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On patent strength, litigation costs, and patent disputes under alternative damage rules[*]

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Abstract

This paper analyzes the effects of two damage rules (Lost Profit *vs* Unjust Enrichment) mainly used by Courts in patent litigations. In our model, the Infringer either is a mere imitator of the Patentee or introduces incremental innovations, and litigation costs are private information such that a pretrial settlement may be better for both litigants. We show that the Unjust Enrichment rule yields less trials than the Lost Profit one. But regarding three main objectives, Patentee's protection, incentives to invest in *R&D*, and social welfare maximization, we find that no rule is better than the other generally speaking. Our model also allows to emphasize how the combination between the size of litigation costs, the negotiation gains and the IPR strength, shapes the incentives to enforce as well infringe a IPR, although in a way specific to each rule.

Keywords: intellectual property, probabilistic patents, patent litigations, incremental innovations, pretrial negotiations, legal costs, imperfect competition.

JEL classification: 03, L1, L4, D8, K2, K4.

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1 Introduction

The former literature on innovation and *R&D* investments (see the survey by Reinganum, 1989) took as granted that Intellectual Property Rights afford a perfect legal protection to their holders. It is now well known that this view has weak empirical support. Lanjouw and Lerner (1998) and Bessen and Meurer (2005) first reported the dramatic increase in the number of patent litigation cases over the two last decades. Based on all litigated patents cases, Allison and Lemley (1998) found a figure close to 50% for the plaintiffs' rate of success, associated with high rates of reversal of judgments on appeal. It seems also that many patentees forgive to sue their infringer, given the delays and costs of litigation in patent cases (Gallini, 2002; Lanjouw and Schankerman, 2001; Pagano and Rossi, 2009).

In contrast, the literature of the last decade recognizes that IPR are *probabilistic*, i.e. there is a high risk that the litigated IPR be found invalid at trial. Several papers dealing with the issue of patent litigations (including Anton and Yao, 2003, 2006; Aoki and Hu, 1999; Choi, 2009; Crampes and Langinier, 2002; Henry and Turner, 2010; Schankerman and Scotchmer, 2001; Shapiro, 2003) have thus found that IP laws have no deterrence effect on infringement (at equilibrium) under alternative assumptions regarding the strategic interactions between innovative firms and their imitators (i.e. quantity vs price competition, or vertical relationships) or the legal environment (i.e. use of preliminary injunctions, role of alternative damage rules, limits of antitrust and competition law). Moreover, this results of zero deterrence has been found both for drastic innovations (the cost advantage to the innovative firm allows to potentially monopolize a market to the extent that no infringement occurs; Choi, 2009) and non drastic ones (the market power of the innovators increases but not in such a way that competing firms will exit; Anton and Yao, 2006). Finally, as regards to social welfare and/or incentives to innovate, existing works found that the effects of IPR are only channeled through the impacts on production costs and/or products differentiation at the market stage (Anton and Yao, 2003, 2006; Choi, 2009); in short, the usual determinants of behaviors at the litigation stage (Spier 2007) seem to play no role on social welfare, surprisingly. This contrasts with the empirical literature and the ongoing debate touching to the increasing social costs of intellectual property laws (Gallini, 2002; Lanjouw and Schankerman, 2001; Pagano and Rossi, 2009), associated with the patent litigation explosion (Lanjouw and Lerner (1998) and Bessen and Meurer (2005)).

A first innovative feature of this paper is that it tackles with two restrictive assumptions usually adopted in this literature which partly explains the zero deterrence result. First, the literature neglects the existence and strategic role of legal costs borne both by the patentee and the infringer

at trial[1]. They are seen as unavoidable fixed costs incurred by litigants, and assumed to be equal to zero for simplicity. Second, it assumes that a trial occurs for sure, although with an uncertain judgment, soon as infringement has been detected. Thus it ignores[2] that a settlement may be preferable to a trial when the court prevails for the plaintiff only with a probability less than one (IPR are probabilistic). In our paper in contrast, we combine a model of settlement litigations in the *Law & Economics* tradition (see the surveys by Spier, 2007; Daughety and Reinganum, 2012) with a model of oligopolistic competition. In this set up, we analyze how the choice between two damage rules, Lost Profit *versus* Unjust Enrichment[3], affects both the social welfare and the incentives to innovate.

