nation-states differ because they do not share the same values (*e.g.*, the death penalty, or abortion). In these cases, legal convergence seems to be impossible, unless values change. We leave the exploration of this last possibility as a topic for further reasearch.

References

[1] Armour J., Deakin S., Sarkar P. , Siems M. and A. Singh (2008), "Shareholder Protection and Stock Market Development: An Empirical Test

of the Legal Origins Hypothesis", World Economy & Finance, Research Programme, Working Paper Series.

[2] Armour J., Deakin S., Lele P. and M.H. Siems (2009), "How do Legal Rules Evolve ? Evidence From a Cross-Country Comparison of Shareholder, Creditor and Worker Protection", European Corporate Governance Institute Law Working Paper No. 129/2009.

[3] Alonso, R., Dessein, W. and N. Matouschek (2008), "When Does Coordination Require Centralization", *American Economic Review*, 98:1, 145-179.

[4] Balas A., La Porta R., Lopez-de-Silanes F. and A. Shleifer (2009), "The Divergence of Legal Procedures", *American Economic Journal: Economic Policy*, August, 138-162.

[5] Baniak, A. and P. Grajzl (2010), "Interjurisdictional Linkages and the Scope for Interventionist Legal Harmonization", Working Paper.

[6] Carbonara E., and F. Parisi (2008), "The Paradox of Legal Harmonization", *Public Choice*, 132 (3-4), pp. 367-400.

[7] Carbonara E., and F. Parisi (2009), "Choice of Law and Legal Evolution: Rethinking the Market for Legal Rules", *Public Choice*, Vol.. 139, ISS. 3: 461-492.

[8] Casella A. (2001), "Product Standards and International Trade. Harmonization through Private Coalitions", *Kyklos* , 54, 243-264.

[9] Crettez, B. and R. Deloche (2006), "On the unification of legal rules in the European Union", *European Journal of Law and Economics*, 3, 203-214.

[10] Deffains, B. (2001), "Competition between legal systems: A Comparitive Law and Economics Perspective", in *Law and Economics in Civil Law Countries*, B. Deffains and T. Kirat eds, JAI Press, New York.

[11] de Cruz (1999), *Comparative Law in a Changing World*, London: Cavendish Publishing.

[12] Garoupa N. and. Ogus, A. (2006), "A Strategic interpretation of legal transplants", *Journal of Legal Studies*, **35**, 339-363.

[13] Gomez Pomar (2008), "The Harmonization of Contract Law through European Rules: a Law and Economics Perspective", *Indret: Review on the Analysis of Law*, 2.

[14] Halberstam, D. and M. Reimann (2010), "Federalism and Legal Unification: A Comparative Empirical Investigation of 20 Systems", University of Michigan Law School, Workin Paper No. 186.

[15] Herings PJ.J., and A.J. Kanning (2008), "Harmonization of private law on a global level", *International review of law and economics*, 28, pp. 256-262.

[16] Kramer X. (2008), "A major step in the harmonization of procedural law in Europe: the European small claims procedure", in A.W. Jongbloed (ed.), *The XIIIth World Congress of Procedural Law: The Belgian and Dutch reports*, Antwerpen: Intersentia.

[17] La Porta R., F. Lopez de Silanes, A. Shleifer and R. Vishny (1998), "Law and finance", *Journal of Political Economy*, December, 1113-1156.

[18] Legrand, P. (1996), "European legal systems are not converging", *International and Comparative Law Quarterly*, 45, pp. 52-81.

[19] Legrand, P. (1997), "The impossibility of legal transplants", *Maastricht Journal of European and Comparative Law*, 111-119.

[20] Loeper, A. (2008), "Coordination in Federal Systems", Working Papers.

[21] Loeper, A. (2009), "Coordination in Heterogeneous Federal Systems", Working Papers.

[22] Marciano, A. and. Josselin (2002), *The Economics of Harmonizing European Law*, Edward Elgar, Pub., Cheltenham, Northampton.

[23] Mattei, U. (1994), "Efficiency in legal transplants: an essay incomparative law and economics", *International Review of Law and Economics*, 14, 1, 3-19.

[24] McMillan, J. (1986), *Game Theory in International Economics*, Harwood, UK.

[25] Merryman, Clark, Haley (1994), *The Civil Law Tradition : Europe, Latin America and East Asia*, Charlottesville, Virginia., Michie Co., 1994.

[26] Monheim Helstroffer J. and M. Obidzinski (2010), "Optimal Discretion in Asylum Lawmaking", *International Review of Law and Economics*, 30, 86-97.

[27] Ogus A., and N. Garoupa (2006), "Conflict or Credibility: A Strategic Interpretation of Legal Transplants", *Journal of Legal Studies*, 35-2, pp. 339-363.

[28] Ogus, A. (1999), "Competition between national legal systems: a contribution of economic analysis to comparative law", *International Comparative Law Quarterly*, 48, 2, 405-418.

