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# Revisiting the hedonic price method to assess the implicit price of environmental quality with market segmentation

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**Abstract**

The article highlights the role of heterogeneity in the formation of hedonic prices. The article distinguishes between continuous and groupwise heterogeneity. The distinction helps understanding two important points. First, the analysis of market equilibrium with groupwise heterogeneity makes explicit the role of participation and incentives compatibility constraints for groups of buyers. The case of continuous heterogeneity may be thought of as a limit case of groupwise heterogeneity when the number of groups goes to infinity and their masses go to zero. The hedonic price curve is then obtained as the solution of a differential equation resulting from a market clearing condition. Second, the article outlines that submarkets emerge from market equilibrium only in the case of groupwise heterogeneity. The existence of submarkets means that the hedonic price function is continuous but the implicit price of characteristics is discontinuous at endogenous threshold values separating submarkets. Major implications for the valuation of environmental quality follow on. Based on numerical simulations, the article gives some insights into the way significant biases and drawbacks in the estimation of the implicit price of environmental quality can arise if the usual two steps procedure is implemented.

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# 1 Introduction

An extensively used method to assess the value of the environment when facing localized and potentially multiple sources of pollution and/or amenity is the hedonic price method. The key idea of the hedonic price method is that arbitrages made by economic agents in their choice of location indirectly reveal their preferences for local environmental characteristics. The theoretical foundations of the hedonic price concept are due to Rosen (1974) who was the first to elicit how market clearing in the presence of a continuum of differentiated goods on the supply side and a continuum of heterogeneous agents on the demand side leads to a functional relation between the characteristics of goods and their price, namely the hedonic price function. The slope of the hedonic price function with respect to a given characteristic is typically interpreted as the implicit price of this characteristic. Since the theoretical contribution of Rosen (1974), the development of econometric methods has generated an extensive literature of empirical contributions that aim at estimating the implicit price of various pollutions and amenities. For instance Boyle and Kiel (2001) present a comparative analysis of 37 studies dealing with environmental externalities such as air and water pollution. Simons and Saginor (2006) conduct a meta-analysis of 75 environmental contaminations due to leaking underground storage tanks, superfund sites, landfills, power lines power-plants and environmental amenities generated by views, beach access, park and riparian area proximity. The main stream of this

literature focuses on the correct statistical treatment of peculiar features of data used to implement the hedonic price method. The use of spatial econometrics is more specifically illustrative of this trend (see, among others, Pace and Gilley (1997); Anselin (1998); Basu and Thibodeau (1998); Pace et al. (1998); Dubin et al. (1999); Gillen et al. (2001); LeSage and Pace (2004)). Nevertheless, a crucial though generally neglected point that has to be examined in order to correctly implement the method is the definition of the relevant market and the identification of submarkets.

Following the Marshalian tradition, the relevant market is broadly defined as the market that gathers all goods which are sufficiently interdependent in terms of cross price effects.[1] Indeed, for the price difference observed at market equilibrium between two houses to be interpreted *ceteris paribus* as the implicit value of their difference in terms of environmental quality, it should be the case that residents are able to switch from one house to the other one to take advantage of a gap between the price difference and their private valuation of the difference of environmental quality. Most empirical studies assume that administrative boundaries of jurisdictions, and more specifically that of cities, correctly approximate the relevant market. Whether this assumption is statistically validated or not is seldom tested. However, as suggested in this article, a simple test could rely on the continuity of the hedonic price function. The identification of submarkets has received more attention in the literature.

As stressed by Watkins (2001), alternative approaches of submarkets are encountered in the literature. All of them rely on a broad definition of submarkets, pointing out that some source of heterogeneity in terms of the characteristics of goods or in terms of tastes and income of residents may justify that the estimates of all or part of the hedonic price function parameters differ for each element of a partition of the relevant market. The matter is that, according to the microeconomic theory, heterogeneity is consubstantial to any hedonic price approach. We clearly lack theoretical basis for the analysis of submarkets. The motivation of this paper is thus to address two important questions. Firstly, what type of heterogeneity more specifically underlines the existence of submarkets? Secondly, do the existence of submarkets underpines the standard econometric treatment of hedonic prices and the associated environmental valuation method? We argue that groupwise heterogeneity of agents generates the existence of submarkets, not heterogeneity along a continuum. Due to the key role plaid by implicit prices, we highlight important consequences for the valuation of environmental quality. The case of environmental amenities in touristic and recreational areas is more specifically of interest. Indeed, two distinct groups of residents generally coexist in such areas. The first group gathers residents with high income and whom main objective is to benefit from the amenity. The second group gathers residents with lower income and who seek a job and thus settle in the area essentially because of the presence of

the first group of residents. The two groups of residents compete on the same housing market but one may reasonably expect that dwellings with high environmental quality, for instance dwellings close to the waterfront in seaside resorts or with a nice view in mountain resorts, will be bought essentially by residents of the first group while dwellings in the vicinity of the area with high environmental quality but without necessarily benefiting from this quality will essentially be bought by residents of the second group. As a result, the housing market is probably partitioned in two submarkets associated with two groups of residents that differ, among other, in terms of income. A correct assessment of the willingness to pay of the whole population for environmental quality then requires to distinguish the two submarkets when estimating the hedonic price function. From a theoretical and econometric point of view, a striking consequence compared to the usual two steps procedure (see Freeman (2003) or Palmquist (2005)) used to assess the willingness to pay for environmental quality in a hedonic price approach is that the hedonic price function is defined piecewise, is continuous over all the relevant market but is not continuously differentiable. More specifically, the implicit price exibits discontinuities at endogenous thresholds of the environmental variable that correspond to a switch from one submarket to another one. Econometric consequences of these properties for estimating hedonic price functions and using them for environmental valuation are twofold. Firstly, the hedonic price function can not be estimated separatly

submarket by submarket. This results from the high dependency that exists between the different segments of the hedonic price curve associated to submarkets and from the fact that thresholds between these segments are not known *a priori* but are endogeneous. Secondly, disregarding discontinuities in the slope of the hedonic price curve at threshold levels of the environmental variable that separate submarkets may induce important biaises in the estimation of the implicit price of the environmental quality.

The model of hedonic price developped in this paper is complementary to that of Rosen (1974) in the sense that groupwise heterogeneity is considered on the demand side instead of heterogeneity along a continuum. It may be thought of as a dual approach of sorting models developped, among others, by Epple and Platt (1998); Epple and Sieg (1999); Kuminoff (2009). Indeed, sorting models focus on groupwise heterogeneity on the supply side. More specifically, it is assumed in sorting models that a continuum of heterogenous agents has to choose among houses that differ in terms of a public local good that takes discrete levels generally associated to different jurisdictions. By essence, the discrete nature of the local public good is not consistent with the definition of a marginal price. Discrete choice econometric models have to be used in order to estimate sorting models. Their econometric treatment and their use to infer willingness to pay or willingness to receive for a change in the environmental quality substantially depart from that of the standard hedonic price approach.

The model of hedonic price developped in this paper deals with vertical differentiation. Gabszewicz and Thisse (1979) show that vertical differentiation follows on from heterogeneity of agents in terms of their marginal rate of substitution between the quality of the differentiated good and the other goods, which in turn may result from either a difference in their level of income or a difference in terms of tastes. In the hedonic literature the case of vertical differentiation is study, among others, by Ekeland et al. (2004) in a model that follows the traditional approach of Rosen (1974) and by Epple and Sieg (1999) in a sorting model. By contrast, horizontal differentiation is inspired by Lancaster (1966) theory of consumption and follows on from heterogeneity of agents in terms of their marginal rate of substitution between attributes of houses. In the presence of horizontal differentiation, the aggregation of attributes of a same house generates a different index of housing services for at least two consumers. In his seminal paper Rosen (1974) claims to build on the analysis of demand for horizontally differentiated goods as presented in Lancaster (1966) but actually provides details for the computation of the hedonic price function for the case of vertical differentiation. For their part Kuminoff (2006) and Bayer et al. (2005) introduce horizontal differentiation in sorting models, assuming that the differentiation concerns individual preferences about public goods provided by communities. These public goods can be thought as housing attributes such as school quality or air quality. Levels of these goods can also be assumed to be identical

throughout the territory of the community if its size is relatively small. The emphasis on vertical differentiation made in this article primarily responds to pedagogical concerns. Indeed, it eases graphical analysis thanks to the one dimensional differentiation of houses.[2]

The paper proceeds as follows. Section 2 sets the general assumption of the model with a special attention devoted to the Spence-Mirrlees single crossing property of individuals bid curves. Section 3 details the analysis of market equilibrium in the presence of groupwise heterogeneity. It also paves the way for the discussion of practical consequences made on the section 5. Section 4 recalls the case of heterogeneity along a continuum on both the supply side and the demand side of the housing market. It essentially aims at presenting the traditional model of Rosen (1974) in a way that makes it directly comparable with the model characterised by groupwise heterogeneity and presented in the previous section. Section 5 discusses how important are the consequences for environmental valuation. Estimation results based on simulated data are compared to exact results obtained from the theoretical model. It is stressed that estimation biaises and drawbacks may arise and are more specifically detrimental when trying to infer the marginal price of environmental quality. The last section concludes.

## 2 General setting

In this section we present the general assumptions on the demand and supply sides of the housing market, namely assumptions common to the cases of

continuous *versus* discrete heterogeneity of consumers. In the spirit of Gabszewicz and Thisse (1979) we assume that vertical differentiation follows on from the assumption that all individuals have the same preferences about bundles of goods and only differ in their choices because of a difference in their income level.

### *2.1 Demand side*

On the demand side, each consumer $n$ is assumed to allocate her income $R_n$ between the purchase of a house, characterized by an index of housing services level $H$, and the purchase of a Hicksian composite good, $X$, (which serves as the *numeraire*) representing all the other goods. Income is the only source of heterogeneity between consumers. The functionnal form of utility and the values of its parameters are the same for all consumers and, thus, all consumers have the same tastes. The total utility level $U(H, X)$ for consumer $n$ depends on the index $H$ of housing services level and on the quantity $X$ of the Hicksian composite good. The utility function is supposed to be of class $\mathcal{C}^2$, increasing on its arguments and concave.

The individual budget constraint for agent $n$ is

$$X + p(H) = R_n, \tag{1}$$

where $p(H)$ is the price of a house with an index of housing services $H$.[3]

**Definition 1.** The constrained utility function *of an individual $n$ is:*[4]

$$V(H, P, R_n) \equiv U(H, R_n - P).$$

We can then define Rosen's bid function for individual $n$ by using the constrained utility function:

**Definition 2.** Rosen's bid function *for individual $n$, thereafter denoted $E_n$, is implicitly defined by :*

$$V(H, E_n, R_n) = \bar{u}_n, \tag{2}$$

*where $\bar{u}_n = V(\bar{H}, \bar{P}, R_n)$ stands for the level of utility attained by the individual $n$ at her current location with the level of housing services $\bar{H}$ and the price $\bar{P}$.*

Rosen's bid function for individual $n$ indicates the maximal amount of money $E_n$ the individual would be willing to pay for a house with a level $H$ of housing services given the characteristics, the price and thus the utility level attained at her current location (Rosen, 1974).