Another innovative feature of our paper is linked to the fact that we enrich the set of strategies available to firms investing in *R&D* and competing for the market, such that several decisions pertaining either to the Patentee or to the Infringer are considered as endogenous: the decision to infringe and to enforce a patent, the decision to settle or litigate the case, as well as the decision to produce the good. We assume that at the pretrial negotiations stage, the legal litigants' costs are private information. Such an assumption may be seen as an analytical short-cut (see also Chopard et ali, 2010) capturing that parties opposed in a litigation may initially be unequally endowed in terms of skill or ability to predict the verdict at trial. For example, parties will experience *ex post* differences in legal expenditures as long as, regarding the technology of information production, there are significant differences in the marginal products of the individual effort. As long as the characteristics of the technology of information production are private information, these investments in information acquisition are not observable by the other party. Finally, we focus on the case of a non drastic innovation[4], and assume Cournot competition between the Potential Infringer and the Patent Holder.

We show that our alternative damage rules may entail different selection effects at the pretrial stage, thus inducing a different probability of trial/settlement. This in turn may exert different incentives to infringe as well as enforce a patent. As a major consequence, the comparisons in welfare or in incentives to innovate, may become not easy to perform; or may be non sense since depending on the rule, each equilibrium may be supported but different values of the patent strength. More specifically, under the assumptions of a linear demand and constant marginal

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[1]Two noticeable exceptions are Meurer (2000) and Ottoz and Cugno (2012), who analyze the influence of fee shifting on patent litigations.

[2]The exception is Crampes and Langinier (2002) but they consider Nash negotiations, i.e. the probability of trial is trivially either 1 or 0.

[3]These two alternative damage rules are currently used by Courts both in the U.S. and in Europe; see Choi (2009) and Elkin-Koren and Salzberger (2000) for a discussion of the doctrines.

[4]We consider the case of a drastic innovation in a companion paper, Chopard, Cortade and Langlais (2013).

production costs, we find that only weak patents are potentially enforceable under the Lost Profit rule, in the sense that only patents weak enough will be associated with a positive harm in case of infringement (see also Anton and Yao, 2006). In contrast, under the Unjust Enrichment rule, the Patentee may renounce to enforce his patent when it is weak enough, despite the existence of an effective harm. We also show that the Lost Profit rule yields a probability of trial which is always at least as high as the Unjust Enrichment one. Regarding the objectives of Patentee's protection, *R&D* incentives, and social welfare maximization, we find that generally speaking, no rule is better than the other. When the patent is weak enough, the Patentee's profit is higher under the Lost Profit rule. When the patent is strong enough, social welfare is higher (smaller) under the Lost Profit (Unjust Enrichment) rule when the Patentee is more (respectively less) efficient than the Infringer.

Section 2 introduces the basic framework with non drastic innovations and private information on the patentee's legal costs, discusses the properties of the equilibria obtained when Lost Profit are awarded by Courts or when Unjust Enrichment are applied. Section 3 highlights the performances of the Lost Profit and Unjust Enrichment rules regarding the preservation of firms' profits or the social welfare. Section 4 concludes.

2 A model with non drastic innovations

2.1 Model and assumptions

The game between the Infringer and the Patent Holder has four main stages:

- At stage 1, Nature chooses the type of the Patent Holder c_h. The Infringer only knows that the value of the Patent Holder's cost (labelled Patent Holder's type in the rest of the paper) is a random variable $c_h \in [\underline{c}_h, \bar{c}_h]$ distributed according to a cumulative function $F(c_h)$ and a density $f(c_h)$. In contrast, the Infringer's litigation costs, denoted c_e, are common knowledge[5].

- At stage 2, the Infringer has to decide whether he enters with infringement (chooses *Infringe*), and competes with the Patent Holder (the potential market is a duopoly); or he enters without

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[5]In words, we consider the limit case where individual legal expenditures have no effect on the plaintiff's probability of prevailing at trial. This is a simplification in order to focus on the role of informational asymmetries resulting from the parties' skill in predicting a given outcome at trial, requiring, all else equal, different amount of investments in information production.

infringement (chooses *Non Infringe*) such that the Patent Holder produces the output y_h^N and earns π_h^N, while the Infringer produces the output y_e^N and earns π_e^N.

