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characterization and economic implications

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Abstract

The aim of this paper is to propose a behavioral characterization of individuals who underestimate probability modifications and to characterize this behavior in the standard preferences representation models under risk (Expected utility, Dual theory, Rank Dependant Utility Theory and MaxMin Expected Utility). Our main results are the following. Underreaction to probability modifications is in general independent from standard risk aversion and prudence. In models involving probability transformation functions, it is characterized by the slope of the probability transformation function. In the MaxMin Expected utility model under risk, it is related to the weights of the maximal and minimal consequences in the preferences representation function. Considering a simple prevention decision, consisting in the reduction of the probability of a monetary loss, we show that individuals who underreact to probability modifications invest less in prevention than individuals who objectively evaluate these modifications. Underreaction to probability modification is thus a possible explanation for low investment in prevention.

Keywords: probability perception, non expected utility, prevention

JEL code: D81

1 Introduction

The relation between risk attitudes and prevention decisions have been broadly discussed in the literature. Standard risk attitudes have a well determined and intuitive impact on self insurance decision (in the terminology of Ehrlich, Becker 1972) but an ambiguous impact on self protection. Indeed neither standard risk aversion, nor prudence are able to explain completely these decisions: when individuals are expected utility maximizers, an increase in risk aversion does not always increase the prevention level and prudence can decrease the prevention level (see for instance Jullien et alii 1999 and Eeckhoudt, Gollier 2005).

These results are mainly due to the fact that the two types of prevention do not correspond to the same risk transformation: self insurance reduces risk in the sense of second order stochastic dominance, while self protection corresponds to two first order stochastic dominance (FSD) transformations: a first one modifying the probabilities of the different outcomes and corresponding to a FSD improvement, and a second one, decreasing the outcomes and corresponding to a FSD deterioration. Consequently, self protection decisions are influenced by the individuals' sensitivity to probability modifications that is not captured by the existing risk attitude definitions. More precisely, an individual who underestimates the probability modifications resulting from prevention is likely not to invest much in prevention. Characterizing individuals who underestimate probabilities modifications can thus allow to better understand self protection decisions but other decisions involving probability modifications too.

The aim of this paper is to propose a behavioral characterization of individuals who are underreactive to probability modifications (these individuals will be called fatalists in the following) and to characterize this behavior in different preferences representation models under risk (Expected utility, Dual theory, Rank Dependant Utility Theory and MaxMin Expected Utility).

Our main results are the following. Underreaction to probability modifications is in general independent from standard risk aversion and prudence. In models involving probability transformation functions, it is characterized by the slope of the probability transformation function. In MaxMin Expected utility model under risk, it is related to the weights of the maximal and minimal consequences in the preferences representation function.

Considering a standard self protection decision, we show that individuals who un-

derestimate probability modifications invest less in self protection than individuals who objectively evaluate these modifications. Fatalism is thus a possible explanation for low investment in prevention.

The paper proceeds as follows. Section 2 presents the settings and the behavioral characterization of under (and over) reaction to probability modifications. Section 3 characterizes these behaviors in different preferences representation models. In section 4, we determine the implications of our results in a problem of optimal effort choice when effort can modify the probability of a monetary loss.

2 Behavioral characterization of underreaction to probability modifications

We consider a general problem of decision making under risk. Let $(S, \mathscr{A}, P)$ be a probability space where S is a set of states of nature, $\mathscr{A}$ is a $\sigma-$algebra of events (subsets of S) and P a $\sigma-$additive probability measure. $\mathscr{C}$ is a set of consequences with $\mathscr{C} \subset \mathbf{R}$. The set of decisions, denoted by $\mathscr{X}$, is composed of all bounded real random variables from S to $\mathscr{C}$, defined on the probability space $(S, \mathscr{A}, P)$. For $X \in \mathscr{X}$, F_X will denote the cumulative distribution function of $X : F_X(x) = P(X \leq x)$, $x \in \mathbf{R}$.

The set of finite probability distributions (lotteries) is denoted by $\mathscr{L}$ and a generic element of $\mathscr{L}$ is denoted $L = (x_1, p_1; ...; x_n, p_n)$ where consequences are ranked in an increasing order $x_1 \leq ... \leq x_n$, $p_i \geq 0$ and $\sum_{i=1}^{n} p_i = 1$.

We assume that any decision maker has a preference relation on $\mathscr{X}$ which is assumed to be a weak order (i.e. reflexive, transitive and complete relation). This preference relation is denoted $\succsim$, with $\succ$, the strict preference and ˜, the indifference. Individuals preferences on $\mathscr{X}$ induce preferences on the real numbers that we assume compatible with the relation $\geq$ that is, for any $x, y \in \mathbf{R} : x \geq y \Rightarrow \delta_x \succsim \delta_y$ (where δ_x denotes a r. v. taking as unique value the consequence x). Individuals preferences are also assumed to respect the first order stochastic dominance i.e. $\forall X, Y \in \mathscr{X}$, $F_X(x) \leq F_Y(x)$, $\forall x \in \mathbf{R} \Longrightarrow X \succsim Y$.