By the theorem of implicit functions we obtain from (2) that:[5]

$$\frac{\partial E}{\partial H} = \frac{\partial U/\partial H}{\partial U/\partial X}\Big|_{X=R_n-P} > 0, \tag{3}$$

$$\frac{\partial E}{\partial \bar{u}_n} = -\frac{1}{\partial U/\partial X}\Big|_{X=R_n-P} < 0, \tag{4}$$

$$\frac{\partial^2 E}{\partial H^2} = \frac{U_X^2 U_{HH} - 2U_X U_H U_{X\,H} + U_H^2 U_{XX}}{U_X^3}\Big|_{X=R_n-P} < 0, \tag{5}$$

Because $U(\cdot)$ is concave, the bid function $E(\cdot)$ is also concave in $H$. Equations (3) and (5) mean that the bid function is increasing in $H$ but at

a decreasing rate. The slope $\partial E/\partial H$ is tightly linked to the marginal rate of substitution between the index of housing services level $H$ and money (represented by the Hicksian composite good $X$) and may be thought of as the implicit marginal value which consumer $n$ attributes to $H$ given her current utility level and her income.

In space $(H, P)$ the implicit function theorem defines, for each individual $n$, a family of individual bid functions parameterized by the reference utility level $\bar{u}_n \in DU_n$, where $DU_n$ stands for the set of utility levels attainable given the prices that prevail on the housing market and the income of consumer $n$:

$$DU_n = \{u(H, R_n - P) \in \mathbb{R} \ : \ \{P, H\} \in [0, R_n] \times \mathbb{R}_+\} \tag{6}$$

We use the notation $E_n = E(H|R_n, \bar{u}_n)$ to denote the bid function of individual $n$ associated to the reference utility level $\bar{u}_n$.[6]

The corresponding bid curves in space $(H, P)$ are formally identical to iso-constrained utility curves. Moreover, the further from the abscisa the curve is located the higher the corresponding utility level is. Bid curves correspond to a higher reference utility level when they lie to the bottom (toward the axis for $H$). Therefore the utility maximization behavior of consumers is equivalent to the minimization of their bid function.

We assume that the constrained utility function, and thus the individual

bid function, satisfies the Spence-Mirrlees condition with respect to $R_n$:

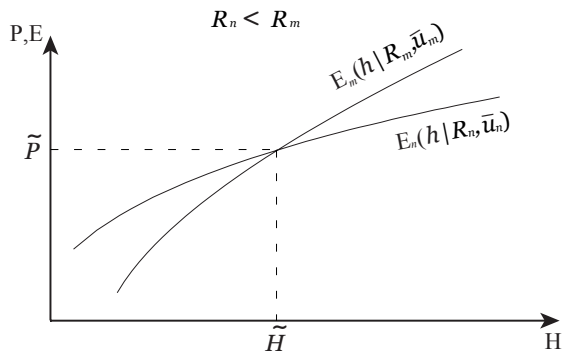
$$\begin{cases} \frac{\partial V}{\partial P} \neq 0 \\ \frac{\partial}{\partial R_n}\left(-\frac{\partial V/\partial H}{\partial V/\partial P}\right) > 0, \ \forall R_n \end{cases} \Leftrightarrow \begin{cases} \left.\frac{\partial U}{\partial X}\right|_{X=R_n-P} \neq 0 \\ \frac{\partial}{\partial R_n}\left(\frac{\partial E}{\partial H}\right) > 0, \ \forall R_n \end{cases} \tag{7}$$

Condition (7) involves that individual bid curves satisfy a single-crossing property (Milgrom and Shannon, 1994; Edlin and Shannon, 1998) in space $(H, P)$.[7] Figure 1 illustrates this point. Let consider two arbitrary bid curves $E_n(H|R_n, \bar{u}_n)$ and $E_m(H|R_m, \bar{u}_m)$, associated to two individuals $n$ and $m$ with different incomes $R_n < R_m$. The single-crossing condition means that, whoever $n$ and $m$ and whatever $\bar{u}_n$ and $\bar{u}_m$, the two bid curves cross only once. If $(\tilde{H}, \tilde{P})$ denotes the crossing point, then we have:

$$E_m(H|R_m, \bar{u}_m) - E_n(H|R_n, \bar{u}_n) < 0, \quad \forall\, H < \tilde{H} \tag{8}$$

$$E_m(H|R_m, \bar{u}_m) - E_n(H|R_n, \bar{u}_n) > 0, \quad \forall\, H > \tilde{H} \tag{9}$$

Figure 1: Single-crossing property of individual bid curves



### *2.2 Supply side*

We limit our analysis to the short-term case in the sense that the distribution of goods on the supply side is fixed and does not adjust to the economic environment. The focus on short-term analysis is motivated by two arguments. Firstly, adjusting the attributes of a house is timely and costly so that one may reasonably consider that transactional data used in empirical studies of hedonic prices rather correspond to a short term equilibrium than to a long term equilibrium. Secondly, the assumption of fixed attributes of houses is consistent with the assumption that these attributes are exogenous explanatory variables commonly used for econometric purposes in most empirical studies of hedonic prices. We assume that there is a continuum of atomistic and competitive sellers homogeneous in their tastes and other parameters. Each of them sells one house characterized by an index of housing services level $H$. The index $H$ is distributed on the interval $[H_{min}, H_{max}]$ with a density function $\phi(H)$.[8] It is additionally assumed that there exist a "*outside the market*" alternative in the sense that it is always possible for a buyer to opt for $H = H_{out} < H_{min}$ with exogenous price $P_{out}$.

Due to the focus on short-term analysis, the behavior of sellers is substantially simplified compared to the long-term analysis proposed by Rosen (1974). Sellers no longer have to determine the optimal level of housing services $H$ they want to offer.

In other to go one step further on the characterization of market equi-

librium it is now required to distinguish between continuous and groupwise heterogeneity on the demand side. We first analyze the case of groupwise heterogeneity and then re-examine the standard case of continuous heterogeneity to better highlight the impact of groupwise heterogeneity.

## 3 Groupwise heterogeneity on the demand side

Though the focus of the paper on short term analysis *a priori* reduces the problem of sellers to its simplest form, the assumption of groupwise heterogeneity on the demand side requires to make explicit important constraints sellers have to satisfy when describing market equilibrium. Therefore, this section first introduces a general definition of the hedonic equilibrium of the market for houses and then derives important propositions that follow on. The existence and uniqueness of an equilibrium hedonic price function if then shown. It is more specifically outlined that there exist a recursive construction method of the hedonic price curve.

### *3.1 Definition and characterization of an hedonic equilibrium*

In the presence of groupwise heterogeneity of consumers, there are $J$ groups of consumers ranked in the increasing order of income $R_j$, which is the variable that differentiates one group from the others: $R_1 < R_2 < ... < R_J$. $\theta_j$ (with $\sum_1^J \theta_j = 1$) denotes the proportion of the population of consumers that belong to a same group $j$.

Consumers within a group have the same income, the same utility func-

tion and the same budget constraint. As a result, they also have identical bid functions. Let $u_j^{(k)}$ stand for the utiity level reached by buyers of group $j$ when they benefit from a level of housing services $H_k$ and pay $P_k$ for it. We denote $E_j^{(k)}\left(H|R_j, u_j^{(k)}\right)$ the corresponding bid function of individuals from group $j$ parameterised by the reference utility level $u_j^{(k)}$. The objective of any buyer from group $j$ is to select the combination $\{H_k, P_k\}$ of the level and price of housing services that is affordable to her, including the "outside the market" alternative, given her level $R_j$ of income and that gerenates the highest utility level. In space $(H, P)$, such a combination makes buyers from group $j$ stay on their common lowest possible bid curve. The corresponding optimization program may thus be written as:

$$\min_{u_j^{(k)} \in DU_j} E_j^{(k)}(H|R_j, u_j^{(k)}) \tag{10}$$

where $DU_j$ has been defined in (6) and already incorporates the budget constraint.

Though each consumer is "hedonic price taker", all consmers within a same group have similar choices. Therefore, sellers act as if they were directly facing groups of consumers. Accordingly, each seller $s$ attempts to sell her house to a buyer from a targeted group that she chooses optimally. More precisely, the optimal choice of each seller may be decomposed in two steps. In a first step a target group is predefined for each seller. Each seller then seeks the maximum price she can charge to buyers from this group subject to two types of constraints as regards the behaviour of buyers. The first

type of constraints is referred to as participative constraints and reflects the trade-off made by buyers between purchasing a house on the market or choosing the "outside the market" alternative. The second type of constraints is referred to as incentive compatibility constraints and reflects the trade-off between the different goods offered on the market that buyers are facing. The corresponding optimization program for a seller $s$ selling a house characterised by a level of housing services $H_s$ is

$$\max P_s$$

subject to the participation constraint of the target group $j$ which is formally defined by

$$V(H_s, P_s, R_j) \geq V(H_{out}, P_{out}, R_j)$$

and to the following set of incentives compatibility constraints for the target group $j$

$$V(H_s, P_s, R_j) \geq V(H_l, P_l, R_j) \quad \forall H_l \neq H_s$$

Note that the price that solves this program is actually the optimal response of seller $s$ to prices $P_l$ chosen for the other competing goods by their sellers. As these prices are treated as exogenous variables in the optimal program of seller $s$ we can think of sellers as multiple Stackelberg leaders competing "*à la Bertrand*". Let $P_s^{(j)}$ be the resulting maximum price that seller $s$ can charge to buyers form the target group $j$. The second step of the optimization program reflecting the objective of such a seller is then to select the optimal

target group among the $J$ groups of buyers:

$$\max_{j \in \{1,\ldots,J\}} P_s^{(j)}$$

Given these optimization programs characterising the behaviour of buyers and sellers we are now able to define a hedonic equilibrium of the market:

**Definition 3.** Hedonic equilibrium. *An allocation of houses for sale on the market among buyers constitutes a short term hedonic equilibrium of the housing market if and only if it satisfies the following three conditions:*

*Condition 1 (Optimal choice of buyers): None of the buyers is able to find another house that is budgetary affordable to her and generates a higher utility level.*

*Condition 2 (Optimal choice of sellers): None of the sellers is able to sell at a higher price without violating the participation constraint or one of the incentives compatibility constraints of the optimally targeted group of buyers.*

*Condition 3 (Market clearing): There is no excess supply and no excess demand on the market for all levels of the index of housing services $H$.*[9]

The set of prices that fulfill these conditions characterizes the Hedonic Price Function. If $P_s^*$ denote the equilibrium price chosen by the seller of a house with a level $H_s$ of housing serivices, if $u_j^* = U(H_s, R_j - P_s^*)$ stand for the utility level reached at equilibrium by buyers of the group $j$ that is optimally targeted by seller $s$ and $E_j^*(H|R_j, u_j^*)$ is the associated bid

function, then we can formally define the hedonic price function as follows:

**Definition 4.** *For any level of the index $H$ of housing services, the* Hedonic price function *yields the highest of the individual bids associated to the equilibrium situation:*

$$\forall H \qquad P(H) = \max_{j \in \{1,\ldots,J\}} E_j^*(H|R_j, u_j^*) \tag{11}$$

Said another way, the hedonic price curve is the upper enveloppe of equilibrium bid curves of all groups of buyers.