[29] Ripstein L., and Kobayashi B. (1996), "Evolution and Uniformity", *Economic Inquiry*, 34, pp. 464,

[30] Rodrik (2004), "Globalization and growth — looking in the wrong places", *Journal of Policy Modeling*, 26(4), pp. 513-517.

[31] Siems M. (2008), *Convergence in Shareholder Law* Cambridge University Press.

[32] Smits, J. (2002), "How to predict the differences in uniformity between different areas of a future european private law ? An evolutionary approach", in Marciano and Josselin (2002), 50-70.

[33] Trachtman J. (2008), *The Economic Structure of International Law*, Harvard University Press, Cambridge Ma.

[34] Wagner G. (2002), "The Economics of Harmonization: The Case of Contract Law", *Common Market Law Review,* 39*,* 995-1023

A Proofs

Proof of Proposition 1

Proof. Let us prove the first assertion. It suffices to prove for the case where $\max\{\overline{x}_t^1, \overline{x}_t^2\} < \overline{x}_{t-1}^1$ (the other case being symmetrical to this one). Let us assume first that: $\overline{x}_t^1 < \overline{x}_t^2 < \overline{x}_{t-1}^1$. Assumption 1 implies that $U^1(x^1, \overline{x}_t^2) + V^1(x^1, \overline{x}_{t-1}^1)$ is increasing for all $x^1 < \overline{x}_t^2$. Thus, $\overline{x}_t^1$ cannot be an equilibrium choice. The same reasoning applies for the case where $\overline{x}_t^2 < \overline{x}_t^1 < \overline{x}_{t-1}^1$. Thus, the only remaining possibility is $\overline{x}_t^1 = \overline{x}_t^2$. Now it is clear that both nation-states will always choose the same strategies from date t on.

Let us prove the second assertion. Let $\overline{x}_t^2 = \max\{\overline{x}_t^1, \overline{x}_t^2\}$ be such that: $\overline{x}_{t-1}^1 \leq \overline{x}_t^2 \leq \overline{x}_{t-1}^2$. Then, we necessarily have $\overline{x}_{t-1}^1 \leq \overline{x}_t^1 \leq \overline{x}_t^2$. Indeed, by assumption 1, if $x^1 < \overline{x}_{t-1}^1$, $U^1(x^1, \overline{x}_t^2) + V^1(x^1, \overline{x}_{t-1}^1)$ is increasing whereas if $x^1 > \overline{x}_t^2$, $U^1(x^1, \overline{x}_t^2) + V^1(x^1, \overline{x}_{t-1}^1)$ is decreasing. Thus, we must have $\overline{x}_t^1 \in [\overline{x}_{t-1}^1, \overline{x}_t^2]$. If $\overline{x}_t^1 = \max\{\overline{x}_t^1, \overline{x}_t^2\}$, one must have: $\overline{x}_{t-1}^1 \leq \overline{x}_t^1 \leq \overline{x}_t^2 \leq \overline{x}_{t-1}^2$ by the same reasoning as above. But since $\overline{x}_t^2 \leq \overline{x}_t^1$ by assumption, we must have complete convergence: $\overline{x}_t^1 = \overline{x}_t^2$.

Let us suppose finally that convergence is not realized in a finite amount of time. By induction, we must have $\overline{x}_{s-1}^1 \leq \overline{x}_s^1 \leq \overline{x}_s^2 \leq \overline{x}_{s-1}^2$ for all $s \geq t$. Then the sequences $(\overline{x}_t^1)_t$ and $(\overline{x}_t^2)_t$ are respectively non-decreasing and non-increasing. As they are respectively bounded above and below they both converge to some limits $\overline{x}^1$ and $\overline{x}^2$ which satisfy: $\overline{x}^1 \leq \overline{x}^2$.

□

Proof of Proposition 2

Proof. The only thing which needs to be proven is that at a steady-state $\overline{x}^1 = \overline{x}^2$. But, by continuity, taking limits in the the first-order necessary conditions gives:

$$U_1^1(\overline{x}^1, \overline{x}^2) + V_1^1(\overline{x}^1, \overline{x}^1) = 0 \tag{61}$$

$$U_1^2(\overline{x}^2, \overline{x}^1) + V_1^2(\overline{x}^2, \overline{x}^2) = 0 \tag{62}$$

By assumption 1, we have $V_1^1(\overline{x}^1, \overline{x}^1) = V_1^2(\overline{x}^2, \overline{x}^2) = 0$. Thus, one gets:

$$U_1^1(\overline{x}^1, \overline{x}^2) = U_1^2(\overline{x}^2, \overline{x}^1) = 0. \tag{63}$$

Again, by assumption 1, this proves that: $\overline{x}^1 = \overline{x}^2$. Thus legal convergence is achieved at steady-state. □