- At stage 3, the Patent Holder chooses either *Accommodate* to adapt the entry of the Infringer (thus, they produce their duopoly outputs and earn their duopoly profits, respectively y_e^A, π_e^A for the Infringer and y_h^A, π_h^A for the Patent Holder), or *Litigate* such that the Patent Holder produces the output y_h^L and earns at least the market profit π_h^L, while the Infringer produces the output y_e^L and earns at best π_e^L, since the case may be defended at trial or be settled.

- At stage 4, the pretrial bargaining process takes place: the Infringer makes a (take-it-or-leave-it) *Settlement offer* L to the Patent Holder, corresponding to a price for the patent agreement (or fees for the normal use of the patent). On the one hand, if the Patent Holder chooses *Accept*, they settle amicably their dispute - and they earn their duopoly profits up to the cost and price of licensing, respectively $u_e(L) = \pi_e^L - L$ for the Infringer and $u_h(L) = \pi_h^L + L$ for the Patent Holder. On the second, if the Patent Holder chooses *Reject*, then a trial occurs. The Court sets for the Patent Holder with probability $\theta \in (0,1)$, i.e. claims that the patent is valid; the verdict consists in a damage that the Infringer must pay to the Patent Holder, denoted as D; then, each party earns its duopoly profits minus legal cost incurred at trial, respectively $\pi_e^L - D - c_e$ for the Infringer and $\pi_h^L + D - c_h$ for the Patent Holder. In contrast, the Court sets for the Infringer with probability $1 - \theta$; in this event, each party earns its duopoly profits minus legal cost incurred at trial, respectively $\pi_e^L - c_e$ for the Infringer and $\pi_h^L - c_h$ for the Patent Holder.

For the sake of simplicity, we will also assume that :

Assumption 1: the ratio $\frac{1-F}{f}$ is decreasing on $[\underline{c}_h, \bar{c}_h]$, and satisfies: $\left(\frac{1-F}{f}\right)_{|\underline{c}_h} > c_e + \bar{c}_h$ and $\left(\frac{1-F}{f}\right)_{|\bar{c}_h} < c_e + \underline{c}_h$.

which allows to avoid technical developments regarding the issue of existence and uniqueness of the solution at the pretrial stage, which are not central here.

Assumption 2: the market demand is linear with a demand price given by: $P = a - b(y_h + y_e)$, where $a, b > 0$ and y_h, y_e denoting the quantity produced by the Patent Holder and the Infringer, respectively.

Assumption 3: the technology of production entails constant marginal costs of production, respectively: $\bar{k} < a$ (before innovation occurs, for both firms), $k_h < \bar{k}$ (for the patent holder, after innovation) and $k_e < \bar{k}$ (for the infringer).

Note that we do not impose on a priori ground whether $k_h < k_e$ or $k_h > k_e$. This allows us to capture two opposite situations: on the one hand the (maybe imperfect) imitation of the patent ($k_h \leq k_e$), and on the other, the case of incremental innovation ($k_h > k_e$) by the Infringer. Note that the levels of $R\&D$ expenditures required by each case are likely to be very different (higher in the latter case than in the former). However, we do not consider those (fixed) costs in the present model.

We assume that at equilibrium, we have:

Assumption 4: $\theta D - \bar{c}_h > 0$ for $D \in \left\{ D_{LP} = \pi_h^N - \pi_h^L, D_{UE} = \pi_e^L - \pi_e^N \right\}$.

which holds both under the Lost Profit rule (D_{LP}) and under the Unjust Enrichment rule (D_{UE}). Thus, the credibility issue of the Patent Holder's threat associated with the decision to sue at the pretrial bargaining stage is beyond the scope of our paper (see Meurer (1989)).

Finally, we introduce a upper bond for the Infringer's legal cost:

Assumption 5: $c_e < \pi_e^A - \pi_e^N$.

which implies that the Infringer's legal cost is bounded above. This assumption is useful in order to rule out cases of deterrence which are not essential here. For example, assumption 5 prevents that the size of the legal costs has a deterrent effect on patent infringement despite a very small probability of losing at trial (i.e. in the neighborhod of $\theta = 0$; see the formal justification in appendix).