By summarizing definitions 3 and 4, the following equations define the hedonic equilibrium with vertical differentiation of houses and groupwise heterogeneity of buyers:

$$\forall j \in \{1, ..., J\} \qquad \max_{u_j^{(k)} \in DU_j} E_j(H|R_j, u_j^{(k)}), \qquad H \in \mathrm{Dh}_j, \tag{12}$$

$$V(H, E_j(H|R_j, u_j^{(k)}), R_j) \geq V(H_{out}, P_{out}, R_j), \tag{13}$$

$$V(H, E_j(H|R_j, u_j^{(k)}), R_j) \geq V(H_i, E_i(H|R_i, u_j^{(k)}), R_j) \tag{14}$$

$$\forall H_i \in \mathrm{Dh}_i, \; i \neq j,$$

$$\int_{Dh_j} \phi(H) \mathrm{d}h = \theta_j. \tag{15}$$

The latter condition formally expresses Condition 3 in Definition 3. $Dh_j$ (respectively $Dh_i$) is the set of housing services levels characterising houses sold to buyers from group $j$ (respectively group $i$). By definition of the hedonic price function we know that

$$Dh_j = \left\{ H \in [H_{\min}, H_{\max}] \; : \; P(H) = E_j^*(H|R_j, u_j^*) \right\}$$

We now proceed with some elementary propositions as regards the characteristics of the equilibrium.

**Proposition 5.** *Let $(H_n^*, P_n^*)$ be the purchasing equilibrium of individual $n$ with the income $R_n$ and let $E_n^*(H|R_n, u_n^*)$ be the equilibrium bid function of this individual. Then*

$$P(H_m^*) \geq E_n^*(H_m|R_n, u_n^*) \qquad \forall m \neq n$$

*Proof.* Assume that there exists a house with a level of housing services $H_m \in [H_{\min}, H_{\max}]$ such that we have

$$P(H_m) < E_n^*(H_m|R_n, u_n^*). \tag{16}$$

Define $u_n^{(m)}$ as the utility level reached by individual $n$ if located in the house with $H_m$ as the level of housing services. Let $E_n^{(m)}(H|R_n, u_n^{(m)})$ denote the bid function of individual $n$ that passes through point $(H_m, P(H_m))$. Then we have $E_n^{(m)}(H|R_n, u_n^{(m)}) < E_m^*(H|R_n, u_n^*)$, $\forall H$.

According to the graphical properties of bid curves, and more specifically the fact that the lower a bid curve on the graph the higher the reference utility level, we known that $u_n^* < u_n^{(m)}$. This inequality also involves that $V_n(H_m, P(H_m), R_n) > V_n(H_n^*, P_n^* R_n)$ and, by consequence, that the incentives compatibility constraint (14) is violated. Therefore, $(H_n^*, P_n^*)$ does not characterise the equilibrium. We deduce that

$$\forall m \neq n, \quad P(H_m) \geq E_n^*(H_m|R_n, u_n^*).$$

QED □

Proposition 5 means that the price of houses that are purchased by other consumers are located either above or on the same equilibrium bid curve of individual $n$ (the bid curve passing through point $(H_n^*, P_n^*)$).

Based on the proposition 5, the next proposition reduces the problem to a locational choice of groups of consumers:

**Proposition 6.** *At the short term hedonic equilibrium individuals belonging to a same group are located on the same bid curve. This bid curve is referred to as the group bid curve.*

*Proof.* Cf. appendix A.1. □

The next proposition directly follows on from the single-crossing property of bid curves discussed in the first section:

**Proposition 7.** [Complete sorting] *Consider two houses* $\{1, 2\}$ *such that* $H_1 < H_2$ *and two individuals* $\{n, m\}$ *such that* $R_n < R_m$*. At equilibrium house* 1 *is necessarily bought by consumer* $n$ *and house* 2 *by consumer* $m$*.*

*Proof.* Cf. appendix A.2. □

Proposition 7 means that individuals from groups with a higher income level necessarily buy houses with a higher level of the index of housing services. The short term hedonic equilibrium thus implies a complete sorting of

consumers.[10] By combining propositions 7 and 6, we obtain the next proposition that states a key characteristic of the equilibrium in the presence of groupwise heterogeneity on the demand side:

**Proposition 8.** [Segmentation] *The short term hedonic equilibrium on the housing market induces a segmentation of the interval* $[H_{\min}, H_{\max}]$ *of offered housing services levels in as many subintervals as groups of buyers with thresholds values* $H_j$ *separating these subintervals obtained from the following conditions:*

$$\int_{H_{j-1}}^{H_j} \phi(H)dH = \theta_j,\ j \in \{1, 2, ..., J\},\ H_0 = H_{\min} \text{ and } H_J = H_{\max} \quad (17)$$

*Proof.* By Proposition 6, at equilibrium, all individuals from a same group stand on a single identical equilibrium bid function. Proposition 7 then implies that housing services levels of goods bought by consumers from a same group form a segment on the interval $[H_{\min}, H_{\max}]$. Let $[H_{j-1}, H_j]$ be the segment of housing services levels of houses bought by a group $j$. Condition 4 in Definition 3 implies that the density of supplied levels of $H$ in the subinterval $[H_{j-1}, H_j]$ equals the density of group $j$ in the total population of consumers, which is formalized by (17). □

Finally, the last proposition gives a somewhat intuitive but nevertheless important characteristic:

**Theorem 9.** *At the short term hedonic equilibrium the hedonic price function is continuous on the interval* $[H_{\min}, H_{\max}]$.

*Proof.* Cf. appendix A.3. □

### *3.2 Construction of the hedonic price function*

To demonstrate the existence and uniqueness of a short term hedonic equilibrium we first determine which participative and incentive compatibility constraints of the different groups are binding. As regards incentive compatibility constraints we first deduce from the definition of the constrained utility function, the definition of the bid function, the single-crossing property of bid functions and Proposition 7, that satisfying inequality (14) that formalizes incentives compatibility constraints for a group $j$ is also equivalent to satisfying the two following set of constraints:

$$\begin{cases} V(H, E_j(H|R_j, u_j^{(k)}), R_j) \geq V(H_{i-1}, E_i(H_{i-1}|R_i, u_i^{(k)}), R_j), & \forall i \ : R_j < R_i \\ V(H, E_j(H|R_j, u_j^{(k)}), R_j) \geq V(H_i, E_i(H_i|R_i, u_i^{(k)}), R_j), & \forall i \ : R_j > R_i \end{cases} \tag{18}$$

We are then able to show the following result:

**Proposition 10.** *At the short term hedonic equilibrium of the housing market, for any group $j$, only incentives compatibility constraints with adjacent segments are binding.*

*Proof.* Cf. appendix A.4. □

In order to fully characterize the equilibrium we now have to determine the price for at least one of the levels of $H$ in $[H_{\min}, H_{\max}]$. By Proposition 7, houses with a level of housing services inside the first segment $[H_{\min}, H_1]$ are

bought at equilibrium by consumers of the group $j = 1$. The maximal price consumers from this group are willing to pay is given by the bid function when the group's participative constraint is just binding. Let adopt the notation $CP_j(H) \equiv E_j(H|R_j, u_j^{(0)})$ for the bid function of group $j$ when its participative constraint is just binding. We have

$$V(H, CP_j(H), R_j) = V(H_{out}, P_{out}, R_j) \equiv u_j^{(0)}.$$

and the bid curve $CP_j$ passes through the point $(H_{out}, P_{out})$. We are then able to show the following result as regards participation constraints:

**Proposition 11.** *At the short term hedonic equilibrium of the housing market, the sole participation constraint which is binding is that of the group with the lowest income level.*

*Proof.* Cf. appendix A.5. □

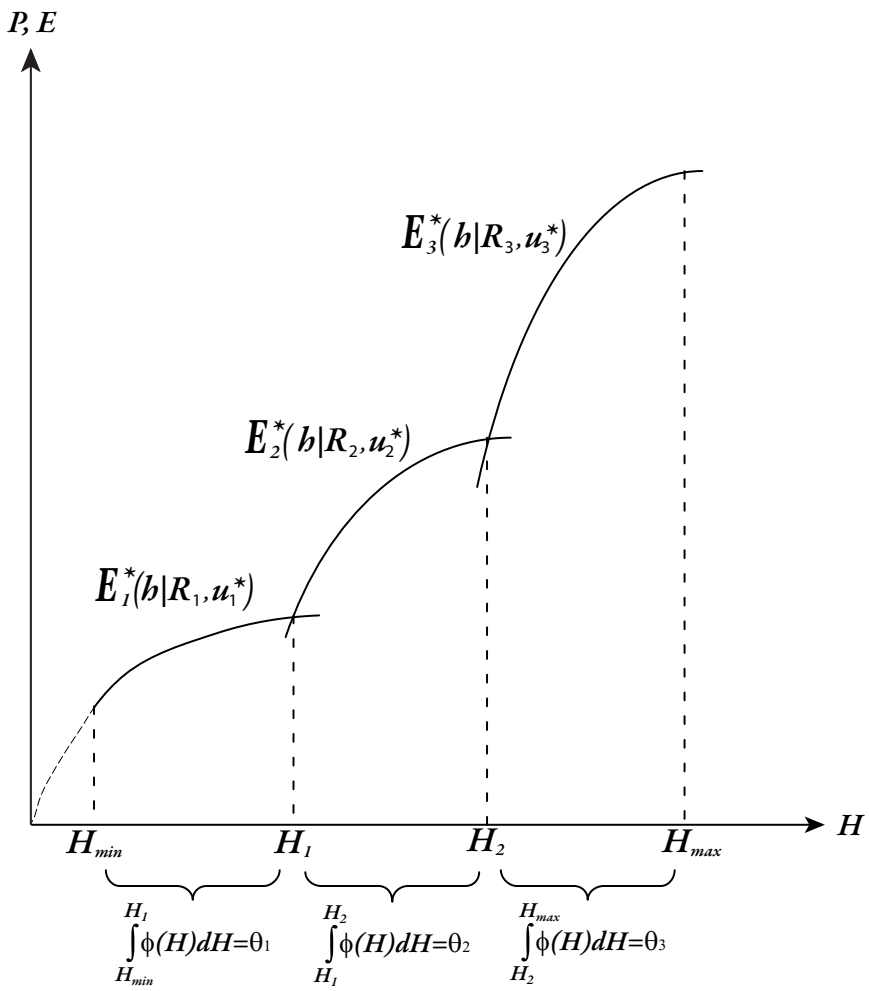
Propositions 10 and 11 suggest an iterative method to obtain the hedonic price function. According to proposition 11, the first group stays on the bid function corresponding to its participative constraint and its members all reach the same utility level $u_1^* = u_1^{(out)} = V(H_{out}, P_{out}, R_1)$. The hedonic price function on the interval $[H_{min}, H_1]$ coincides with the bid curve of the 1st group and is defined as $P^* = CP_1(H)$. The threshold value $H_1$ is obtained from equation (17) for $j = 1$:

$$\int_{H_{min}}^{H_1} \phi(H)dH = \theta_1$$

Proposition 10 implies that for each next segment $[H_{j-1}, H_j]$ ($j \in \{2, ..., J\}$) the equilibrium hedonic price coincides with the following equilibrium group bid function: $P_j^* = E_j^*(H|R_j, u_j^*)$, such that the incentive compatibility constraint of group $j$ with respect to the previous group $(j-1)$ is just binding:

$$V(H, E_j(H|R_j, u_j^*), R_j) = V(H_{j-1}, E_{j-1}(H_{j-1}|R_{j-1}, u_{j-1}^*), R_j). \qquad (19)$$

Figure 2: Iterative Construction of the Hedonic Price in the Segmentation model



The iterative procedure described above, generates an equilibrium hedonic price curve which corresponds to the upper envelope of the equilibrium group bid functions (Figure 2). By construction all buyers find a house and all houses have a buyer. This result is ensured by the systematic verification of the condition given by equation (17). By construction, consumers of the same group are indifferent between their respective purchases because they stay on the same group's bid function. Moreover, individuals of each group do not gain from purchasing a house bought by individuals from other groups. Indeed, except on the segment of housing services occupied by their group, the hedonic price is everywhere strictly superior to their individual (or equivalently group) equilibrium bid curve. Purchasing such a house would lead to a loss of satisfaction. The situation shown on Figure 2 satisfies all incentive compatibility constraints. The single-crossing property ensures that all participative constraints are satisfied as long as it is biding for the first group. Condition 2 in the definition 3 of a hedonic equilibrium is also satisfied because none of the sellers is able to increase her selling price.

