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Towards a clean vehicle fleet: from households' valuation
of fuel efficiency to policy implications

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Abstract

This paper investigates household behaviour with regard to vehicle fuel efficiency. We propose to approach the Willingness to Pay (WTP) for better fuel efficiency through the Hicksian compensating variation in income. Specifically, we distinguish the Willingness to Pay or to Accept (WTA) buying a more fuel-efficient car from the theoretical WTP for a reduction in fuel consumption without changing one's car. Then by assuming that the household has to replace its car, we estimate a WTP for the cleanest car

We also analyse what effect a fuel tax and/or a feebate scheme (e.g. a bonus-malus scheme) have on the WTP for the cleanest car and on the driven mileage. We find that the WTP for the cleanest car decreases following the implementation of a fuel tax. To the contrary, a feebate system leads to an increase in this WTP. But we also find that reducing the market price of the new vehicle (i.e. through a bonus) is not worthwhile in the light of the rebound effect. However, a fuel tax – as soon as it exceeds a certain level – is able to nullify the rebound effect.

Key words: fuel efficiency, willingness to pay, fuel tax, feebate scheme, rebound effect.

Résumé

Le comportement des ménages au regard de l'efficacité énergétique de leur véhicule est examiné dans ce papier. Nous proposons d'approcher le Consentement à Payer (CAP) pour une meilleure efficacité énergétique du véhicule par la variation de revenu compensatoire. Plus précisément, nous distinguons le consentement à payer ou à recevoir (CAR) pour l'achat d'un véhicule moins consommateur de carburant du CAP théorique pour une réduction de la consommation de carburant sans devoir changer de véhicule. Puis, en assumant que le ménage est contraint de changer de véhicule, nous estimons le CAP pour l'achat du véhicule le plus efficient.

Les impacts d'une taxe sur le carburant et/ou d'un système de bonus-malus sur le CAP pour l'achat du véhicule le plus efficient et sur les distances parcourues en véhicule particulier sont également discutés. Nous trouvons que le CAP pour l'achat du véhicule le plus efficient diminue avec l'instauration de la taxe. Au contraire, l'instauration d'un système de bonus-malus conduit à une hausse du CAP pour l'achat du véhicule le plus efficient. Toutefois, diminuer le prix de marché d'un véhicule (i.e. avec un bonus) augmente le risque d'effet rebond. Cependant, une taxe sur le carburant, pourvu qu'elle soit suffisamment élevée, permet d'annuler l'effet rebond.

Mots clés: efficacité énergétique, consentement à payer, taxe sur le carburant, bonus-malus, effet rebond.

1 Introduction

There is a common understanding that climate change is one of the main global challenges of the 21st century and that its impacts will be more serious than previously thought (Wang and al 2009). Specifically, it is broadly admitted that such impacts are threatening the welfare of human beings (Shi and Lai, 2012).

Rapidly increasing traffic and a high dependency on fossil fuels have made transport a crucial issue with regard to the action required to fight climate change. Indeed, the transport sector is the second largest source – after the power sector – of global carbon dioxide emissions, with nearly a quarter of the total amount (IEA, 2010). Moreover, emissions due to transport have increased by 1.7% per year on average since 2000, with fuel for road transportation predominating (IEA, 2013). Passenger vehicles play a significant role, since they account for around 12% of man-made CO_2 emission in Europe (GMID, 2010), 20% in the United States (EPA, 2010) and 5% worldwide (IEA, 2010). The relative weight of passenger vehicles is likely to continue. The IEA (2011) expects the number of cars worldwide to have doubled, to almost 1.7 billion, by 2035, and Schafer and Victor (2000) predict that absolute mobility by car will increase by 260% by 2050.

With reference to Schipper's ASIF scheme, GHG emissions of transport can be tackled through four main levers: transport Activity, modal Share, the energy Intensity and the carbon intensity of Fuel (Schipper and al., 2000). Focussing on passenger vehicles on the one hand, and on household's behaviour on the other, considerations with respect to modal share and carbon intensity do not enter the scope of this research work.

With regard to capital goods (e.g. vehicles, appliances, and so on) a standard approach used in a lot of papers consists in investigating the replacement decision. Papers that address the vehicle replacement decision from a fleet manager point of view argue that due to recently implemented emissions standards, new diesel vehicles are cleaner than the existing ones ("I" in ASIF), so that replacement can be cost effective, even before the end of the car lifespan (Gao and Stasko, 2009). The idea for this article arises from the need to study the replacement decision from the household's point of view, that is to say starting with a utility-maximizing framework, rather than a cost-minimizing framework (i.e. used in the literature dealing with the fleet manager point of view). Such analysis is justified since the purchase of a new vehicle by households – what we are actually interested in – consists in most cases of a replacement decision rather than an investment decision. This conclusion is based on the high average age of a new vehicle purchaser (around 50 years in Europe, Observatoire Cetelem, 2014) which makes us believe that these purchasers have already got a second-hand vehicle.

Another important consideration is car use ("A" in ASIF). Actually, as well as the choice of transport mode ("S" in ASIF), car purchase behaviour (e.g. vehicle size and performance choices) and car use are decisive factors in reducing energy consumption (Schipper, 2011). The emphasis on demand-side phenomena is also justified by the fact that transportation accounts for a significant proportion of household expenditure – more than 10% on average in Europe in 2010 – within which vehicle purchase and use play a large part (European Commission, 2012). The figure is even higher in the United States, where the average household spends about 10% of its annual income on vehicle transport alone (BEA, 2012).

In this paper we conduct a static-comparative analysis of a basic model of consumer behaviour: a representative agent makes optimal decisions on driving (i.e. a continuous choice) and vehicle ownership (i.e. a discrete choice) in a one-period utility-maximizing framework. Actually, the decision of owning a vehicle and that of using a vehicle are interrelated (West, 2004) and have to be jointly modelled under an integrated framework to estimate reductions in fuel consumption due to either changes in fleet composition or reductions in vehicle miles travelled (Bento and al., 2006). Train (1985) provides an early application of the indirect utility

approach – initiated by Dubin and McFadden (1984) who study the residential electric appliance holdings and consumption – on car ownership study. But, by solely considering households that already own a vehicle, we only address the 'replacement decision' (i.e. to purchase or not a *new* car? And, if so, which vehicle to purchase?) and we do not explain the choice of owning a vehicle. This way, we differ from existing works.

Specifically, we approach household's Willingness to Pay (WTP) for a vehicle with better fuel efficiency using compensating variation, and thus starting from a utility-maximising framework. Then we analyse the impacts of two different pricing regulation tools on the WTP and on the rebound effect. They are a fuel tax and a feebate scheme. While the first policy tool consists of an immediate cost when facing a new trip, and thus affects the decision on driving, the second one consists either of a lower purchase price (e.g. bonus) or of a higher purchase price (ie.g. malus), and thus affects the purchase decision.

The remainder of the paper is organized as follows. Section 2 presents the model and gives the results. Section 3 discusses their policy implications. Section 4 offers our conclusions.

2. The model

Consider a household owning only one vehicle ("vehicle a"), with a corresponding average fuel consumption per kilometre of ck_a. The household has two options: keeping their current car or buying a new one. Now consider two cars for sale of a given model: vehicle b with a fuel consumption of ck_b and vehicle c with a fuel consumption of ck_c. The fuel consumption figures are such that: $ck_c < ck_b < ck_a$. A further assumption is that vehicle c (the most fuel efficient) has a higher purchase price than vehicle b ($P^{v_c} > P^{v_b}$), implying that a more efficient vehicle generally embodies more expensive technology.

Consider a utility function with the following attributes: first, the mileage travelled by passenger car (m) and, second, a composite good (C)[1] (Muthukrishnan, 2010; Wei, 2013, Zhu and al, 2013). We use the following utility function $U = \sqrt{C} + \theta\sqrt{m}$ with θ a preference parameter ($\theta > 0$). Taking the square root function allows the WTP being dependent on the fuel price[2]. Yet, in line with intuition, the WTP varies with car use (Mandell, 2009), and the latter depends on transportation cost (including the fuel cost). U'_c and U'_m are positive and U''_c and U''_m are negative, as stated in consumer theory.

Variables (m, C and I that denotes the household's income) are specified on a single period. Its duration corresponds to the payback period required by car buyers of a vehicle with better fuel efficiency. This period is relatively short (around three years, ITF, 2010), what allows us to assume that fuel price is constant. Things being what they are, a dynamic model would not have brought things any further.

Then let p_f denote the fuel price, and P^{v_b} and P^{v_c} the market prices of vehicles b and c. The composite good's market price is normalized to 1. If the motorist buys a new car, we assume that he simultaneously sells his current vehicle, so that he always owns only one vehicle. The resale value is δP^{v_a} (with δ the depreciation rate). Thus the budget constraints are:
- If the motorist keeps his car (situation a): $I = m\,p_f\,ck_a + C$. (By assuming that the payback period is over, the purchase expenditure no longer appears in the budget constraint),
- If the motorist buys vehicle b (situation b): $I - P^{v_b} + \delta P^{v_a} = m\,p_f\,ck_b + C$,

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[1] Note that the composite good includes transport services, thus assuming that the motorist has alternatives to using his own car.
[2] This is not the case using the function $U = \ln C + \theta \ln m$

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- If the motorist buys vehicle c (situation c): $I - P^{v_c} + \delta P^{v_a} = m \, p_f c k_c + C$.

Given this, the motorist chooses the mileage (m) and the consumption of other goods and services (C) that maximise his utility, under the constraint that such expenditures cannot exceed his disposable income (cf. Table 1).

Table 1: Expressions of the optimal mileage and utility functions: comparison of the three situations

	Mileage (m^*)	Utility (U)
Situation a: Keeps his vehicle a	$\dfrac{I\theta^2}{p_f c k_a (p_f c k_a + \theta^2)}$	$\sqrt{I\left(1 + \dfrac{\theta^2}{p_f c k_a}\right)}$
Situation b: Buys vehicle b	$\dfrac{(I - P^{v_b} + \delta P^{v_a})\theta^2}{p_f c k_b (p_f c k_b + \theta^2)}$	$\sqrt{(I - P^{v_b} + \delta P^{v_a})\left(1 + \dfrac{\theta^2}{p_f c k_b}\right)}$
Situation c: Buys vehicle c	$\dfrac{(I - P^{v_c} + \delta P^{v_a})\theta^2}{p_f c k_c (p_f c k_c + \theta^2)}$	$\sqrt{(I - P^{v_c} + \delta P^{v_a})\left(1 + \dfrac{\theta^2}{p_f c k_c}\right)}$

From Table 1, it can be seen that the mileage travelled by passenger car is – as expected – a decreasing function both of the fuel consumption per kilometre and of the fuel price. Actually, the role played by the product "$p_f c k$" in this approach is particularly noteworthy. It depicts the price of one kilometre travelled by passenger car, or the transportation cost per kilometre. This implicitly means that the transportation cost is assumed to be limited to the short-run variable transport cost, i.e. the fuel cost, since it guides consumer decisions (De Borger and Mayeres, 2007). Indeed, other costs associated with driving a car (parking fees, road tolls, and so on) do not affect the choice of vehicle (Mandell, 2009). The fuel price and fuel consumption per kilometre are two independent parameters, though their variations modify the price per kilometre travelled by car. That said, the Hicksian compensating variation method – estimating the change in income consented to by a consumer in order not to give up the idea of a price change – clearly fits to estimate the WTP for a reduction of the fuel consumption per kilometre. Moreover, it follows from the quasi-linear property of our utility function that using either the compensating variation or the equivalent variation leads to the same results.