The behavioral definition of underreaction to probability modifications we propose is based on the comparative approach, used by Yaari (1969) for defining risk aversion and by Ghirardato and Marinacci (2002) for defining ambiguity aversion.

The direct approach (see for instance Arrow (1965), Pratt (1964)) first defines risk

aversion, as a preference for certain outcomes with respect to random ones and then deduces from this a measure for comparing the risk aversion of two individuals: individual 1 is more risk averse than individual 2 if he is ready to pay more for exchanging a random outcome for a certain one. The limit of this approach is that it requires the (arbitrary) definition, for any decision, of a benchmark risk free decision. For decision X, this risk free decision is $\delta_{E(X)}$: the decision giving outcome $E(X)$ with certainty.

The "comparative" approach does not require this kind of assumptions. It starts with a comparative notion of risk aversion and then uses this comparative ranking to obtain an absolute notion of risk aversion by identifying some decision makers as "risk neutral".

Consider two individuals with preference relations $\succsim^1$ and $\succsim^2$. 1 is more risk averse than 2 if, for any $x \in \mathscr{C}$ and any $X \in \mathscr{X}$, $x \succsim^2 X \Rightarrow x \succsim^1 X$. To simplify notations, from now, we note x instead of δ_x for $x \in \mathscr{C}$.

An individual will be called risk neutral if he is an expected value maximizer, that is if for any $X \in \mathscr{X}$, $E(X) \sim X$.

An individual will be called risk averse if he is more risk averse than a risk neutral individual.

The advantage of the comparative approach is to be based on two very intuitive primitive assumptions: constant acts are riskless and expected value maximization corresponds to risk neutrality.

We will use this comparatively founded approach for the definition of the underreaction to probability modifications.

Under (or over)-reaction to probability modifications is related to a perception of probability modifications that differs from its objective (numerical) values. To relate this probability perception to prevention decisions, we consider probability modifications that decrease the probability of low outcomes.

The individuals' under (or over)-reaction to probability modifications will be captured via their choices between lotteries X and Y defined as follows:

$X = (x_{inf}, p; x_{sup}, 1 - p)$ and $Y = (x_{inf} - a, p - \varepsilon; x_{sup} - a, 1 - p + \varepsilon)$ with $x_{inf}, x_{sup}, x_{inf} - a, x_{sup} - a \in \mathscr{C}$ with $x_{inf} < x_{sup}$ and $p, \varepsilon, p - \varepsilon \in [0, 1]$.

Lottery Y is obtained from lottery X by two first stochastic dominance transformations in opposite directions. Indeed, if we introduce an "intermediate" lottery $X' = (x_{inf}, p - \varepsilon; x_{sup}, 1 - p + \varepsilon)$, it is obvious that $X'FSDX$ and thus for any rational individual, $X' \succsim X$. Moreover $X'FSDY$ and thus $X' \succsim Y$.

Comparative underreactivity to probability modifications is then defined as follows:

Definition 1. *Comparative fatalism* *Consider two individuals 1 and 2 with respective preferences $\succsim^1$ and $\succsim^2$ and lotteries X and Y such that $X = (x_{inf}, p; x_{sup}, 1 - p)$ and $Y = (x_{inf} - a, p - \varepsilon; x_{sup} - a, 1 - p + \varepsilon)$. Individual 1 is said to be **more fatalist** than individual 2 if, for any $x_{inf}, x_{sup}, a, p, \varepsilon \in \mathscr{C}$ with $x_{inf} < x_{sup}$ and $p, \varepsilon, p - \varepsilon \in [0, 1]$:*

$$X \succsim^2 Y \Rightarrow X \succsim^1 Y$$

To obtain an absolute notion of "fatalism" or "underreactivity" to probability modifications, it is necessary now to identify individuals who "correctly" or "objectively" evaluate probability modifications. These individuals are naturally those whose preferences verify the independence axiom that guarantees the linearity of the preferences representation function with respect to probabilities and thus corresponds to an objective assessment of probability modifications. Individuals who do not under or over react to probability modifications, and who we will call "realists" are then expected utility maximizers.

Definition 2. *Realism* *Consider an individual with preferences $\succsim$. This individual is called **realist** if his preferences respect the independence axiom, that is if, for any $X, Y, Z \in \mathscr{C}$ and $\lambda \in [0, 1]$,*

$$X \succsim Y \Rightarrow \lambda X \oplus (1 - \lambda)Z \succsim \lambda Y \oplus (1 - \lambda)Z$$

where $\oplus$ denotes the mixture operation.

It is now possible to give a definition of "absolute" underreactivity to probability modifications (fatalism).

Definition 3. *Fatalism* *An individual is fatalist if he is more fatalist (in the sense of Definition 1) than a realist.*

In the following our objective is to give a characterization of the fatalists and to examine the relation between fatalism and standard risk attitudes as risk aversion and prudence.

3 Characterization in different models

In this section, we consider the usual preferences representation models under risk: Expected Utility model, Dual Theory, Rank Dependent Utility and the MaxMinExpected utility model (Cohen 1992) and characterize fatalism for all of them.