Proof of Proposition 3

Proof. Let us assume that the status quo is not chosen (otherwise the proposition is obviously satisfied). Let us also assume first that $\tilde{x}_t^1 \neq \tilde{x}_t^2$. Then, we must have $x_{t-1}^1 \leq \tilde{x}_t^2$. Indeed, suppose that $\tilde{x}_t^2 < x_{t-1}^1$. Then, by assumptions

2 and 3 $f(x_t^1) \equiv U^1(x_t^1, \tilde{x}_t^2) + U^2(\tilde{x}_t^2, x_t^1) + V^1(x_t^1, x_{t-1}^1) + V^2(\tilde{x}_t^2, x_{t-1}^2)$ is increasing with respect to x_t^1 when $x_t^1 \leq \tilde{x}_t^2$ and decreasing when $x_t^1 \geq x_{t-1}^1$. Thus $\tilde{x}_t^1$ is in $[\tilde{x}_t^2, x_{t-1}^1]$. Since $\tilde{x}_t^1 \leq x_{t-1}^1 < x_{t-1}^2$, by assumptions 2 and 3 $g(x_t^2) \equiv U^1(\tilde{x}_t^1, x_t^2) + U^2(x_t^2, \tilde{x}_t^1) + V^1(\tilde{x}_t^1, x_{t-1}^1) + V^2(x_t^2, x_{t-1}^2)$ is increasing if $x_t^2 \leq \tilde{x}_t^1$, and decreasing if $x_t^2 \geq x_{t-1}^2$. Thus $\tilde{x}_t^2 \in [\tilde{x}_t^1, x_{t-1}^2]$. Hence we have: $\tilde{x}_t^2 \leq \tilde{x}_t^1 \leq \tilde{x}_t^2$. Therefore $\tilde{x}_t^1 = \tilde{x}_t^2$, which is a contradiction. Thus, we have $x_{t-1}^1 \leq \tilde{x}_t^2$ and it follows easily that $x_{t-1}^1 \leq \tilde{x}_t^1 \leq \tilde{x}_t^2$. By using the same reasoning as above we may show that $\tilde{x}_t^1 \leq x_{t-1}^2$. As a consequence, one has: $x_{t-1}^1 \leq \tilde{x}_t^1 \leq \tilde{x}_t^2 \leq x_{t-1}^2$. We now conclude the proof by analyzing the case where $\tilde{x}_t^1 = \tilde{x}_t^2$. Let us show that $x_{t-1}^1 \leq \tilde{x}_t^1$. If not, we have $\tilde{x}_t^1 = \tilde{x}_t^2 < x_{t-1}^1$. But, due to assumption 3 it would then be better to choose $\tilde{x}_t^1 = \tilde{x}_t^2 = x_{t-1}^1$. Therefore, we have $x_{t-1}^1 \leq \tilde{x}_t^1$. By symmetry, we have $\tilde{x}_t^1 = \tilde{x}_t^2 < x_{t-1}^2$ and we are done. □

Proof of Proposition 4

Proof. Let us first consider a date t such that $x_{t-1}^1 \neq x_{t-1}^2$ (this could be $t = 0$). If the status quo is chosen at date $t-1$, it is clear that it remains chosen at any future date and legal unification is never chosen. Now, assume that the status quo is not chosen at date $t-1$ and that legal unification is chosen at date t, *i.e.*, $\tilde{x}_t^1 = \tilde{x}_t^2$. The optimality conditions are:

$$U_1^1(\tilde{x}_t^1, \tilde{x}_t^2) + U_2^2(\tilde{x}_t^2, \tilde{x}_t^1) + V_1^1(\tilde{x}_t^1, x_{t-1}^1) = 0 \tag{64}$$
$$U_2^1(\tilde{x}_t^1, \tilde{x}_t^2) + U_1^2(\tilde{x}_t^2, \tilde{x}_t^1) + V_1^2(\tilde{x}_t^2, x_{t-1}^2) = 0 \tag{65}$$

Then from (64)-(65), assumptions 2 and 3, it follows that: $U_1^1(\tilde{x}_t^1, \tilde{x}_t^2) = U_2^2(\tilde{x}_t^2, \tilde{x}_t^1) = 0$ and $U_2^1(\tilde{x}_t^1, \tilde{x}_t^2) = U_1^2(\tilde{x}_t^2, \tilde{x}_t^1) = 0$. Thus: $V_1^1(\tilde{x}_t^1, x_{t-1}^1) = V_1^2(\tilde{x}_t^2, x_{t-1}^2) = 0$. As a result, $\tilde{x}_t^1 = x_{t-1}^1$ and $\tilde{x}_{t-1}^2 = x_{t-1}^2$. But this implies that the status quo holds, which is a contradiction. Hence $\tilde{x}_t^1 \neq \tilde{x}_t^2$. Since we have assumed that $x_0^1 \neq x_0^2$, it follows by induction that $\tilde{x}_t^1 \neq \tilde{x}_t^2$ for all t. This proves that legal unification is never chosen along a sequence of cooperative equilibria.