**Theorem 12.** *In the presence of vertical differentiation, groupwise heterogeneity of buyers as regard their income and with standard assumptions about monotony of the utility function and single-crossing property of bid functions, the hedonic equilibrium price exists and is unique.*

*Proof.* The hedonic price function construction ensures the existence of the equilibrium and its uniqueness because the bid functions are monotonous

(increasing in $H$) and the single-crossing property of bid functions is satisfied. $\square$

## 4 Continuous heterogeneity on the demand side.

As stressed in its title, Rosen's article (Rosen, 1974) deals with perfect competition on both the demand side and the supply side. Perfect competition implies that buyers and sellers act as *hedonic price takers*. Though it does not necessarily require heterogeneity of buyers and sellers along a continuum, this case is nevertheless considered as the "natural" reference case in the literature on hedonic prices. In order to highlight the role of this additional assumption, at least as regards the demand side, we now analyze market equilibrium in this specific context. The focus on short-term equilibrium makes our model slightly different from that of Rosen (1974) but reinforces the parallel with the case of groupewise heterogeneity presented in the previous section.

### *4.1 Continous heterogeneity as a limit case*

So far we have analysed the hedonic equilibrium in the context of groupwise heterogeneity on the demand side and continuous heterogeneity on the supply side. Groupwise heterogeneity requires to identify $J \in \mathbb{N}$ groups of buyers who are homogeneous within each group. The proportion of the population of buyers that belongs to a specific group $j \in \{1, ..., J\}$ was denoted $\theta_j$ and it was required that $\theta_j \in [0, 1]$ and $\sum_1^J \theta_j = 1$. This context is

not incompatible with the assumption of a high number $J$ of groups, each group gathering a very small proportion $\theta_j$ of the total population of buyers. What our analysis of groupwise heterogeneity on the demand side induces is just that if $J \to \infty$ and $\theta_j \to 0 \ \forall j \in \{1, ..., J\}$, consecutive groups $j$ and $j+1$ will be located on very narrow intervals $[H_{j-1}, H_j]$ and $[H_j, H_{j+1}]$ with $H_0 = H_{\min}$ and $H_J = H_{\max}$. This property directly follows on from Proposition 8. In the limit case where $J \to \infty$ and $\theta_j \to 0 \ \forall j \in \{1, ..., J\}$ each group is thus located on an infinitesimal interval. Moreover, according to the iterative construction of the hedonic curve discussed in the previous section, at the hedonic equilibrium each bid curve overlaps the hedonic price curve only on this infinitesimal interval and is strictly lower elsewhere. Graphically, the optimal locational choice of a group in this limit case is found reasoning as if the hedonic price curve was given to the group and then seeking the lowest of the group's bid curve that admits at least one common point with the hedonic price curve. This observation raises two important points. Firstly, invoking the fact that the continuous case corresponds to the limit case where $J \to \infty$ and $\theta_j \to 0 \ \forall j \in \{1, ..., J\}$, we know that all propositions that apply in the context of groupwise heterogeneity have an equivalent in the context of continuous heterogeneity. Secondly, we can analyse the case of continuous heterogeneity on the demand side as if the hedonic price function $p(H)$ was known to all buyers and treated as exogeneous by all buyers. Said another way, buyers are considered as "*hedonic*

*price takers*". Keeping in mind these two important points we now turn to the direct analysis of the case of continuous heterogeneity on the demand side.

### *4.2 Distribution of demand*

We assume that the income of consumers is continuously distributed on the interval $[R_{min}, R_{max}]$ with a density function $\psi(R)$. Consumers are hedonic price takers and maximize their utility function subject to their budget constraint:

$$\max_{H,X} U(H, X) \tag{20}$$

$$\text{s.t.} \quad X + p(H) = R \tag{21}$$

The optimal choice $\{H^*, X^*\}$ of housing services and consumption level solves the first order condition associated to the optimisation program (20)-(21). This first order condition may be arranged in such a way that it makes explicit that the marginal bid equals the marginal hedonic price at equilibrium:

$$p_H(H^*) = \frac{\partial U / \partial H}{\partial U / \partial X}\Big|_{(H^*,\, R-p(H^*))} \tag{22}$$

By the theorem of implicit functions, equation (22) yields the optimal level of housing services as a function of individual income:

$$H^* = f(R) \tag{23}$$

After some rearrangements, the second order condition for this choice of housing services to be truly optimal states that

$$\frac{\left[\frac{\partial^2 U}{\partial H^2} - \frac{\partial^2 U}{\partial H \partial X} p_H\right] \frac{\partial U}{\partial X} - \left[\frac{\partial^2 U}{\partial H \partial X} p_H - \frac{\partial^2 U}{\partial X^2} (p_H)^2\right] \frac{\partial U}{\partial X}}{\left(\frac{\partial U}{\partial X}\right)^2} < p_{HH} \tag{24}$$

where all first order and second order derivatives are evaluated at point $(H^*, R - p(H^*))$ characterising the solution to the first order condition for an individual with income $R$. We can thus substitute the right hand side of (22) to $p_H$ in the second term in brackets at the numerator of (24). This yields the following condition

$$\frac{\left[\frac{\partial^2 U}{\partial H^2} - \frac{\partial^2 U}{\partial H \partial X} p_H\right] \frac{\partial U}{\partial X} - \left[\frac{\partial^2 U}{\partial H \partial X} - \frac{\partial^2 U}{\partial X^2} p_H\right] \frac{\partial U}{\partial H}}{\left(\frac{\partial U}{\partial X}\right)^2} < p_{HH}$$

which indicates that the curvature of the individual bid curve has to be lower than the curvature of the hedonic price curve, at least in the neighborhood of the optimal choice (23). According to the discussion of the limit case of groupwise heterogeneity with $J \to \infty$ and $\theta_j \to 0 \; \forall j \in \{1, ..., J\}$ made above, the second order condition (24) is satisfied by definition and construction of the equilibrium hedonic price function. Note also that in our discussion of the equilibrium with groupwise heterogeneity on the demand side, the respective position of bid curves and hedonic price curves were resulting from incentives compatibility constraints. Similarly, the first and second order conditions associated to program (20)-(21) state that a buyer with income $R$ as no incentives to choose an other house than a house characterised by $H^*$ as defined in (23). Note finally that the interpretation of the continuous

case as the limit case of groupwise heterogeneity with $J \to \infty$ and $\theta_j \to 0$ $\forall j \in \{1, ..., J\}$ implies that, as long as the single property of bid curves is satisfied, $f(R)$ is monotone increasing (See Proposition 7). Therefore, expression (23) can be inverted in order to obtain the level of income of individuals who optimally choose a given level $H^*$ of housing services as defined in (22):

$$R = g(H^*, p(H^*), p_H(H^*)) \tag{25}$$

If furthermore $g(\cdot)$ is differentiable with respect to all its arguments, its differential is given by:

$$dR = [g_1(\cdot) + g_2(\cdot)p_H + g_3(\cdot)p_{HH}]\, dH \tag{26}$$

The two functions $f(\cdot)$ and $g(\cdot)$ are key to obtain the density of housing services that are demanded by consumers in any interval of values $[H_k, H_l]$. Knowing the density of income $\psi(R)$, equation (25) indicates that the density of housing services demanded inside the interval of values $[H_k, H_l]$ also corresponds to the density of consumers with income inside the interval $[R_k, R_l]$ with $R_k = g(H_k, p(H_k), p_H(H_k))$ and $R_l = g(H_l, p(H_l), p_H(H_l))$. This density is initially defined by

$$\int_{g(H_k, p(H_k), p_H(H_k))}^{g(H_l, p(H_l), p_H(H_l))} \psi(R) dR \tag{27}$$

Nevertheless, we are rather interesting in writing this density in terms of the level of housing services rather than in terms of income. With this aim

in view we proceed with a change of variable to finally obtain the density in the following form:

$$\int_{H_k}^{H_l} \kappa(H) dH \tag{28}$$

with

$$\kappa(H) = [g_1(\cdot) + g_2(\cdot)p_H + g_3(\cdot)p_{HH}]\, \psi\,(g(\cdot)) \tag{29}$$

The density function $\kappa(H)$ plays a role similar to that of the demand function for homogeneous goods. Its exact form depends on the hedonic price function which must be shaped in such a way that market clearing occurs. We now turn to this point.

### *4.3 Market equilibrium*

On the market for a homogeneous good, market clearing is ensured by a specific value of the price. This specific value makes the quantity demanded by buyers just coincide with the quantity supplied by sellers. Accordingly, the equilibrium price is a value of the price that solves the equation obtained by setting the demand function equal to the supply function. On the market for houses with an index of housing services $H$, continuously distributed on the interval $[H_k, H_l]$, what we seek to characterize market equilibrium is not a value of the price but a functional relation between the index of housing services and the price. Indeed, for market clearing to occur it is required that, over any subinterval $[H_k, H_l]$ included in $[H_{min}, H_{max}]$, the

density of demand $H$ exactly coincides with the density of supply. This yields Definition 13:

**Definition 13.** Hedonic equilibrium. *An equilibrium on the housing market exists if there is a function $p(H)$ such that*

$$\phi(H) = \kappa(H) \; \forall \, H, \tag{30}$$

*A function $p(H)$ satisfying this condition is called a* hedonic price function.