Let V denote a preferences representation function under risk.
V is such that for any $X_1, X_2 \in \mathscr{X}$

$$X_1 \succsim X_2 \Leftrightarrow V(X_1) \geq V(X_2).$$

In the following, V will take different forms according to the preferences representation model we consider.

To focus on the perception of probability and probability modifications, we will in all the following compare individuals who have **the same attitude towards certain outcomes, that is, the same utility function.**

3.1 Expected Utility

In this subsection, we assume that $\succsim$ verifies the axioms of the expected utility model, then: $V(X) = Eu(X)$ for any $X \in \mathscr{X}$ with $u : \mathscr{C} \to \mathbf{R}$ and $u' > 0$. The expected utility preferences representation function is linear with respect to probabilities. A probability modification is then evaluated "objectively" and independently on the initial probabilities. The following proposition proves that all expected utility maximizers are "realists".

Proposition 1. *An individual with preferences representation function $V_{EU}(X) = Eu(X)$ is realist for any u with $u' > 0$.*

Proof. Expected utility preferences satisfying the independence axiom, the result follows directly from the definition of realism. □

3.2 Rank dependant utility

The Rank Dependent Utility model (RDU) is a generalization of the Expected Utility model, proposed and axiomatized by Quiggin (1982) and Yaari (1987). This model weakens the independence axiom which leads to a non linear treatment of probabilities,

in addition to a non linear treatment of outcomes. This non linearity in probabilities allows moreover a possible relation between probabilities and outcomes.

The corresponding preferences representation function writes as follows where φ is a probability transformation function and u is the standard utility function. For a continuous random variable X:

$$V_{RDU}(X) = -\int_{\mathscr{C}} u(x)d\varphi\left[1 - F_X(x)\right] \tag{1}$$

with $\varphi : [0,1] \to [0,1], \varphi(0) = 0$ and $\varphi(1) = 1; \varphi(p)$ is assumed continuous, differentiable and increasing on $]0,1[$, $u'(x) \geq 0$.

For a simple, two outcomes lottery $Z = (z_{\inf}, q; z_{\sup}, 1 - q)$ with $z_{\inf} < z_{\sup}$,

$$V_{RDU}(Z) = u(z_{\inf}) + \varphi(1 - q)\left[u(z_{\sup}) - u(z_{\inf})\right] \tag{2}$$

3.2.1 Dual theory: $u(x) = x$

We first consider the case of linear utility for outcomes ($u(x) = x$), corresponding to the Dual Theory (Yaari (1987)).

Proposition 2. *Consider two individuals with preferences represented by the Dual Theory ($u(x) = x$) who differ only by their probability transformation functions. Individual 1 is more fatalist than individual 2 if and only if $\varphi_1'(p) \leq \varphi_2'(p)$ for any $p \in]0, 1[$.*

Proof. From Definition 1, individual 1 is more fatalist than individual 2 if and only if $X \succsim^2 Y \Rightarrow X \succsim^1 Y$ with $X = (x_{inf}, p; x_{sup}, 1-p)$ and $Y = (x_{inf} - a, p - \varepsilon; x_{sup} - a, 1 - p + \varepsilon)$. From (2),

$X \succsim^2 Y \Leftrightarrow V_{RDU}^2(X) - V_{RDU}^2(Y) \geq 0.$

If $u(x) = x$,

$V_{RDU}^2(X) - V_{RDU}^2(Y) = a - [\varphi_2(1 - p + \varepsilon) - \varphi_2(1 - p)](x_{\sup} - x_{\inf}).$

A linear approximation in the neighborhood of $1 - p$ gives:

$V_{RDU}^2(X) - V_{RDU}^2(Y) \geq 0 \Leftrightarrow a - \varphi_2'(1 - p)\varepsilon(x_{\sup} - x_{\inf}) \geq 0.$

$V_{RDU}^2(X) - V_{RDU}^2(Y) \geq 0 \Rightarrow V_{RDU}^1(X) - V_{RDU}^1(Y) \geq 0$ for any $a, p, \varepsilon, x_{\inf}, x_{\sup}$

$\Leftrightarrow V_{RDU}^1(X) - V_{RDU}^1(Y) \geq V_{RDU}^2(X) - V_{RDU}^2(Y)$

$\Leftrightarrow [\varphi_2'(1 - p) - \varphi_1'(1 - p)]\varepsilon(x_{\sup} - x_{\inf}) \geq 0 \Leftrightarrow \varphi_1'(1 - p) \leq \varphi_2'(1 - p)$ for any $p \in]0, 1[$. $\qquad\square$

An individual with RDU preferences will be realist only if his probability transformation function is linear, that is, if $\varphi(p) = p$ because in this case his preferences are represented by an expected utility.

In the following, we characterize fatalism in "absolute" terms.