Let us now show that the status quo is always chosen at a finite date. If this is false, Proposition 3 shows that there is more and more harmonization across legal systems. Moreover, as $(\tilde{x}_t^1)_t$ and $(\tilde{x}_t^2)_t$ are respectively non-decreasing and non-increasing bounded sequences, each converge to a limit. One may prove that these limits are equal as in the preceding section. As a consequence, by continuity the sequence of utility levels of both nation-states tends to a limit. But then there is a time at which the gain for acting and not choosing the status quo is strictly lower than M. From this date on, it is therefore optimal to choose the status quo. This contradicts the assumption that the status quo is never chosen. □

Proof of Proposition 5

Proof. The assertion follows easily since by assumption 1, $V^1(x, x_{t-1}^1) + V^2(x, x_{t-1}^2)$ is increasing if $x < x_{t-1}^1$ and decreasing if $x_{t-1}^2 < x$. □

Proof of Proposition 6

Proof. By assumption, we have:

$$U^1(\overline{x}_t^1, \overline{x}_t^2) + V^1(\overline{x}_t^1, x_{t-1}^1) \geq U^1(x_t^u, x_t^u) + V^1(x_t^u, x_{t-1}^1) \tag{66}$$
$$U^2(\overline{x}_t^2, \overline{x}_t^1) + V^2(\overline{x}_t^2, x_{t-1}^2) \geq U^2(x_t^u, x_t^u) + V^1(x_t^u, x_{t-1}^2) \tag{67}$$

As by assumption 3 we have $U^i(x_t^u, x_t^u) = U^i(\overline{x}_t^i, \overline{x}_t^i) \geq U^i(\overline{x}_t^i, \overline{x}_t^j)$, $i \neq j$, $i = 1, 2$, the preceding inequalities imply:

$$V^i(\overline{x}_t^i, x_{t-1}^i) \geq V^1(x_t^u, x_{t-1}^i), \; i = 1, 2. \tag{68}$$

□

Proof of Proposition 7

Proof.

• Non-cooperative choices

At any non-cooperative equilibrium, one must satisfy:

$$Z'(x_t^1 - x_t^2) + Z'(x_t^1 - x_{t-1}^1) = 0 \tag{69}$$
$$Z'(x_t^2 - x_t^1) + Z'(x_t^2 - x_{t-1}^2) = 0 \tag{70}$$

Using the symmetry property, one obtains:

$$Z'(x_t^1 - x_t^2) = -Z'(x_t^1 - x_{t-1}^1) = -Z'(x_t^2 - x_{t-1}^1) = Z'(x_t^2 - x_{t-1}^2) \tag{71}$$

By the same symmetry property we also get:

$$Z'(x_{t_1}^1 - x_t^1) = Z'(x_t^2 - x_{t_1}^2) \tag{72}$$

Thus, we have:

$$x_{t-1}^1 - x_t^1 = x_t^2 - x_{t-1}^2 \iff x_t^1 + x_t^2 = x_{t-1}^1 + x_{t-1}^2 \tag{73}$$

Moreover, we also have:

$$Z'(x_t^1 - x_t^2) = Z'(x_{t-1}^1 - x_t^1) \tag{74}$$

So:

$$x_t^1 - x_t^2 = x_{t-1}^1 - x_t^1 \iff x_t^1 = \frac{x_{t-1}^1 + x_t^2}{2} \tag{75}$$

Combining (73) and (75), one arrives at:

$$x_t^1 = \frac{2x_{t-1}^1 + x_{t-1}^2}{3} \tag{76}$$
$$x_t^2 = \frac{x_{t-1}^1 + 2x_{t-1}^2}{3} \tag{77}$$

Since: $x_t^1 - x_t^2 = \frac{x_{t-1}^1 - x_{t-1}^2}{3}$, by the symmetry property, the utilities of the two nation-states are similar and equal to:

$$\mathcal{U}_t^{nc} = U(\frac{x_{t-1}^1 - x_{t-1}^2}{3}) + U(\frac{x_{t-1}^2 - x_{t-1}^1}{3}) = 2U(\frac{x_{t-1}^1 - x_{t-1}^2}{3}) \quad (78)$$

• Legal unification cooperative equilibrium

We assume that the two-nations states cooperate and decide to unify their legal rules. By this, we mean that: $x_t^1 = x^2 = x$. The optimal choice of x is the solution of:

$$\max_x Z(x - x_{t-1}^1) + Z(x - x_{t-1}^2) \quad (79)$$

The necessary and sufficient condition can be written as follows:

$$Z'(x - x_{t-1}^1) + Z'(x - x_{t-1}^2) = 0 \quad (80)$$

Using this equation and the symmetry property one has:

$$Z'(x - x_{t-1}^1) = -Z'(x - x_{t-1}^2) = Z'(x_{t-1}^2 - x) \quad (81)$$

This proves that:

$$x - x_{t-1}^1 = x_{t-1}^2 - x \iff x_t^u = \frac{1}{2}(x_{t-1}^1 + x_{t-1}^2) \quad (82)$$