Before proceeding to the analysis, note that here we only consider *screening* games. The reason is that when legal costs are private information, their signalling value typically does not exist if basic rules of legal costs allocation are chosen by Courts, such as the American rule which is considered here[6]. However, a complete analysis of fee shifting is an issue beyond the scope of our paper. We focus rather on the effects of two specific damage rules.

2.2 Equilibria under the Lost Profit rule

Under the **Lost Profit rule,** we have $D_{LP} = \left[\pi_h^N - \pi_h^L \right]$, which implies that at the trial stage, the expected outcome for any Patent Holder $c_h \in [\underline{c}_h, \bar{c}_h]$ is :

$$
\begin{aligned}
u_h(c_h) &= \pi_h^L + \theta \left[\pi_h^N - \pi_h^L \right] - c_h \\
&= \theta \pi_h^N + (1 - \theta) \pi_h^L - c_h
\end{aligned}
$$

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[6]See Chopard and ali (2010).

while for the Infringer, it is defined as :

$$u_e(c_e) = \pi_e^L - \theta\left[\pi_h^N - \pi_h^L\right] - c_e$$

The central results in this case are provided in the next proposition:

Proposition 1 $1/\ D_{LP} \geq 0 \Leftrightarrow \pi_h^N \geq \left(\pi_h^L\right)_{LP}$ iff $\theta \leq \theta_{LP} \equiv 3\left(\frac{\bar{k}-k_e}{a-2k_h+\bar{k}}\right)$.

$2/$ If $\theta \in (0, \tilde{\theta})$, with $\tilde{\theta} \leq \theta_{LP}$, then the equilibrium under the Lost Profit rule is such that:

- the Infringer chooses Infringe and produces $y_e^L = \frac{a-2k_e+k_h-\theta(a-k_h)}{b(3-\theta)}$, and at the pretrial stage he makes a licensing demand whose value is $L^* = \theta\left[\pi_h^N - \left(\pi_h^L\right)_{LP}\right] - c_h^* > 0$, where c_h^* is defined according to $\left(\frac{1-F}{f}\right)_{|c_h^*} = c_h^* + c_e$.

- the Patent Holder chooses Litigate and produces $y_h^L = \frac{a-2k_h+k_e}{b(3-\theta)}$, and then at the pretrial stage he chooses Reject if his type is $c_h \in [\underline{c}_h, c_h^*)$, or Accept if his type is $c_h \in [c_h^*, \bar{c}_h]$.

See the proof in appendix. We highlight here the basic principles driving the incentives to enforce or infringe the patent. To begin with, note that θ_{LP} is the value of the patent strength satisfying $D_{LP} = \pi_h^N - \left(\pi_h^L\right)_{LP} = 0$. Thus, the restriction $\theta < \theta_{LP}$ is required in order to have $D_{LP} > 0$, i.e. positive lost profit at equilibrium[7]. In this case, proposition 1 says that the Patent Holder always enforces his right under the Lost Profit rule whatever his type, and either there is a trial, or a licensing agreement is reached. The incentives to defend the patent run as follows. Given that any type c_h may settle for a licensing price $L^* > 0$, and given that at the market stage we have (under $\theta < \theta_{LP}$) $\pi_h^N > \left(\pi_h^L\right)_{LP} > \pi_h^A$, then the Patentee reaches (at least) the minimum expected benefit $u_h(L^*)$ when he chooses Litigate such that: $u_h(L^*) = \pi_h^L + \theta\left[\pi_h^N - \left(\pi_h^L\right)_{LP}\right] - c_h^* > \pi_h^A$; thus, whatever his type, the Patent Holder is always better off enforcing his right rather than choosing Accommodate.