Proposition 3. *Consider an individuals with preferences represented by the Dual Theory ($u(x) = x$). This individual is fatalist if and only if $\varphi'(p) \leq 1$ for any $p \in]0, 1[$.*

Proof. From Definition 3, an individual is fatalist if he is more fatalist than a realist. A realist is characterized by $\varphi(p) = p$. Using the results of the previous proposition, it follows directly that an individual is fatalist if and only if $\varphi'(p) \leq 1$. □

Remark 1. *Concerning the form of the probability transformation curve of a fatalist, it should have discontinuities in 0 and/or in 1, otherwise the condition $\varphi'(p) \leq 1$ can not be satisfied for any $p \in]0, 1[$. Note that continuity of the probability transformation functions is not required in all rank dependant theories because discontinuities at 0 and 1 are empirically interesting (see Wakker 2010).*

3.2.2   General RDU preferences: $u'' < 0$

In this section, it is assumed that the individual utility function is no more linear, but concave ($u'' < 0$). In this case, the evaluation of the cost a is no more independent from $x_{\inf}$ and $x_{\sup}$ and depends on the concavity of the utility function. Consequently, the choice between X and Y will not only depend on the perception of probability modifications, but also on the perception of the probabilities themselves.

Proposition 4. *Consider two individuals with preferences represented by the RDU model who differ only by their probability transformation functions ($u_1(x) = u_2(x) = u(x)$). Their utility function $u(x)$ is assumed to be strictly concave. Individual 1 is more fatalist than individual 2 if $\varphi_1(p) \leq \varphi_2(p)$ and $\varphi'_1(p) \leq \varphi'_2(p)$ for any $p \in]0, 1[$.*

Proof. From Definition 1, individual 1 is more fatalist than individual 2 if and only if $X \succsim^2 Y \Rightarrow X \succsim^1 Y$ with $X = (x_{inf}, p; x_{sup}, 1-p)$ and $Y = (x_{inf} - a, p - \varepsilon; x_{sup} - a, 1 - p + \varepsilon)$. From (2),

$X \succsim^2 Y \Leftrightarrow V^2_{RDU}(X) - V^2_{RDU}(Y) \geq 0$

$V^2_{RDU}(X) - V^2_{RDU}(Y) =$

$u(x_{\inf}) - u(x_{\inf} - a) + \varphi_2(1-p) \left[u(x_{\sup}) - u(x_{\inf}) \right] - \varphi_2(1 - p + \varepsilon) \left[u(x_{\sup} - a) - u(x_{\inf} - a) \right].$

A linear approximation in the neighborhood of $1 - p$ gives:

$$V_{RDU}^2(X) - V_{RDU}^2(Y) \geq 0 \Leftrightarrow$$

$$u(x_{\text{inf}}) - u(x_{\text{inf}} - a) + \varphi_2(1 - p)\left[(u(x_{\text{sup}}) - u(x_{\text{inf}})) - (u(x_{\text{sup}} - a) - u(x_{\text{inf}} - a))\right]$$

$$-\varepsilon\varphi_2'(1 - p)\left[u(x_{\text{sup}} - a) - u(x_{\text{inf}} - a)\right] \geq 0.$$

$$V_{RDU}^2(X) - V_{RDU}^2(Y) \geq 0 \Rightarrow V_{RDU}^1(X) - V_{RDU}^1(Y) \geq 0 \text{ for any } a, p, \varepsilon, x_{\text{inf}}, x_{\text{sup}} \Leftrightarrow$$

$$V_{RDU}^1(X) - V_{RDU}^1(Y) \geq V_{RDU}^2(X) - V_{RDU}^2(Y) \Leftrightarrow$$

$$[\varphi_1(1 - p) - \varphi_2(1 - p)]\left[(u(x_{\text{sup}}) - u(x_{\text{inf}})) - (u(x_{\text{sup}} - a) - u(x_{\text{inf}} - a))\right]$$

$$-\varepsilon\left[\varphi_1'(1 - p) - \varphi_2'(1 - p)\right]\left[u(x_{\text{sup}} - a) - u(x_{\text{inf}} - a)\right] \geq 0$$

From the concavity of u, it follows that $(u(x_{\text{sup}}) - u(x_{\text{inf}})) - (u(x_{\text{sup}} - a) - u(x_{\text{inf}} - a)) < 0$ and thus $V_{RDU}^2(X) - V_{RDU}^2(Y) \geq 0 \Rightarrow V_{RDU}^1(X) - V_{RDU}^1(Y) \geq 0$ if $\varphi_1(1-p) - \varphi_2(1-p) \leq 0$ and $\varphi_1'(1 - p) - \varphi_2'(1 - p) \leq 0$ for any $p \in]0, 1[$. $\qquad\square$

As in the Dual Theory case, an individual with RDU preferences will be realist only if its probability function is linear, that is, if $\varphi(p) = p$ because in this case his preferences are represented by expected utility.

In the following, we characterize fatalism in "absolute" terms.