As a result, the utilities of the two nation-states are similar and equal to:

$$\mathcal{U}_t^u = U(\frac{x_{t-1}^1 + x_{t-1}^2}{2} - x_{t-1}^1) = U(\frac{x_{t-1}^2 - x_{t-1}^1}{2}) \quad (83)$$

□

Remark. We cannot have general results for the case of cooperation with coordination costs, unless the status quo is chosen, then the values of both nation-states objectives are the same and equal to:

$$\mathcal{U}_t^h = Z(x_{t-1}^1 - x_{t-1}^2)$$

Otherwise, the legal systems chosen by the nation-states solve:

$$\max_{x_t^1, x_t^2} 2Z(x_t^1 - x_t^2) + Z(x_t^1 - x_{t-1}^1) + Z(x_t^2 - x_{t-1}^2)$$

The necessary first-order conditions are:

$$2Z'(x_t^1 - x_t^2) + Z'(x_t^1 - x_{t-1}^1) = 0 \quad 2Z'(x_t^2 - x_t^1) + Z'(x_t^2 - x_{t-1}^2) \quad = 0 \quad (84)$$

The only result that one can have from here is that:

$$x_t^1 + x_t^2 = x_{t-1}^1 + x_{t-1}^2.$$

Proof of Proposition 8

Proof. From Proposition 7, the condition $\mathcal{U}_t^u > \mathcal{U}_t^{nc}$ is:

$$-\sqrt{a + \frac{\lambda^2}{4}} > -2\sqrt{a + \frac{\lambda^2}{9}}. \tag{85}$$

where $\lambda = x_{t-1}^2 - x_{t-1}^1$. This above condition reduces to:

$$3a + \frac{7}{36}\lambda^2 > 0. \tag{86}$$

which is always satisfied. $\square$

Proof of Proposition 9

Proof. Let us first prove that $\min_i x_{t-1}^i < \min_i \overline{x}_t^i$ (it is easy to adapt the reasoning to show that $\max_i \overline{x}_t^i < \max_i x_{t-1}^i$). If the statement of the Proposition is false, there is an integer $k \in \{1, \cdots, n\}$ such that: $\overline{x}_t^k = \min_i \overline{x}_t^i \leq \min_i x_{t-1}^i$. But in a Nash equilibrium, we have:

$$\frac{\partial L^k}{\partial x^k}(\overline{x}_t^k, \overline{x}_t^{-k}) + \frac{\partial V^k}{\partial x^k}(\overline{x}_t^k, x_{t-1}^k) = 0. \tag{87}$$

As $\overline{x}_t^k \leq \min_i x_{t-1}^i$,

$$\frac{\partial V^k}{\partial x^k}(\overline{x}_t^k, x_{t-1}^k) \geq 0. \tag{88}$$

It then follows that:

$$\frac{\partial L^k}{\partial x^k}(\overline{x}_t^k, \overline{x}_t^{-k}) \leq 0. \tag{89}$$

There are then two cases.

- First case: $\overline{x}_t^k < \min_{j \neq k} \overline{x}_t^j$.

Then $\frac{\partial L^k}{\partial x^k} > 0$. This is a contradiction.

- Second case: $\overline{x}_t^k = \min_{j \neq k} \overline{x}_t^j \leq \min_i x_{t-1}^i \leq x_{t-1}^k$.

This implies that there exists an integer j such that $\overline{x}_t^j = \overline{x}_t^k \leq \min_i x_{t-1}^i \leq \min\{x_{t-1}^j, x_{t-1}^k\}$. But by assumption, $x_{t-1}^j \neq x_{t-1}^k$. If $\overline{x}_t^k < \overline{x}_{t-1}^k$, we have: $\frac{\partial V^k}{\partial x^k} > 0 \Rightarrow \frac{\partial L^k}{\partial x^k} < 0$. As $\overline{x}_t^k = \min_{i \neq k} \overline{x}_t^i \Rightarrow \frac{\partial L^k}{\partial x^k} \geq 0$. This is a contradiction. If $\overline{x}_t^j < x_{t-1}^j$, the same argument applies. Thus we necessarily have $\min_i x_{t-1}^i < \min_i \overline{x}_t^i$.