Two questions are addressed in the two following subsections. We first wonder if the motorist is willing to pay to benefit from better fuel efficiency (cf. subsection 2.1.). If this is not the case, and by considering the purchase decision as an exogenous constraint – that is not explained by the model – we wonder how much a motorist is ready to pay for the least consuming vehicle (cf. subsection 2.2.).

2.1. To benefit – or not – from better fuel efficiency

Here we only consider the situations a and b. The question addressed here is whether the motorist is – or is not – willing to pay for the purchase of a less consuming car. The answer depends on the comparison of the utility of keeping his current vehicle and that of buying a new vehicle (De Palma and Kilani, 2008). It is tantamount to saying that the answer is given by the sign of the Hicksian Compensating Variation (HCV) in income. Taking the initial utility as a

reference, if the HCV is positive it consists of a Willingness to Pay (WTP), and if it is negative, it consists of a Willingness to Accept (WTA).

Proposition 1. *The Hicksian Compensating Variation in income for the purchase of the vehicle b, termed* $X^{v_a \to v_b}$*, is given by:*

$$X^{v_a \to v_b} = \frac{ck_a - ck_b}{ck_a} \frac{I\theta^2}{p_f \, ck_b + \theta^2} - P^{v_b} + \delta P^{v_a} \tag{1}$$

Proof. See Appendix.

Yet, the first term consists of the WTP for a reduction in fuel consumption from ck_a to ck_b without buying vehicle *b*.

Proof. The compensating variation for the reduction in fuel consumption from ck_a to ck_b, termed $X^{ck_a \to ck_b}$, is such that: $V\big(I, ck_a, p_f, \theta\big) = V\big(I - X^{ck_a \to ck_b}, ck_b, p_f, \theta\big)$. Or:

$$\sqrt{I\left(1 + \frac{\theta^2}{p_f \, ck_a}\right)} = \sqrt{(I - WTP^{ck_a \to ck_b})\left(1 + \frac{\theta^2}{p_f \, ck_b}\right)}$$

We obtain:

$$WTP^{ck_a \to ck_b} = \frac{ck_a - ck_b}{ck_a} \frac{I\theta^2}{p_f \, ck_b + \theta^2} \tag{2}$$

It follows that the motorist is not willing to pay for the purchase of a less consuming vehicle ($X^{v_a \to v_b} < 0$) if the net car purchase expenditure exceeds the amount (see eq. 2) he is willing to pay to benefit from the reduction in fuel consumption per kilometre. In that case, the motorist does express a Willingness to Accept (WTA) purchasing vehicle *b*. For the sake of simplifying interpretations, we use the absolute value of the HCV when it consists of a WTA. We thus have:

$$WTA^{v_a \to v_b} = P^{v_b} - \delta P^{v_a} - \frac{ck_a - ck_b}{ck_a} \frac{I\theta^2}{p_f \, ck_b + \theta^2} \tag{3}$$

Figure 1 below illustrates this situation.

Figure 1: Graphical representation of the WTA for the purchase of a less consuming vehicle
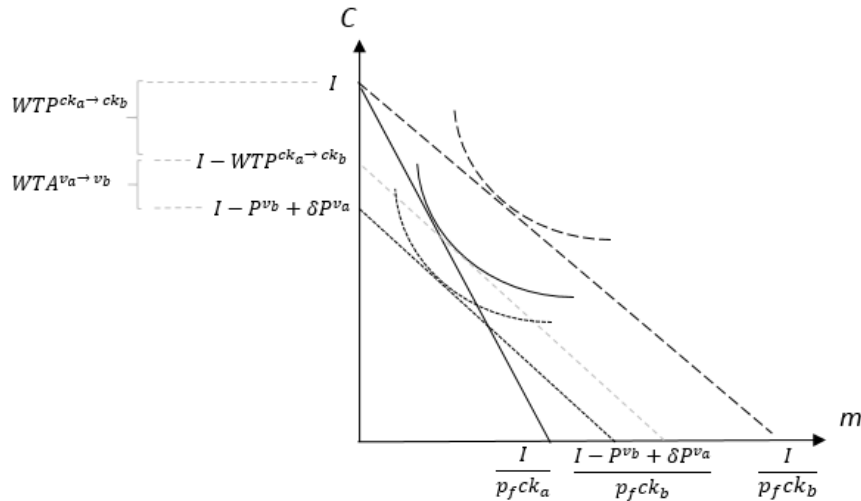
The straight lines are the budget constraints. The faint line refers to a budget constraint with the current vehicle and a fuel consumption equal to ck_a. The dashed line illustrates a budget constraint with the new

fuel consumption (ck_b) but without the purchase of vehicle b. The dotted line refers to a situation with both the new fuel consumption (ck_b) and the net purchase car expenditure ($P^{v_b} - \delta P^{v_a}$).

Since the amount the motorist is willing to pay to benefit from the reduction in fuel consumption per kilometre (see eq. 2) is function of the household's income, we have the following Proposition.

Proposition 2. *The motorist does express a WTP for the purchase of a less consuming vehicle as soon as his income is µ times higher than the net car purchase expenditure, with:*

$$\mu = \frac{ck_a\left(p_f\, ck_b + \theta^2\right)}{(\,ck_a - ck_b)\theta^2} \tag{4}$$

Proof. See Appendix.

2.2. To pay – or not – for the least consuming vehicle

In this section, we focus on the situation where the net car purchase expenditure exceeds the WTP for a theoretical reduction of fuel consumption per kilometre so that the motorist is not willing to pay for the purchase of a less consuming vehicle. Indeed, households are more likely to keep their current car than to change it (Berkovec and Rust, 1984).

This said, we assume that the motorist is obliged to change car and thus consider only situations *b* and *c*. This constraint is considered exogenous and is not explained by the model. Only the choice amongst the two vehicles offered for sale is thus addressed. Here we are able to determine his WTP for vehicle *c* – the least consuming vehicle.

Since the consumer purchases only one vehicle, his inverse demand function is a single point (i.e. his WTP for the first unit). Consequently the household's WTP for vehicle *c* ($WTP^{v_a \to v_c}$) is exactly the maximum market price of the vehicle *c* ($P_{max}^{v_c}$). We will break down the maximum market price of vehicle *c* in what follows.

When the market prices of the two vehicles are the same – remember that we assume $P^{v_b} \leq P^{v_c}$ –, the purchase of vehicle *c* produces greater utility than when purchasing vehicle *b*, which is less efficient. Hence the WTA purchasing vehicle *c* is lower than the WTA purchasing vehicle *b*. Moreover, by using (3), the WTA for the purchase of a new car is an increasing function of the market price of that vehicle and is independent of the market price of the other vehicle for sale. Thus in the Cartesian coordinate system (P^{v_c}, $WTA^{v_a \to v_c}$), the WTA for the purchase of vehicle *c* ($WTA^{v_a \to v_c}$) is an increasing straight line (with slope equal to 1) and the WTA for the purchase of vehicle *b* ($WTA^{v_a \to v_b}$) is a horizontal straight line such that the WTA for vehicle *c* is lower than for the vehicle *b*, when the two market prices are equal (see Figure 2).

That said, and given that $P^{v_b} < P^{v_c}$, the difference between the two WTAs ($WTA^{v_a \to v_b} - WTA^{v_a \to v_c}$) decreases with an increase of vehicle *c*'s market price. In other words, the difference between the two WTAs is maximum when the market prices are the same.

The difference between the two WTAs shows the amount the motorist does not receive when purchasing vehicle *c* rather than vehicle *b*. Hence it could be considered as an amount the motorist is willing to pay for the purchase of vehicle *c* in place of vehicle *b*. In other words, the

motorist is indifferent as to whether he purchases vehicle b at its market price or purchases vehicle c at a price equal to the market price of vehicle b plus the difference between the two WTAs (see Figure 2). To go further, the difference between the two WTAs can also be interpreted as the households' WTP for improved fuel economy[3]. Studies of the latter WTP (i.e. in most cases discrete choice models or hedonic regressions) are reviewed in Greene (2010).

Vehicle c's market price such that the motorist is indifferent between the two vehicles is vehicle c's market price above which the motorist prefers buying vehicle b rather than vehicle c, in other words the maximum market price of the vehicle c.

Figure 2: Breakdown of the maximum market price of vehicle c
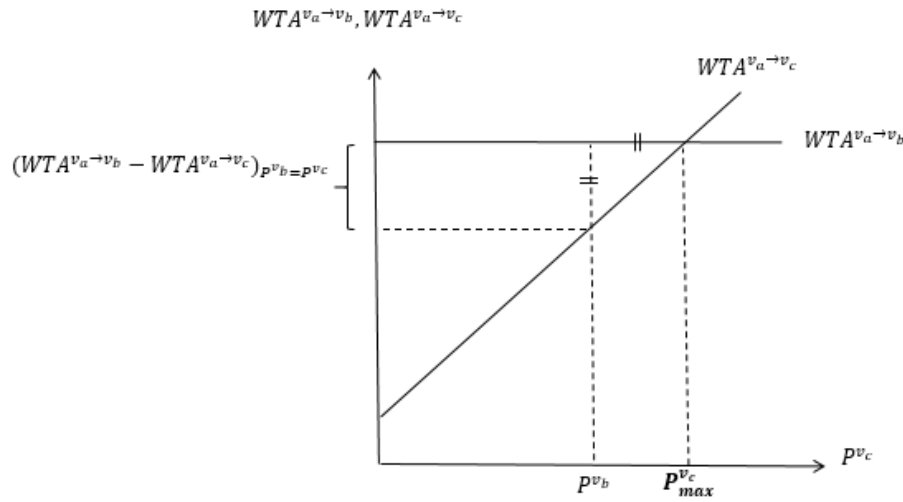
Given that the household's WTP for vehicle c ($WTP^{v_a \to v_c}$) is exactly the maximum market price of the vehicle c ($P_{max}^{v_c}$), we have the following Proposition.