Proposition 5. *Consider an individuals with RDU preferences and concave utility function ($u'' < 0$). This individual is fatalist if $\varphi(p) \leq p$ and $\varphi'(p) \leq 1$ for any $p \in]0, 1[$.*

Proof. From Definition 3, an individual is fatalist if he is more fatalist than a realist. A realist is characterized by $\varphi(p) = p$. Using the results of the previous proposition, it follows directly that an individual is fatalist if $\varphi(p) \leq p$ and $\varphi'(p) \leq 1$. $\qquad\square$

The previous results show that fatalism is related to the slope of the probability transformation function that measures, in the RDU theory, the individuals assessment of probability modifications, a smaller slope corresponding to a lower reactivity to probability modifications as it appears in Figure 1. The results also show that risk aversion and fatalism are different concepts. Indeed, in the RDU model weak risk aversion is characterized by $\varphi(p) < p$ and strong risk aversion, by a convex function φ. In the Dual theory, fatalism is completely independent from risk aversion because

no condition on $\varphi(p)$ is involved. Consequently, fatalists can be risk averse as well as risk lovers and for comparing the fatalism of two individuals no condition on their risk aversion is required. This is due to the fact that, due to the linearity of the utility function, the gap between the two outcomes is perceived as identical in X and Y (and equal to a).
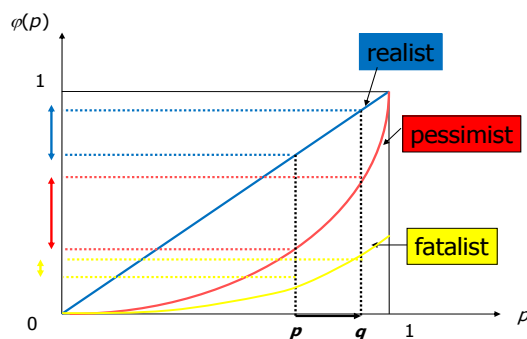

Figure 1: Fatalism, realism and pessimism

In the RDU theory, $\varphi(p)$ (and thus risk aversion) matters for fatalism. To understand this, it is necessary to recall that the choice between X and Y results from the individuals assessment of a trade off between a "gain" in terms of probabilities and a "loss" in terms of outcomes. For all individuals with linear utility functions, the loss in terms of outcomes is equally evaluated (equal to a) and does not depend on the probability transformation function. When the utility function is concave, this is no more the case, the gap between the outcomes is $u(x_{\mathrm{inf}}) - u(x_{\mathrm{inf}} - a)$ for the lower outcome, and $u(x_{\mathrm{sup}}) - u(x_{\mathrm{sup}} - a)$ for the higher one. The global evaluation of the loss in terms of outcomes depends then on the probability transformation function, more precisely, this loss is evaluated as higher for more risk averse individuals. To summarize, risk averse individuals with flat probability transformation functions are fatalists because they both underreact to probability modifications, and overestimate loss in outcomes. Note that, to compare the fatalism of two individuals, they do not need to be both risk averse, only comparative risk aversion being required. Figure 2 illustrates this point.

Fatalism appears as an observed characteristic of individual preferences towards risk.
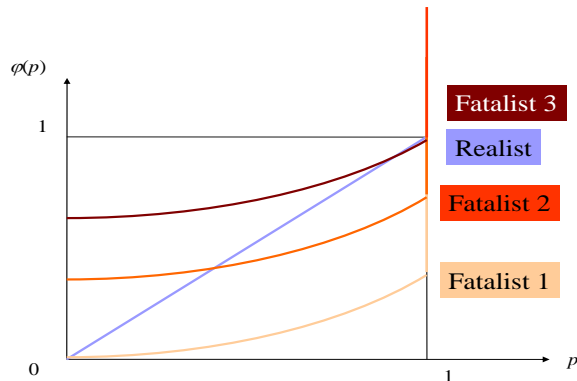
Figure 2: Comparative fatalism

Indeed, experimental studies (see for instance Wu, Gonzales 1996, Bruhin et alii 2010) often conclude to inverse S-shaped probability transformation functions (first concave and then convex), which are compatible with "local" fatalism for probabilities in an interval away from the boundaries, 0 and 1. More precisely, Bruhin et alii (2010) elaborate a typology of individual behavior towards risk and find three types of individuals which probability tranformation functions are represented in Figure 3. It appears that two of the three types seem fatalist for probabilities between 0.1 and 0.9.

3.3   The MaxMin Expected utility model

The MaxMin Expected utility model proposed by Cohen (1992) permits to take into account the security and potential factors identified by Loomes (1986) as the sources of many of the observed violations of the Expected utility model. In this model, a decision (characterized by random variable) is evaluated as a weighted sum of its expected utility and the best and worst possible outcomes. Individual preferences are then characterized by a standard utility function and two parameters α and β corresponding to the weight associated to the worst and to the best outcomes. For a decision $X \in \mathscr{X}$ this gives the following value function:

$$V_{MMEu}(X) = (1 - \alpha - \beta)Eu(X) + \alpha \min_{s \in S} u(X(s)) + \beta \max_{s \in S} u(X(s)) \qquad (3)$$
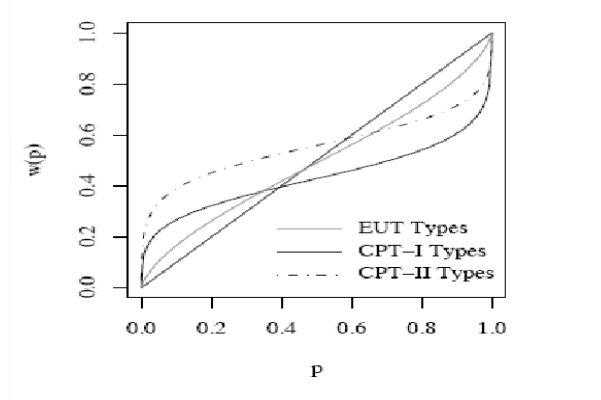
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Figure 3: Probability transformation functions in Bruhin et alii (2010)

with $0 \leq \alpha + \beta \leq 1$.