Substituting (29) to $\kappa(H)$ in this definition, we obtain that the equilibrium hedonic price function is a solution of the following differential non linear equation:

$$\phi(H) = \left[g_1(\cdot) + g_2(\cdot)p_H + g_3(\cdot)p_{HH}\right] \psi\left(g(\cdot)\right) \tag{31}$$

Because (31) is a second order differential equation, two additional conditions are required to determine the two constants of integration and fully characterized the hedonic price function. For this purpose, we take advantage of the interpretation of the continuous case as the limit case of groupwise heterogeneity with $J \to \infty$ and $\theta_j \to 0 \; \forall j \in \{1, ..., J\}$. Indeed, according to Proposition 7, we know that the group with the lowest level of income $R_{\min}$ buys houses with the lowest level $H_{\min}$ of housing services. Combining with the first order condition (22) we thus have a first initial condition[11]:

$$p_H(H_{\min}) = \frac{\partial U / \partial H}{\partial U / \partial X}\Big|_{(H_{\min},\, R_{\min} - p(H_{\min}))} \tag{32}$$

Furthermore, according to Proposition 11, we know that at the equilibrium of the market the group with the lowest income $R_{\min}$ is just indifferent between buying the house with $H_{\min}$ or opting for the "outside the market" alternative with $H = H_{out}$ paid at price $P_{out}$. This yields the second initial condition:

$$U(H_{min}, R_{min} - p(H_{min})) = U(H_{out}, R_{min} - P_{out}) \tag{33}$$

The differential equation (31) is obtained by assuming that the standard conditions for the theorem of implicit functions to apply are verified, by assuming that the function $f(R)$ defined by (23) is monotone and, finally, by assuming that the function $g(\cdot)$ defined by (25) is differentiable with respect to all its arguments, which requires assumptions about the utility function. Moreover, the analysis of the solution of the problem (31)-(32)-(33) involves additional conditions about the density functions on the demand side and on the supply side. Even if all necessary conditions are fulfilled, the high non linearity of the differential equation (31) often makes it impossible to provide an analytical solution, except for very special and simple cases. We detail in Appendix B two illustrations of this point: firstly, the case of a Cobb-Duglas specification for the utility function with continuous and uniform distributions of both the income of buyers and the index of housing services and, secondly, the case of a CES specification of the utility function with similar assumptions as regards the distribution of income and the

distribution of the index of housing services. The analytical solution can be obtained only in the first case. In the second case, in spite of the commonly used specification of the utility function, the analytical solution for the hedonic price function cannot be obtained.

## 5 Implications of groupwise heterogeneity for environmental valuation

### *5.1 Segmentation of the housing market*

A striking feature of the hedonic price curve generated by the model in the presence of groupwise heterogeneity on the demand side is that it is continuous but defined by a different function on each segment, the bid function of the group of buyers associated to the segment. The overall interval $[H_{\min}, H_{\max}]$ of the index of housing services describes the initial set of goods that any household consider for his locational choice. In this sense we may designate the interval $[H_{\min}, H_{\max}]$ as the relevant market. Nevertheless, market equilibrium implies that a given household belonging to group $j$ in terms of income will effectively consider only a subset of goods corresponding to the interval $[H_{\mathrm{j}-1}, H_{\mathrm{j}}]$. Moreover, a change in preferences for group $j$ induces a change of the hedonic price curve for all groups $l > j$ but not for groups $i < j$. The interdependence of prices from one interval or segment of values for $H$ to another one is thus not reciprocal. In this sense, segments $[H_{\mathrm{j}-1}, H_{\mathrm{j}}]$ can be thought of as submarkets. As states the following theorem, the existence of submarkets is tightly linked to discontinuities of the

marginal hedonic price function at threshold levels of $H$ separating segments of the corresponding curve.

**Theorem 14.** *In the presence of vertical differentiation and groupwise heterogeneity of buyers as regard their income, the marginal hedonic price function is discontinuous between submarkets.*

*Proof.* 1°. Within each segment, the marginal hedonic price function corresponds to the equilibrium marginal bid of the segment:

$$\forall j \in \{1, ..., J\},\ \forall H \in (H_{j-1}, H_j)\,,\ \frac{\mathrm{d}P_j}{\mathrm{d}H} = \frac{\mathrm{d}E_j^*(H|R_j, u_j^*)}{\mathrm{d}H} \tag{34}$$

2°. Since $R_j - R_{j+1} < 0$ and $u_j^* - u_{j+1}^* < 0$, $\forall j \in \{1, ..., J-1\}$, the Spence-Mirlees condition (7) implies that:

$$\frac{\mathrm{d}E_j^*(H_j|R_j, u_j^*)}{\mathrm{d}H} < \frac{\mathrm{d}E_{j+1}^*(H_j|R_{j+1}, u_{j+1}^*)}{\mathrm{d}H}. \tag{35}$$

Thus, even if the hedonic price function is continuous on the whole interval $[H_0, H_J]$, the marginal hedonic price function is discontinuous at each threshold value of $H$ between two submarkets. QED □

Major implications for the environmental valuation follow on from theorem 14. Indeed, when the environmental quality varies from one house to another, the index of housing services level changes with this quality. The implicit price of the environmental quality, used at the second stage of the hedonic estimation procedure to determine the demand for environmental quality, is obtained as the marginal hedonic price. By using the estimation of

the hedonic price curve with a functional form not only continuous but also continuously differentiable, the "usual" two stage procedure of the hedonic valuation method neglects the existence of submarkets and the discontinuities of marginal price associated with it. Consequently, in the presence of a groupwise heterogeneity of consumers, the "usual" econometric approach has two drawbacks. Firstly, it leads to a misspecification of the hedonic price function. Indeed, because the procedure is based on the assumption of atomiticy of each agent on the demand side, characteristics of buyers may not be used as explanatory variables in the first step of the "usual" procedure consisting in the estimation of the hedonic price function.[12] But, as we have shown just above, in the presence of groupwise heterogeneity on the demand side the hedonic price curve is defined piece-wise on the basis of bid functions of individuals which belong to a same group and, by definition, thus have identical characteristics. As a result, individual characteristics that define homogeneous groups of buyers affect the form of the hedonic price curve. Secondly, there is a risk of error in the evaluation of demand for environmental quality. The discontinuity of the marginal hedonic price function invalidates the use of continuous functional forms for the regression to be implemented in the second step of the usual hedonic price valuation method.

### *5.2 Illustration for a CES utility function*

In order to illustrate how important the consequences of groupwise heterogeneity and segmentation may be, we proceed with a test based on numerical simulations. In a first step, we calibrate the model and examine the impact of considering less groups gathering more homogeneous households each. In a second step, we generate random draws of the index of housing services and associated prices and then estimate an *ad hoc* Box-Cox specification for these simulated data[13]. Comparison of exact and estimated results for hedonic prices and marginal prices serves as a basis for discussing the relevance of taking into account groupwise heterogeneity for environmental valuation.

We start calibration with the supply side of the housing market. There is little observation that helps calibrating the distribution of the index $H$ of housing services. We have some information about the distribution of prices but, by construction, this distribution results from heteregeneity on both the supply side and the demand side of the market and does not correctly reflects heterogeneity on the supply side alone. Note that, however, $H$ is an index and consequently its absolute value does not make sense. What makes sense is the general shape of the distribution of the index over the range of possible values. We thus somewhat arbitrarily assume that $H$ is distributed over $\mathbb{R}_+$ (so that $H_{\min} = 0$ and $H_{\max}$ goes to infinity) according to a Gamma distribution with 25 as the shape parameter and 0.4 as the scale parameter. Among distribution functions defined on $\mathbb{R}_+$, the Gamma distribution is

one of the most flexible. Whereas the log-normal distribution has a strictly positive mode, the Gamma distribution may exibit a mode equal to either zero or a positive value. In the absence of data on the observed distribution of housing services, we choosed a calibration that yields a limited asymetry of the graph of the partial density function and reflects the presence of numerous medium quality goods on the market and some low and high quality goods in comparable proportions.

Calibration on the demand side is easier. Information from national accounts on the mean and median Gross Disposable Revenue per household constitutes a first useful element to calibrate the distribution of income. Indeed, under the usual assumption that income is log-normaly distributed, the two parameters of the distribution can be identified thanks to the mean and median values. The mean and median Gross Disposable Revenues per household respectively amount to 34540 Euros and 28740 Euros in 2009 for France[14]. With the aim to focus on groupwise heterogeneity, we discretise the distribution of income in the following way. When considering $J$ groups of households, we consider the $J$ percentiles of the initial continuous distribution and affect all the density of a given percentile $j \in \{1, ..., J\}$ to the middle value of the corresponding range of values for income. For the last percentile, we truncate the upper bound to the $999^{th}$ percentile. This method generates groups of identical density, which is only one peculiar case our model deals with and eases the visual comparison between theoretical

and simulated marginal hedonic prices. The demand demand side is also featured by a direct utility function with parameters to be estimated. We specify a CES direct utility function written as:

$$U(H, X) = \left[\alpha H^{\sigma} + (1-\alpha)X^{\sigma}\right]^{\frac{1}{\sigma}},$$

with $\sigma \in ]0,1[$[15] and $\alpha \in [0,1[$ the two parameters. The motivation for a CES specification rather than, for instance, a Cobb-Douglas specification is that the income of households affects their utility and their bid curve even when the "outside the market" alternative is defined by $H_{out} = 0$ and $P_{out} = 0$. Parameters $\alpha$ and $\sigma$ play a crucial role in determining the share of his income a household devote to financing the acquisition of his house or flat. Therefore, these two parameters are calibrated so that the average budget per household devoted yearly to real estate acquisitions according to the model corresponds to the average budget reported by the Institut National de la Statistique et des Etudes Economiques for France. This budget amounts to 6652 Euros for 2009 and may be generated by setting $\alpha$ and $\sigma$ to respectively 0.80278 and 0.3 when the log-normal distribution of income is discretised in $J = 50$ groups following the discretisation method presented just above. The average ratio between the yearly budget devoted to real estate and income then amounts to 0.192587 and the elasticity of substitution between the agregate Hicksian good and the index of housing services is 1.25.

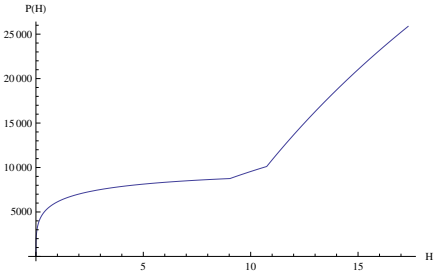
Figure 3 shows the general shape of the hedonic price function (on the

left hand side) and the marginal hedonic price function of housing services (on the right hand side) that are generated by the model with the calibration of parameters described above and four different numbers of groups ($J = 3$, $J = 10$, $J = 20$ and $J = 50$ from top to bottom). There is no major differences in the general shape of the hedonic price function when considering these four cases. The hedonic price function exibits similar ranges of values of $H$ for which it is locally concave or convex and it is generally hard to visually identify the thresholds separating the different segments of the housing market. The only exceptions are the two extreme segments associated to the left and right tails of the distribution of $H$. Indeed, whatever the number of groups $J$ considered, a change in the slope of the hedonic curve clearly appears at values of $H$ that are identified as the first and last thresholds in the graphs of the marginal hedonic price function. Note that in the case $J = 3$ there is only one segment between the two extreme segments so that all segments are visually detected (graphics 3(a)). By contrast, discontinuities of the marginal hedonic price curve clearly appears for each of the four considered cases for $J$ and at all thresholds values of the index $H$ of housing services. Nevertheless, the magnitude of discontinuities tends to decrease as the number of segments increases. The case with $J = 50$ groups may even be considered as close to the case of a continuous distribution of incomes given that only very small drops of the marginal hedonic price are observed at thresholds levels of $H$ separating the different segments of the

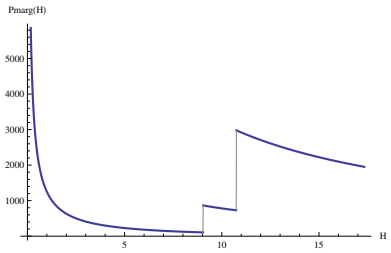
housing market, at least those which are not at one extreme or the other of the curve (graphics 3(h)). Conversely, the case with $J = 3$ groups exhibits important drops of the marginal hedonic price curve (graphics 3(b)). These contrasted graphical results suggest that disregarding the existence of market segmentation in econometric works would not necessarily affect the overall quality of regression but could have dramatic consequences when inferring the marginal price of housing services, more especially when group-wise heterogeneity involves a few number of groups. We go one step further into the investigation of how critical are the consequences of disregarding market segmentation by implemented an econometric test based on simulated data.