Let us come now to the incentives to infringe the patent. The Infringer's expected total benefit associated with the decision Infringe (and given the strategies of the Patent Holder according to his type) is:

$$\begin{aligned}U_e\left(L^*; c_h^*\right) &= \left(1 - F\left(c_h^*\right)\right)\left(\left(\pi_e^L\right)_{LP} - L^*\right) + F\left(c_h^*\right)\left(\left(\pi_e^L\right)_{LP} - \theta\left[\pi_h^N - \left(\pi_h^L\right)_{LP}\right] - c_e\right) \\ &= \left(\pi_e^L\right)_{LP} - \theta\left[\pi_h^N - \left(\pi_h^L\right)_{LP}\right] - c^*\end{aligned}$$

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[7]In other words, $\theta \geq \theta_{LP}$ implies that infringement is not harmful to the Patentee.

where $c^* = F(c_h^*) c_e - (1 - F(c_h^*)) c_h^* = c_e - (1 - F(c_h^*)) (c_e + c_h^*)$ is the difference between the Infringer's legal cost c_e and the expected gains of the negotiation $(1 - F(c_h^*)) (c_e + c_h^*)$ (the probability of settlement, times the negotiation gains). Note that c^* may be either positive or negative; moreover, it is increasing in c_e. Thus, the Infringer chooses *Infringe* as long as $U_e(L^*; c_h^*) > \pi_e^N$ or equivalently:

$$\left(\left(\pi_e^L\right)_{LP} - \pi_e^N\right) - \theta\left[\pi_h^N - \left(\pi_h^L\right)_{LP}\right] > c^*$$

Strictly speaking[8], when $c^* \leq 0$, it is sufficient that $\theta < \theta_{LP}$, for having $U_e(L^*; c_h^*) > \pi_e^N$; in contrast when $c^* > 0$, the condition must be strengthened: it must be that $\theta < \tilde{\theta}$, where $\tilde{\theta} < \theta_{LP}$ satisfies $U_e(L^*; c_h^*) = \pi_e^N$.

Note the specific role of legal costs under the Lost Profit rule. On the one hand, we find that they induce a screening effect between high Patentee's types (settling for L^*) and low ones (going to trial) at the pretrial stage; but under assumption 4 (credibility of claims), the amount paid by the Patentee in case of trial is not that high to make him better off in accommodating patent infringement. As a result, patent infringement is always litigated under the Lost Profit rule. On the other hand, the influence of legal costs on the Infringer side depends on the strength of the patent. When the Infringer's legal cost is smaller than the expected gains of the negotiation ($c^* \leq 0$) and the patent strength is small enough, i.e. $\theta \leq \theta_{LP}$, infringement is never deterred; in contrast when the Infringer's legal cost is larger than the expected gains of the negotiation ($c^* > 0$) and the strength of the patent is large enough, i.e. $\theta > \tilde{\theta}$, the Infringer is deterred. To sum up, enforcement is never deterred under the Lost Profit, whereas infringement may be deterred in case of strong patents associated with large legal costs (as compared to the expected gains of the negotiation) on the Infringer side.

Finally, the output decisions are also worth to mentioned, since the feedback effect of the trial stage on the competition stage may be easily understood. As also noticed in the literature (Anton and Yao, 2006; Choi, 2009), under the Lost Profits rule the feed-back effect of the trial stage on the market stage results of an externality effect that this damage rule imposes to the Infringer which modifies its best response function specifying the quantity produced. This point is made clearer, coming to the first order conditions $\frac{\partial}{\partial y_h} u_h(c_h) = 0$ and $\frac{\partial}{\partial y_e} u_e(c_e) = 0$, since they write as :

$$(1 - \theta) \frac{\partial \pi_h^L}{\partial y_h} = 0$$

$$\frac{\partial \pi_e^L}{\partial y_e} + \theta \frac{\partial \pi_h^L}{\partial y_e} = 0$$

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[8]See in appendix: at $\theta = \theta_{LP}$, we have $\pi_h^N = \left(\pi_h^L\right)_{LP}$ and $\left(\pi_e^L\right)_{LP} = \pi_e^N$.

Along the equilibrium path where the Patent Holder chooses *Litigate* (either the case is settled, or there is a trial), the best response function of the Patentee is not affected (same FOC) by the occurrence of a trial, while the best response function of the Infringer shows that the Infringer incentives to produce will be reduced ($\frac{\partial \pi_e^L}{\partial y_e} + \theta \frac{\partial \pi_h^L}{\partial y_e} < \frac{\partial \pi_e^L}{\partial y_e}$). This externality also operates when the case is settled, since the "take-it-or leave-it" offer corresponds to the expected outcome at trial of the marginal Patentee c_h^*.

