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# Asymmetries in Rent-Seeking*

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## Abstract

In rent-seeking contests, players are seldom identical to one another. In this chapter, we examine the rent-seeking literature that explores the effects of specific forms of asymmetry between contestants. We consider Tullock's rent-seeking contests involving two players who differ in strength (marginal returns to effort), motivation (valuations of the sought-after rent) and cunning (bargaining power). We study the combined interaction of these three possible forms of asymmetry in rent-seeking. We examine how these asymmetries affect the rent-seeking contest and investigate the effect of ex post trading opportunities on the players' efforts, on probabilities of winning and on the social costs of rent-seeking.

Keywords: rent-seeking games, returns to effort, asymmetric rents, asymmetric strength, tradable rents.

JEL Codes: D72.

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# 1 Introduction

In real world competitive contests, players are rarely identical to one another. Just like other competitive scenarios, rent-seeking contests are seldom symmetric. Players, more often than not, exhibit non-trivial differences with respect to their competitive abilities, resources and costs, motivation, bargaining power, etc. The existing rent-seeking literature has analyzed the effects of players' asymmetric characteristics in isolation. In this chapter, we wish to review the existing literature, contributing to the understanding of the effect of various asymmetries in rent-seeking. We consider the interaction between various forms of asymmetries, and we organize ideas around three relevant characteristics: strength, motivation and cunning. We use the term *strength* to refer to a player's marginal returns to effort: effort by a strong player increases his probability of winning more than the effort of a weak player. *Motivation* refers to a player's valuation of the prize at stake: motivated players value the price more than unmotivated players. Both strength and motivation have relevance for the rent-seeking phase of the game. When rents are tradable, the players enter a second phase in which they can negotiate over who gets the rent. Typically, the motivated player buys the rent from his unmotivated counterpart. In this phase, strength plays no role and is replaced by *cunning*, a player's ability to strike a better deal: his negotiating abilities and bargaining power.

This chapter is organized as follows. In section 2, we discuss various examples of asymmetries in the existing literature. Literature on non-tradable rents has studied the effect of players' asymmetries in strength and motivation (or both), while the literature on tradable rents has focused on the role of cunning for players with asymmetric motivation. In section 3, we look at some of the interactions between various asymmetries, introducing a two-player rent-seeking model with and without opportunity for resale. We study the players' efforts and probabilities of winning and the social costs of rent-seeking. In section 4, we compare the results of the rent-seeking games discussed in the previous sections. More specifically, in section 4.1 we consider whether the introduction of ex post trading opportunities exacerbates or mitigates the effect of asymmetries between players, looking at the effect of ex post trade on ex ante incentives to exert effort in the primary contest. In section 4.2,in order to set some landmarks for the study of the combined effects of strength, motivation and cunning, we introduce the notions of *balanced rent-seeking* and *fair negotiations*. In balanced rent-seeking, one player's strength perfectly offsets the other-player's motivation. With fair negotiations, a player's cunning in the negotiation phase compensates his lack of strength in the rent-seeking phase. In section 5, we conclude with some additional analysis and policy considerations.

## 2 Asymmetries in the literature

Tullock's (1980) seminal paper and much of the early rent-seeking literature considered a wide array of rent-seeking contest in which contestants were identical to one another. It is only with more recent contributions starting in the late 1980s that the effects of specific forms of asymmetry between the contestants began to draw the attention of rent-seeking scholars.[1]

In the basic rent-seeking setting, players compete for the appropriation of a rent, which cannot be transferred ex post between players. In this chapter, we shall refer to this underlying assumption as "nontradable rent."The literature that assumes non-tradable rents focuses its attention on the players' asymmetries in strength and motivation. The reason for this focus is that when rents are nontradable, the players' motivation is fully at work: a high-valuing player who loses the contest cannot subsequently acquire the rent from the winner. Without an ex post trading opportunity, however, differences in the parties' bargaining power have no effect on rent-seeking incentives. Within this body of literature, Allard (1988) is the first contribution to introduce the possibility of differences among players in a rent-seeking game, while an earlier contribution by Harris and Vickers (1985) studied sequential investments by two asymmetric parties in a race. Differences in the parties' strength are generally modeled as parameters added to the rent-seeking return functions or cost functions. These asymmetric parameters can affect the effectiveness of the parties' efforts, denoted by the coefficients preceding their effort levels, and the returns to scale in their efforts, denoted by the exponents of the parties' effort levels. Baik (1994 and 2004) is an example of the former formulation of the problem, in which different strength is modeled as a parameter (under the form of a coefficient that multiplies the effort of a party), so that the share of party A is $\frac{sA}{sA+B}$ with$s > 0$. In this model players exhibit different strength, as well as different motivation, which is modeled by allowing the value of the rent to vary between the players. One of Baik's results is that more strength can compensate the effect of more motivation.[2] The model developed in Section 3 is an example of asymmetries in the parties' returns to scale, denoted by the exponents of the parties' effort levels, such that the share of party A is $\frac{A^a}{A^a+B^b}$ where $a \neq b$. A third way in which differences in the parties' strength have been depicted in the rent-seeking literature can be found in Ryvkin (2007), who models differences in strength as a difference in cost parameter: asymmetric players face different (constant) marginal costs of effort. Finally,

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[1]Literature on non-tradable rents has studied the effect of players' asymmetries in strength (Allard, 1988; Kohli and Singh, 1999; Singh and Wittman, 2001; Bennour, 2009) and motivation (Hillman and Riley, 1989; Leininger , 1993; Hurley, 1998; Nti, 1998 and 1999; Matros, 2006) or both (Baik, 1994 and 2004; Nitzan, 1994; Stein, 2002), while the literature on tradable rents has focused on the role of cunning for players with asymmetric motivation (Suen, 1989; Dari-Mattiacci, Onderstal and Parisi, 2009).

[2] Kohli and Singh (1999) model a rent-seeking contest with players with different strength, as in Baik (1994), combine asymmetry in strength with endogenous rent formation and multiple periods. Bennour (2009) modelsdifferences in the parties' strength as in Baik (1994), analyzing the likelihood of conflict in a predator-prey setting. Singh and Wittman (2001)present a model with asymmetric strength, in which the information about the strength is private. Their focus is more closely related to the literature on contest design and auctions than to the standard rent-seeking literature. Stein and Rapoport (2004) study asymmetric contests with two stages.

Cornes and Hartley (2005) model different strength, with a general implicit function of the effort, to study existence and uniqueness of equilibria and rent dissipation.