One thousand values of the index of housing services have been drawn from the Gamma probability distribution with 25 as the shape parameter and 0.4 as the scale parameter. The hedonic price functions generated with a discretisation in respectively $J = 3$, $J = 10$, $J = 20$ and $J = 50$ groups of households of the log-normal distribution of incomes and represented on the left side in Figure 3 have been used to compute the prices associated to each random draw of the index of housing services. The sets of combinations of housing services index and price obtained for each of the four discretisation levels have then served as datasets to estimate a Box-Cox specification of the hedonic price function. The Box-Cox specification is a functional form commonly used in the econometric literature on hedonic prices for houses. It

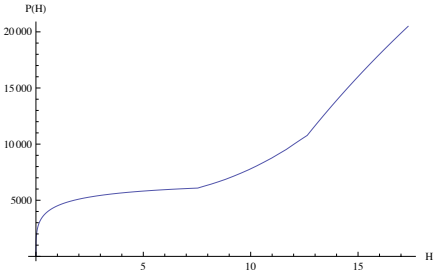
Figure 3: Hedonic price function and implicit prices. Numerical application



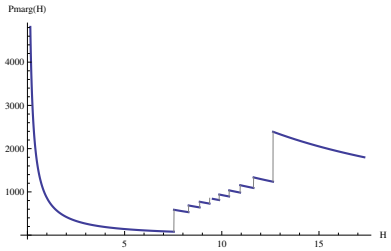

(a) HPF, 3 groups.



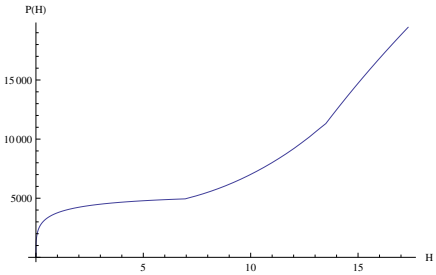

(b) Marginal HPF, 3 groups



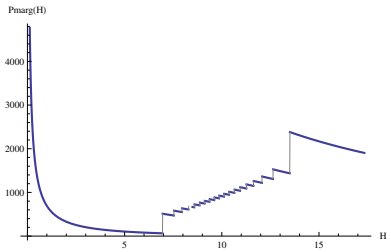

(c) HPF, 10 groups.



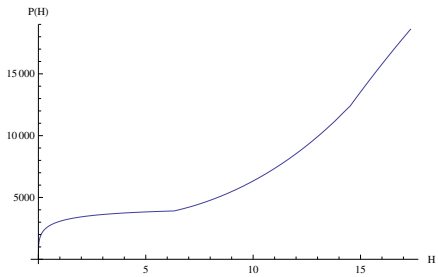

(d) Marginal HPF, 10 groups



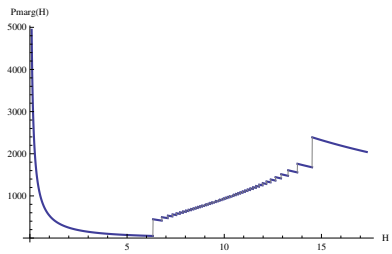

(e) HPF, 20 groups.




(f) Marginal HPF, 20 groups




(g) HPF, 50 groups.

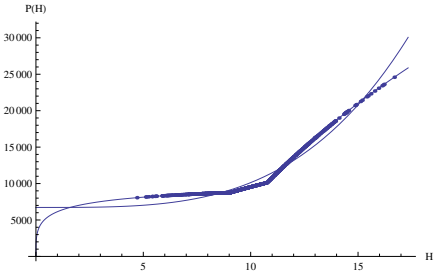



(h) Marginal HPF, 50 groups

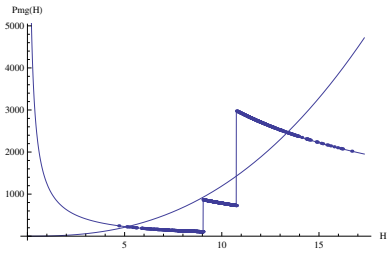
is known as one of the most flexible functional forms, which is an important feature in the context of our test. No error terms have been added to observations so that estimation errors are entirely imputable to the use of an *ad hoc* though flexible specification of the hedonic price curve. The performed estimations initially allowed for a Box-Cox transformation of both the explained variable and the regressor. Nevertheless, the estimated parameter of the Box-Cox transformation for the explained variable was systematically very close to unity so that only the Box-Cox transformation of the index of housing services revealed to be relevant.

The estimated hedonic price functions are superposed to the real hedonic price functions in the left hand side of Figure 4. There are drawn as simple thick curves whereas real hedonic curves are completed by points corresponding to the randomly generated observations. As indicated by the distribution of points along the real hedonic price curves, all segments are correctly covered by observations. This results from the choice of discretisation method which implies that all groups gather the same density of households and thus, as induced by the theoretical model, the same density of houses. As shown by the series of graph in the left hand side of Figure 4, the higher the number of groups considered, the better the approximation of the real hedonic price curve by the estimated Box-Cox curve. The major divergence concerns the two extreme parts of the hedonic price curve and is not of great importance because no observation point lies on these parts.

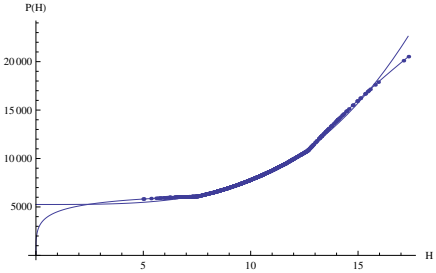
Figure 4: Estimation of Hedonic price function and implicit prices.



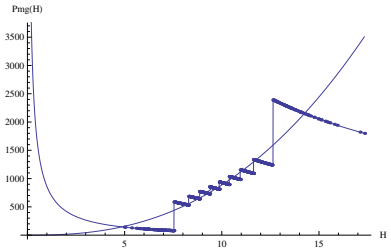

(a) HPF, 3 groups.



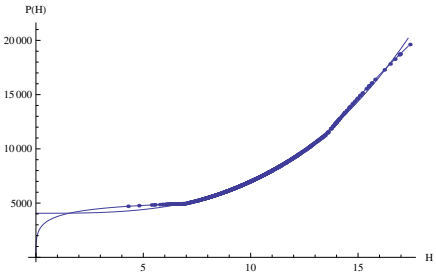

(b) Marginal HPF, 3 groups



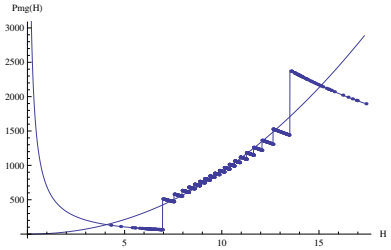

(c) HPF, 10 groups.



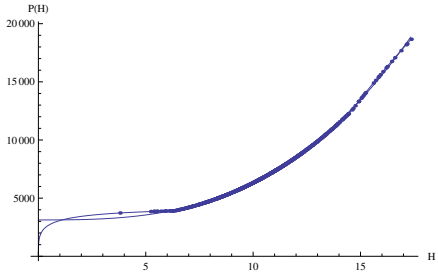

(d) Marginal HPF, 10 groups



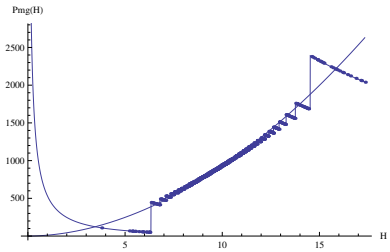

(e) HPF, 20 groups.




(f) Marginal HPF, 20 groups




(g) HPF, 50 groups.




(h) Marginal HPF, 50 groups

The afford mentioned divergence thus rather corresponds to extrapolation errors than to estimation errors. Similar graphs for the marginal hedonic price of housing services are displayed on the right hand side of Figure 4. The discontinuous curve with superposed observed points corresponds to the real marginal hedonic price curve. Unsurprisingly, the gap between real and estimated marginal prices is of higher magnitude when the number of groups is low and tends to zero (at least in the middle part of the curve) when the number of groups approaches $J = 50$. Nevertheless, the gap remains high even with a large number of groups for observations belonging to the two groups at the extreme left and the three groups at the extreme right. Though such observations are less numerous as the number of groups is increased they are clearly identified as outliers in the regression. The approximation error reaches one third of the real marginal price for a large number of observations in the case with $J = 3$ groups. Thus, important risks of error exist in the usual two steps hedonic price method for environmental valuation in the presence of groupwise heterogeneity involving a small number of groups on the demand side.

## 6 Conclusion

The article is a first insight into the impact of groupwise heterogeneity on the theoretical fundations of hedonic price functions on the one hand and on the use of hedonic price function for applied environmental valuation on the other hand. It explores a symetric or dual approach to sorting models

by focusing on the case of groupwise heterogeneity on the demand side and continuous heterogeneity on the sppuly side. A key advantage of such a dual approach for environmental valuation is that the concept of implicit price or marginal hedonic price of environmental characteristics is still relevant. From a theoretical point of view, the analysis of market equilibrium with groupwise heterogeneity on the demand side makes the role of participation and incentives compatibility constraints for groups of buyers explicit. The case of continuous heterogeneity may be thought of as a limit case of groupwise heterogeneity when the number of groups of houselholds goes to infinity and their respective masses go to zero. A direct consequence of this approach is that the determination of the hedonic price function proceeds iteratively and that each iteration induces a non marginal change of the implicit price of housing services. From an empirical point of view, the article highlights that a standard estimation method of the hedonic price function may correctly fit the data in the presence of groupwise heterogeneity on the demand side but that implicit prices inferred from such an estimation may be substantially biaised. The article thus argues in favor of further econometric works investigating reliable statistical tests of the presence of groupwise heterogeneity on the demand side and suggesting alternative estimation methods that correctly deal with this phenomena.

## Acknowledgments

The authors are indebted to N. Kuminoff for valuable comments on a prelimary version of this paper. All remaining errors are our own.

## Notes

[1]"*Sufficiently interdependent in terms of cross price effects*" has to be understood as with "*cross price elasticities significantly different from zero or a given negative threshold*" in statistical tests. Note that the interdependent goods have to be substitutes and may thus exibit negative cross price elasticities.

[2]The case of horizontal differentiation of houses with groupwise heterogeneity of consumers is studied in Maslianskaïa-Pautrel (2012).

[3]Note that we consider that perfect information rules out the eventuality of different prices for houses with a same index of housing services. In this sense, we anticipate on the existence of a hedonic price function that will be demonstrated more formally latter on.

[4]Contrary to the indirect utility function, the constrained utility function does not assume an optimization behavior. It is obtained from direct utility of the individual $n$ by just substituting the quantity $X$ of the numeraire by the maximum quantity $R_n - P$ that the individual can afford given her individual budget constraint and the price $P$ associated with a given level $H$ of the index of housing services.