2.3   Equilibria under the Unjust Enrichment rule

Under the **Unjust Enrichment rule,** we have $D_{UE} = \left[\pi_e^L - \pi_e^N \right]$, which implies that at the trial stage for any Patent Holder $c_h \in [\underline{c}_h, \bar{c}_h]$ we have:

$$
\begin{aligned}
u_h(c_h) &= \pi_h^L + \theta D - c_h \\
&= \pi_h^L + \theta \left[\pi_e^L - \pi_e^N \right] - c_h
\end{aligned}
$$

and for the Infringer, it comes:

$$
\begin{aligned}
u_e(c_e) &= \pi_e^L - \theta \left[\pi_e^L - \pi_e^N \right] - c_e \\
&= \theta \pi_e^N + (1 - \theta) \pi_e^L - c_e
\end{aligned}
$$

The central results in this case are provided in the two next propositions. A first difference with the previous case is worth to notice: D_{UE} is always positive. The second main difference is that an equilibrium may exist associated with the same qualitative features as under the Lost Profit rule, but it can be that other kinds of equilibria also arise where at least some Patent Holder types have an incentive to choose *Accommodate* rather than *Litigate* the case.

Proposition 2 *If $\theta \in [\theta'_{UE}, \theta''_{UE}]$, where $\theta''_{UE} \leq 1$, the equilibrium obtained under the Unjust Enrichment rule is such that:*

- the Infringer chooses Infringe and produces $y_e^L = \frac{a - 2k_e + k_h}{b(3 - \theta)}$, and at the pretrial stage he makes a licensing demand whose value is $L^{**} = \theta \left[\left(\pi_e^L \right)_{UE} - \pi_e^N \right] - c_h^* > 0$.

- the Patent Holder chooses Litigate and produces $y_h^L = \frac{a - 2k_h + k_e - \theta(a - k_e)}{b(3 - \theta)}$, and then at the pretrial stage he chooses Reject if his type is $c_h \in [\underline{c}_h, c_h^*)$, or Accept if his type is $c_h \in [c_h^*, \bar{c}_h]$.

See the proof in appendix. The incentives to enforce the patent or to infringe it now run as follows. On the one hand, the equilibrium of proposition 2 requires to hold that[9] $u_h(L^{**}) = \left(\pi_h^L\right)_{UE} + \theta\left[\left(\pi_e^L\right)_{UE} - \pi_e^N\right] - c_h^* > \pi_h^A$ (where c_h^* is still defined as under the LP rule), where we now have $\left(\pi_h^L\right)_{UE} < \pi_h^A$ and $\left(\pi_e^L\right)_{UE} > \pi_e^N$. Let us assume that there exists a θ'_{UE} for which $u_h(L^{**}) = \pi_h^A$. Then in order that the Patentee chooses *Litigate* at equilibrium whatever his type, it must be that $\theta > \theta'_{UE}$, i.e. the strength of the Patent Holder's case must be strong enough.

On the other hand, the best decision of the Infringer at the initial node is *Infringe* if: $U_e\left(L^{**}; c_h^*\right) > \pi_e^N$, which writes now as:

$$(1 - \theta)\left[\left(\pi_e^L\right)_{UE} - \pi_e^N\right] > c^*$$

where $c^* \gtrless 0$. Once more (see in appendix), we may distinguish between the case where $c^* \leq 0$ and the case where $c^* > 0$. When $c^* \leq 0$, then it can be shown that $U_e\left(L^{**}; c_h^*\right) > \pi_e^N \ \forall \theta \in]0, 1]$; in contrast when $c^* > 0$, the condition must be strengthened: $U_e\left(L^{**}; c_h^*\right) > \pi_e^N$ only if $\theta < \theta''_{UE}$, where $\theta''_{UE} < 1$ satisfies $U_e\left(L^{**}; c_h^*\right) = \pi_e^N$.