The rent-seeking literature has considered the effects of asymmetric players' motivation on rent-seeking behavior, by modeling players with different valuations of the sought-after rents. Most notably, Hillman and Riley (1989) show that asymmetric valuations may inhibit the participation of low-valuing contenders. Their application of the model focuses on political influence contests and provides a possible explanation of several stylized facts in the empirical literature, including the fact that only a small numbers of players actively participate in lobbying and thatpolitical markets to exercise political influence are similarly concentrated.[3] Leininger (1993) formulated a model with players with different rent valuations, focusing on the effect of sequential moves. Stein (2002) and Matros (2006) subsequently considered the effect of differences in rent valuations on rent-seeking expenditures, in the context of an *N*-player model. In Matros (2006), valuations of the players varied across players. The author showed that one of the results of Tullock's (1980) analysis did not hold when asymmetries in valuation were taken into consideration. Similar to the conventional result, total rent dissipation increased in *N* and individual winning probabilities decreased in *N*. Differently from the conventional result, individual rent-seeking efforts could under certain conditions actually increase(rather than decrease, as in Tullock (1980)), with the addition of a player. Stein (2002) models rent-seeking contests in which players exhibit different motivation and strength. Strength is modeled as in Baik (1994), and extends the results to consider *N*-player contests.[4] The paper shows that rent dissipation increases in the valuation of each player. A player's expenditure increases in his own valuation. Both players' efforts increase in the valuation of the highest-valuing player. An increase in a player's valuation makes his probability of winning and his payoff increase, but makes the probability of winning and the payoff of the other player decrease. A similar effect of strength is revealed. Nti (2004) develops a model with asymmetric valuations, showing that efforts may be reduced when players with different rent valuations participate in a contest.[5] Nti approaches the problem from the point of view of contest design—aimed at maximizing contestants' effort, rather than minimizing rent-dissipation—hence suggesting the use of value-weighted contest (discounting the effort of the high-valuation player) in order to induce him to make greater effort.[6]

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[3] Nitzan's (1994) survey of rent-seeking models includes a review of the prior literature on asymmetric valuations and strength.

[4] Runkel (2006) also models a rent-seeking contest in which players exhibit different valuations of the rent and different strength, but strength is modeled with an implicit function, not with parameters of Tullock's standard return function.

[5] Nti (1998 and 1999) had previously developed a model with asymmetric valuations, considering the existence of equilibria in pure strategy.

[6] There is a wide array of related literature that falls outside the traditional framing of the rent-seeking problem, but which is worthy of mention. Donohue and Levitt (1998) model a contest with asymmetric players and imperfect information where parties are allowed to exit the contest. They do not use Tullock success function, but interestingly highlight the effect of an exit option on players' strategies. Hirshleifer (2001) similarly analyzes several models in which asymmetries between players are important, but often uses a different success function from Tullock's.

The effect of differences in the parties' valuations is not limited to parties' rent-seeking incentives. When valuations are asymmetric, a new dimension of the rent-seeking context acquires relevance: allocative efficiency. In the absence of ex post markets, when a low-valuing contestant appropriates the rent, an allocative loss arises. Hurley (1998) modeled rent-seeking contests with players with different valuations, introducing the notion of contest efficiency as an alternative to the traditional measure of rent dissipation. When examining contest with asymmetric valuations, the criteria for evaluating the outcomes of the contest should also include contest efficiency—defined as the measure of the maximum obtainable benefit captured by the contestants. The allocative gains may offset rent dissipation, such that a contest may be more efficient notwithstanding the greater rent-seeking expenditures incurred by the parties.

In our analysis, we shall extend the standard model to look at the effects of ex post transfer opportunities on the parties' rent-seeking incentives. We shall refer to this alternative setting as "tradable rents." The literature on tradable rents has instead focused on the role of cunning for players with asymmetric motivation. Suen (1989) considers differences in strength (modeled as a cost of exerting effort) and valuations, but the model is not a traditional Tullock contest but a model of waiting; the model also includes a trading phase where trading does not necessarily reduce dissipation. Dari-Mattiacci, Onderstal and Parisi (2009) analyze the effect of ex post market in Tullock's traditional rent-seeking game by allowing the parties' valuations to differ. In the first stage, contestants compete to appropriate the rent; in the second stage, if a misallocation occurs (i.e., the low-valuing contestant wins the rent), the winner can resell the rent to the loser. The authors focus on the ex ante effects of a secondary market on efforts, payoffs, rent-dissipation and rent-misallocation, showing that tradable rents have a double-edge effect. Ex post markets correct possible misallocations but may exacerbate rent dissipation. In some situations, the increase in rent dissipation more than offsets the allocative advantage of the ex post trade—the availability of a secondary market might actually reduce welfare. In this chapter, we will build on their setup to study the interaction of strength, motivation and cunning in the case of asymmetric players with resale opportunities.

The results of the asymmetric rent-seeking literature (and the interaction of different forms of asymmetry) are important for the understanding of real life contests among asymmetric players, when the players differ in more than one characteristic. We study how these asymmetries affect the rent-seeking game, comparing the outcomes obtained in the presence of secondary markets, or lack thereof, where rents can be reallocated after the contest (tradable versus non-tradable rents). In particular, we focus on the players' efforts and probabilities of winning and on the social costs of the rent-seeking game. With non-tradable rents we consider rent-dissipation—the sum of the players' efforts—and rent misallocation—the probability that the unmotivated, low-valuing player wins the contest times the difference in the valuations. With tradable rents, misallocation disappears and only rent dissipation constitutes a social cost. We look at whether opening trade increases or reduces the total social costs. The three characteristics that we consider play different roles in a rent-seeking contest. Strength and cunning are advantages that become relevant at different stages of the game. Strength is relevant in the rent-seeking stage (primary market), while cunning is relevant in the reallocation stage (secondary market). Motivation instead is an advantage that consists of the larger surplus that is obtainable by the parties. The effect of motivation is relevant in both stages of the game, although its effect differs depending on whether trade is allowed or not. To put the model used in this chapter in the context of the existing literature, we should note that although strength is usually modeled as Baik (1994), as a multiplier of the effort of one of the parties, we use Tullock's exponent. Further, asymmetry in valuation is usually said to reduce dissipation (Hillman and Riley, 1989); we show that, by adding the rent misallocation loss, the maximum total social loss remains constant, with and without asymmetry. We finally show that the maximum total social loss is less with tradable rents in general.

## 3 Rent-seeking with asymmetric players

In the traditional Tullock rent-seeking game,[7] two parties compete to appropriate a rent of common value $V > 0$ (which motivates the parties to play) by investing each a nonnegative amount of effort denoted by $A$(he) and $B$ (she), respectively. The rent is shared according to the now classic Tullock success function $Q_A = \frac{A^r}{A^r+B^r}$ for party A and $Q_B = \frac{B^r}{A^r+B^r}$ for party B. We can alternatively interpret these shares as probabilities to obtain the entire rent. The exponent $r > 0$ indicates the parties' returns to effort (their strength). With $r < 1$ parties have decreasing returns to effort, which implies that the party who exerts more effort obtains a less than proportional increase in his or her share of the rent; that is, if A invests more than B, then the ratio of their shares in the rent is $\frac{Q_A}{Q_B} = \left(\frac{A}{B}\right)^r < \frac{A}{B}$, giving A a less than proportional return to his or her effort. With $r = 1$ parties have constant returns to effort, and the rent is shared proportionally to the parties investments and $\frac{Q_A}{Q_B} = \frac{A}{B}$. Finally, with $r > 1$, parties have increasing returns to effort so that the party exerting more effort obtains a more than proportional advantage; if for instance A exerts more effort than B, we have $\frac{Q_A}{Q_B} = \left(\frac{A}{B}\right)^r > \frac{A}{B}$. The parties' return-to-effort exponent determines their incentives to exert effort and hence the amount of resources that will be dissipated in total in the rent-seeking contest. The standard result is that total rent dissipation—that is, the sum of the parties' investments in rent-seeking—will be equal to $\frac{r}{2}V$, that is, it will increase in the parties' return to effort and in their valuation of the rent.[8] Thus, stronger (higher $r$) and more motivated (higher $V$) parties exert more effort. Since the sought-after rent has the same value for both contestants, ex post resale opportunities are not considered in this framework.

This basic framework considers identical parties and will be extended in three directions in order to consider as many dimensions along which parties can differ. The first dimension concerns the parties' strength. While the traditional Tullock game assumes that parties have the same strength (return-to-effort exponent) $r$, we will consider here parties with different levels of

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[7] Tullock (1980 and 1985); Tollison (1982); Rowley, Tollison, and Tullock (1988); Lockard and Tullock (2001).

[8] The usual condition is $0 < r \leq 2$, which guarantees that the sum of the parties' efforts does not exceed the value of the rent and hence gives pure-strategy solutions.

strength $a$ and $b$, respectively. Therefore, the rent will be shared according to the parties' strength, and Tullock's success functions become $S_A = \frac{A^a}{A^a+B^b}$ and $S_B = \frac{B^b}{A^a+B^b}$. We will say that A is stronger than B if $a > b$ and vice versa.

Note also that with different levels of strength, the interpretation of the exponents $a$ and $b$ as returns to effort is not as straightforward as with $r$. In fact, in this case the ratio of the parties' shares is $\frac{S_A}{S_B} = \frac{A^a}{B^b}$ and it is no longer true that the share in the rent of the party investing more is more or less than proportional to his or her effort depending on whether that party's strength is greater or less than 1. In fact, this relationship also depends on the other party's strength in a rather complex way. To preserve the traditional interpretation it is therefore necessary to examine how the ratio of the parties' share in the rent changes if effort changes. For instance, if party A increases his of her effort, the ratio changes in his or her favor at a decreasing rate if $a < 1$, at a constant rate if $a = 1$, and at an increasing rate if $a > 1$. These results hold symmetrically true for party B.[9] Therefore, we can speak of the parties' asymmetric levels of strength in the same way as in the traditional Tullock framework.

The second departure from the traditional framework consists of allowing the parties to have different valuations for the rent, so that party A values the rent at $V_A$ and party B values it at $V_B$, that is, they have different motivations to exert effort. Introducing differences in motivation has two sizable effects. Motivation changes the parties' incentives to exert effort in a similar way as strength does, so that we expect strong motivated parties to be willing to invest more than weak unmotivated contestants. Yet, motivation has also an additional and possibly more radical effect on the rent-seeking game. Since parties attach different values to the rent, it is in theory possible—and even socially desirable—that a low-valuing winner of the contest will sell the rent to the high-valuing loser. Differences in valuations open the door to a secondary market for rents, which cannot be conceived in the traditional, homogeneous-valuation framework.

Analyzing the case of ex post resale of the rent makes it possible to introduce a third dimension of asymmetry: the parties' cunning, that is, their ability to obtain a more favorable price during the negotiations. This asymmetry can be easily represented by the resale price $P$. By hypothesis, the price will have to lie somewhere between the higher valuation—which determines the maximum price that the high-valuing party is willing to pay for the rent—and the lower valuation—which determines the minimum price that the low-valuing party is willing to accept. While strength and motivation were introduced as absolute characteristics of the parties, cunning is necessarily a relative notion: a high price means that the low-valuing seller is more cunning—a

[9] Formally, we have $\frac{\partial}{\partial A}\left(\frac{A^a}{B^b}\right) = \frac{aA^{a-1}}{B^b} > 0$ and $\frac{\partial^2}{\partial A^2}\left(\frac{A^a}{B^b}\right) = (a-1)\frac{aA^{a-2}}{B^b}$, which is less than, equal to or greater than zero depending on whether $a < 1$, $a = 1$ or $a > 1$, respectively. The same holds symmetrically for B: $\frac{\partial}{\partial B}\left(\frac{B^b}{A^a}\right) = \frac{bB^{b-1}}{A^a} > 0$ and $\frac{\partial^2}{\partial B^2}\left(\frac{B^b}{A^a}\right) = (b-1)\frac{bB^{b-2}}{A^a}$. The case with $a = b = r$ can be seen as a special case of this more general framework. See Dari-Mattiacci and Parisi (forthcoming) for a discussion of the possible interpretations of the returns to effort in rent-seeking games.

more seasoned negotiator—than the high-valuing buyer and, vice versa, a low price indicates that the high-valuing party has the upper hand during the negotiation.

The presence of a trading phase after the rent-seeking phase of the game is important for social welfare as well as for the parties' incentives. Knowing that he can buy the rent ex post might reduce the incentives of the high-valuing party, while making the low-valuing party implicitly more motivated by the resale opportunity. In fact, the availability of trade removes the asymmetry in motivation and replaces it with an asymmetry in cunning, which operates in a markedly different way. To anticipate one of the aspects of rent-seeking with resale, parties with different valuations play a symmetric game due to the fact that the rent is now implicitly valued at its resale price by both contestants. In the following, we study the consequences of asymmetries in strength, motivation and cunning for the parties and for society. In the analysis we focus exclusively on the characteristics of pure-strategy Nash equilibria. Each section will be closed with a proposition summarizing the main results. All propositions are restricted to pure-strategy equilibria.[10] We start with non-tradable rents.

## 3.1 Non-tradable rents

We consider two rent-seeking contestants, A and B, who differ in their strength and motivation. Without loss of generality, throughout this chapter we assume $V_A > V_B$, i.e., we assume that player A values the sought-after rent more than player B and is thus the more motivated player. Instead, we allow either player to be stronger than the other; with $a > b$ the more motivated player A is also stronger; with $a = b$ players have equal strength; and with $a < b$ the less motivated player B is stronger than the more motivated A. The contestants earn the following expected payoffs from the game: $U_A = \frac{A^a}{A^a+B^b} V_A - A$ and $U_B = \frac{B^b}{A^a+B^b} V_B - B$, respectively. We denote the parties' probabilities of winning as $S_A = \frac{A^a}{A^a+B^b}$ and $S_B = \frac{B^b}{A^a+B^b}$, so that we can rewrite the parties' payoffs more concisely as $U_A = S_A V_A - A$ and $U_B = S_B V_B - B$. The parties choose their levels of rent-seeking efforts independently of each other in order to maximize their payoffs. The first order conditions for this maximization problem yield the following implicit relationships between the parties' equilibrium levels of effort and their strength and motivation—an asterisk denotes equilibrium values:

$$
\begin{aligned}
A^* &= a S_A^* S_B^* V_A \\
B^* &= b S_A^* S_B^* V_B
\end{aligned}
\tag{1}
$$

[10] On mixed-strategy equilibria, see Dari-Mattiacci, Langlais, Lovat, and Parisi (2007) and references therein.

By some simple manipulations of (1) we have[11] $S^*_A S^*_B = \frac{A^*}{aV_A} = \frac{B^*}{bV_B} \leq \frac{1}{4}$, which confirms a well-known result in the symmetric case: in a pure-strategy equilibrium, the players' efforts are bounded by $A^* \leq \frac{a}{4} V_A$ and $B^* \leq \frac{b}{4} V_B$. Yet, levels of effort satisfying first order conditions constitute a pure-strategy equilibrium only if they lead to nonnegative payoffs for the parties, since it is always possible for a party to exert no effort and earn a payoff equal to zero.[12] This condition[13] yields an upper boundary for the parties' shares in equilibrium:

$$S^*_A \leq \frac{1}{b}$$
$$S^*_B \leq \frac{1}{a} \tag{2}$$

Since $S_A = 1 - S_B$, these conditions imply that in a pure-strategy equilibrium the parties' shares cannot be excessively far away from each other. In a sense, for parties to play a pure-strategy equilibrium the outcome must be somewhat fair. The conditions above are satisfied for any value of $A$ and $B$ if both$a$ and $b$ are less than 1,that is, when both parties have decreasing marginal returns to effort (no economies of scale in rent-seeking effort) the first order conditions are sufficient to identify a pure-strategy equilibrium.[14] Yet, if this is not the case, some outcomes that satisfy the first-order conditions lead to too extreme a sharing of the rent and do not constitute a pure-strategy equilibrium.[15]

A player's equilibrium payoff increases in his own valuation and strength but decreases in the valuation and strength of the other player;[16] hence the conditions above put some limits to the

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[11] The inequality follows from the fact that $S^*_A + S^*_B = 1$ and hence the maximum of $S^*_A S^*_B$ is reached when players face equal probability of winning, $S^*_A = S^*_B = \frac{1}{2}$.

[12] $\frac{\partial U_A}{\partial A} = \frac{a}{A} S_A S_B V_A - 1 = 0$ and $\frac{\partial U_B}{\partial B} = \frac{b}{B} S_A S_B V_B - 1 = 0$. An alternative way to define Nash equilibria is to assume that there are minimum levels of the parties' efforts and to look for mixed-strategy equilibria. See Dari-Mattiacci, Langlais, Lovat, and Parisi (2007).

[13] Using (1) and noting that $S_A = 1 - S_B$, we have that payoffs are nonnegative($U_A \geq 0$ and $U_B \geq 0$) if and only if $S^*_A V_A \geq a S^*_A S^*_B V_A \Leftrightarrow S^*_B \leq \frac{1}{a}$ and $S^*_B V_B \geq b S^*_A S^*_B V_B \Leftrightarrow S^*_A \leq \frac{1}{b}$.

[14] Yet, this condition does not guarantee the existence of values of the parties' efforts that satisfy the first order conditions. For the case in which A is stronger and obtains the rent with a greater probability than B, the general condition for the existence and uniqueness of a pure-strategy Nash equilibrium is $\left(\frac{aV_A}{4}\right)^a \geq \left(\frac{bV_B}{4}\right)^b$, if $b < a \leq 1$; if $a > 1$, an additional condition needs to be satisfied, which depends on two subcases: with $b < 1 < a$, the additional condition is $(bV_B)^b a^{2(a-b)} < (aV_A)^a (a-1)^{a-b-1}$; with $1 < b < a$, the additional condition is $(bV_B)^b b^{2(a-b)} > (aV_A)^a (b-1)^{a-b-1}$. The proof is involved and is available with the authors.

[15] Note that the conditions imposed by the nonnegative-payoff restriction are more stringent than the second order conditions, which, therefore, never bind. The second derivatives of the parties' payoffs are $\frac{\partial^2 U_A}{\partial A^2} = -\frac{a}{A^2} \frac{A^a B^b}{A^a + B^b} (aA^a -$ *aBb+Aa+BbVA* and*∂2UB∂B2=−bB2AaBbAa+Bb3bBb−bAa+Aa+BbVB*. Some simple manipulations yield $\frac{b+1}{b-1} > \frac{S^*_A}{S^*_B} > \frac{a-1}{a+1}$, which can also be rewritten as $S^*_A < \frac{b+1}{2b}$ and $S^*_B < \frac{a+1}{2a}$. Note that these two conditions are always satisfied if the conditions in (2) are satisfied.

[16] This is a straightforward consequence of the envelope theorem. See also Stein (2002).

values of the strength parameters. Since the sum of the parties shares in the rent is equal to 1, the conditions in (2) can be simultaneously satisfied only if $\frac{1}{a}+\frac{1}{b}\geq 1$, which can be rewritten as

$$ab \leq a+b \tag{3}$$

The condition in (3) represents an absolute limit to the admissible values of the strength parameters that can support a pure-strategy equilibrium. If the parties' strength parameters sum up to less than $4(a+b\leq 4)$, then the condition in (3) is always satisfied. However, above this threshold ($a+b>4$), the condition in (3) can be satisfied only if there is a large asymmetry between the parties, that is, if $a$ and $b$ are sufficiently different. This point is illustrated by Figure 1, where the admissible values are contained in the region below the curve. The straight lines depict three cases. The line$a+b=2$, lies entirely within the region, indicating that the product of any combination of $a$ and $b$ summing up to 2 will be less than 2, hence satisfying the condition in (3). The case of $a+b=4$ represent the boundary case. Above this value, as for instance with $a+b=6$, there are values of $a$ and $b$ whose product exceeds their sum.For instance, with$a=3$ and $b=3$, we have $ab>a+b$.In this case the necessary condition in (3) can only be satisfied if the values of $a$ and $b$are sufficiently different, thus never for $a=b$. For instance, with $a=5$ and $b=1$, we have $ab<a+b$, as required by condition (3). The interpretation of this result is that the stronger the parties are the more they must differ in strength in order for a pure-strategy equilibrium to emerge.

FIGURE 1

Yet, values below the curve only satisfy a necessary condition on the strength parameters. This condition is not sufficient. In general, not all the points below the curve in Figure 1 correspond to pure-strategy equilibria. Outside the unit square in Figure 1,the existence of an equilibrium will depend both on the parties' motivation and on the first-order conditions. Instead, inside the unit square, the existence of an equilibrium only depends on the first order conditions because the equilibrium payoffs will always be nonnegative irrespective of the parties' valuations.[17]

*Proposition 1*: With non-tradable rents, an increase in a player's motivation or strength increases that player's payoff and reduces the payoff of his or her opponent.

## 3.2 Tradable rents

The difference in the parties' valuations of the rent creates trading opportunities. If resale is feasible, rents that are appropriated by an unmotivated (low-valuing) player will be transferred to the motivated (higher-valuing) player. The possible contract space for the ex post transfer of the rent is bounded by the players' respective threat points: player B's minimum willingness to

[17] A more precise characterization of the existence conditions yields some additional restrictions on the parameters for both strength and motivation. See note 14.

accept, $V_B$, and player A's maximum willingness to pay, $V_A$. The actual price paid for the rent,$P$, will be determined by the players' cunning, which results among other things from their ability to negotiate and their bargaining power. The presence of a secondary market affects not only the final allocation of the rent but also players' strategies, since ex post trading opportunities change the ex ante stakes of the game.

Player A expects to obtain the rent if he succeeds in the rent-seeking contest and to obtain the difference between his valuation and the price to be paid to buy the rent from B if she succeeds in the rent-seeking contest. Therefore, the payoff of party A is $U_A = S_A V_A + S_B(V_A - P) - A = S_A P - A + (V_A - P)$. As before, $S_A$ and $S_B$ can be interchangeably interpreted as shares in the rent or as probabilities of success. Note that the term in brackets is a fixed amount, which does not change with the effort exerted by player A and hence does not affect his incentives. Therefore, player A plays the game as if the rent had a value equal to the price $P$. Party B also takes into account the fact that, if she succeeds in the contest, she will not keep the rent but rather she will sell it to A and obtain the price $P$. Therefore, the payoff of party B is$U_B = S_B P - B$. Also party B plays the game as if the rent had a value equal to $P$. The presence of a secondary market makes the differences in motivation irrelevant for the parties' incentives. Yet, cunning affects the price at which the rent will be sold ex post and hence implicitly determines the overall stakes of the rent-seeking contest. Quite intuitively, if a player's strength or cunning increases, that player's payoff will increase to the detriment of his or her opponent's payoff.[18]

As before, the first order conditions[19] determine an implicit relationship between the levels of effort exerted by the parties and the strength and cunning parameters of the game. Differently from motivation, cunning has the same effect on both parties. Here we indicate equilibrium values with a double asterisk:

$$\begin{aligned} A^{**} &= a S_A^{**} S_B^{**} P \\ B^{**} &= b S_A^{**} S_B^{**} P \end{aligned} \tag{4}$$

from which we can obtain again a boundary condition, $S_A^{**} S_B^{**} = \frac{A^{**}}{aP} = \frac{B^{**}}{bP} \leq \frac{1}{4}$, so that the players' levels of effort in a pure-strategy equilibrium are bounded by $A^{**} \leq \frac{a}{4} P$ and $B^{**} \leq \frac{b}{4} P$.Second order conditions and the conditions for nonnegative payoffs are as in the previous section.

*Proposition 2*: With tradable rents, an increase in a player's strength or cunning increases that player's payoff and reduces the payoff of his or her opponent.

[18] As in the previous section, this follows from the envelope theorem.

[19] The first-order conditions are $\frac{\partial U_A}{\partial A} = \frac{a}{A} S_A S_B P - 1 = 0$ and $\frac{\partial U_B}{\partial B} = \frac{b}{B} S_A S_B P - 1 = 0$.

# 4 Effort in asymmetric contests

In this section we further study the parties' choices of efforts and theirprobability of winning the contest.

## 4.1 Non-tradable rents

We have already seen in the previous section that an increase in motivation or strength increases a player's payoff and reduces the payoff of his or her opponent. Similarly, an increase in motivation or strength makes a player's effort increase relative to the effort of the other party: from (1) we obtain the following relationship between the players' efforts $\frac{A^*}{B^*} = \frac{aV_A}{bV_B}$, which also indicates that in equilibrium the player exerting more effort must have an advantage in terms of strength, motivation[20] or both. Strength and motivation have a comparable impact on the parties' decisions to invest in rent-seeking effort, so that a party's lack of motivation might be compensated by a greater strength and vice versa.[21] Note also that the ratio of the parties' idiosyncratic rent dissipation—that is, the ratio of the portion of the parties' idiosyncratic valuations that is invested in the rent-seeking contest—is equal to the ratio of the strength parameters $\frac{A^*}{V_A}\frac{V_B}{B^*} = \frac{a}{b}$, that is, it does not depend on the parties' valuations,[22] so that an increase in motivation will not change the parties' relative idiosyncratic dissipation, while an increase in a player's strength will.

Yet, these results concern the parties' relative efforts and say very little about their absolute levels of rent-seeking investments. So be sure, the ratio of the parties' rent-seeking efforts might remain unchanged even if both parties' efforts went up or if both went down. An inspection of the parties' first order conditions reveals some interesting patters of behavior. If a party's valuation of the rent increases, that party has stronger incentives to exert effort[23] and hence his or her level of effort will tend to increase. The incentives of the other party depend on his or her position in the game: a party that is winning with a high probability will tend to increase his or her effort, while a losing party will tend to give up and reduce his or her effort.[24] This effect will feed back into the contestant's effort level, so that the final outcome is difficult to determine in a general way. The effect of an increase in strength on the absolute levels of the parties' effort is even more involved, as it also depends on whether the returns to effort are increasing or decreasing.[25]

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[20] See also Nti (1999).

[21] See also Baik (1994).

[22] Baik (2004, 687) finds that the rent dissipation ratios are independent of the strength parameters. This is due to a different way of modeling strength, thanks to which the strength parameter cancels out in the first order conditions.

[23] The first-order conditions in footnote 12 increase in $V_A$ and $V_B$, respectively.

[24] We have $\frac{\partial^2 U_A}{\partial A \partial B} = \frac{ab}{AB} S_A S_B (S_A - S_B) V_A$ and $\frac{\partial^2 U_B}{\partial B \partial A} = \frac{ab}{AB} S_A S_B (S_B - S_A) V_B$, where the sign depends on whether $\frac{S_A}{S_B}$ is greater or less than one.

[25] We have $\frac{\partial^2 U_A}{\partial A \partial a} = \frac{1}{A} S_A S_B (1 - a(S_A - S_B) \log A) V_A$ and $\frac{\partial^2 U_A}{\partial A \partial b} = \frac{a}{A} S_A S_B (S_A - S_B)(\log B) V_A$ for party A, and $\frac{\partial^2 U_B}{\partial B \partial b} = \frac{1}{B} S_A S_B (1 - b(S_B - S_A) \log B) V_B$ and $\frac{\partial^2 U_B}{\partial B \partial a} = \frac{b}{B} S_A S_B (S_B - S_A)(\log A) V_B$ for party B.

If a party's effort increases due to a boost in motivation or strength, it is not a priori clear whether his or her probability of winning the contest will also improve. In fact, in equilibrium, we have $\frac{S_A^*}{S_B^*} = \frac{(A^*)^a}{(B^*)^b}$. Since $S_A^* = 1 - S_B^*$, a relative improvement in a party's success probability corresponds to an absolute increase is such probability. Yet, a relative increase in a party's effort does not necessarily translate in an increase in his or her probability of winning since the final effect will also depend on the values of the strength parameters.

*Proposition 3*: With non-tradable rents, an increase in a player's motivation or strength increases the ratio of that player's effort relative to the effort of his or her opponent; an increase in a player's strength also increases the ratio of that player's idiosyncratic dissipation relative to the idiosyncratic dissipation of his or her opponent; the effects on the player's success probabilities are ambiguous.

## 4.2 Tradable rents

We shall now consider how the results derived above are changed when players have ex post trading opportunities. As discussed in the previous section, with tradable rents subjective valuations of the rent are rendered immaterial by the presence of resale opportunities. Both players, whether they subjectively value the prize or not, face the same stakes in the rent-seeking contest given by the resale value of the prize in the secondary market, $P$. From (4) the relationship between the players' efforts thus becomes $\frac{A^{**}}{B^{**}} = \frac{a}{b}$, which indicates that, when a player exerts more effort, this is because of his advantage in terms of strength, not because of his motivation. Motivation does not affect effort in this case, because the possibility of trade makes the game symmetric with respect to the stakes. Ceteris paribus, high-valuing and low-valuing players will exert the same amount of effort in the game. It is clear that, in this case, only strength can drive differences in effort and a stronger player will thus exert relatively more effort.

Note however that the ratio of the players' idiosyncratic dissipation is now $\frac{A^{**}}{V_A}\frac{V_B}{B^{**}} = \frac{aV_B}{bV_A}$, which suggests that a player is willing to invest a relatively larger share of the expected prize if his or her strength is large relative to his or her valuation. This is due to the fact that resale opportunities are more profitable for higher valuing players and therefore lower-valuing players will focus more heavily on the initial rent-seeking phase of the game rather than rely on the trading phase and vice versa.

The success probability of the stronger player might however be lower than the success probability of his or her opponent, as in the case of non-tradable rents, since we have $\frac{S_A^{**}}{S_B^{**}} = \frac{(A^{**})^a}{(B^{**})^b}$. Cunning, determining the price of the rent at the resale phase, will determine the absolute levels of the parties' effort but does not have an effect on a party's effort relative to his or her opponent.

*Proposition 4*: With tradable rents, an increase in a player's strength increases the ratio of that player's effort relative to the effort of his or her opponent; an increase in a player's strength also increases the ratio of that player's idiosyncratic dissipation relative to the idiosyncratic dissipation of his or her opponent; an increase in a player's motivation decreases the ratio of that player's idiosyncratic dissipation relative to the idiosyncratic dissipation of his or her opponent; cunning does not affect relative efforts and idiosyncratic dissipations; the effects on the player's success probabilities are ambiguous.

# 5 The social cost of asymmetric rent-seeking

## 5.1 Non-tradable rents

In the presence of asymmetries, the social cost of rent-seeking consists of the traditional rent dissipation and an additional cost, rent misallocation, due to the fact that the parties value the rent differently.[26] Dissipation costs are given by the sum of the players' efforts in equilibrium. Misallocation costs arise when players have different valuations of the rent, and are given by the difference in the players' valuations weighted by the probability that the lower-valuing player actually appropriates the rent. Using (1), these costs can thus be written as follows in the no-trade scenario:

$$\begin{aligned} D^* &= A^* + B^* = S_A^* S_B^* \,(aV_A + bV_B) \\ M^* &= S_B^* \,(V_A - V_B) \end{aligned} \tag{5}$$

Since the less motivated (lower-valuing) player B does not necessarily exert a lower level of effort to appropriate the rent, nor is she less likely to win the contest than her motivated (higher-valuing) opponent, rent misallocation costs can be particularly relevant. Increasing asymmetries in strength can have ambiguous effects on social welfare when motivation is also at stake. Consider rent dissipation. If A is stronger, then there is more weight on $V_A$, which is larger, thus we have more dissipation. However, the players' probabilities of winning become more divergent due to the fact that A will invest more in the contest and, hence, the term $S_A^* S_B^*$ becomes smaller, thus reducing dissipation. Moreover, if player A is stronger, we have a smaller $S_B^*$, thus misallocation is reduced. Asymmetry with respect to motivation also has ambiguous effects when players with asymmetric strength are involved. A large difference in the players' valuations leads to a decrease in $S_A^* S_B^*$, hence reducing dissipation, but exacerbating misallocation costs.

It is interesting to expand on the notion of full dissipation. When the rent is commonly valued, full dissipation occurs when the sum of the parties' efforts equals the value of the rent. In our setting, however, the rent is valued differently by the two parties, so that an additional social loss will derive from the misallocation of the rent, occurring when the low-valuing party B wins the contest. Yet, as in the traditional setting, also with asymmetric valuations the total social loss

[26] Hurley (1998) introduced the notion of context efficiency, which measures the ability of the highest-value player to obtain the rent, net of the total rent dissipation. Rent misallocation is a somewhat analogous concept, but it only focuses on the allocation of the rent, while leaving aside considerations of the resources spent to obtain it, which are captured by the notion of rent dissipation.

is bounded in equilibrium. Since in equilibrium the parties must earn nonnegative payoffs, the sum of the parties' investments in rent-seeking cannot exceed the sum of their expected gains from rent-seeking and we must have: $D^* = A^* + B^* \leq S_A^* V_A + S_B^* V_B$ . Since the rent misallocation is $M^* = S_B^*(V_A - V_B)$, summing up, we have that the total social loss cannot exceed the highest rent valuation: $D^* + M^* \leq V_A$.[27]

Within this overall limit, both dissipation and misallocation are also individually bounded. Using (2), rent dissipation can be at most a weighted average of the parties valuations, $D^* \leq \frac{aV_A + bV_B}{ab} \leq V_A$. Similarly, the rent misallocation is also bounded: $M^* \leq \frac{1}{a}(V_A - V_B) \leq V_A$.

The worst-case scenario in which the highest rent valuation is entirely lost due to the rent-seeking game arises when both parties obtain a payoff equal to zero.[28] We have seen above[29] that this can be the case only if $S_A^* = \frac{1}{b}$ and $S_B^* = \frac{1}{a}$. Since the sum of the parties' shares is necessarily equal to 1, then we must have that $\frac{1}{a} + \frac{1}{b} = 1$, which can be rewritten as $ab = a + b$. This means that the total social cost of rent-seeking will be equal to the highest rent valuation only along the curve in Figure 1. Looking at Figure 1, it is also clear that this condition can be verified only if $ab \geq 4$ and if both $a$ and $b$ are greater than 1.Also the opposite is true: if a point lies on the curve in Figure 1, then it must be the case that the parties' payoffs are both zero parties spend $A^* = \frac{V_A}{b}$ and $B^* = \frac{V_B}{a}$ in rent-seeking.[30]Any other value of the strength parameters that supports a pure-strategy equilibrium will leave some social value untouched.

This result generalizes the traditional full-dissipation result in Tullock's framework. In fact setting $a = b = r$, the condition becomes $r^2 = 2r$, which yields $r = 2$, as in the traditional analysis. The fact that the asymmetry in the parties' valuations does not appear in the definition of the maximal social cost of rent-seeking should not be surprising. In fact, such difference enters the calculus of the total social loss in two offsetting ways. On the one hand, rent dissipation is somewhat diminished by the fact that valuations are asymmetric, in that the sum of the parties' efforts will not reach the highest valuation. On the other hand, rent misallocation comes about precisely because of the asymmetric valuations, picking up the remaining loss and bringing the total social loss to a level that, at the limit, equals the highest valuation.

*Proposition 5*: With non-tradable rents, the total social cost of rent-seeking is equal to the highest valuation of the rent if and only if the product of the parties strength is equal to their sum.

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[27] Compare to Hillman and Riley (1989)'s result that asymmetries in rent valuations reduce the rent dissipation. We add that if one considers rent misallocation, asymmetries do not affect the maximal social loss.

[28] To clarify, $D^* + M^* = V_A$ occurs if and only if $D^* = S_A^* V_A + S_B^* V_B$.

[29] See note 13 and accompanying text.

[30] This is because, if $\frac{1}{a} + \frac{1}{b} = 1$ en the conditions in (2) must hold strictly. The parties' rent-seeking expenditures are derived substituting (2) into (1).

## 5.2 Tradable rents

Ex post trading opportunities will correct any initial misallocation of the rent, bringing misallocation costs to zero, but retain the potential for dissipation through rent-seeking expenditures:

$$D^{**} \;=\; A^{**} + B^{**} = S_A^{**} S_B^{**} (a+b) P$$
$$M^{**} \;=\; 0 \qquad (6)$$

Also in this case, the rent dissipation (and hence the total social loss of rent-seeking) is bounded and cannot exceed the price$D^* \leq P$. Yet, although the opportunity to trade rents after the initial contest eliminates possible misallocation costs, it can exacerbate rent dissipation. This is due to the fact that resale opportunities raise the stakes for low-valuing contestants making the contest more symmetrical, and thus increasing the value of $S_A S_B$. As it will be seen below, the net effect of ex post trading opportunities on social welfare is ambiguous. We could repeat here what already observed in the previous section about the total social loss. Yet, the problem here is simpler, in that the rent misallocation is always zero and social loss is confined to rent dissipation. Thus, full dissipation occurs when $D^{**} = V_A$, which can occur only if $P = V_A$.The results obtained in the previous section need to be slightly adapted to the new setting:

*Proposition 6*: With tradable rents, the total social cost is equal to the highest valuation of the rent if and only if the product of the parties strength is equal to their sum and if the resale price is equal to the valuation of the high-valuing party.

# 6 Trading away asymmetries

## 6.1 The effect of trade on effort

The possibility to trade the won prize in a secondary market aligns the players' motivation. Although they attach different subjective valuations to the prize, both players engage in the rent-seeking context facing similar stakes. Low-valuing players know that rents can be sold after the contest at price $P$; similarly, high-valuing players know that rents can be bought; thus, both payers play the rent-seeking game facing symmetric stakes, $P$, rather than asymmetric stakes $V_A$ and $V_B$. The effect of this aligned motivation on the players' efforts can be studied by comparing (1) and (4), which gives $\frac{A^*}{B^*} = \frac{aV_A}{bV_B} > \frac{a}{b} = \frac{A^{**}}{B^{**}}$. Yet, this inequality does not offer any determinate answer to the question under consideration. In fact, it may be the case that $A^* > B^*$without trade and $A^{**} < B^{**}$ with trade, satisfying the inequality but leaving open the question as to the effect of ex post trade on parties' ex ante efforts. To answer this question, let us first define an indifference criterion with respect to efforts:$\frac{A^*}{B^*} = \frac{B^{**}}{A^{**}}$. If this equality is satisfied, opening trade triggers a perfect switch in the players' effort choices. Without trade the motivated player would exert more effort, while with trade the unmotivated player would exert more effort, perfectly inverting the ratio between the efforts. The equality is satisfied if the following holds:[31]

$$\frac{V_A}{V_B} = \left(\frac{b}{a}\right)^2 \tag{7}$$

The latter condition implies that we must have $a < b$, given that $V_A > V_B$. This means that such a reversal of effort strategies will occur only in rent-seeking games where a strong but unmotivated player plays against a weak but motivated player. Without trade the motivated, weak player exerts more effort, inasmuch as the rent-seeking contest provides for him the only opportunity for appropriating the sought-after rent. Once trade opportunities are introduced, this relationship could be reversed. Trade opportunities remove motivation-driven asymmetries, leaving only strength-driven differences between the players' incentives. Whether the resulting effort levels are more or less asymmetric when trade is introduced depends on the relative importance of motivation-driven and strength-driven incentives on the parties' strategies. We can restate the problem in light of the above considerations, distinguishing three possible cases.

First, with $\frac{V_A}{V_B} > \left(\frac{b}{a}\right)^2$, player A's advantage in motivation weighs more heavily than player B's advantage in strength. In this case, trade removes the greater of the two sources of asymmetry, leading to a convergence between the parties' efforts levels. This case includes two scenarios. In the first scenario, player A has greater motivation as well as greater strength than player B. Trade leads to a convergence between the players' efforts by removing one of the two advantages for player A. In the second scenario, we might observe player A with greater motivation and player B with greater strength, with a heavily dominating effect of motivation-driven incentives. By introducing trade and eliminating the effect of motivation, we would replace the large asymmetry driven by motivation with a smaller asymmetry driven by strength, leading again to a convergence between the parties' effort levels.

Second, with $\frac{V_A}{V_B} = \left(\frac{b}{a}\right)^2$, we have the special case considered in (7) where the two sources of asymmetry perfectly balance one another. In this case, introducing trade leads to an inversion of the ratio of the players' efforts.

Finally, with $\frac{V_A}{V_B} < \left(\frac{b}{a}\right)^2$, player B's advantage in strength generates a greater asymmetry than player A's advantage in motivation. Without trade, these two advantages operate in opposite and partially offsetting directions. Trade removes the advantage in motivation, upsetting this balance, thus increasing the asymmetry between the players' efforts.

Concluding, the effect of trade on the players' success probabilities is ambiguous because it depends on the absolute values of the player's efforts and on their strength levels as we have seen in the previous section.

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[31] Note that $\frac{A^*}{B^*} = \frac{B^{**}}{A^{**}} \Leftrightarrow \frac{aV_A}{bV_B} = \frac{b}{a} \Leftrightarrow \frac{V_A}{V_B} = \left(\frac{b}{a}\right)^2$.

*Proposition 7:* Trade reduces (increases) the asymmetry of the rent-seeking contest and leads to a convergence (divergence) between the players' rent-seeking efforts if and only if a player's advantage in motivation is greater (less) than the other player's squared advantage in strength. In the special case where a player's advantage in motivation is equal to the other player's squared advantage in strength, trade keeps the asymmetry of the rent-seeking contest unchanged, but inverts the ratio of players' rent-seeking efforts. The effect of trade on the success probabilities is ambiguous.

## 6.2 The effect of trade on the social loss of rent-seeking

In general, trade does not necessarily reduce rent dissipation,[32] but put a lower limit on the maximal total social loss that can materialize, which is $V_A$ without trade and only $P \leq V_A$ with trade.In this section we introduce the two notions, which will be useful to study the effect of trade on the social loss of rent-seeking. As it was observed above, strength, motivation and cunning play different roles in asymmetric rent-seeking contests. Strength and motivation are only relevant in the rent-seeking phase, while cunning is only relevant in the resale phase when secondary markets are introduced. Unlike strength, which is always relevant in the rent-seeking phase of the game, regardless of the presence of resale opportunities, motivation is only relevant in the rent-seeking phase when resale opportunities are unavailable. In order to refine our understanding of the combined effects of strength, motivation and cunning, we shall now introduce the notions of *balanced rent-seeking* and *fair negotiations*. In balanced rent-seeking, one player's strength perfectly offsets the other player's motivation. With fair negotiations, a player's cunning in the negotiation phase compensates his lack of strength in the rent-seeking phase. Both notions crucially depend of the asymmetry in motivation.

The notion of balanced rent-seeking applies to the rent-seeking phase and is characterized by $aV_A = bV_B$. In a balanced contest the players' exert the same levels of efforts notwithstanding their differences in both strength and motivation. In these situations, a player's advantage in one dimension is balanced by a disadvantage in the other dimension. The notion of fair negotiations considers instead the correlation between a player's disadvantage in the rent-seeking phase and his advantage in the ex post trading phase. Imposing a restriction on the trading phase, a contest with fair negotiations is characterized by $P = \frac{aV_A+bV_B}{a+b}$, that is, the price is a weighted average of the players' valuations, where the weights are given by the strength exponents. The condition of fair negotiations is verified when a player's advantage in strength translates into a disadvantage in bargaining power and vice versa. In fact, it is in a player's interest that the price be close to the other player's valuation and not his own. If player A is strong (large $a$) in the rent-seeking phase, then he will be disadvantaged in the trading phase, since the price will be relatively high and in fact close to $V_A$. Conversely, if player B is strong (large $b$) in the rent-seeking phase, then she will be disadvantaged in the trading phase, since the price will be relatively low and close to $V_B$.

[32] See also Suen (1989) showing that trading does not necessarily reduce rent dissipation.

In a balanced contest, the players' efforts without trade are the same, $A^* = B^*$ and the stronger player always wins with a higher probability. Since we are assuming that A is more motivated, then B must be stronger for the contest to be balanced, thus we have $\frac{a}{b} < 1$ and hence $\frac{S^*_A}{S^*_B} < 1$. With trade, the players' efforts depend on their relative strength, with the stronger player exerting more effort, $\frac{A^{**}}{B^{**}} < 1$,[33] and hence winning more often, $\frac{S^{**}_A}{S^{**}_B} < \frac{S^*_A}{S^*_B} < 1$.[34]

If the contest is characterized by fair negotiations, we can write the ratio of the rent dissipations with and without trade as follows:

$$\frac{D^*}{D^{**}} = \frac{S^*_A S^*_B}{S^{**}_A S^{**}_B}\frac{aV_A + bV_B}{(a+b)P} = \frac{S^*_A S^*_B}{S^{**}_A S^{**}_B}$$

Note that, if the context is balanced, we have $\frac{S^*_A S^*_B}{S^{**}_A S^{**}_B} > 1$.[35]Thus, combining fair negotiations with a balanced context, we obtain the following result: rent dissipation decreases with trade. We can further compare total social loss in the two scenarios. Recall that in the case of rent-seeking without trade, total social loss is given by the sum of dissipation and misallocation costs, whereas in the case of rent-seeking with trade misallocation losses are eliminated, yielding total social loss equal dissipation.[36] When the parties' motivation, strength and cunning are balanced, while trade decreases dissipation, the total social loss might be greater or less than without trade.

*Proposition 8*: Trade reduces the maximal social loss of rent-seeking. In a balanced contest with fair negotiations trade also reduces dissipation.

## 7 Conclusions

Rent-seeking contests, more often than not, involve players who differ with respect to one or more characteristic. In this chapter we marked a pathfinder through the relevant literature on asymmetric rent-seeking, considering the effect of asymmetries between the parties' competitive abilities, motivation, and bargaining power. These characteristics play different roles and become relevant at different stages of a rent-seeking contest. Strength, for example, acquires relevance in the rent-seeking stage, while cunning becomes relevant in the reallocation stage. Motivation is an advantage (larger surplus that is obtainable from the rent) that plays different roles on the parties'

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[33] As it is intuitive, in a balanced contest, we have $\frac{V_A}{V_B} < \left(\frac{b}{a}\right)^2$: trade increases the asymmetry of the contest in terms of incentives to exert effort.

[34] See Proposition 2.

[35] Note that $\frac{S^{**}_A}{S^{**}_B} < \frac{S^*_A}{S^*_B} < 1$ implies that the difference between the winning probabilities increases with trade. Since the winning probabilities sum up to 1, their product decreases if the difference between the two increases, this implies $\frac{S^*_A S^*_B}{S^{**}_A S^{**}_B} > 1$.

[36] The ratio of the total social losses in the two scenarios is $\frac{D^*+M^*}{D^{**}} = \frac{S^*_B}{S^{**}_A S^{**}_B}\frac{V_A(1+aS^*_A)-V_B(1-bS^*_A)}{(a+b)P}$. By simulating the results it is possible to verify that the ratio can be greater or less than 1.

incentives. The effect of motivation differs depending on whether rent is tradable ex post or not and the bargaining ability of the parties. We look at whether the possibility of tradable rents increases or reduces the total social cost of rent-seeking. This bird's eye view on asymmetric rent-seeking problems provides a valuable lens through which to analyze real life conditions of rent-seeking competition.

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# Figures

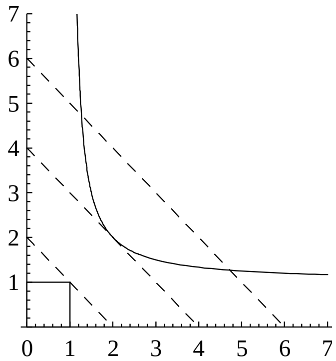


*Figure 1*: Necessary condition for pure-strategy equilibria with asymmetric strength (the thick curve depicts$ab = a + b$, while the dashed lines depict $a + b = 2$, $a + b = 4$, and $a + b = 6$, respectively).