[5]Inequality (5) comes from the assumption of concavity of the utility function, which implies that the Hessian of $U(\cdot)$ is bounded.

[6]The subscript $n$ in the reference utility level points that the family of bid functions of each individual is parameterized by its own reference utility level.

[7]Thereafter, by abuse of language we will present condition (7) as "*the single-crossing condition of Spence-Mirrlees*".

[8]Note that, as long as we analyze vertical differentiation, it does not matter how the index of housing services $H$ is deduced from the set of attributes of houses. Note also that only the case of variability along a continuum is examined as regards the index of housing services. The reason for this is that not only important intrinsic characteristics (living area for instance) but also important environmental characteristics (typically the distance to a source of pollution or a source of amenity) are themselves continuous variables.

[9]Condition 3 in Definition 3 implies that, at market equilibrium, the density of demand for $H$ exactly fits the density of supply. Note that, whereas the density of supply is exogenous because of the focus on short term equilibrium, it is not the case of the density of demand. Indeed, what is exogenous on the demand side is the density of income which is only one of the determinants of the density of demand expressed as a density of the index of housing services demanded by buyers conditionally on market conditions.

[10]Maslianskaïa-Pautrel (2012) shows that in the case of horizontal differentiation, a complete sorting of consumers does not always result from the short term hedonic equilibrium.

[11]We could equivalently use the condition that buyers with the higher income level buy houses with the highest level of housing services. These two conditions are just two different ways to specify that is a sorting of buyers on the basis of their income.

[12]These characteristics must however be introduced in the estimation of the demand curve of the second stage (cf. Rosen, 1974, p.50).

[13]All simulations and estimations are implemented with *Mathematica*® 7.

[14]Data collected from the website of the Institut National de la Statistique et des Etudes Economiques (`http://www.insee.fr/fr/bases-de-donnees/`)

[15]For all the assumptions the theoretical model developped in the previous sections to be satisfied we only consider the case of an elasticity of substitution that exceeds one. See Appendix B.2 for more details on this point.

# Appendices

## A Proofs of Propositions and Theorems

### A.1 Proof of Proposition 6

Let $n$ and $m$ be two consumers belonging to a same group $j$:

$$V^n(H, P, R_j) = V^m(H, P, R_j) = V_j(H, P, R_j), \qquad \forall H,\ \forall P$$

Assume that, at market equilibrium, $P_n^* = E_n^*(H_n^*|R_j, u_j^n)$, $P_m^* = E_m^*(H_m^*|R_j, u_j^m)$ and $E_n^*(H_n^*|R_j, u_j^n) > E_m^*(H_n^*|R_j, u_j^m)$. Given that the reference utility level increases with the bids of consumers, we have $u_j^n < u_j^m$. It follows on that $V^n(H_n^*, P_n^*, R_j) < V^m(H_m^*, P_m^*, R_j)$ and, as a consequence, the incentives compatibility constraint (14) for individual $n$ is violated.

Following the same rationale, we are able to show that if at market equilibrium $E_n^*(H_n^*|R_j, u_j^n) < E_m^*(H_n^*|R_j, u_j^m)$ then the incentives compatibility constraint (14) for individual $m$ is also violated.

We conclude that, at market equilibrium we have

$$E_n^*(H|R_j, u^n) \equiv E_m^*(H|R_j, u^m) \equiv E_j^*(H|R_j, u_j^*), \qquad \forall n, m \in j, \forall H. \tag{A.1}$$

and we thus obtain Proposition 6. QED

### A.2 Proof of Proposition 7

Assume that Proposition 7 does not apply. Then, at market equilibrium, consumer $n$ would buy house 2, characterised by the combination of housing services and price $\{H_2, P_2\}$, whereas individual $m$ would buy house 1, characterised by the combination of housing services and price $\{H_1, P_1\}$. According to Proposition 5 we should have:

$$P_1 \geq E_n^{(2)}(H_1|R_n, u_n^{(2)}), \tag{A.2}$$

$$\text{with } E_n^{(2)}(H_2|R_n, u_n^{(2)}) = P_2. \tag{A.3}$$

If $E_m^{(1)}(H|R_m, u_m^{(1)})$ stands for the bid function of individual $m$ associated to market equilibrium, then:

$$E_m^{(1)}(H_1|R_m, u_m^{(1)}) > E_n^{(2)}(H_1|R_n, u_n^{(2)}) \tag{A.4}$$

For $E_m^{(1)}(H|R_m, u_m^{(1)})$ and $E_n^{(1)}(H|R_n, u_n^{(1)})$ so that $E_n^{(1)}(H_1|R_i, u_n^{(1)}) = P_1$ (i.e. the bid curve of $n$ passes through point $(H_1, P_1)$), the single crossing property and the fact that $R_n < R_m$ imply that:

$$\frac{dE_m^1}{dH}\Big|_{(H_1,P_1)} > \frac{dE_n^1}{dH}\Big|_{(H_1,P_1)}, \tag{A.5}$$

As $(H_1, P_1)$ is the only crossing point between the bid curves $E_n^{(1)}$ and $E_m^{(1)}$, combining with inequality (A.5) we obtain:

$$E_m^{(1)}(H|R_m, u_m^{(1)}) > E_n^{(1)}(H|R_n, u_n^{(1)}),\ \forall H > H_1 \tag{A.6}$$

Proposition 5 implies

$$E_n^{(1)}(H|R_n, u_n^{(1)}) > E_n^{(2)}(H|R_n, u_n^{(2)}),\ \forall H \in [H_{\min}, H_{\max}] \tag{A.7}$$

so that it follows on from (A.6) and (A.7) that

$$E_m^{(1)}(H_2|R_n, u_n^{(1)}) > E_n^{(2)}(H_2|R_n, u_n^{(2)}), \tag{A.8}$$

As (A.8) is in contradiction with Proposition 5 we conclude that Proposition 7 actually applies. QED

## *A.3 Proof of Theorem 9*

1°. According to Propositions 6 and 8, the hedonic price curve overlaps the bid curve of group $j$ over all the interval of values $]H_{j-1}, H_j[$ of the index of housing services. The continuity of the hedonic price function on the interval thus directly follows on from the continuity of the group's bid function.

2°. Assume that, at market equilibrium, there is a contact between groups $j$ and $j+1$ and let $H_j$ denote the corresponding level of the index of housing services.

Assume moreover that the hedonic price function is discontinous at this contact value of the index of housing services:

$$\exists H_j \ : \ E_j^*(H_j|R_j, u_j^*) \neq E_{j+1}^*(H_j|R_{j+1}, u_{j+1}^*) \tag{A.9}$$

Then, combining Proposition 7 with the fact that $R_j < R_{j+1}$ yields

$$E_j^*(H_j|R_j, u_j^*) < E_{j+1}^*(H_j|R_{j+1}, u_{j+1}^*).$$

Let set $E_j^*(H_j|R_j, u_j^*) = P^*(H_j)$. Then $P^*(H_j) < E_{j+1}^*(H_j|R_{j+1}, u_{j+1}^*)$, so that Proposition 5 is not satisfied. As a consequence, we obtain that

$$\forall j \in \{1, ..., J-1\} \ : \ E_j^*(H_j|R_j, u_j^*) = E_{j+1}^*(H_j|R_{j+1}, u_{j+1}^*). \tag{A.10}$$

Therefore 1° et 2° imply that the equilibrium hedonic price function is continuous and Theorem 9 is proven. QED

## *A.4 Proof of Proposition 10*

1°. By continuity of the equilibrium hedonic price function (Theorem 9) we know that:

$$\forall j \in \{1, ..., J-1\} \ : \ P_j^*(H_j) = P_{j+1}^*(H_j). \tag{A.11}$$

We deduce that

$$\forall j \in \{1, ..., J-1\} \ : \ E_j^*(H_j|R_j, u_j^*) = E_{j+1}^*(H_j|R_{j+1}, u_{j+1}^*), \tag{A.12}$$

and therefore

$$\forall j \in \{1, ..., J-1\} \ :$$

$$V(H_j, E_j^*(H_j|R_j, u_j^*), R_j) = V(H_j, E_{j+1}^*(H_j|R_{j+1}, u_{j+1}^*), R_j); \tag{A.13}$$

$$V(H_j, E_{j+1}^*(H_j|R_{j+1}, u_{j+1}^*), R_{j+1}) = V(H_j, E_j^*(H_j|R_j, u_j^*), R_{j+1}). \tag{A.14}$$

By definition of the bid function of group $j$ we can write:

$$V(H, E_j^*(H|R_j, u_j^*), R_j) = u_j^*, \ \forall H, \tag{A.15}$$

(A.13) and (A.15) imply that

$$\forall j \in \{1, ..., J-1\}, \quad V(H, E_j^*(H|R_j, u_j^*), R_j) = V(H_j, E_{j+1}^*(H_j|R_{j+1}, u_{j+1}^*), R_j), \tag{A.16}$$

$$\forall H \in [H_{j-1}, H_j].$$

Similarly, by definition of the bid function of group $j+1$ we can write:

$$V(H, E^*_{j+1}(H|R_{j+1}, u^*_{j+1}), R_{j+1}) = u^*_{j+1},\ \forall H, \tag{A.17}$$

(A.14) and (A.17) then imply that

$$\forall j \in \{1, ..., J-1\}\ :\ V(H, E^*_{j+1}(H|R_{j+1}, u^*_{j+1}), R_{j+1}) = V(H_j, E^*_j(H_j|R_j, u^*_j), R_{j+1}),$$
$$\forall H \in [H_j, H_{j+1}]\,.$$

or equivalently,

$$\forall j \in \{2, ..., J\}\ :\ V(H, E^*_j(H|R_j, u^*_j), R_j) = V(H_{j-1}, E^*_{j-1}(H_{j-1}|R_{j-1}, u^*_{j-1}), R_j), \tag{A.18}$$
$$\forall H \in [H_{j-1}, H_j]\,.$$

Equations (A.16) and (A.18) just state that the incentives compatibility constraints of group $j$ with the adjacent groups are biding. The next step is to prove that they there are the only ones that are binding.

2°. Assume that the incentive compatibilty constraint of group $j$ with another group than groups $j-1$ and $j+1$ is also biding. Let index $i$ denote this group and assume that $i < j-1$. By assumption we have:

$$V(H, E^*_j(H|R_j, u^*_j), R_j) = V(H_i, E^*_i(H_i|R_i, u^*_i), R_j),\ \forall H \in [H_{j-1}, H_j]\,. \tag{A.19}$$

(A.18) and (A.19) then induce

$$(H_{j-1}, E^*_{j-1}(H_{j-1}|R_{j-1}, u^*_{j-1})) \in E^*_j(H_i|R_j, u^*_j) \tag{A.20}$$
$$(H_i, E^*_i(H_i|R_i, u^*_i)) \in E^*_j(H_i|R_j, u^*_j) \tag{A.21}$$

Applied to the bid curves $E^*_{j-1}(H_{j-1}|R_{j-1}, u^*_{j-1})$ and $E^*_j(H_j|R_j, u^*_j)$, the single crossing property implies that:

$$E^*_j(H_i|R_j, u^*_j) < E^*_{j-1}(H_i|R_{j-1}, u^*_{j-1}),\ \text{because } H_i < H_{j-1} \tag{A.22}$$

(A.21) and (A.22) finally imply that

$$P^*(H_i) = E^*_i(H_i|R_i, u^*_i) < E^*_{j-1}(H_i|R_{j-1}, u^*_{j-1}). \tag{A.23}$$

which is in contradiction with Proposition 5 so that it cannot be the case that $i < j-1$.