Thus under the Unjust Enrichment rule, Patentee's legal costs also entail a screening effect at the pretrial stage between high (now settling for $L^{**} > L^*$) and low (going to trial) Patentee's types; but now, the amount paid by the *marginal* Patentee (indifferent between a settlement and a trial) in case of trial may be high enough to make him better off in accommodating patent infringement, despite the credibility of claims (assumption 4): patent infringement is now litigated only when patent strength is large enough ($\theta > \theta'_{UE}$). On the other hand, the effect on the Infringer is qualitatively similar to the Lost Profit one: when the Infringer's legal cost is smaller than the expected gains of the negotiation ($c^* \leq 0$), infringement is never deterred whatever the patent strength; in contrast when the Infringer's legal cost is larger than the expected gains of the negotiation ($c^* > 0$), the Infringer is deterred if the strength of the patent is large enough, i.e. $\theta > \theta''_{UE}$.

The consequences for the outputs at equilibrium may also be easily understood. With the Unjust Enrichment rule (see also Anton and Yao, 2006; Choi, 2009), the externality effect is now shifted on the Patent Holder and do modify its best response function. The first order conditions $\frac{\partial}{\partial y_h} u_h(c_h) = 0$ and $\frac{\partial}{\partial y_e} u_e(c_e) = 0$ write by symmetry as :

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[9]Under assumption 4, we still have $\theta\left[\left(\pi_e^L\right)_{UE} - \pi_e^N\right] - c_h^* > 0$. However under the Unjust Enrichment rule, the inequality $\left(\pi_h^L\right)_{UE} < \pi_h^A$ now holds (see in the appendix).

$$\frac{\partial \pi_h^L}{\partial y_h} + \theta \frac{\partial \pi_e^L}{\partial y_h} = 0$$

$$(1-\theta)\frac{\partial \pi_e^L}{\partial y_e} = 0$$

Along a path where the Patent Holder chooses *Litigate* (either the case is settled, or there is a trial), the best response function of the Infringer now is not affected (same FOC) by the occurrence of a trial, while the best response function of the Patentee shows that his incentives to produce will be reduced ($\frac{\partial \pi_h^L}{\partial y_h} + \theta \frac{\partial \pi_e^L}{\partial y_h} < \frac{\partial \pi_h^L}{\partial y_h}$).

However, alternative equilibria may be obtained for weaker patents, that we describe in the following proposition:

Proposition 3 *Assume that*[10] $\theta < \theta'_{UE}$; *if there exists a* $\tilde{c}_h \in [\underline{c}_h, c_h^*)$ *defined by*[11] $u_h(\tilde{c}_h) = \pi_h^A$, *two alternative equilibria may be obtained under the Unjust Enrichment rule:*

i) The first kind of equilibrium holds when $\left(\pi_h^L + \pi_e^L\right)_{UE} > \pi_h^A + \pi_e^A$, *and is such that:*

- the Infringer chooses Infringe and produces $y_e^L = \frac{a-2k_e+k_h}{b(3-\theta)}$, *and at the pretrial stage he makes a licensing demand whose value is* $\tilde{L} = \pi_h^A - \left(\pi_h^L\right)_{UE} > 0$.

- the Patent Holder chooses Litigate and produces $y_h^L = \frac{a-2k_h+k_e-\theta(a-k_e)}{b(3-\theta)}$, *and then at the pretrial stage he chooses Reject if his type is* $c_h \in [\underline{c}_h, \tilde{c}_h)$ *or Accept if his type is* $c_h \in [\tilde{c}_h, \bar{c}_h]$.

ii) The second kind of equilibrium holds when $\left(\pi_h^L + \pi_e^L\right)_{UE} < \pi_h^A + \pi_e^A$, *and is such that:*

- the Infringer chooses Infringe and produces $y_e^L = \frac{a-2k_e+k_h}{b(3-\theta)}$ *conditional on* $c_h \leq \tilde{c}_h$, *or* $y_e^A = \frac{a-2k_e+k_h}{3b}$ *conditional on* $c_h > \tilde{c}_h$, *and then at the pretrial stage he may make any licensing demand*[12] $\hat{L} \in [0, \tilde{L})$.