For a simple two outcomes lottery $Z = (z_{\inf}, q; z_{\sup}, 1 - q)$ with $z_{\inf} < z_{\sup}$,

$$V_{MMEu}(Z) = [(1 - \alpha - \beta)q + \alpha] \, u(z_{\inf}) + [(1 - \alpha - \beta)(1 - q) + \beta] \, u(z_{\sup}) \qquad (4)$$

The associated (linear) probability transformation function $((1 - \alpha - \beta)p + \beta))$ is represented in Figure 4.

In the following, we characterize comparative and absolute fatalism in this model, first for linear utility functions, and then for concave ones.

3.3.1 The MaxMin Expected gain model: $u(x) = x$

Proposition 6. *Consider two individuals 1 and 2 with $MaxMinEu$ preferences and linear utility function. Individual 1 is more fatalist than individual 2 if and only if $\alpha_1 + \beta_1 > \alpha_2 + \beta_2$.*
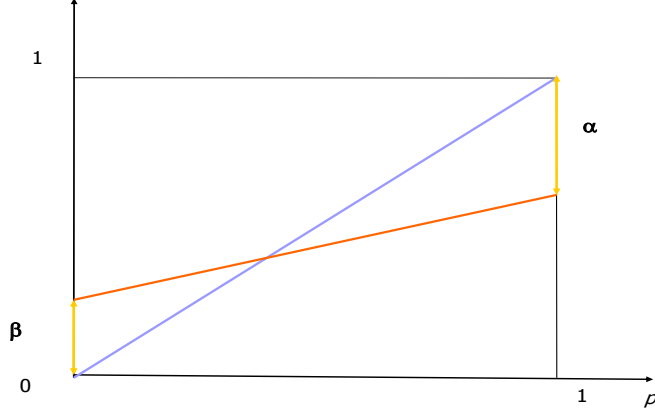
Proof

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Figure 4: Probability transformation in the MaxMinEu Model

Consider lotteries X and Y of Definition 1:

$V_{MMEu}(X) = (1 - \alpha - \beta)[px_{\inf} + (1 - p)x_{\sup}] + \alpha x_{\inf} + \beta x_{\sup}$

$V_{MMEu}(Y) = (1 - \alpha - \beta)[(p - \varepsilon)(x_{\inf} - a) + (1 - p + \varepsilon)(x_{\sup} - a)] + \alpha(x_{\inf} - a) + \beta(x_{\sup} - a)$

For an individual with $MMEu$ preferences $X \succsim Y \Leftrightarrow a \geq (1 - \alpha - \beta)\varepsilon(x_{\sup} - x_{\inf})$.

From Definition 1, an individual 1 if more fatalist than 2 if and only if $X \succsim^2 Y \Rightarrow X \succsim^1 Y$ or equivalently, if and only if:

$a \geq (1 - \alpha_2 - \beta_2)\varepsilon(x_{\sup} - x_{\inf}) \Rightarrow a \geq (1 - \alpha_1 - \beta_1)\varepsilon(x_{\sup} - x_{\inf})$ which is true whenever $(1 - \alpha_2 - \beta_2) \geq (1 - \alpha_1 - \beta_1)$.

The independence axiom is verified in the MMEu model if individuals evaluate decisions according to their expected utility, that is, for $\alpha = \beta = 0$. Consequently, realists in this model are individuals who do not take into account explicitly the maximal and minimal outcomes of decisions for their evaluation. These individuals do not transform probabilities.

Let us now characterize "absolute" fatalism in this model.

Proposition 7. *An individual with MaxMinEu preferences is fatalist if and only if* $\alpha + \beta > 0$.

Proof. Follows directly from the previous proposition and from the fact that a realist is characterized by $\alpha + \beta = 0$. $\qquad\square$

The previous results show that in the MaxMinEu model fatalism depends on parameters α and β. More precisely, the higher is the sum of these parameters, the higher is the fatalism. This is justified by the fact that the sum of these parameters determines i) the departure from expected utility, which corresponds to realism, and ii) the slope of the probability transformation function in this model. The higher $\alpha + \beta$ is, the lower is the slope of the probability transformation line which corresponds to an underreaction to probability modifications. Note that pessimism and optimism, measured here by α and β do not influence fatalism, and an individual who considers only the worst outcome of a decision ($\alpha = 1$) will be fatalist, as well as an individual who considers only the best one ($\beta = 1$).

In the following we consider the case of concave utility function.