Suppose finally that $\min_i \overline{x}_t^i = \max_i \overline{x}_t^i$. By assumption, there must be some k such that: $x_{t-1}^k < \overline{x}_t^k$. Then as:

$$\frac{\partial L^k}{\partial x^k}(\overline{x}_t^k, \overline{x}_t^{-k}) + \frac{\partial V^k}{\partial x^k}(\overline{x}_t^k, x_{t-1}^k) = 0, \tag{90}$$

we must also have:

$$\frac{\partial V^k}{\partial x^k}(\overline{x}_t^k, x_{t-1}^k) < 0, \tag{91}$$

and thus:

$$\frac{\partial L^k}{\partial x^k}(\overline{x}_t^k, \overline{x}_t^{-k}) > 0. \tag{92}$$

However, since $\overline{x}_t^k \geq \overline{x}_t^i$, $\forall i$, we must have:

$$\frac{\partial L^k}{\partial x^k}(\overline{x}_t^k, \overline{x}_t^{-k}) \leq 0. \tag{93}$$

This is a contradiction and we are done. $\square$

B Study of the mixed absolute-value-quadratic case

• Non-cooperative equilibria

In the first place, it is useful to introduce the next two functions:

$$\phi_-(x_t^1) \equiv x_t^1 - x_t^2 - \frac{\theta}{2}(x_t^1 - x_{t-1}^1)^2 \quad \phi_+(x_t^1) \quad \equiv x_t^2 - x_t^1 - \frac{\theta}{2}(x_t^1 - x_{t-1}^1)^2 \tag{94}$$

If we let $\phi(x_t^1) \equiv - \mid x_t^1 - x_t^2 \mid - \frac{\theta}{2}(x_t^1 - x_{t-1}^1)^2$, then:

$$\phi(x_t^1) = \begin{cases} \phi_-(x_t^1) \text{ if } x_t^1 \leq x_t^2 \\ \phi_+(x_t^2) \text{ if } x_t^1 \geq x_t^2 \end{cases} \tag{95}$$

We observe that $\phi_-(x_t^1)$ realizes its maximum at $x_t^1 = 1/\theta + x_{t-1}^1$, while $\phi_+(x_t^1)$ realizes its maximum at $-1/\theta + x_{t-1}^1$.

We are now study the maxima of $\phi(.)$.

1. If $x_t^2 \leq -1/\theta + x_{t-1}^1$, then $\phi_-(.)$ realizes its maximum at $x_t^1 = x_t^2$ and we have: $V_- = \phi_-(x_t^2) = -(\theta/2)(x_t^2 - x_{t-1}^1)^2$. The function $\phi_+(.)$ realizes its maximum at $x_t^1 = -(1/\theta) + x_{t-1}^1$ and we have: $V_+ = x_t^2 - x_{t-1}^1 + (1/2\theta)$. Now $V_- \geq V_+ \iff 0 \geq \theta\mu^2 + 2\mu + 1/\theta$ (where $\mu \equiv x_t^2 - x_{t-1}^1$. It easy to see that in fact we always have $V_+ \geq V_-$ and then it is optimal to choose $x_t^1 = -(1/\theta) + x_{t-1}^1$.
2. If $(-1/\theta) + x_{t-1}^1 < x_t^2 \leq (1/\theta) + x_{t-1}^1$, both ϕ_- and ϕ_- realize their maxima at x_t^2 and we get: $V_- = V_+ = -(\theta/2)(x_t^2 - x_{t-1}^1)^2$.
3. If $1/\theta + x_{t-1}^1 < x_t^2$, $\phi_-(.)$ realizes its maximum at $x_t^1 = 1/\theta + x_{t-1}^1$ and thus $V_- = \phi_-(1/\theta + x_{t-1}^1) = x_{t-1}^1 - x_t^2 + 1/(2\theta)$. The function $\phi_+(.)$ realizes its maximum at $x_t^1 = x_t^2$ and we have: $V_+ = -(\theta/2)(x_t^2 - x_{t-1}^1)^2$. We have $V_- \geq V_+ \iff \theta\mu^2 - 2\mu + 1/\theta \geq 0$. It is easy to see that this is always true so that the optimal choice of x_t^1 is $x_t^1 = 1/\theta + x_{t-1}^1$.

From here one gets the best-response function (16) readily. It is easy but tedious to get the different possible Nash equilibria.

• Cooperative Harmonization with fixed costs

In this case, the difficulty mainly lies in the computation of the cooperative equilibrium (without coordination costs). It is easy to check that the maximum of:

$$2(x_t^2 - x_t^1) - \frac{\theta}{2}(x_t^1 - x_{t-1}^1)^2 - \frac{\theta}{2}(x_t^2 - x_{t-1}^2)^2 \tag{96}$$

under the constraint $x_t^1 \geq x_t^2$ is reached when:

$$x_t^1 = x_t^2 = x = \frac{x_{t-1}^1 + x_{t-1}^2}{2} \tag{97}$$

Now, it is also easy to see that the maximum of

$$2(x_t^1 - x_t^2) - \frac{\theta}{2}(x_t^1 - x_{t-1}^1)^2 - \frac{\theta}{2}(x_t^2 - x_{t-1}^2)^2 \tag{98}$$

under the constraint $x_t^2 \geq x_t^1$ is reached at:

$$x_t^1 = x_t^2 = x = \frac{x_{t-1}^1 + x_{t-1}^2}{2}$$

when $x_{t-1}^2 - x_{t-1}^1 \leq 4/\theta$, and at:

$$x_t^1 = \frac{2}{\theta} + x_{t-1}^1 \tag{99}$$

$$x_t^2 = -\frac{2}{\theta} + x_{t-1}^2 \tag{100}$$

otherwise. Equation (29) follows then easily.