Proposition 3. *The WTP for vehicle c[4] is given by:*

$$WTP^{v_a \to v_c} = P^{v_b} + (WTA^{v_a \to v_b} - WTA^{v_a \to v_c})_{P^{v_b} = P^{v_c}} \tag{5}$$

Using the expressions of $WTA^{v_a \to v_b}$ and $WTA^{v_a \to v_c}$ and since $P^{v_b} = P^{v_c}$, it can be rewritten as follows:

$$WTP^{v_a \to v_c} = P^{v_b} + I \frac{p_f \, ck_a + \theta^2}{ck_a} \left(\frac{ck_b}{p_f \, ck_b + \theta^2} - \frac{ck_c}{p_f \, ck_c + \theta^2} \right) \tag{5 bis}$$

We now address how the different variables of interest affect the WTP for vehicle c. In this regard, Figure 3 below represents the direction of variation of the WTP following a change of one of the variables of interest by highlighting the way the two WTAs for the purchase of

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[3] Common to the WTP for improved fuel economy on the one hand and the difference between the two WTAs on the other is that both amounts result from a comparison of how many households are willing to pay for different new vehicles. On the contrary the *theoretical* WTP for a reduction in fuel consumption (calculated in subsection 2.1. above) results from a comparison between the current vehicle and a new one.

[4] If other vehicles are offered for sale with lower fuel consumption per kilometre than that of vehicle c (i.e. $.. < ck_n < .. < ck_c < ck_b < ck_a$), and using the same approach as previously for the two vehicles, the WTP for each vehicle would be $WTP^{v_a \to v_n} = P^{v_b} + (WTA^{v_a \to v_b} - WTA^{v_a \to v_n})_{P^{v_b} = P^{v_n}}$.

vehicles b and c evolve. Note that the impact of the fuel price on the WTP for the purchase of vehicle c is examined through the analysis of the implementation of a fuel tax (see section 3).

The mains results are summarized in Proposition 4 below.

> **Proposition 4.** *The WTP for vehicle c is:*
> i)   *a decreasing function of the fuel consumption of vehicle a,*
> ii)   *an increasing function of the fuel consumption of vehicle b,*
> iii)   *a decreasing function of the fuel consumption of vehicle c,*
> iv)   *under the assumption that $ck_b + ck_c > ck_a$, an increasing function of the relative preference for mobility by passenger car.*

> **Proof.** See Appendix.

Note that the condition that fuel consumption is such that $ck_b + ck_c > ck_a$ is sufficient but not necessary. However, by considering homogeneous vehicles and given the rate of technological progress on the one hand and the average length of car ownership on the other, we may assume that this condition is met.

Figure 3: Direction of variation of the WTP for vehicle c following changes in the fuel consumption (ck_a, ck_b, ck_c) and relative preference for mobility by passenger car (θ)
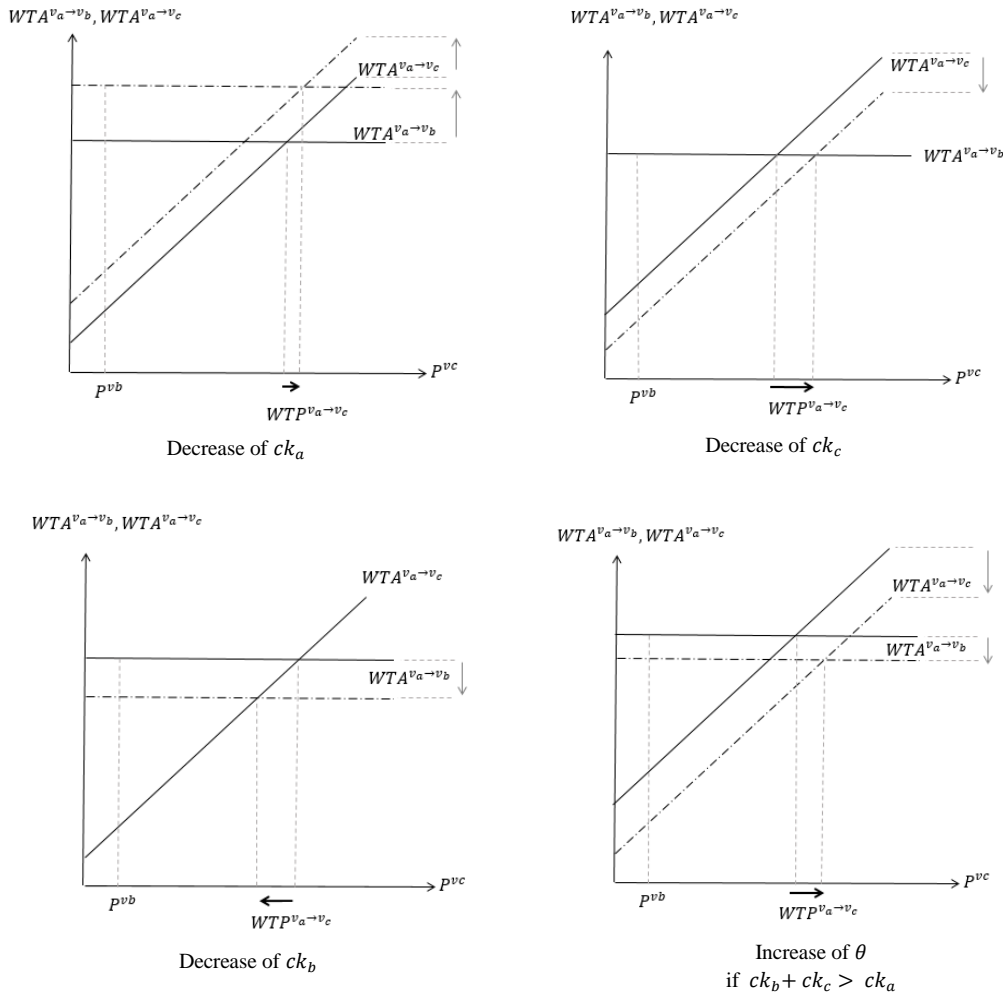
3. Policy implications

In this section, we consider only pricing tools. More specifically, we will here analyse the impact of a fuel tax (expressed in euros and termed τ) on the one hand, and a feebate[5] scheme (e.g. a bonus-malus scheme) on the other. Specifically, let B^{v_c} denote the bonus amount (expressed in euros) that is granted when purchasing vehicle c, and M^{v_b} the malus amount (i.e. the penalty expressed in euros) that is charged when purchasing vehicle b.

We can already emphasize that a feebate scheme consists of change in purchase prices. Hence, it has a direct impact on the purchase decision (i.e. the discrete choice). And, although automobile purchase pricing schemes are claimed not to present the driver with the correct incentive for mileage choice (Santos and al, 2010), they make the household's disposable income (i.e. after the car purchase) increase (i.e. through a bonus) or decrease (i.e. through a malus) and thus are expected here to also have an impact on the mileage (i.e. the continuous choice).

As for a fuel tax, it consists of an immediate cost when facing a new trip. Hence, it both affects the decision on driving, and influences vehicle choice (Jacobsen, 2012). The fact that the purchase decision varies with a fuel tax can be explained by the dependency of the WTP on car use (Mandell, 2009).

Impacts both on the WTA of households that are not willing to purchase a new car (see subsection 3.1.) and on WTP for the least consuming vehicle of households that have to purchase a new car (see subsection 3.2.) are analysed in what follows. Then the risk of rebound effect is addressed in subsection 3.4. Tables 2 below list the budget constraints and corresponding optimal mileages and utilities in the four following situations: without policy tools, with a fuel tax, with a feebate scheme, and with a combination "fuel tax + feebate scheme".

Table 2a: Comparison of the four possible situations involving a fuel tax and a feebate scheme: expressions of the budget constraint

	Budget constraints		
	Situation a	Situation b	Situation c
Without policy tools	$I = m\,p_f\,ck_a + C$	$I - P^{v_b} + \delta P^{v_a} = m\,p_f\,ck_b + C$	$I - P^{v_c} + \delta P^{v_a} = m\,p_f\,ck_c + C$
With a fuel tax	$I = m(p_f + \tau)ck_a + C$	$I - P^{v_b} + \delta P^{v_a} = m(p_f + \tau)ck_b + C$	$I - P^{v_c} + \delta P^{v_a} = m(p_f + \tau)ck_c + C$
With a feebate scheme	$I = m\,p_f\,ck_a + C$	$I - P^{v_b} + \delta P^{v_a} - M^{v_b} = m\,p_f\,ck_b + C$	$I - P^{v_c} + \delta P^{v_a} + B^{v_c} = m\,p_f\,ck_c + C$
With a fuel tax and a feebate scheme	$I = m(p_f + \tau)ck_a + C$	$I - P^{v_b} + \delta P^{v_a} - M^{v_b} = m(p_f + \tau)ck_b + C$	$I - P^{v_c} + \delta P^{v_a} + B^{v_c} = m(p_f + \tau)ck_c + C$

[5] It combines fees and rebates.

Table 2b: Comparison of the four possible situations involving a fuel tax and a feebate scheme: expressions of the optimal mileage

	Optimal mileage		
	Situation a	Situation b	Situation c
Without policy tools	$\dfrac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)}$	$\dfrac{(I - P^{v_b} + \delta P^{v_a})\theta^2}{p_f ck_b(p_f ck_b + \theta^2)}$	$\dfrac{(I - P^{v_c} + \delta P^{v_a})\theta^2}{p_f ck_c(p_f ck_c + \theta^2)}$
With a fuel tax	$\dfrac{I\theta^2}{(p_f + \tau)ck_a\big[(p_f + \tau)ck_a + \theta^2\big]}$	$\dfrac{(I - P^{v_b} + \delta P^{v_a})\theta^2}{(p_f + \tau)ck_b\big[(p_f + \tau)ck_b + \theta^2\big]}$	$\dfrac{(I - P^{v_c} + \delta P^{v_a})\theta^2}{(p_f + \tau)ck_c\big[(p_f + \tau)ck_c + \theta^2\big]}$
With a feebate scheme	$\dfrac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)}$	$\dfrac{(I - P^{v_b} + \delta P^{v_a} - M^{v_b})\theta^2}{p_f ck_b(p_f ck_b + \theta^2)}$	$\dfrac{(I - P^{v_c} + \delta P^{v_a} + B^{v_c})\theta^2}{p_f ck_c(p_f ck_c + \theta^2)}$
With a fuel tax and a feebate scheme	$\dfrac{I\theta^2}{(p_f + \tau)ck_a\big[(p_f + \tau)ck_a + \theta^2\big]}$	$\dfrac{(I - P^{v_b} + \delta P^{v_a} - M^{v_b})\theta^2}{(p_f + \tau)ck_b\big[(p_f + \tau)ck_b + \theta^2\big]}$	$\dfrac{(I - P^{v_c} + \delta P^{v_a} + B^{v_c})\theta^2}{(p_f + \tau)ck_c\big[(p_f + \tau)ck_c + \theta^2\big]}$