3.3.2 The general MaxMin Expected utility model: $u''(x) < 0$

Proposition 8. *Consider two individuals 1 and 2 with MaxMinEu preferences such that $u_1 = u_2 = u$ with $u'' < 0$. Individual 1 is more fatalist than individual 2 if $\beta_1 \leq \beta_2$ and $\alpha_1 + \beta_1 \geq \alpha_2 + \beta_2$.*

Proof. Consider lotteries X and Y of Definition 1.

For an individual with $MaxMinEu$ preferences and $u'' < 0$, $X \succsim Y \Leftrightarrow$

$\Delta V(\alpha, \beta) = [(1 - \alpha - \beta)p + \alpha] A + [(1 - \alpha - \beta)(1 - p) + \beta] B - (1 - \alpha - \beta)\varepsilon C \geq 0$
with $A = u(x_{\text{inf}}) - u(x_{\text{inf}} - a)$, $B = u(x_{\text{sup}}) - u(x_{\text{sup}} - a)$, $C = u(x_{\text{sup}} - a) - u(x_{\text{inf}} - a)$.

An individual 1 is then more fatalist than 2 if and only if:

$$\Delta V_1(\alpha, \beta) - \Delta V_2(\alpha, \beta) \geq 0$$

or equivalently:

$$[(\alpha_1 - \alpha_2)(1 - p) - p(\beta_1 - \beta_2)] (A - B) + (\alpha_1 - \alpha_2 + \beta_1 - \beta_2)\varepsilon C \geq 0 \qquad (5)$$

$A > B$ from the concavity of u and thus the previous expression is positive for $\beta_1 \leq \beta_2$ and $\alpha_1 + \beta_1 \geq \alpha_2 + \beta_2$. $\qquad \square$

Corollary 1. *Consider two individuals with MaxMinEu preferences and such that $u_1 = u_2 = u$ with $u'' < 0$ and $\beta_1 = \beta_2$. Individual 1 is more fatalist than individual 2 if and only if $\alpha_1 \geq \alpha_2$.*

Proof. If $\beta_1 = \beta_2$, condition (5) writes $(\alpha_1 - \alpha_2)\left[(A - B) + \varepsilon C\right] \geq 0$ where $A > B$ from the concavity of u and thus is satisfied if and only if $\alpha_1 \geq \alpha_2$. □

As in the case of linear utility function, realism is characterized here by $\alpha = \beta = 0$.

Proposition 9. *An individual with $MaxMinEu$ preferences is fatalist if $\beta = 0$ and $\alpha > 0$.*

Proof. A direct consequence of Proposition 8 □

When the utility function is concave, the characterization of fatalism depends not only on the sum $\alpha + \beta$ but also on the value of the parameter β that should not be too high. The intuition is the same as for fatalism in RDU model with concave utility function. Recall that the choice between X and Y results from the individuals assessment of a trade off between a "gain" in terms of probabilities and a "loss" in terms of outcomes. In the MaxMinEu model, the evaluation of the "gain" in terms of probabilities depends on $\alpha + \beta$ and not on the specific values of α and β. Concerning the evaluation of the "loss" in terms of outcomes, the impact of α and β are different and more precisely, this loss is considered as low for individuals with high β. Consequently, a sufficient condition for fatalism (preference for X) is a high value for $\alpha + \beta$ and a low value for β.

4 The impact of fatalism on self-protection decisions

In this section, we determine the impact of fatalism on optimal self protection decisions.

We consider an agent endowed with wealth w who faces a risk of loss L. The probability of loss $p(e)$ is a decreasing function of the effort e that is expressed in monetary equivalent. We assume that the characteristics of the loss and of prevention technology are such that the optimal level of effort is strictly positive. The following propositions determine the impact of fatalism on optimal prevention levels for individuals with RDU preferences and MaxMinEu preferences.

Proposition 10. *Consider two individuals with preferences represented by the RDU model who differ only by their probability transformation functions, $u_1(x) = u_2(x) = u(x)$. Their utility function $u(x)$ is assumed to be strictly concave. If individual 1 is more fatalist than individual 2, $\varphi_1(p) \leq \varphi_2(p)$ and $\varphi'_1(p) \leq \varphi'_2(p)$ for any $p \in]0,1[$, then the optimal level of prevention of individual 2 is higher than that of individual 1, $e_1^* < e_2^*$.*

Proof. For an individual with RDU preferences, utility function u and probability transformation function φ, .the optimal level of effort is solution of the following optimization problem:

$$Max_e \ V_{RDU}(e) \tag{6}$$

where

$$V_{RDU}(e) = u(w - L - e) + \varphi(1 - p(e))[u(w - e) - u(w - L - e)]$$

The first order condition of this problem writes:

$$-p'(e)\varphi'(1-p(e))[u(w-e) - u(w-L-e)] - [1 - \varphi(1-p(e))]u'(w-L-e) - \varphi(1-p(e))u'(w-e) = 0 \tag{7}$$

We assumed at the beginning of the section that the second order condition for an interior solution $(e > 0)$ is verified.