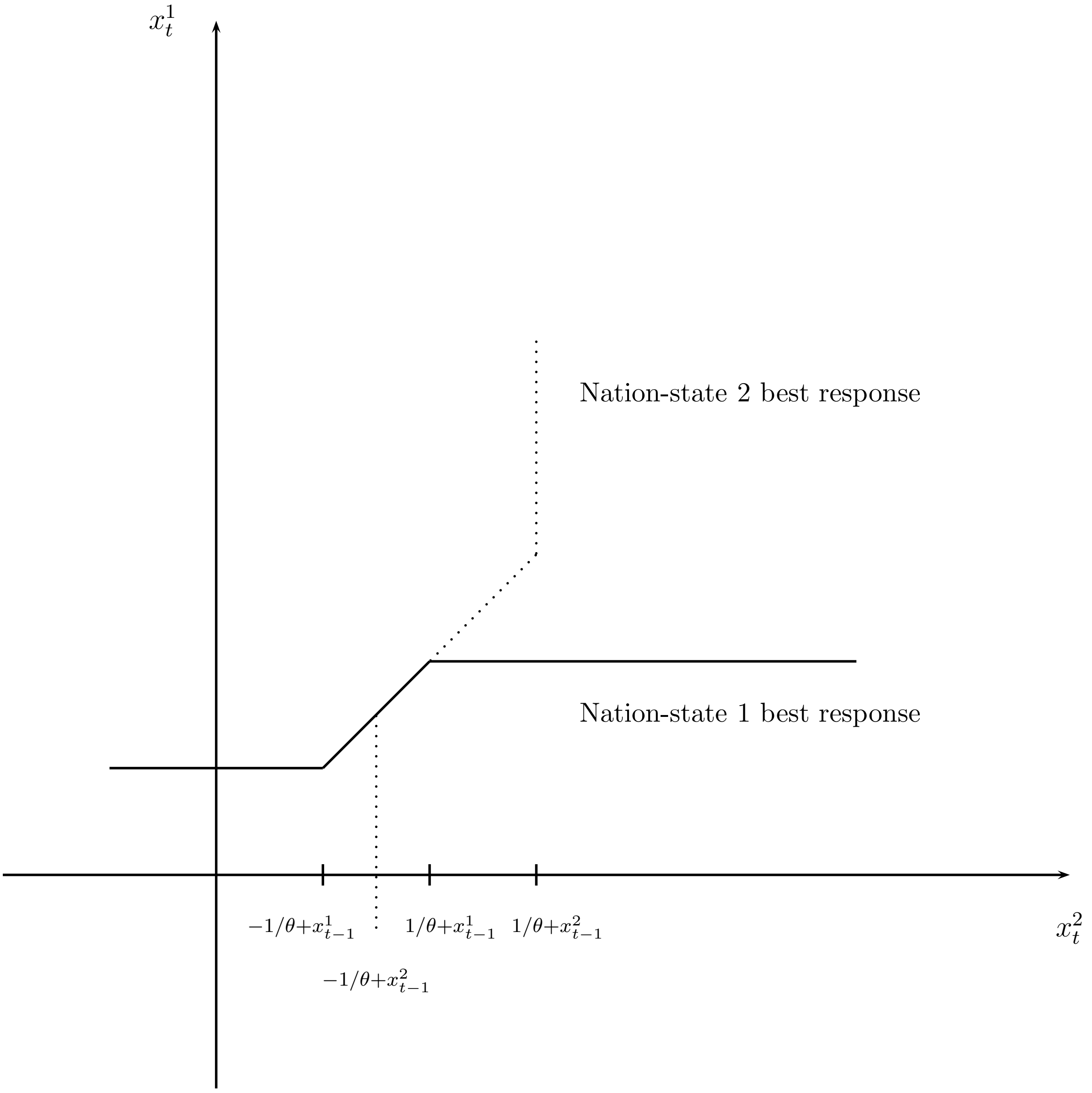

Figure 1: The case of multiple equilibria with $x_t^1 = x_t^2$.