Similarly, if $i > j+1$, it follows on from the single crossing property that

$$P^*(H_i) = E_i^*(H_i|R_i, u_i^*) < E_{j+1}^*(H_i|R_{j+1}, u_{j+1}^*). \tag{A.24}$$

which is also in contradiction with 5. Thus it cannot be the case that $i > j+1$.

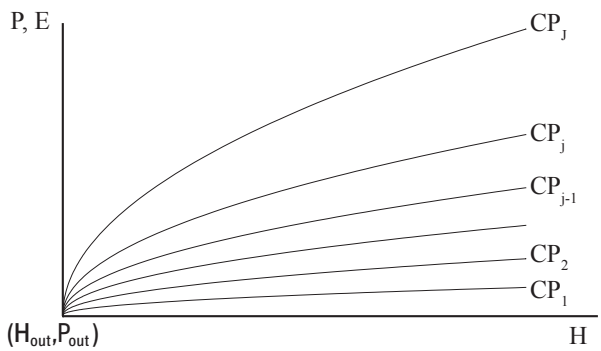
According to 1° and 2°, only incentives compatibility constraints with adjacent groups are biding at the hedonic equilbrium. This is what Proposition 10 states.

## *A.5 Proof of Proposition 11*

The participation constraint induces that for each group the equilibrium bid curve is either located below the bid curve associated with the "outside the market" alternative or just corresponds to this specific bid curve. By construction, for all groups, the bid curve passing through point $(H_{out}, P_{out})$ corresponds to the participation constraint. A first consequence of the single crossing property is that point $(H_{out}, P_{out})$ is their common crossing point. Given that $H_{out} < H_{min}$, a second consequence of the single crossing property is that the higher the index of a group, the higher its participation constraint on the interval $[H_{min}, H_{max}]$ (cf. figure 5, p. 52):

$$\forall H, \quad CP_1(H) < CP_2(H) < ... < CP_J(H) \tag{A.25}$$

Figure 5: Participation constraints of the different groups of consumers




Income is the only source of heterogeneity among consumers. Groups are ordered and indexed according to the increasing level of income.

1°. Assume that the participation constraint for a group $j > 1$ is biding at market equilibrium. Accordingly, we have:

$$P^*(H) = CP_j(H),\ \forall H \in [H_{j-1}, H_j]\ :\ V(H, CP_j(H), R_j) = V(H_{out}, P_{out}, R_j). \tag{A.26}$$

(A.25) and (A.26) induces that

$$P^*(H) > CP_1(H),\ \forall H \in [H_{j-1}, H_j] \tag{A.27}$$

As the equilibrium bid curve is located below or just corresponds to the participation constraint for each group, we can write that for group 1:

$$E_1^*(H|R_1, u_1^*) \leq CP_1(H),\ \forall h \in [H_{min}, H_1] \tag{A.28}$$

(A.27), (A.28) and the single crossing property applied to participation constraints of groups 1 and $j$ implies that $E_1^*(H|R_1, u_1^*) < E_j^*(H|R_j, u_j^{(0)}),\ \forall H \in [H_{j-1}, H_j]$. This is in contradiction with Proposition 5. Thus, participation constraints for other groups than group 1 cannot be biding at equilibrium.

2° Assume now that the particpation constraint is not biding for group 1 at equilibrium. By assumption, the equilibrium bid curve $E_1^*$ for group 1 then satisfies the following inequality:

$$V(H, E_1^*(H|R_1, u_1^*), R_1) > V(H_{out}, P_{out}, R_1),\ \forall H \in [H_{min}, H_1]\,. \tag{A.29}$$

It follows on that $P^*(H) < CP_1(H),\ \forall H \in [H_{min}, H_1]$, which is in contradiction with the assumption that sellers extract the maximum surplus from buyers at equilibrium subject to the group participative constraint (conditions (12) and (13)).

1° and 2° imply that, as stated in Proposition 11, the participation constraint of group 1 is the only one that is biding at equilibrium. QED

# B Solving for the equilibrium hedonic price function in Rosen's model

## *B.1 The Cobb-Douglas case*

The optimal choice of a buyer with income $R$ solves the following maximisation program:

$$\max_H U(H) = H^{\beta}(R - p(H))^{1-\beta} \tag{B.1}$$

where $\beta$ is an exogenous preference parameter. $\beta$ satisfies $0 < \beta < 1$ and takes the same value for all buyers.

The corresponding first order condition can be writen as:

$$p_H(H) = \frac{\beta}{1-\beta}\frac{R - p(H)}{H} \tag{B.2}$$

The right hand side of (B.2) is the slope of the bid function. It is increasing with respect to income $R$ so that the single crossing property of bid curves is satisfied. It follows on that the optimal level of the index of housing services is an increasing function $f(R)$ of income so that we are able to define the reciprocal function $R = g(H)$:

$$g(H) = \frac{1-\beta}{\beta}\, p_H(H)\, H + p(H) \tag{B.3}$$

The optimal level of the index of housing services that is demanded by buyers is thus distributed on the interval $[g^{-1}(R_{min}), g^{-1}(R_{max})]$ with the following density function:

$$\kappa(H) = \psi(g(H))\, g_H(H) \tag{B.4}$$

As we assume that income is continously and uniformly distributed on the interval $[R_{min}, R_{max}]$, the corresponding density function for income is:

$$\psi(R) = \frac{1}{R_{max} - R_{min}}$$

The resulting density function $\kappa(H)$ for the index of housing services on the demand side can then be written as

$$\kappa(H) = \frac{1}{R_{max} - R_{min}}\left[\frac{1-\beta}{\beta} H\, p_{HH}(H) + \frac{1}{\beta} p_H(H)\right] \tag{B.5}$$

On the supply side, the index of housing services is assumed to be continously and uniformly distributed on the interval $[H_{min}, H_{max}]$. The corresponding density functions reads:

$$\phi(H) = \frac{1}{H_{max} - H_{min}} \tag{B.6}$$

The equilibrium hedonic price function that ensures market clearing then solves the following condition:

$$H\, p_{HH}(H) + \frac{1}{1-\beta} p_H(H) - \frac{\beta}{1-\beta} \frac{R_{max} - R_{min}}{H_{max} - H_{min}} = 0,\ \forall H \in [H_{min}; H_{max}] \tag{B.7}$$

The analytical form of the general solution to (B.7) can be written as:

$$p(H) = \beta \frac{R_{max} - R_{min}}{H_{max} - H_{min}} H + \frac{1-\beta}{\beta} C_1\, H^{-\frac{\beta}{1-\beta}} + C_2, \tag{B.8}$$

where $C_1$ and $C_2$ are two unkwown constants of integration. In order to fully characterise the solution to (B.7), we use the two initial conditions given in (32)-(33) and solve them with respect to $C_1$ and $C_2$. We then obtain:

$$\begin{aligned} C_1 =& \frac{\beta}{1-\beta} \left[ H_{out}^{\frac{\beta}{1-\beta}} P_{out} + R_{min} \left( H_{min}^{\frac{\beta}{1-\beta}} - H_{out}^{\frac{\beta}{1-\beta}} \right) \right] + \beta \frac{H_{min}^{\frac{1}{1-\beta}} R_{max}}{H_{max} - H_{min}} - \\ & \frac{\beta}{1-\beta} H_{min}^{\frac{\beta}{1-\beta}} R_{min} \frac{H_{max} - \beta H_{min}}{H_{max} - H_{min}} \end{aligned} \tag{B.9}$$

$$C_2 = \frac{H_{max} R_{min} - H_{min} R_{max}}{H_{max} - H_{min}} \tag{B.10}$$

The analytical expression of the hedonic price function is then obtained from (B.8) by replacing $C_1$ and $C_2$.

## *B.2 The CES case*

The optimal choice of a buyer with income $R$ solves the following maximisation program:

$$\max_H U(H) = [\alpha H^\sigma + (1-\alpha)(R - p(H))^\sigma]^{\frac{1}{\sigma}} \tag{B.11}$$

where $\alpha$ and $\sigma$ are two preference parameters satisfying $0 < \alpha < 1$ and $\sigma \neq 1$. As consumers are homogenous in terms of preference, $\alpha$ and $\sigma$ takes identical values for all of them.

The first order condition associated to the optimal choice of a consumer with income $R$ may be written as:

$$p_H(H) = \frac{\alpha}{1-\alpha}\left[\frac{R-p(H)}{H}\right]^{1-\sigma} \tag{B.12}$$

As in the Cobb-Douglas case, the right hand side of (B.12) is the slope of the bid function. It is increasing with respect to income $R$ if and and only if $\sigma < 1$, a condition that is assumed to be satisfied thereafter. As a result, the single crossing property of bid curves is satisfied. It follows on that the optimal level of the index of housing services is an increasing function $f(R)$ of income so that we are able to define the reciprocal function $R = g(H)$:

$$g(H) = \left(\frac{\alpha}{1-\alpha}\right)^{-\frac{1}{1-\sigma}} [p_H(H)]^{\frac{1}{1-\sigma}}\ H + p(H) \tag{B.13}$$

The distribution of income is also assume to be the same than in the Cobb-Douglas case. We are then able to obtain the density function characterising the distribution of the index of housing services on the demand side :

$$\begin{aligned}\kappa(H) =& \psi(g(H))\, g_H(H) = \\ =& \frac{1}{R_{max}-R_{min}}\left[\left(\frac{\alpha}{1-\alpha}\right)^{-\frac{1}{1-\sigma}} (p_H(H))^{\frac{\sigma}{1-\sigma}}\left[\frac{1}{1-\sigma}H\, p_{HH}(H) + p_H(H)\right] + p_H(H)\right]\end{aligned} \tag{B.14}$$

Under the assumption that the distribution of the index of housing services is similar to that used in the Cobb-Douglas case, the equilibrium hedonic price function that ensures market clearing then solves the following conditions:

$$\begin{aligned}&\frac{1}{1-\sigma}H\, p_{HH}(H) + p_H(H) + \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\sigma}} [p_H(H)]^{\frac{1-2\sigma}{1-\sigma}} - \\ &- \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\sigma}} [p_H(H)]^{-\frac{\sigma}{1-\sigma}}\ \frac{R_{max}-R_{min}}{H_{max}-H_{min}} = 0,\ \forall H \in [H_{min}; H_{max}]\end{aligned} \tag{B.15}$$

$$p_H(H_{min}) = \frac{\alpha}{1-\alpha}\left[\frac{R_{min}-p(H_{min})}{H_{min}}\right]^{1-\sigma} \tag{B.16}$$

$$[\alpha H_{min}^{\sigma} + (1-\alpha)(R_{min}-p(H_{min}))^{\sigma}]^{\frac{1}{\sigma}} = [\alpha H_{out}^{\sigma} + (1-\alpha)(R_{min}-P_{out})^{\sigma}]^{\frac{1}{\sigma}} \tag{B.17}$$

Unfortunatly, because of the high non linearity of equation (B.15) no analytical solution can be found and we have to rely exclusively on numerical solutions.

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