$e_1^* < e_2^* \Leftrightarrow \left. \dfrac{dV_{RDU}^1(e)}{de} \right|_{e=e_2^*} < 0$ which is equivalent to the condition $\left. \dfrac{dV_{RDU}^1(e)}{de} \right|_{e=e_2^*} <$

$\left. \dfrac{dV_{RDU}^2(e)}{de} \right|_{e=e_2^*}$.

From (7), $\left. \dfrac{dV_{RDU}^1(e)}{de} \right|_{e=e_2^*} < \left. \dfrac{dV_{RDU}^2(e)}{de} \right|_{e=e_2^*} \Leftrightarrow$

$$-p'(e_2^*)[\varphi'_1(1 - p(e_2^*)) - \varphi'_2(1 - p(e_2^*))][u(w - e_2^*) - u(w - L - e_2^*)]$$
$$+ [\varphi_2(1 - p(e_2^*)) - \varphi_1(1 - p(e_2^*))][u'(w - e_2^*) - u'(w - L - e_2^*)] < 0$$

From $u'' < 0$ and $p' < 0$ it follows that the previous inequality is true for $\varphi'_1 < \varphi'_2$ and $\varphi_1 < \varphi_2$. $\qquad \square$

    This proposition shows that, contrarily to risk aversion, which impact on self protection is ambiguous, the impact of fatalism is well determined: an increase in fatalism leads to a lower level of self protection. This result is obtained under the assumption

of RDU preferences.

The following proposition addresses the same question for MaxMinEu preferences and allows to obtain similar results.

Proposition 11. *Consider two individuals with preferences represented by the MaxMinEu model who differ only by their parameters α and β, $u_1(x) = u_2(x) = u(x)$. Their utility function $u(x)$ is assumed to be strictly concave. If individual 1 is more fatalist than individual 2, $\beta_1 \leq \beta_2$ and $\alpha_1 + \beta_1 \geq \alpha_2 + \beta_2$, then the optimal level of prevention of individual 2 is higher than that of individual 1, $e_1^* < e_2^*$.*

Proof. For an individual with MaxMinEu preferences, utility function u and parameters α and β, the optimal level of effort is solution of the following optimization problem:

$$Max_e \; V_{MMEu}(e) \tag{8}$$

where

$$V_{MMEu}(e) = [(1 - \alpha - \beta)p(e) + \alpha]\, u(w - L - e) + [(1 - \alpha - \beta)(1 - p(e)) + \beta]\, u(w - e)$$

The first order condition of this problem writes:

$$
\begin{aligned}
&-p'(e)(1 - \alpha - \beta)\left[u(w - e) - u(w - L - e)\right] \\
&- [(1 - \alpha - \beta)p(e) + \alpha]\, u'(w - L - e) - [(1 - \alpha - \beta)(1 - p(e)) + \beta]\, u'(w - e) = 0
\end{aligned}
\tag{9}
$$

We assumed at the beginning of the section that the second order condition for an interior solution $(e > 0)$ is verified.

$e_1^* < e_2^* \Leftrightarrow \left. \left| \dfrac{dV_{MMEu}^1(e)}{de} \right| \right|_{e=e_2^*} < 0$ which is equivalent to the condition $\left. \left| \dfrac{dV_{MMEu}^1(e)}{de} \right| \right|_{e=e_2^*} <$

$\left. \left| \dfrac{dV_{MMEu}^2(e)}{de} \right| \right|_{e=e_2^*}$.

From (7), $\left. \left| \dfrac{dV_{MMEu}^1(e)}{de} \right| \right|_{e=e_2^*} < \left. \left| \dfrac{dV_{MMEu}^2(e)}{de} \right| \right|_{e=e_2^*} \Leftrightarrow$

$$
\begin{aligned}
&-p'(e_2^*)\left[\alpha_2 + \beta_2 - \alpha_1 - \beta_1\right]\left[u(w - e_2^*) - u(w - L - e_2^*)\right] \\
&+ \left[(\beta_2 - \beta_1)\,p(e_2^*) - (\alpha_2 - \alpha_1)\,(1 - p(e_2^*))\right]\left[u'(w - e_2^*) - u'(w - L - e_2^*)\right] < 0
\end{aligned}
$$

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From $u'' < 0$ and $p' < 0$ it follows that the previous inequality is true for $\beta_1 \leq \beta_2$ and $\alpha_1 + \beta_1 \geq \alpha_2 + \beta_2$. □

The previous results suggest that fatalism is an important risk attitude characterization for understanding self protection decisions. Identifying fatalists in the population by objective characteristics can allow for a better targeting of public prevention policies. More precisely, special incentive mechanisms have to be designed for people who underreact to probability modifications and by the same way, underestimate prevention efficiency.

5 Concluding Remarks

In this paper, we proposed a behavioral characterization of sensitivity to probability modifications and showed that this sensitivity can reveal useful for better understanding individual prevention decisions. Enforcement of incentive measures to promote prevention has become an important concern for public authorities as well as for insurance compagnies in many developped countries. The underestimation of probability modifications allows to explain low levels of prevention. Our results can be used to better target public prevention policies. One perspective for future research is the design of optimal prevention policies in an economy where individuals have different risk perceptions.

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