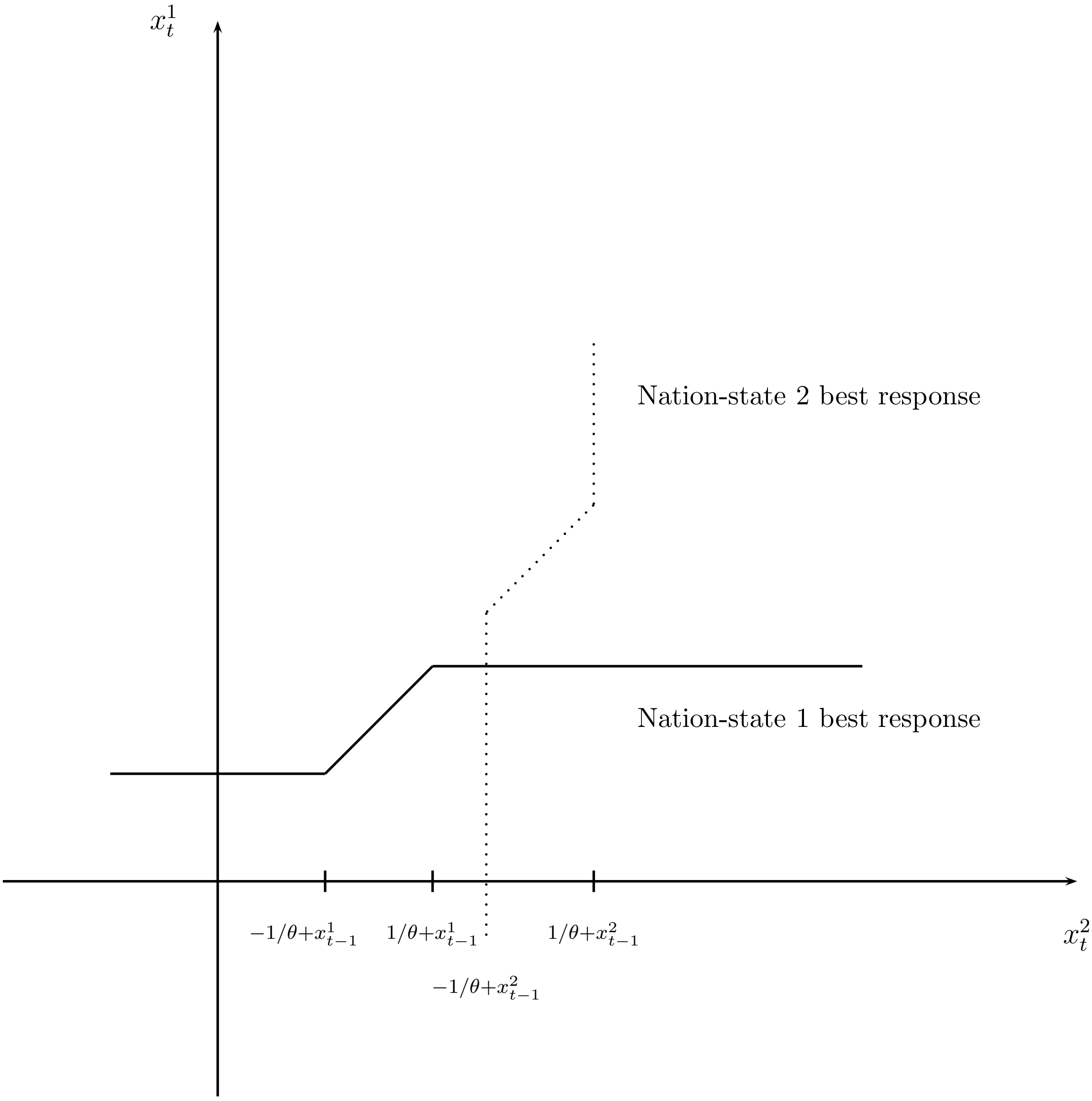

Figure 2: The case with a unique equilibrium with $x_t^1 \neq x_t^2$.

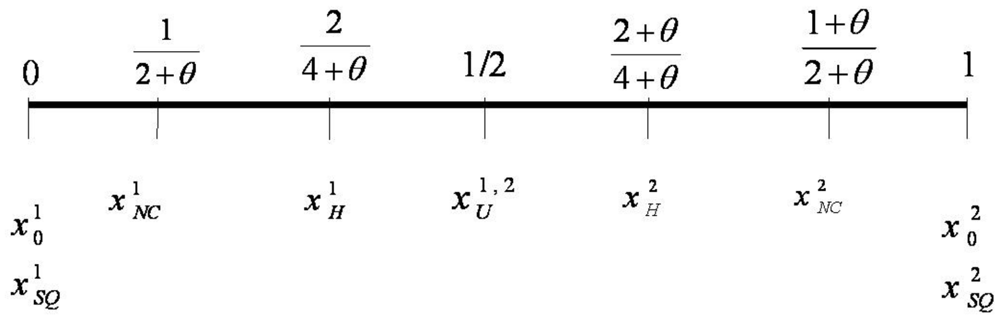

Figure 3:

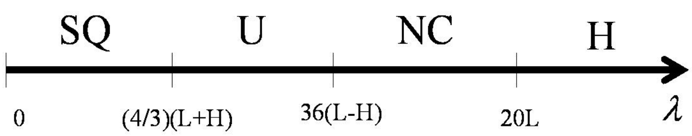

Figure 4: