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Abstract. The Ricardian dynamics study the evolution of distribution when demand increases. The successive marginal methods in agriculture are determined by means of a rule which has never been stated explicitly by Ricardo's commentators. But the rule also allows us to point at two limits of Ricardo's construction, first in his attempt to get rid of rent in the analysis of distribution, second in the working of the dynamics themselves. Similarly, the identification of a productivity and a profitability criterion is at the basis of Sraffa's mistake in his reconstruction of the theory of rent, whereas post-Sraffian formalizations have abandoned Ricardo's dynamic approach.

Keywords. Rent, Ricardian dynamics, Ricardo, Sraffa

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The Ricardian dynamics are among the most classical pieces of economic analysis: Ricardo (1817) combined parsimonious assumptions with a deep analysis of the extension of cultivation, from which he drew dramatic conclusions on the evolution of distribution between rents and profits and a plea for free trade. Consider three classes in society: the workers, who earn wages, the landowners, who rent lands to farmers, and the capitalists, who invest capital in industry (or, in the case of farmers, in agriculture), pay the workers and receive profits on their advances. It is assumed that transfers of capital from less to more profitable sectors lead to the uniformity of the rates of profits across sectors. Ricardo examined the impact of the long-run dynamics of capitalism on distribution. The real wage may fluctuate in the short run depending on economic conjuncture, but in the long run the Malthus law exerts a strong pressure against a significant increase above a socially defined minimum subsistence level: we may assume that the long-run real wage is constant and represented by a basket of commodities, with bread as its main component. Therefore, the development of industry and the employment of more workers require the production of more and more wheat and, in a closed economy, poorer and poorer lands are cultivated. In contrast with the extension of cultivation on a given quality of land, which has no incidence on values, the cultivation of a land of a lower grade requires a rise in the price of corn, that must be sufficient to make the investment on the rather bad quality of land as profitable as in any other sector of the economy. But, then, the owners of the more fertile lands, on which the production costs are lower, are in a position to demand a rent of their farmers: the level of the rent compensates for the gap between the production costs on the marginal land, the last to be cultivated, and those on the intramarginal lands of better qualities which were cultivated before.

As the marginal methods pay no rent, the prices are determined by the conditions of production in industry and on the marginal lands. That ‘separation property’ allows Ricardo to discard rent from the analysis of value, as if there were no scarce resources. Then the trade-off property between wages and profits, a basic law of distribution established in the absence of lands, also holds in their presence. In the long run, the rents surge because of the increasing gap between the worst and the best lands. Constant wages and increasing rents lead to a squeeze of profits: the uniform rate of profits falls, there is no more incentive to invest and the economy

reaches a steady state. An alternative to the cultivation of new lands may be to let more capitalistic, also more productive, agricultural methods operate on the best lands. Taking that possibility into account does not change the conclusions, however, as the principle according to which "the capital last employed pays no rent" (*Principles*, Chapter II) applies to intensive as well as to extensive cultivation.

It is often admitted that, under the retained hypotheses, which ignore technical progress, Ricardo's description of the dynamics is correct and general. Our aim is to study their analytical grounds in more details. Apart Ricardo (1817), we shall also refer to Sraffa (1960) who developed an explicit formalization of prices and rents, and to post-Sraffian economists who discovered a certain number of unexpected phenomena in rent theory. Useful as they may be, most of these modern studies miss the central point of Ricardo's construction, viz. the dynamics themselves, and do not examine the law of succession of marginal methods when demand increases. We thus start from Ricardo and explain the nature of that law, which is governed by the ‘minimum rule’ (Section 1). Ricardo did not see, however, some logical consequences of the rule. We examine two of them: first, the working of the dynamics requires a condition to guarantee that the physical and the value sides of an equilibrium both pull in the same direction (Section 2). Second, from a methodological point of view, the validity of Ricardo's attempt to get rid of rent in the analysis of value and distribution is dubious (Section 3).

The analysis holds independently of the distribution variable chosen as exogenous, be it the real wage (Ricardo) or the uniform rate of profits (Sraffa). For reasons which will be explained, calculations are much simpler under the given rate of profits hypothesis. Even if this departure from Ricardo's own hypothesis is unfortunate from a historical perspective, we first retain that assumption, then check that the phenomena we point at hold under the alternative hypothesis (Section 4). Next, we apply the same device to Sraffa's approach and put into light the source of Sraffa's mistake in his analysis of production with land, then conclude with a brief survey of post-Sraffian studies on rent (Section 5).

The main points are illustrated by means of numerical examples in one-commodity and, in the Appendix, a two-commodity model. Clearly enough, the apparent paradoxes with regard to Ricardo's construction which occur in simple models also hold in general multisector models.

1. THE MINIMUM RULE

When the present marginal land becomes fully cultivated, cultivation can be extended on a new land provided that the price of corn increases sufficiently to justify that investment. Which new land, and by means of which new method? For Ricardo, a given grade of land is characterized by its physical qualities, which are described by the amounts of physical inputs, land and labor needed for the production of corn. Ricardo classified the lands 1, 2, 3, etc., according to their grade. The order of cultivation is defined *a priori*, but that classification relies on the costs of production on the various lands, normal profits included. Ricardo's note attached to the last word of Chapter II shows the way to calculate the price of corn when its cultivation is extended to a land of lower quality. The price of corn when land 2 is marginal is higher than when land 1 is marginal, and it is clear from Ricardo's example that cultivation has been extended to land 2 rather than to land 3 because the price of corn is smaller on land 2. In other words, when land 1 has become fully cultivated, the price of corn rises up to the minimum level which allows for the extension of cultivation on another land. Less immediate and more instructive is the choice between an extension of cultivation on another land or its intensification on the same land, because the issue is uncertain. Ricardo gave the following hint: "It often, and, indeed, commonly happens, that before No. 2, 3, 4, or 5, or the inferior lands are cultivated, capital can be employed more productively on those lands which are already in cultivation" (*Principles*, Chapter II). ‘Before’ has a chronological, also a logical meaning: the cultivation of other lands would certainly become profitable for a sufficient rise in the price of corn but, in a given situation, the rise stops at the first value making some new agricultural method profitable, given the adjustment in rent. Indeed, if the rise went beyond that minimum level, the profitability of the corresponding method would continue to increase and would exceed the ruling rate of profits. That is what we dub the minimum rule.

A numerical example in a one-commodity model illustrates the rule and the dynamics. Let corn be initially cultivated on land 1 by means of method (1):

$$\frac{5}{11}\text{ qr. corn} + 1\text{ d. labor} + 1\text{ acre land}_1 \rightarrow 1\text{ qr. corn} \qquad (1)$$

(the physical units called ‘quarter’, ‘day’ and ‘acre’ are conventional). Assume that the given uniform rate of profits is r = 10% (that rate will be maintained in all examples). The rent is zero as long as land is not fully cultivated and, for a wage paid *post factum* (Sraffa's hypothesis) and set equal to one, the price p of corn is equal to 2. Before considering any alternative method, we know that, when land 1 will be fully cultivated, the price of corn will rise from 2 to 2 + h (h > 0) and that the landowner will get a positive rent per acre $\rho = \rho(h)$: "A rent is paid because corn is high" (*Principles*, Chapter II). The price-and-rent equation associated with method (1)

$$(1 + 10\%)\frac{5}{11}(2 + h) + 1 + \rho = 2 + h$$

shows that $\rho = 0.5h$. Suppose now that the increasing demand can be met either by operating a more intensive method of cultivation (2) on land 1 (more inputs and more production per acre, and a positive rent):

$$\frac{6}{11}\text{ qr. corn} + 3\text{ d. labor} + 1\text{ acre land}_1 \rightarrow 1.6\text{ qr. corn} \qquad (2)$$

or by extending the cultivation to land 2 (more inputs and same production par acre, but a zero rent) by method (3):

$$\frac{8}{11}\text{ qr. corn} + 1\text{ d. labor} + 1\text{ acre land}_2 \rightarrow 1\text{ qr. corn} \qquad (3)$$

These methods are not profitable at the initial price p = 2 but their profitability improves with the price of corn, therefore with the level of h. The level of h_2 of h for which method (2) becomes profitable is the solution of

$$(1 + 10\%)\frac{6}{11}(2 + h) + 3 + 0.5h = 2 + h$$

hence h_2 = 2. A similar calculation shows that the extensive method becomes profitable at h_3 = 3. Therefore, the price of corn rises from p = 2 to p' = 2 + h_2 = 4 and the intensive method on land 1 is operated before an extension to land 2.

In Chapter II of the *Principles*, Ricardo adopts the farmers' point of view and sets forth the minimum rule in terms of price as a maximum rule in terms of product: a given amount of capital is invested where the product is maximum. Whatever its formulation, the rule is a specific application of the general law of the one and minimum price induced by competition. This does not explain why the interactions of the rule and the dynamics are hardly mentioned in the economic literature on Ricardo and why their consequences have been ignored.

2. FAILURE OF THE DYNAMICS

A long-term equilibrium satisfies two types of constraints: on the quantity side, the operated methods must meet the given level of final demand; on the value side, they must yield the ruling rate of profits. The basic remark for the study of the dynamics is: starting from a given equilibrium, its limit is determined by the quantity side, whereas the reconstruction of a new equilibrium as defined by the minimum rule starts from the value side. Ricardo failed to notice some dramatic effects of that double determination.

Let the demand for corn increase and suppose that the present equilibrium reaches its physical limit. If the incoming method consists in extending the cultivation of corn on a new land, the productive system does meet the evolution of demand. Otherwise, the issue is unclear. To illustrate the point, let us return to the above corn model with methods (1)-(2)-(3) and introduce another method of cultivation on land 1:

$$0.1 \text{ qr. corn} + 1.03 \text{ d. labor} + 1 \text{ acre land}_1 \rightarrow 0.62 \text{ qr. corn} \qquad (4)$$

The same calculations as above show that method (4) becomes profitable at $h_4 = 1$, before methods (2) or (3): the price of corn cannot rise further, otherwise method (4) would pay extra profits. But the net productivity of that method (0.52 qr. per acre) is smaller than that of method (1) it replaces progressively (6/11 qr., i.e. about 0.54 qr. per acre): the introduction of the new method reduces the net product and does not help to meet a higher demand. The reason of the phenomenon is clear: the choice of the incoming method is based on its profitability, not on its productivity, and a contradiction here appears between the value and the quantity side of the equilibrium problem. In that case, the Ricardian dynamics fail.

Two comments are in order. First, it might be shown that the dynamics always work when the rate of profits is zero, because of the duality property which then holds between the quantity side and the value side of the model (golden rule). Second, a demand corresponding to an intermediate productivity, say 0.53 quarters per acre, can be met by two distinct equilibria: one consists of method (1) used alone on 53/54th of land 1, the other of the joint use of methods (1) and (4), each on half on the soil. Each of these equilibria is sustained by a specific price-and-rent vector ($p_c = 2$, $\rho = 0$ for the first equilibrium, $p_c = 2 + h_4 = 3$, $\rho = 0.5h_4 = 0.5$ for the second): the failure of the dynamics at some level of demand gives birth to a multiplicity of equilibria at a slightly smaller level.

3. FAILURE OF THE METHOD

We now assume that the dynamics work, so that the critique developed here is independent of that of Section 2.

The minimum rule also applies to multisector models, but its above statement is incomplete. In Chapter II of the *Principles* Ricardo stressed the effects of a rise in the price of corn on rents, but he was also aware that this rise also triggers other price changes. The phenomenon is analyzed at length in the chapters of the *Principles* devoted to the effects of taxation ("By raising the price of raw produce, the prices of all commodities into which raw produce enters would be raised", Chapter IX). Such rises, with changes in relative prices, are necessary to maintain the uniformity of the rates of profits among the operated methods. We shall refer to them as the 'compensating variations' in prices and rents. Their calculation obeys the same principles as for the change in rents (an example is given in the Appendix) and, following an increase h in the price of corn, leads to the determination of an increasing price $p_i = p_i(h)$ for commodity i alongside with increasing rent(s) $\rho = \rho(h)$. The exact statement of the minimum rule is that the new operated method is the first which becomes profitable when the price of corn increases, the compensating variations for rents and prices being taken into account.

What are the consequences of an increase of the price of corn on the profitability of an industrial method, say in a corn-iron model in which each commodity enters into the production of the other? By construction, the compensating variations are such that the presently operated iron method maintains its rate of profits. As for a non-operated iron method, two effects work in opposite directions: the cost of production increases, the value of the product too, therefore the evolution of the rate of profits is unclear. But suppose that the new iron method I_1 we consider is corn saving with regard to the presently operated method I_0. As the weight of corn in the costs of I_1 is smaller than for method I_0, the second effect dominates and the profitability of the iron method I_1 increases with h. Depending on numerical data, method I_1 may reach the normal rate of profits before any agricultural method and is the incoming method. Then the new long-term equilibrium is characterized by the coexistence of two iron methods and the dynamics see the progressive substitution of the corn-saving method I_1 for I_0, therefore the economy

does produce more corn. The phenomenon at stake is similar to that of intensive cultivation, but the substitution of methods here takes place in industry instead of agriculture.

It is an essential feature of Ricardo's approach to get rid of rent by considering the conditions of production of marginal capital, which pays no rent. It is that separation property which explains the architecture of the *Principles*: Chapter I ("On Value") is concerned by the determination of values and prices of the reproducible commodities. The main exception does not concern "wines of a peculiar quality, which can be made only from grapes on a particular soil", but land, which is not produced by men. In Chapter II ("On Rent"), Ricardo reduces the study a productive system with land to another without land, made of the industrial methods and the marginal agricultural methods. The properties of single-product systems (which, more than one century later, will be proved formally by means of the Perron-Frobenius theorem) then apply. Wages and profits are determined in a simple productive system, as if there was no scarce resource. In a second time, rents are reintroduced and determined as differential costs.

Is the separation property universal? The long-term equilibrium we consider admits two operated iron methods whereas land is fully cultivated by means of a unique agricultural method. A similar two-step procedure applies: the prices of corn and iron are first determined, then the level of rent is obtained by comparing the production costs and the price of corn. However, in the first step of the procedure, the price equations are those associated with two methods producing the same good, so that the standard properties of single-product systems do not hold. In particular, the trade-off property between wages and profits cannot be expected, as illustrated in the Appendix. We dubbed ‘non-Ricardian' an equilibrium of this type, with no marginal agricultural method. Ricardo's analytical strategy fails in the unavoidable presence of non-Ricardian equilibria.

The distinction between Ricardian and non-Ricardian equilibria can be defined precisely. Let an agricultural method be called marginal if it is operated only on a part of a given quality of soil. That soil itself may be cultivated either partly (extensive cultivation) or totally (intensive cultivation), in which case at least two methods are used jointly. Then the land itself is called marginal. By contrast, the intramarginal

lands are totally cultivated by means of a unique method. For any type of equilibrium, be it Ricardian or non-Ricardian, the equality

card(operated methods) = *card*(commodities) + *card*(fully cultivated lands)

(*card* is for cardinal and indicates the number of elements of a set) holds flukes apart. That equality explains why, for a given rate of profits, the prices and rents are uniquely determined when the operated methods are known. For a Ricardian equilibrium, the equality splits into two:

card(marginal methods) = *card*(agricultural goods) + *card*(marginal lands)

card(operated industrial methods) = *card*(industrial goods)

Are non-Ricardian equilibria exceptional? From a logical point of view, one example suffices to criticize Ricardo's methodology. From a formal point of view, the above splitting becomes unlikely when the numbers of goods and lands are great: there are strong reasons to think that non-Ricardian equilibria are more frequent than Ricardian equilibria in multisector complex economies. With multiple agricultural goods, Ricardian equilibria are even more improbable.

4. GIVEN REAL WAGE

Up to now, it has been assumed that the rate of profits is the independent distribution variable, but the conclusions hold under Ricardo's given real wage hypothesis. For a wage defined in physical terms as a basket (δ_1,..., δ_n) of commodities, the additional equality $\delta_1 p_1$+...+ $\delta_n p_n$ = 1 is introduced and the rate of profits becomes an endogenous magnitude. From a theoretical point of view, the compensating variations can be determined as well. One can first replace each unit of labor by the wage basket, and then labor disappears from the model with augmented input coefficients. The factor of profits is the inverse of dominant root of the extended input matrix and the initial price vector is the associated Perron-Frobenius vector. When corn becomes scarce, its price increases by some scalar h and the compensating variations concern rents and the other prices. This leads to relationships of the type $r = r(h)$, $\rho = \rho(h)$ and $p = p(h)$. For h increasing up from zero, the incoming method is the first among those which are not initially operated whose profitability at prices $p = p(h)$ and rents $\rho(h)$ becomes equal to $r(h)$. In multisector models, explicit calculations are rapidly overwhelming because they deal with Perron-Frobenius characteristics depending on a parameter. However, as far as the objective is only to exhibit either

the failure of the dynamics or non-Ricardian equilibria, numerical examples are easily built.

It turns out that the main technical advantage of Sraffa's hypothesis is that the price equations are linear when the rate of profits is given, with simple formulas for the compensating variations. By contrast, the same system is nonlinear when the unknowns are the prices and the rate of profits. The gap does not affect the economic conclusions.

5. MODERN ANALYSES

The modern revival of Ricardian studies is due to Sraffa (*Production*, 1960). Following Ricardo's approach, Sraffa first introduced a general formalization of prices of production and studied their properties (Chapters I to VI of *Production*), then considered lands and rents (Chapter XI) as part of a general theory of multiple-product industries. It is worth noting that, in spite of the explicit warning of the preface that "no changes in output" are ever considered, Sraffa followed Ricardo's dynamic approach when he studied the effects of "a progressive increase of production" (Section 88). He rejected Ricardo's idea of fertility as an intrinsic quality of lands determining a natural order of cultivation, because the decision to cultivate some land rather than another depends on relative costs, which are sensitive to the distribution of income. Sraffa studied the cases of extensive and intensive cultivation proper and was optimistic as to the possibility to extend the results to several agricultural goods (Section 89).

In his analysis of intensive cultivation, Sraffa specified that both methods are operating side by side on the same land. What are the economic conditions sustaining that coexistence? The price of corn is unique and both methods must be equally profitable. Let the cost per quarter produced be defined as including the cost of material inputs, wages and profits, but excluding rent. Then "the method that produces more corn per acre [shows] a higher cost" (Section 87) and "a progressive increase of production on the land [...] takes place through the gradual extension of the method that produces more corn at a higher unit cost, at the expense of the method that produces less" (Section 88). But the conjunction of both statements does not provide sufficient grounds to the Ricardian dynamics, since the cost order between methods depends on prices and is not an invariant: in the corn model

studied in Section 2, method (2) is more costly when method (1) operates alone, but less costly when they operate simultaneously. Sraffa, who was aware of the pitfalls of capital theory, seems to have forgotten them at that point: more capital does not mean more product. The only surprise, if any, is that the problem occurs even in a simple corn model with land.

The post-Sraffian studies on rent started with Quadrio-Curzio (1966). They were initially devoted to the analysis of simple cases, with a unique agricultural product: Montani (1975) showed that, for a given rate of profits and extensive cultivation proper, the order of cultivation coincides with that of the decreasing wages paid by the agricultural methods for a zero rent, a property in line with Ricardo's dynamic approach. (Note that the definition of extensive cultivation retained in post-Sraffian studies includes the unexpected hypothesis that the industrial methods are given. The hidden reason of that restriction is that non-Ricardian equilibria are thus discarded.)

In contrast with Ricardo, Sraffa did not refer explicitly to demand (he mentioned the short supply of lands as the cause of a change of methods) but introduced the notion of ‘requirements for use'. A given demand vector was later explicitly introduced, first in the formalization of the notion of equilibrium for multiple-product systems (Steedman, 1976; Schefold, 1978), then to deal with the scarcity constraints on lands. Either under the influence of Sraffa's preface or of that formalization, Ricardo's dynamic approach was abandoned and most post-Sraffian papers on rent dealt with the solutions of a system of equations (or, equivalently, of inequalities with complementarity relationships) for a given demand basket. These equations do include a cost-minimization property, but the minimum rule and Ricardo's idea that a small increase in demand is dealt with by introducing one new marginal method are ignored. A partial exception is Saucier (1981), who explained why the net product of corn may increase thanks to a substitution of method in industry. He dubbed that phenomenon, which we call non-Ricardian equilibrium, ‘external differential rent'. Beyond the post-Sraffian stream of thought, the same static approach, characterized by a symmetric treatment of the quantity side and the value side, is adopted by all studies of Ricardo’s works, even those which, like Morishima’s (1989), are analytically oriented.

Post-Sraffian contributions led to a partial rejection of Sraffa's optimistic conclusions and put forward a number of unexpected phenomena and difficulties. Salvadori (1986) established a general existence result for a given demand vector, but D'Agata (1982; see also Freni, 1991) provided examples of multiple equilibria. Starting from geometrical considerations, Erreygers (1990, 1995) elaborated an algebraic criterion for local uniqueness which compares the sign of two determinants. As noticed as the end of Section 2, the failure of the dynamics gives rise to a local multiplicity of equilibria, and therefore another interpretation of the Erreygers criterion is that it ensures the local working of the Ricardian dynamics. In a study of intensive rent proper for a given real wage, Bidard (2010) returned to the dynamic approach by reintroducing the idea that the incoming method is commanded by the value side. Bidard (2012) provides an analytical study of the dynamics and draws a close parallel between Ricardo's method and the Lemke (1965) parametric algorithm used by mathematicians to solve linear complementarity problems.

A return to Ricardo's original approach does enrich the results drawn from a static analysis: if, for a given level of demand, multiple equilibria are found, the ultimate reason is that the dynamics fail somewhere on a path which leads from low levels of demand to the present level. Similarly, an unexpected distribution of income for a given demand is due the presence of non-Ricardian equilibria on that trajectory.

6. CONCLUSION

Ricardo's message can be interpreted in a weak sense: the price of a resource goes to infinity if it becomes scarcer and scarcer and admits no substitute, with significant effects on distribution. The Ricardian dynamics are more precise and ambitious, and Ricardo's views on the problem explain the structure of the *Principles*: Chapter I deals with the value of reproducible commodities and allows him to establish the basic trade-off property between wages and profits in the absence of lands. Chapter II explains why rents are inessential (only from an analytical point of view!) and can be ignored by considering the conditions of production in industry and the marginal conditions in agriculture. But Ricardo did not draw the consequences of an exclusive determination of the incoming marginal method by a profitability criterion. When they are taken into account, the smooth path of the dynamics may become rather chaotic, with either some steps during which the expected laws of distribution do not apply or

a failure of the dynamics themselves. A century and half later, Sraffa, in spite of some critiques, approved most of Ricardo's conclusions. A return to the dynamics also explains the apparent paradoxes discovered by post-Sraffians in a static framework. That case exemplifies an instance in which contemporary analytical studies would have benefited from a closer reading of the original works.

APPENDIX

The Appendix illustrates the statements of Section 3 in a bisector model. The rate of profits is r = 10%, wages are paid *post factum* and labor is chosen as numeraire.

1. Compensating variations

Let there be a corn-iron model in which the present marginal agricultural method (5) and the operated industrial method (6) are described in physical terms as

$$\frac{3}{11}\text{ qr. corn} + \frac{3}{11}\text{ t. iron} + 0.4\text{ d. labor} + 1\text{ acre land}_1 \rightarrow 1\text{ qr. corn} \qquad (5)$$

$$\frac{4}{11}\text{ qr. corn} + \frac{5}{11}\text{ t. iron} + 0.1\text{ d. labor} \rightarrow 1\text{ t. iron} \qquad (6)$$

As long as land 1 is not fully cultivated, the rent is zero and the prices in terms of wage are $p_c = 1$ for corn and $p_i = 1$ for iron. When land 1 becomes fully cultivated, let the price of corn rise up to $p_c = 1 + h$ for some positive scalar h considered as a parameter. The rent ρ becomes positive and the price of iron rises. The relationships between these values are given by Sraffa's price equations

$$(1 + 10\%)\left(\frac{3}{11}p_c + \frac{3}{11}p_i\right) + 0.4 + \rho = p_c$$

$$(1 + 10\%)\left(\frac{4}{11}p_c + \frac{5}{11}p_i\right) + 0.1 = p_i$$

from which there follows that the ruling rate of profits is maintained for both methods if

$$p_c = 1 + h,\ p_i = 1 + 0.8h,\ \rho = 0.46h \qquad (7)$$

2. Determination of the incoming method

Let a (potential) intensification method (8) on land 1 and a (potential) extension method (9) on land 2 be respectively described as

2 qr. corn + 1 t. iron + 1 d. labor + 1 acre land$_1$ → 4 qr. corn (8)

0.3 qr. corn + 0.3 t. iron + 1 d. labor + 1 acre land$_2$ → 1 qr. corn (9)

These methods yield less than the ruling rate of profits at the prices associated with the initial equilibrium, but their profitability improves with the rise in the price of corn. For prices and rent given by relations (7), the intensive method reaches the normal rate of profits for h_8= 0.65 and the extensive method for h_9= 1.62 > h_8. The minimum rule leads to the introduction of the intensive method.

3. A non-Ricardian equilibrium

Let us moreover introduce another iron method (10):

$\frac{2}{11}$ qr. corn + $\frac{5}{11}$ t. iron + 0.4 d. labor → 1 qr. iron (10)

The input coefficients show that method (10) is corn saving with regard to method (6). For prices given by (7), its profitability improves with h and reaches the ruling rate of profits at h_{10} = 0.5, before methods (8) and (9). The minimum rule leads to the introduction of method (10). The prices are then determined by both iron methods

$$(1+r)\left(\frac{4}{11}p_c+\frac{5}{11}p_i\right)+0.1=p_i$$

$$(1+r)\left(\frac{2}{11}p_c+\frac{5}{11}p_i\right)+0.4=p_i$$

At r = 10%, the solution of these equations are $p_c = 1 + h_{10} = 1.5$ and $p_i = 1 + 0.8h_{10} = 1.4$. But, for a moving rate of profits, it turns out that the price of corn in terms of wage is a decreasing function of r

$$p_c = 1.5\frac{1.1}{1+r}$$

therefore the real wage in terms of corn and the rate of profits vary in the same direction.

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