Table 2c: Comparison of the four possible situations involving a fuel tax and a feebate scheme: expressions of the utility functions

	Utility functions		
	Situation a	Situation b	Situation c
Without policy tools	$\sqrt{I\left(1 + \dfrac{\theta^2}{p_f ck_a}\right)}$	$\sqrt{(I - P^{v_b} + \delta P^{v_a})\left(1 + \dfrac{\theta^2}{p_f ck_b}\right)}$	$\sqrt{(I - P^{v_c} + \delta P^{v_a})\left(1 + \dfrac{\theta^2}{p_f ck_c}\right)}$
With a fuel tax	$\sqrt{I\left(1 + \dfrac{\theta^2}{(p_f + \tau)ck_a}\right)}$	$\sqrt{(I - P^{v_b} + \delta P^{v_a})\left(1 + \dfrac{\theta^2}{(p_f + \tau)ck_b}\right)}$	$\sqrt{(I - P^{v_c} + \delta P^{v_a})\left(1 + \dfrac{\theta^2}{(p_f + \tau)ck_c}\right)}$
With a feebate scheme	$\sqrt{I\left(1 + \dfrac{\theta^2}{p_f ck_a}\right)}$	$\sqrt{(I - P^{v_b} + \delta P^{v_a} - M^{v_b})\left(1 + \dfrac{\theta^2}{p_f ck_b}\right)}$	$\sqrt{(I - P^{v_c} + \delta P^{v_a} + B^{v_c})\left(1 + \dfrac{\theta^2}{p_f ck_c}\right)}$
With a fuel tax and a feebate scheme	$\sqrt{I\left(1 + \dfrac{\theta^2}{(p_f + \tau)ck_a}\right)}$	$\sqrt{(I - P^{v_b} + \delta P^{v_a} - M^{v_b})\left(1 + \dfrac{\theta^2}{(p_f + \tau)ck_b}\right)}$	$\sqrt{(I - P^{v_c} + \delta P^{v_a} + B^{v_c})\left(1 + \dfrac{\theta^2}{(p_f + \tau)ck_c}\right)}$

Note that there are two ways of writing conditions pertaining to the policy tools to make the WTA decrease, to make the WTP increase, or to neutralise the rebound effect. Either we consider a given feebate scheme, and we determine which fuel tax is needed, or − what we propose to do − we consider a given fuel tax, and we determine which feebate scheme is required. Our choice can be explained by the relative flexibility of a feebate scheme. In fact, an appealing feature of a feebate scheme is its potential neutrality on government finances that is achievable provided that the feebate scheme is flexible enough (Santos and al, 2010).

3.1. Analysis of WTA

When a motorist is not willing to pay for the purchase of a less consuming car (i.e. he does express a willingness to accept), and is not obliged to change car, the policy tool the public authorities have to set up to induce him to purchase a new car is such that his WTA taking into account the policy tool becomes zero.

In view of this discussion, what remains unclear is whether the WTA for the purchase of a less consuming vehicle is reduced when there is either a fuel tax or a feebate scheme. The impacts

on the WTA with regard to a fuel tax, a bonus, a malus, and a combination "fuel tax + bonus" or "fuel tax + malus" are summarized in Propositions 5 and 6 below.

Proposition 5. *Compared to a situation without policy tool, the WTA for the purchase of vehicle b,*
i)   *Is higher when fuel is taxed,*
ii)   *Is higher when a malus is charged,*
iii)   *Is higher when fuel is taxed and a malus is charged.*

Proof. See Appendix.

Proposition 6. *Compared to a situation without policy tool, the WTA for the purchase of vehicle c,*
i)   *Is higher when fuel is taxed,*
ii)   *Is lower when a bonus is granted,*
iii)   *Is lower when fuel is taxed and a bonus is granted if:*

$$B^{v_c} > I \frac{ck_c \, \tau \, (ck_a - ck_c)\theta^2}{ck_a[(p_f + \tau) \, ck_c + \theta^2](p_f ck_c + \theta^2)} \tag{6}$$

Proof. See Appendix.

3.2. Analysis of WTP

In the same way, – and now considering a motorist who has to purchase a new car – the question is whether public decision-makers can make the WTP for the cleanest car increase.

The impacts on the WTP of a fuel tax, a feebate scheme, and a combination "fuel tax + feebate scheme" are summarized in Proposition 7 below and discussed in what follows.

Proposition 7. *Compared to a situation without policy tools, the WTP for vehicle c:*
i)   *Under the condition that $ck_b + ck_c > ck_a$, decreases following the implementation of a fuel tax,*
ii)   *Increases following the implementation of a feebate scheme (the increases amounts $B^{v_c} + M^{v_b}$),*
iii)   *Increases .following the simultaneous implementation of a fuel tax and of a feebate scheme if:*

$$B^{v_c} + M^{v_b} > \frac{I\theta^2}{ck_a} * \left\{ (ck_a - ck_c)\left[\frac{1}{p_f ck_c + \theta^2} - \frac{1}{(p_f + \tau)ck_c + \theta^2} \right] \right. $$
$$\left. - (ck_a - ck_b)\left[\frac{1}{p_f ck_b + \theta^2} - \frac{1}{(p_f + \tau)ck_b + \theta^2} \right] \right\} \tag{7}$$

Proof. See Appendix.

The condition pertaining to the amounts of bonus and malus to make the WTP increase following the simultaneous implementation of a fuel tax and a feebate scheme can be explained as follows. As previously said (cf. Propositions 5 and 6), the WTA for the purchase of vehicle *b* increases with a malus whereas the WTA for the purchase of vehicle *c* decreases with a bonus. That said, it can be easily demonstrated from (5bis) that the WTP for vehicle *c* increases following the implementation of a bonus-malus scheme. The increase equals the sum of the

bonus and malus amounts (cf. Figure 4 below). Actually, since the motorist has to purchase either a vehicle *b* or a vehicle *c*, the bonus-malus scheme allows two benefits for a vehicle *c* purchaser: a direct benefit through the bonus and an indirect benefit avoiding paying the malus.

Figure 4: Variation of the WTP for vehicle *c* following the implementation of feebate scheme
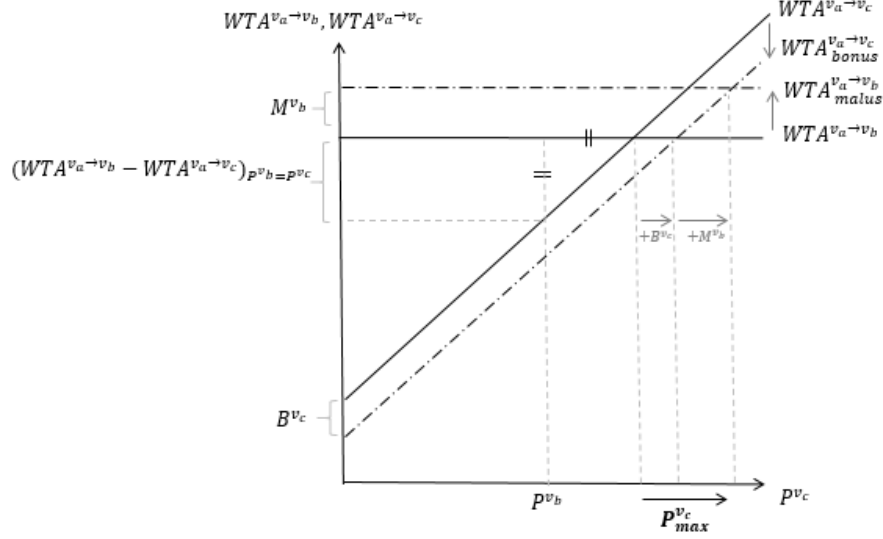
However, the implementation of a fuel tax makes the WTP decrease since the tax's effect on the continuous choice (i.e. the reduction in mileage travelled by passenger vehicle) prevails over the tax's effect on the discrete choice (i.e. the car purchase decision). Hence, the sum of the bonus and malus amounts must equal the decrease of the WTP due to the fuel tax (cf. right-hand term in inequality (7) above) so that the WTP increases when a fuel tax and a feebate scheme are simultaneously implemented.

In view of this discussion, we can emphasize that increasing either the bonus or the malus has the same impact on the WTP for the purchase of vehicle *c*. But, taking into account the normative component of a bonus-malus scheme (that is to say the psychological connotation of punishments and incentives, with the particular higher sensitiveness to losses when facing losses and gains of the same magnitude, see De Hann and al., 2009) would not have led to the same conclusion.

However, even when the WTP increases, as long as it remains below the vehicle *c*'s market price, the motorist still purchases vehicle *b*. The impact of the combination "fuel tax + feebate scheme" on the car purchase decision is addressed in the following subsection.

3.3. Household's purchase decision analysis

To be consistent with Proposition 7, we propose to determine – at a given fuel tax – which feebate scheme makes the WTP for the purchase of vehicle *c* exceed the vehicle *c*'s market price.

> **Proposition 8.** *When a fuel tax and a feebate scheme are simultaneously implemented, the WTP for the purchase of vehicle c is higher than the vehicle c's market price if:*

$$B^{v_c} + M^{v_b} > P^{v_c} - P^{v_b} - I \frac{(p_f + \tau)\, ck_a + \theta^2}{ck_a} \left[\frac{ck_b}{(p_f + \tau)ck_b + \theta^2} - \frac{ck_c}{(p_f + \tau)\, ck_c + \theta^2} \right] \qquad (8)$$

Proof. See Appendix.

This threshold is exactly the difference between the vehicle c's market price and the WTP for the purchase of vehicle c taking into account a fuel tax (by analogy with eq. 5 bis).

But bearing in mind that the public authorities aim to reduce CO_2 emissions, the question is whether implementing simultaneously a fuel tax and a feebate scheme makes households really consume less fuel (see the rebound effect analysis below).

3.4. Rebound effect analysis

The potential change of the mileage travelled by passenger car following the purchase of a new car depends on the balance between two opposing tendencies: higher efficiency on the one hand, which tends to increase the mileage, and net purchase expenditure on the other, which tends to decrease the mileage due to the reduction in the household's disposable income. Thus the so-called rebound effects; i.e. increases in demand induced by efficiency gains, can be taken into account (see Proposition 9).

> **Proposition 9.** *At a given difference between the fuel consumption of the current and the new vehicle (Δ_b for instance with $ck_b = ck_a - \Delta_b$), the rebound effect is observed only when:*
>
> $$P^{v_b} - \delta P^{v_a} < \frac{\Delta_b I p_f}{p_f ck_a + \theta^2} \tag{9}$$

Since both a fuel tax and a feebate scheme affect the optimal mileage (see Table 2b above), the question is whether they can neutralise the rebound effect. The conditions under which the rebound effect is cancelled are summarized through Proposition 10 below and discussed in what follows.

> **Proposition 10.** *The rebound effect is neutralised:*
> i)   *when purchasing vehicle b and:*
> - *solely a fuel tax is implemented, if the tax exceeds the following threshold:*
>
> $$\tau^b_{min} = -\left(p_f + \frac{\theta^2}{2(ck_a - \Delta_b)}\right) + \sqrt{\frac{(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_b)} + \frac{\theta^4}{4(ck_a - \Delta_b)^2}} \tag{10}$$
>
> - *solely a feebate scheme is implemented, if the malus amount exceeds the following threshold:*
>
> $$M^{v_b}{}_{min} = \frac{I \Delta_b p_f}{p_f ck_a + \theta^2} - (P^{v_b} - \delta P^{v_a}) \tag{11}$$
>
> - *both a fuel tax and a feebate scheme are implemented, if the malus amount exceeds the following threshold:*
>
> $$M^{v_b}_{tax\,min} = \frac{I \left[\Delta_b p_f{}^2 - \tau\theta^2 - (ck_a - \Delta_b)(2\tau p_f + \tau^2)\right]}{p_f(p_f ck_a + \theta^2)} - (P^{v_b} - \delta P^{v_a}) \tag{12}$$
>
> ii)   *when purchasing vehicle c and:*
> - *solely a fuel tax is implemented, if the tax exceeds the following threshold:*
>
> $$\tau^c_{min} = -\left(p_f + \frac{\theta^2}{2(ck_a - \Delta_c)}\right) + \sqrt{\frac{(I - P^{v_c} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_c)} + \frac{\theta^4}{4(ck_a - \Delta_c)^2}} \tag{13}$$
>
> - *both a fuel tax and a feebate scheme are implemented, if the bonus does not exceed the following threshold:*

$$B^{v_c}_{tax_{max}} = P^{v_c} - \delta P^{v_a} - \frac{I\left[\Delta_b\, p_f{}^2 - \tau\theta^2 - (ck_a - \Delta_c)(2\tau p_f + \tau^2)\right]}{p_f(p_f ck_a + \theta^2)} \qquad (14)$$

Proof. See Appendix.

Taxing fuel makes the cost per kilometre increase. Then the tendency to increase the mileage owning a less consuming vehicle is reduced. Hence, a fuel tax can neutralise the rebound effect. (see τ^b_{min} in Figure 5 below).

Figure 5: Conditions for the existence of the rebound effect

Note: $\Delta_b = ck_a - ck_b$; $\Delta^* = \frac{(P^{v_b} - \delta P^{v_a})(p_f ck_a + \theta^2)}{I p_f}$
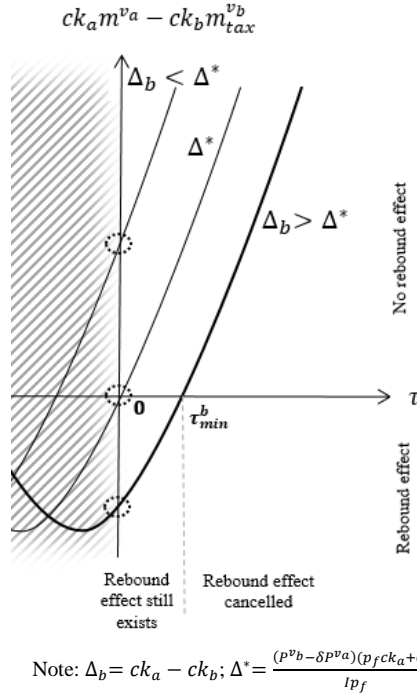
Since, for each situation, the tax threshold depends on the difference between the fuel consumption of the current and the new vehicle on the one hand and on the vehicle's market price on the other, we have the following Proposition.

Proposition 11. *As long as the vehicle c's market price remains lower than the threshold below, the fuel tax required to nullify the rebound effect is higher in situation c than in situation b.*

$$P^{v_c*} = \frac{I\theta^2\left[\theta^2 + (ck_a - \Delta_b)\sqrt{\dfrac{4p_f(ck_a - \Delta_b)(p_f ck_a + \theta^2)(I + \delta P^{v_a} - P^{v_b}) + I\theta^4}{I(ck_a - \Delta_b)}}\right]}{2p_f(p_f ck_a + \theta^2)(ck_a - \Delta_b)^2} + \frac{(I + \delta P^{v_a} - P^{v_b})(\Delta_c - \Delta_b)}{(ck_a - \Delta_b)} + P^{v_b} \qquad (15)$$

Proof. See Appendix.

Such a result highlights the need for public policies to vary according to technological progress. In other words, the greater the technological progress, the more stringent the tax policies have to be, because of such rebound effects. For instance, in response to an increase by 20% in energy efficiency, Brännlund et al. (2007) find that it is necessary to "increase the CO_2 tax by 36% to achieve the same level of CO_2 emissions as before the increase in energy efficiency". Furthermore, it is worth noting that the need for a fuel tax to be introduced alongside an increase in energy efficiency is widely discussed in the empirical literature on the 'fuel intensity standards vs. fuel taxes' debate (see for instance Ajanovic and Haas, 2011; Clerides and Zachariadis, 2008).

Besides, charging a malus when purchasing vehicle b can neutralise the rebound effect; i.e. as soon as the amount of the malus is high enough to make the tendency to decrease the mileage due to the reduction in the disposable income prevail over the tendency to increase the mileage due to higher efficiency. This minimum penalty equals the difference between the net car purchase expenditure above which the rebound effect is neutralised in the absence of policy tools (see eq. 9) and the real-world one. In addition, when fuel is taxed (resulting in a higher cost per kilometre), the tendency to increase the mileage due to higher efficiency is reduced so that the malus required to neutralise the rebound effect is lower. Indeed, the net car purchase expenditure above which the rebound effect is neutralised is lower when fuel is taxed (see Appendix).

Finally, still considering the two tendencies that determine the direction of variation of the mileage, it is straightforward that granting a bonus for the purchase of vehicle c cannot neutralise the rebound effect. However, when fuel is taxed, granting a bonus does not necessarily mean that the rebound effect cannot be neutralised. Actually, the rebound effect is still neutralised as long as the amount the household has really to pay (i.e. the net car purchase expenditure minus the bonus) remains higher than the net car purchase expenditure below which a rebound effect occurs when fuel is taxed. In other words, the bonus has to be such that the tendency to decrease the mileage due to the increase in the cost per kilometre (through the fuel tax) still prevails over the opposite but reduced (thanks to the bonus) tendency to decrease the mileage due to a lower disposable income.

4. Conclusion

This research examines households' willingness to pay (WTP) for an improvement in the fuel efficiency of their vehicle, along with the impact of public policies on this WTP, the car purchase decision, and the rebound effect. To this end, we construct a microeconomic model based on a compensating variation method.

The net car purchase expenditure weighs heavily on the household's budgetary constraint; hence the variation in income that leaves the household's utility unchanged after the purchase of a more efficient vehicle is in fact negative (i.e. as soon as the net car purchase expenditure exceeds a certain percentage of the household's income). In this case, households express a willingness to accept (WTA) instead of a WTP. This WTA therefore appears as a difference to be made good by public policies aiming to provide incentives to purchase more energy-efficient vehicles. With regard to public policies, it emerges that the implementation of a fuel tax as well as of a malus system leads to an increase in household's WTA, whereas a bonus system makes the WTA decrease.

In addition, although a household expresses a WTA to change car, we are able to define a WTP for the least fuel-consuming vehicle offered for sale under the assumption that the household has to purchase a new car. We find that this WTP is the maximum market price of the vehicle. This price is equal to the market price of the more energy consuming vehicle offered

for sale plus the difference between the WTA for the purchase of this latter vehicle and the WTA for the purchase of the least fuel consuming vehicle when the two market prices are the same. Note that a fuel tax leads to a decrease in this WTP and thus cannot lead to the purchase of a more efficient vehicle. This is due to the fact that the reduction in mileage by private car following the implementation of a fuel tax is so important that the tax's effect on the continuous choice (i.e. the driven mileage) prevails over the tax's effect on the discrete choice (i.e. the car purchase decision). To the contrary, a feebate system (e.g. a bonus-malus scheme) leads to an increase in this WTP. Therefore, combining a fuel tax with a feebate scheme can eventually make the WTP increase, and then result in a change in the car purchase decision.

But we also find that reducing the market price of the new vehicle (i.e. through a bonus) is not worthwhile in the light of the rebound effect (i.e. increases in fuel demand induced by efficiency gains). However, a fuel tax – as soon as it exceeds a certain level – is able to nullify the rebound effect. Hence, considering the combination of these two pricing tools, the order of magnitude of the fuel tax on the one hand and that of either the bonus or the malus amount on the other determine whether the household consumes more, or less, fuel.

Way of improving the model so that it better mirrors the real life situation could be to introduce a pure present preference rate – the fact that households tend to give more weight to net expenditure at the moment of the purchase than to the gains over the period of use of the vehicle. It could also be worthwhile to introduce threshold effects between the fuel price and the mileage travelled by car – the fact that an increase in fuel price leads to a reduction in mileage travelled by car is realistic only for kilometres that exceed a certain number of kilometres that consist of travels without alternative transport modes. In that light, distinguish transport services from the other goods or services that enter the utility function would be of great interest. Actually, depending on whether we consider an urban area or a rural one, public transport may or not constitute an alternative to car use, and impact the constraint on car mobility.

Appendixes.

Proof of Proposition 1. The Hicksian compensating variation in income for the purchase of vehicle b, termed $X^{v_a \to v_b}$, is such that:

$$V(I, ck_a, p_f, \theta) = V(I - P^{v_b} + \delta P^{v_a} - X^{v_a \to v_b}, ck_b, p_f, \theta)$$

Or:

$$\sqrt{I\left(1 + \frac{\theta^2}{p_f\, ck_a}\right)} = \sqrt{(I - P^{v_b} + \delta P^{v_a} - X^{v_a \to v_b})\left(1 + \frac{\theta^2}{p_f\, ck_b}\right)}$$

We obtain:

$$X^{v_a \to v_b} = \frac{ck_a - ck_b}{ck_a}\frac{I\theta^2}{p_f\, ck_b + \theta^2} - P^{v_b} + \delta P^{v_a}$$

Proof of Proposition 2. The WTP for the reduction in fuel consumption per kilometre is an increasing function of the household's income (see eq. 2). Therefore, there is an income threshold (termed I^*) above which the WTP for the reduction in fuel consumption per kilometre is higher than the net car purchase expenditure. This income threshold is such that:

$$WTP^{ck_a \to ck_b}(I^*) > P^{v_b} - \delta P^{v_a}$$

From (2) it can be rewritten as follows:

$$\frac{ck_a - ck_b}{ck_a} \frac{I^* \theta^2}{p_f\, ck_b + \theta^2} > P^{v_b} - \delta P^{v_a}$$

We obtain:

$$I^* > (P^{v_b} - \delta P^{v_a}) \frac{ck_a \left(p_f\, ck_b + \theta^2\right)}{(ck_a - ck_b)\theta^2}$$

Or:

$$I^* > \mu (P^{v_b} - \delta P^{v_a})$$

With $\mu = \frac{ck_a \left(p_f\, ck_b + \theta^2\right)}{(ck_a - ck_b)\theta^2}$ and $\mu > 1$.

Proofs of Proposition 4.

i) From (3), we obtain:

$$\frac{\partial WTA^{v_a \to v_b}}{\partial\, ck_a} = -\frac{I\theta^2\, ck_b}{ck_a{}^2 \left(p_f\, ck_b + \theta^2\right)} < 0$$

$$\frac{\partial WTA^{v_a \to v_c}}{\partial\, ck_a} = -\frac{I\theta^2\, ck_c}{ck_a{}^2 \left(p_f\, ck_c + \theta^2\right)} < 0$$

$$\left|\frac{\partial WTA^{v_a \to v_b}}{\partial\, ck_a}\right| - \left|\frac{\partial WTA^{v_a \to v_c}}{\partial\, ck_a}\right| = \frac{I\theta^4 (ck_b - ck_c)}{ck_a{}^2 \left(p_f\, ck_b + \theta^2\right)\left(p_f * ck_c + \theta^2\right)} > 0$$

Using (5), we find:

$$\frac{\partial WTP^{v_a \to v_c}}{\partial\, ck_a} < 0$$

ii) From (3), we obtain:

$$\frac{\partial WTA^{v_a \to v_b}}{\partial\, ck_b} = \frac{I\theta^2 \left(p_f\, ck_a + \theta^2\right)}{ck_a \left(p_f\, ck_b + \theta^2\right)^2} > 0$$

$$\frac{\partial WTA^{v_a \to v_c}}{\partial\, ck_b} = 0$$

Using (5), we have:

$$\frac{\partial WTP^{v_a \to v_c}}{\partial\, ck_b} > 0$$

iii) From (3), we obtain:

$$\frac{\partial WTA^{v_a \to v_b}}{\partial\, ck_c} = 0$$

$$\frac{\partial WTA^{v_a \to v_c}}{\partial\, ck_c} = \frac{I\theta^2 \left(p_f\, ck_a + \theta^2\right)}{ck_a \left(p_f\, ck_c + \theta^2\right)^2} > 0$$

Using (5), we have:

$$\frac{\partial WTP^{v_a \to v_c}}{\partial\, ck_c} < 0$$

iv) From (3), we obtain:

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$$\frac{\partial WTA^{v_a \to v_b}}{\partial \theta} = -\frac{2I\theta p_f(ck_a - ck_b)ck_b}{ck_a(p_f \, ck_b + \theta^2)^2} < 0$$

$$\frac{\partial WTA^{v_a \to v_c}}{\partial \theta} = -\frac{2I\theta p_f(ck_a - ck_c)ck_c}{ck_a(p_f \, ck_c + \theta^2)^2} < 0$$

$$\left|\frac{\partial WTA^{v_a \to v_b}}{\partial \theta}\right| - \left|\frac{\partial WTA^{v_a \to v_c}}{\partial \theta}\right|$$
$$= -\frac{2Ip_f\theta(ck_b - ck_c)\left[2p_f\theta^2 + ck_a ck_b ck_c p_f^2 + (ck_b + ck_c - ck_a)\theta^4\right]}{ck_a(p_f \, ck_b + \theta^2)^2(p_f \, ck_c + \theta^2)^2} < 0$$

Using (5), and $ck_b + ck_c > ck_a$ we have:
$$\frac{\partial WTP^{v_a \to v_c}}{\partial \theta} > 0$$

Proof of Proposition 5.

i)   When fuel is taxed, the WTA for the purchase of vehicle b termed $WTA_{tax}^{v_a \to v_b}$ is such that:
$$V(I, ck_a, p_f + \tau, \theta) = V\left(I - P^{v_b} + \delta P^{v_a} + WTA_{tax}^{v_a \to v_b}, ck_b, p_f + \tau, \theta\right)$$

By analogy with (3), we obtain:

$$WTA_{tax}^{v_a \to v_b} = P^{v_b} - \delta P^{v_a} - \frac{ck_a - ck_b}{ck_a}\frac{I\theta^2}{(p_f + \tau)\,ck_b + \theta^2} > WTA^{v_a \to v_b}$$

ii)   When a malus is charged, the WTA termed $WTA_{malus}^{v_a \to v_b}$ is such that:

$$V(I, ck_a, p_f, \theta) = V\left(I - P^{v_b} + \delta P^{v_a} - M^{v_b} + WTA_{malus}^{v_a \to v_b}, ck_b, p_f, \theta\right).$$
By analogy with (3), we obtain:

$$WTA_{malus}^{v_a \to v_b} = P^{v_b} - \delta P^{v_a} + M^{v_b} - \frac{ck_a - ck_b}{ck_a}\frac{I\theta^2}{p_f ck_b + \theta^2} > WTA^{v_a \to v_b}$$

iii)   When fuel is taxed and a malus is charged , the WTA termed $WTA_{tax\,\&\,malus}^{v_a \to v_b}$ is such that:
$$V(I, ck_a, p_f + \tau, \theta) = V\left(I - P^{v_b} + \delta P^{v_a} - M^{v_b} + WTA_{tax\,\&\,malus}^{v_a \to v_b}, ck_b, p_f + \tau, \theta\right).$$

By analogy with (3), we obtain:

$$WTA_{tax\,\&\,malus}^{v_a \to v_b} = P^{v_b} - \delta P^{v_a} + M^{v_b} - \frac{ck_a - ck_b}{ck_a}\frac{I\theta^2}{(p_f + \tau)\,ck_b + \theta^2} > WTA^{v_a \to v_b}$$

Proof of Proposition 6.

i)   When fuel is taxed, the WTA for the purchase of vehicle c termed $WTA_{tax}^{v_a \to v_c}$ is such that:

$$V(I, ck_a, p_f + \tau, \theta) = V\left(I - P^{v_c} + \delta P^{v_a} + WTA_{tax}^{v_a \to v_c}, ck_c, p_f + \tau, \theta\right)$$

By analogy with (3), we obtain:

$$WTA_{tax}^{v_a \to v_c} = P^{v_c} - \delta P^{v_a} - \frac{ck_a - ck_c}{ck_a}\frac{I\theta^2}{(p_f + \tau)\,ck_c + \theta^2} > WTA^{v_a \to v_c}$$

ii) When a bonus is granted, the WTA termed $WTA_{bonus}^{v_a \to v_c}$ is such that:

$$V(I,\, ck_a, p_f, \theta) = V(I - P^{v_c} + \delta P^{v_a} + B^{v_c} + WTA_{bonus}^{v_a \to v_c},\, ck_c, p_f, \theta).$$

By analogy with (3), we obtain:

$$WTA_{bonus}^{v_a \to v_c} = P^{v_c} - \delta P^{v_a} - B^{v_c} - \frac{ck_a - ck_c}{ck_a}\frac{I\theta^2}{p_f\, ck_c + \theta^2} < WTA^{v_a \to v_c}$$

iii) When fuel is taxed and a bonus is granted, the WTA termed $WTA_{tax\,\&\,bonus}^{v_a \to v_c}$ is such that:

$$V(I,\, ck_a, p_f + \tau, \theta) = V(I - P^{v_c} + \delta P^{v_a} + B^{v_c} + WTA_{bonus}^{v_a \to v_c},\, ck_c, p_f + \tau, \theta).$$

By analogy with (3), we obtain:

$$WTA_{tax\,\&\,bonus}^{v_a \to v_c} = P^{v_c} - \delta P^{v_a} - B^{v_c} - \frac{ck_a - ck_c}{ck_a}\frac{I\theta^2}{(p_f + \tau)\,ck_c + \theta^2}$$

The difference between the WTAs with and without a fuel tax coupled with a bonus is negative if:

$$WTA_{tax\,\&\,bonus}^{v_a \to v_c} < WTA^{v_a \to v_c}$$

Or,

$$P^{v_c} - \delta P^{v_a} - B^{v_c} - \frac{ck_a - ck_c}{ck_a}\frac{I\theta^2}{(p_f + \tau)\,ck_c + \theta^2} < P^{v_c} - \delta P^{v_a} - \frac{ck_a - ck_c}{ck_a}\frac{I\theta^2}{p_f ck_c + \theta^2}$$

We obtain:

$$B^{v_c} > \frac{ck_c\,\tau\,(ck_a - ck_c)I\theta^2}{ck_a\big[(p_f + \tau)\,ck_c + \theta^2\big]\big(p_f ck_c + \theta^2\big)} > 0$$

Proof of Proposition 7.

i) By analogy with (5 bis), the WTP for the purchase of vehicle c with a fuel tax is given by:

$$WTP_{tax}^{v_a \to v_c} = P^{v_b} + I\frac{(p_f + \tau)\,ck_a + \theta^2}{ck_a}\left[\frac{ck_b}{(p_f + \tau)ck_b + \theta^2} - \frac{ck_c}{(p_f + \tau)\,ck_c + \theta^2}\right]$$

The difference between the WTPs for a vehicle c with and without a tax is:

$$WTP_{tax}^{v_a \to v_c} - WTP^{v_a \to v_c} = \frac{I\theta^2}{ck_a} *$$

$$\left\{(ck_a - ck_b)\left[\frac{1}{p_f ck_b + \theta^2} - \frac{1}{(p_f + \tau)ck_b + \theta^2}\right] - (ck_a - ck_c)\left[\frac{1}{p_f ck_c + \theta^2} - \frac{1}{(p_f + \tau)ck_c + \theta^2}\right]\right\}$$

The quantity is zero when no tax is implemented ($\tau = 0$). Hence if the difference between the WTPs decreases when a fuel tax is implemented, the difference is necessarily negative. Accordingly, we look at the derivative of the difference with respect to the fuel tax amount:

$$\frac{\partial\left(WTP_{tax}^{v_a \to v_c} - WTP^{v_a \to v_c}\right)}{\partial\tau} = \frac{I\theta^2}{ck_a}\left[\frac{ck_a - ck_b}{(p_f + \tau)ck_b + \theta^2} - \frac{ck_a - ck_c}{(p_f + \tau)ck_c + \theta^2}\right]$$

Using $ck_b + ck_c > ck_a$:

$$\frac{\partial\left(WTP_{tax}^{v_a \to v_c} - WTP^{v_a \to v_c}\right)}{\partial\tau} < 0$$

ii) By analogy with (5), the WTP for the purchase of vehicle c with a bonus-malus scheme is given by:

$$WTP_{bonus}^{v_a \to v_c} = P^{v_b} + (WTA_{malus}^{v_a \to v_b} - WTA_{bonus}^{v_a \to v_c})_{P^{v_b} = P^{v_c}}$$

Or, using expressions from proof of Propositions 5 and 6:

$$WTP_{bonus}^{v_a \to v_c} = P^{v_b} + \left[P^{v_b} - \delta P^{v_a} + M^{v_b} - \frac{ck_a - ck_b}{ck_a}\frac{I\theta^2}{p_f ck_b + \theta^2}\right]$$
$$- \left[P^{v_b} - \delta P^{v_a} - B^{v_c} - \frac{ck_a - ck_c}{ck_a}\frac{I\theta^2}{p_f ck_c + \theta^2}\right]$$

We obtain:

$$WTP_{bonus-malus}^{v_a \to v_c} = P^{v_b} + M^{v_b} + B^{v_c} + I\frac{p_f ck_a + \theta^2}{ck_a}\left[\frac{ck_b}{p_f ck_b + \theta^2} - \frac{ck_c}{p_f ck_c + \theta^2}\right]$$

The difference between the WTPs for a vehicle c with and without a bonus-malus scheme is:

$$WTP_{bonus-malus}^{v_a \to v_c} - WTA^{v_a \to v_c} = M^{v_b} + B^{v_c} > 0$$

iii) By analogy with (5), the WTP for the purchase of vehicle c with a fuel tax coupled with a bonus-malus scheme is:

$$WTP_{tax\,\&\,bonus-malus}^{v_a \to v_c} = P^{v_b} + (WTA_{tax\,\&\,malus}^{v_a \to v_b} - WTA_{tax\,\&\,bonus}^{v_a \to v_c})_{P^{v_b} = P^{v_c}}$$

Or, using expressions from proof of Propositions 5 and 6:

$$WTP_{tax\,\&\,bonus-malus}^{v_a \to v_c}$$
$$= P^{v_b} + \left[P^{v_b} - \delta P^{v_a} + M^{v_b} - \frac{ck_a - ck_b}{ck_a}\frac{I\theta^2}{(p_f + \tau)ck_b + \theta^2}\right]$$
$$- \left[P^{v_b} - \delta P^{v_a} - B^{v_c} - \frac{ck_a - ck_c}{ck_a}\frac{I\theta^2}{(p_f + \tau)ck_c + \theta^2}\right]$$

We obtain:

$$WTP_{tax\,\&\,bonus-malus}^{v_a \to v_c}$$
$$= P^{v_b} + B^{v_c} + M^{v_b}$$
$$+ I\frac{(p_f + \tau)ck_a + \theta^2}{ck_a}\left[\frac{ck_b}{(p_f + \tau)ck_b + \theta^2} - \frac{ck_c}{(p_f + \tau)ck_c + \theta^2}\right]$$

The difference between the WTPs for a vehicle c with and without policy tools is:

$$WTP_{tax\,\&\,bonus-malus}^{v_a \to v_c} - WTA^{v_a \to v_c} = B^{v_c} + M^{v_b} + \frac{I\theta^2}{ck_a} *$$

$$\left\{ (ck_a - ck_b) \left[\frac{1}{p_f ck_b + \theta^2} - \frac{1}{(p_f + \tau)ck_b + \theta^2} \right] - (ck_a - ck_c) \left[\frac{1}{p_f ck_c + \theta^2} - \frac{1}{(p_f + \tau)ck_c + \theta^2} \right] \right\}$$

The WTP with policy tools is higher than without if:

$$B^{v_c} + M^{v_b} > \frac{I\theta^2}{ck_a}$$
$$* \left\{ (ck_a - ck_c) \left[\frac{1}{p_f ck_c + \theta^2} - \frac{1}{(p_f + \tau)ck_c + \theta^2} \right] \right.$$
$$\left. - (ck_a - ck_b) \left[\frac{1}{p_f ck_b + \theta^2} - \frac{1}{(p_f + \tau)ck_b + \theta^2} \right] \right\}$$

Proof of Proposition 8. The WTP for the purchase of vehicle c when a fuel tax and a bonus-malus scheme are simultaneously implemented becomes higher than the vehicle c's market price if:

$$WTP^{v_a \to v_c}_{tax\,\&\,bonus-malus} > P^{v_c}$$
$$P^{v_b} + B^{v_c} + M^{v_b} + I \frac{(p_f + \tau)\,ck_a + \theta^2}{ck_a} \left[\frac{ck_b}{(p_f + \tau)ck_b + \theta^2} - \frac{ck_c}{(p_f + \tau)\,ck_c + \theta^2} \right] > P^{v_c}$$

We obtain:

$$B^{v_c} + M^{v_b} > P^{v_c} - P^{v_b} - I \frac{(p_f + \tau)\,ck_a + \theta^2}{ck_a} \left[\frac{ck_b}{(p_f + \tau)ck_b + \theta^2} - \frac{ck_c}{(p_f + \tau)\,ck_c + \theta^2} \right]$$

Or (by analogy with (5bis)):

$$B^{v_c} + M^{v_b} > P^{v_c} - WTP^{v_a \to v_c}_{tax}$$

Proof of Proposition 9. Considering for instance situation b (i.e. purchase of vehicle b with $ck_b = ck_a - \Delta_b$), a rebound effect exists when the household's fuel consumption in situation b is higher than in situation a (i.e. keeping their current vehicle a). It can be written as follows:

$$m^{v_a} ck_a - m^{v_b}(ck_a - \Delta_b) < 0$$

$$\frac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)} ck_a - \frac{(I - P^{v_b} + \delta P^{v_a})\theta^2}{p_f(ck_a - \Delta_b)[p_f(ck_a - \Delta_b) + \theta^2]} (ck_a - \Delta_b) < 0$$

We obtain:

$$P^{v_b} - \delta P^{v_a} < \frac{\Delta_b I p_f}{p_f ck_a + \theta^2}$$

Proof of Proposition 10.

i) Considering the purchase of vehicle b
- **With a fuel tax.**

Considering for instance situation b, a rebound effect still exists when the household's fuel consumption in situation b following the implementation of a fuel tax remains higher than in situation a. Note that in order to find the amount of the tax that keeps fuel consumption unchanged despite the greater fuel efficiency of the new vehicle, the baseline situation in which the motorist keeps his current car is considered without a fuel tax. It can be written as follows:

$$ck_a m^{v_a} - ck_b m_{tax}^{v_b} < 0$$

$$\frac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)}ck_a - \frac{(I - P^{v_b} + \delta P^{v_a})\theta^2}{(p_f + \tau)(ck_a - \Delta)[(p_f + \tau)(ck_a - \Delta) + \theta^2]}(ck_a - \Delta_b) < 0$$

$$\tau^2 Ick_a\theta^2(ck_a - \Delta_b)^2 + \tau[2p_f(ck_a - \Delta_b)^2 + (ck_a - \Delta_b)\theta^2] + p_f(ck_a - \Delta)[p_f(ck_a - \Delta_b) + \theta^2]$$
$$- \frac{p_f(ck_a - \Delta_b)(p_f ck_a + \theta^2)(I - P^{v_b} + \delta P^{v_a})}{I} < 0$$

It can been written as: $\alpha\,\tau^2 + \beta\,\tau + \gamma < 0$. The two roots of the polynomial are:

$$Sol1 = -\left(p_f + \frac{\theta^2}{2(ck_a - \Delta_b)}\right) - \sqrt{\frac{(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_b)} + \frac{\theta^4}{4(ck_a - \Delta_b)^2}}$$

$$Sol2 = -\left(p_f + \frac{\theta^2}{2(ck_a - \Delta_b)}\right) + \sqrt{\frac{(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2) * p_f}{I(ck_a - \Delta_b)} + \frac{\theta^4}{4(ck_a - \Delta_b)^2}}$$

Sol1 is negative; only *Sol2* can take positive values.

In addition, *Sol2* is an increasing function of the fuel efficiency gain Δ – what makes the fuel efficiency gain increase when moving to the right in Figure 5.
Proof:

$$\frac{\partial Sol2}{\partial \Delta_b} > 0$$

$$\frac{\frac{(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_b)^2} + \frac{1}{2}\frac{\theta^4}{(ck_a - \Delta_b)^3}}{\sqrt{\frac{4(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_b)} + \frac{\theta^4}{(ck_a - \Delta_b)^2}}} - \frac{\theta^2}{2(ck_a - \Delta_b)^2} > 0$$

$$\frac{(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_b)^2} + \frac{1}{2}\frac{\theta^4 P^2}{(ck_a - \Delta_b)^3}$$
$$> \frac{\theta^2}{2(ck_a - \Delta_b)^2}\sqrt{\frac{4(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_b)} + \frac{\theta^4}{(ck_a - \Delta_b)^2}}$$

Multiplying this inequality first by $(ck_a - \Delta_b)^3$ and then by $\frac{2}{\theta^4}$, we obtain:
$$\frac{2(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f(ck_a - \Delta_b)}{I\theta^4} + 1$$
$$> \sqrt{\frac{4(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f(ck_a - \Delta_b)}{I\theta^4} + 1}$$

Let x and k denote:
$$x = (ck_a - \Delta_b)$$
$$k = \frac{2(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I\theta^4}$$
Then the inequality can be rewritten: $kx + 1 > \sqrt{2kx + 1}$

Raising it to the power of 2, we obtain: $k^2x^2 > 0$. This is always true.
In the end: $\frac{\partial Sol2}{\partial \Delta} > 0$

Since α is positive, the quantity $\alpha\tau^2 + \beta\tau + \gamma$ is negative between the roots, and positive outside. Hence:

- If $Sol2 < 0$, then $\forall \tau$, $\alpha\tau^2 + \beta\tau + \gamma > 0$: the fuel consumed by the household with vehicle b following the implementation of the fuel tax is lower than the fuel consumed by the household with vehicle a: there is no rebound effect.
  Yet, by resolving $Sol2 = 0$, we find that $Sol2 < 0$ as long as Δ_b:

$$\Delta_b < \frac{(P^{v_b} - \delta P^{v_a})(p_f ck_a + \theta^2)}{I p_f} = \Delta^*$$

  This result is consistent with the rebound effect analysis above (cf. Proposition 9).

- If $Sol2 > 0$ ($\Delta_b > \Delta^*$) then:
  o if $0 < \tau < Sol2$, then $\alpha\tau^2 + \beta\tau + \gamma < 0$: the fuel consumed by the household with vehicle b following the implementation of the fuel tax remains higher than the fuel consumed by the household with vehicle a (without a fuel tax). The rebound effect still exists.
  o if $0 < Sol2 < \tau$, then $\alpha\tau^2 + \beta\tau + \gamma > 0$: the fuel consumed by the household with vehicle b following the implementation of the fuel tax becomes lower than the fuel consumed by the household with vehicle a (without a fuel tax). The amount of the tax is high enough to nullify the rebound effect.

Thus $Sol2$ represents the minimum required tax to nullify the unwanted rebound effect. Then:

$$\tau_{min}^b = -\left(p_f + \frac{\theta^2}{2(ck_a - \Delta_b)}\right) + \sqrt{\frac{(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_b)} + \frac{\theta^4}{4(ck_a - \Delta_b)^2}}$$

- **With a feebate scheme.**
In the same way, the rebound effect is neutralised when:

$$m^{v_a} ck_a - m_{malus}^{v_b}(ck_a - \Delta_b) > 0$$

$$\frac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)} ck_a - \frac{(I - P^{v_b} + \delta P^{v_a} - M^{v_b})\theta^2}{p_f(ck_a - \Delta_b)[p_f(ck_a - \Delta_b) + \theta^2]}(ck_a - \Delta_b) > 0$$

We obtain:

$$M^{v_b} > \frac{I \Delta_b p_f}{p_f ck_a + \theta^2} - (P^{v_b} - \delta P^{v_a}) = M^{v_b}{}_{min}$$

In other words, the malus has to equal the difference between the car net purchase expenditure above which there is no rebound effect (see Proposition 9) and the real-world car net purchase expenditure.

- **With the combination "fuel tax + feebate scheme"**
The rebound effect is neutralised when:

$$m^{v_a} ck_a - m_{tax \& malus}^{v_b}(ck_a - \Delta_b) > 0$$

$$\frac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)} ck_a - \frac{(I - P^{v_b} + \delta P^{v_a} - M^{v_b})\theta^2}{(p_f + \tau)(ck_a - \Delta_b)[(p_f + \tau)(ck_a - \Delta_b) + \theta^2]}(ck_a - \Delta_b) > 0$$

We obtain:

$$M^{v_b} > \frac{I\left[\Delta_b\left(p_f+\tau\right)^2 - \tau\theta^2 - ck_a\left(2\tau p_f + \tau^2\right)\right]}{p_f\left(p_f ck_a + \theta^2\right)} - (P^{v_b} - \delta P^{v_a}) = M^{v_b}_{tax\,min}$$

The first part of the right-hand term consists of the net car purchase expenditure above which there is no rebound effect when fuel is taxed. Proof:

$$m^{v_a}ck_a - m^{v_b}_{tax}\left(ck_a - \Delta_b\right) > 0$$

$$\frac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)}ck_a - \frac{(I - P^{v_b} + \delta P^{v_a})\theta^2}{(p_f + \tau)(ck_a - \Delta_b)[(p_f + \tau)(ck_a - \Delta_b) + \theta^2]}(ck_a - \Delta_b)$$
$$> 0$$

We obtain:

$$P^{v_b} + \delta P^{v_a} > \frac{I\left[\Delta_b\left(p_f+\tau\right)^2 - \tau\theta^2 - ck_a\left(2\tau p_f + \tau^2\right)\right]}{p_f\left(p_f ck_a + \theta^2\right)}$$

In addition, this malus threshold ($M^{v_b}_{tax\,min}$) is lower than that above which the rebound effect is neutralised when there is solely the malus (i.e. $M^{v_b}_{min}$). Proof:

$$M^{v_b}_{tax\,min} < M^{v_b}_{min}$$
$$\leftrightarrow$$
$$\frac{I\left[\Delta_b\left(p_f+\tau\right)^2 - \tau\theta^2 - ck_a\left(2\tau p_f + \tau^2\right)\right]}{p_f\left(p_f ck_a + \theta^2\right)} - (P^{v_b} - \delta P^{v_a}) < \frac{I\,\Delta_b\,p_f}{p_f ck_a + \theta^2} - (P^{v_b} - \delta P^{v_a})$$
$$\leftrightarrow$$
$$\Delta_b\left(p_f + \tau\right)^2 - \tau\theta^2 - ck_a\left(2\tau p_f + \tau^2\right) < \Delta_b p_f^2$$
$$\leftrightarrow$$
$$\left(2\tau p_f + \tau^2\right)(\Delta_b - ck_a) - \tau\theta^2 < 0$$

This is always true (with $\Delta_b - ck_a < 0$).

ii)   *Considering the purchase of vehicle c*
• *With a fuel tax*
By analogy with the minimum tax that neutralise the rebound effect when purchasing vehicle b, we have:

$$\tau^c_{min} = -\left(p_f + \frac{\theta^2}{2(ck_a - \Delta_c)}\right) + \sqrt{\frac{(I - P^{v_c} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_c)} + \frac{\theta^4}{4(ck_a - \Delta_c)^2}}$$

• *With a feebate scheme*
The rebound effect is neutralised when:

$$m^{v_a}ck_a - m^{v_c}_{bonus}(ck_a - \Delta_c) > 0$$

$$\frac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)}ck_a - \frac{(I - P^{v_c} + \delta P^{v_a} + B^{v_c})\theta^2}{p_f(ck_a - \Delta_c)[p_f(ck_a - \Delta_c) + \theta^2]}(ck_a - \Delta_c) > 0$$

We obtain:

$$B^{v_c} < (P^{v_c} - \delta P^{v_a}) - \frac{I\,\Delta_c\,p_f}{p_f ck_a + \theta^2}$$

When there is a rebound effect we have: $(P^{v_c} - \delta P^{v_a}) - \frac{I \Delta_c p_f}{p_f ck_a + \theta^2} < 0$ (by analogy with eq. 9). Yet the amount of a bonus is necessarily positive ($B^{v_c} > 0$). Hence, the condition is never met, and a bonus cannot neutralise the rebound effect.

- ***With a combination "fuel tax + feebate scheme"***

The rebound effect is neutralised when:

$$m^{v_a} ck_a - m^{v_c}_{tax \,\&\, bonus}(ck_a - \Delta_c) > 0$$

$$\frac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)} ck_a - \frac{(I - P^{v_c} + \delta P^{v_a} + B^{v_c})\theta^2}{(p_f + \tau)(ck_a - \Delta_c)[(p_f + \tau)(ck_a - \Delta_c) + \theta^2]}(ck_a - \Delta_c) > 0$$

We obtain:

$$B^{v_c} < (P^{v_c} - \delta P^{v_a}) - \frac{I\left[\Delta_c(p_f + \tau)^2 - \tau\theta^2 - ck_a(2\tau p_f + \tau^2)\right]}{p_f(p_f ck_a + \theta^2)} = B^{v_c}_{tax \, max}$$

It can be rewritten as:

$$P^{v_c} - \delta P^{v_a} - B^{v_c} > \frac{I\left[\Delta_c(p_f + \tau)^2 - \tau\theta^2 - ck_a(2\tau p_f + \tau^2)\right]}{p_f(p_f ck_a + \theta^2)}$$

The rebound effect is still neutralised as long as the bonus does not make the amount the household has really to pay (the net car purchase expenditure minus the bonus; i.e. the left-hand term) becomes lower than the net car purchase expenditure below which there is a rebound effect when fuel is taxed (i.e. the right hand term; see proof just below)

Proof: The right-hand term consists of the net car purchase expenditure above which there is no rebound effect when fuel is taxed.

$$m^{v_a} ck_a - m^{v_c}_{tax}(ck_a - \Delta_c) > 0$$

$$\frac{I\theta^2}{p_f ck_a(p_f ck_a + \theta^2)} ck_a - \frac{(I - P^{v_c} + \delta P^{v_a})\theta^2}{(p_f + \tau)(ck_a - \Delta_c)[(p_f + \tau)(ck_a - \Delta_c) + \theta^2]}(ck_a - \Delta_c) > 0$$

We obtain:

$$P^{v_c} + \delta P^{v_a} > \frac{I\left[\Delta_c(p_f + \tau)^2 - \tau\theta^2 - ck_a(2\tau p_f + \tau^2)\right]}{p_f(p_f ck_a + \theta^2)}$$

Proof of Proposition 11. The vehicle c's market price that equals the tax levels required to nullify the rebound effect in situation b on the one hand and in situation c on the other (termed P^{v_c*}) is such that

$$\tau^b_{min} = \tau^c_{min}$$

$$-\left(p_f + \frac{\theta^2}{2(ck_a - \Delta_b)}\right) + \sqrt{\frac{(I - P^{v_b} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_b)} + \frac{\theta^4}{4(ck_a - \Delta_b)^2}}$$

$$= -\left(p_f + \frac{\theta^2}{2(ck_a - \Delta_c)}\right) + \sqrt{\frac{(I - P^{v_c*} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_c)} + \frac{\theta^4}{4(ck_a - \Delta_c)^2}}$$

We obtain:

$$P^{v_c*} = \frac{I\theta^2 \left[\theta^2 + (ck_a - \Delta_b)\sqrt{\dfrac{4p_f(ck_a - \Delta_b)(p_f ck_a + \theta^2)(I + \delta P^{v_a} - P^{v_b}) + I\theta^4}{I(ck_a - \Delta_b)}}\right]}{2p_f(p_f ck_a + \theta^2)(ck_a - \Delta_b)^2}$$
$$+ \frac{(I + \delta P^{v_a} - P^{v_b})(\Delta_c - \Delta_b)}{(ck_a - \Delta_b)} + P^{v_b}$$

Moreover, we have:

$$\frac{\partial \tau_{min}^c}{\partial P^{v_c}} = -\frac{(p_f ck_a + \theta^2)p_f}{2I(ck_a - \Delta_c)\sqrt{\dfrac{(I - P^{v_c} + \delta P^{v_a})(p_f ck_a + \theta^2)p_f}{I(ck_a - \Delta_c)} + \dfrac{\theta^4}{4(ck_a - \Delta_c)^2}}} < 0$$

It follows that, as long as the vehicle c's market price remains lower than P^{v_c*}, the fuel tax required to nullify the rebound effect is higher in situation c than in situation b.

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