- the Patent Holder chooses Litigate and produces $y_h^L = \frac{a-2k_h+k_e-\theta(a-k_e)}{b(3-\theta)}$, *and at the pretrial stage he chooses Reject, if his type is* $c_h \in [\underline{c}_h, \tilde{c}_h)$.

- but he chooses Accommodate and produces $y_h^A = \frac{a-2k_h+k_e}{3b}$ *if his type satisfies* $c_h \in [\tilde{c}_h, \bar{c}_h]$.

See the proof in appendix. Proposition 3 says that under the Unjust Enrichment rule, different kinds of equilibria may arise when the patent is weak $\left(\theta < \theta'_{UE}\right)$. Assume there exists a $\tilde{c}_h \in [\underline{c}_h, c_h^*)$ satisfying $u_h(\tilde{c}_h) = \pi_h^A \Leftrightarrow \tilde{c}_h = \left(\left(\pi_h^L\right)_{UE} - \pi_h^A\right) + \theta\left[\left(\pi_e^L\right)_{UE} - \pi_e^N\right]$, and consider the licensing

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[10] Alternatively, it can be the case that θ'_{UE} does not exist, i.e. there exists no value of θ for which $u_h(L^{**}) = \pi_h^A$.

[11] If $\tilde{c}_h \leq \underline{c}_h$, then any equilibrium may be build such that whatever the settlement offer made by the Infringer at the pretrial stage, every Patent Holder types choose *Accommodate*, while the Infringer chooses *Infringe*. The outputs (y_h^A, y_e^A) are obtained at the market stage.

[12] Indeed, there exists a continuum of equilibria of the third type, each being associated with a specific value of $\hat{L} \in [0, \tilde{L}[$, but yielding the same outcome since they induce the same separation between the Patent Holder types.

price $\tilde{L} = \pi_h^A - \left(\pi_h^L\right)_{UE}$. On the one hand, if the Infringer proposes $\tilde{L}$, there is a separation of the population of Patent Holder's types between those who choose (*Litigate,Reject*) (types $c_h \in [\underline{c}_h, \tilde{c}_h)$) and those who choose (*Litigate,Accept*) (types $c_h \in [\tilde{c}_h, \bar{c}_h]$). On the other hand, if the Infringer proposes any $\hat{L} \in [0, \tilde{L})$, there is a separation between Patentee's types between those who choose (*Litigate,Reject*) ($c_h \in [\underline{c}_h, \tilde{c}_h)$) and those who choose (*Accommodate*) ($c_h \in [\tilde{c}_h, \bar{c}_h]$). The offer $\tilde{L}$ ($\hat{L}$) is part of an equilibrium if $U_e(\tilde{L}; \tilde{c}_h) > (<) U_e(\hat{L}; \tilde{c}_h)$ which requires that the condition $\left(\pi_h^L + \pi_e^L\right)_{UE} > (<) \pi_h^A + \pi_e^A$ holds (see in the appendix).

In a sense, what proposition 3 says is that holding a weak patent is not a sufficient condition for that the Patent Holder forgives to enforce his right under the Unjust Enrichment rule – this is only a necessary condition[13]. In an equilibrium where $\left(\pi_h^L + \pi_e^L\right)_{UE} > \pi_h^A + \pi_e^A \Leftrightarrow \left(\pi_e^L\right)_{UE} - \pi_e^A > \pi_h^A - \left(\pi_h^L\right)_{UE}$, the Patent Holder will always enforce its patent despite its weakness. The condition is not always satisfied given that under the Unjust Enrichment rule we have at the same time $\left(\pi_e^L\right)_{UE} - \pi_e^A > 0$ *and* $\pi_h^A - \left(\pi_h^L\right)_{UE} > 0$. Then the case is litigated only as long as $\left(\pi_e^L\right)_{UE} - \pi_e^A > \pi_h^A - \left(\pi_h^L\right)_{UE} = \tilde{L}$, which means that the Infringer must win more than what the Patent Holder looses, as compared to a situation where the infringement is accommodated. The driving force is that the Infringer has the opportunity to offer a licensing price $\tilde{L}$ which is large enough ($\tilde{L} > \hat{L}$) to be accepted by the Patent Holder in case he has the highest legal costs – the counterpart is that whatever his type, the Patent Holder always enforces his right. In contrast, to induce the Patent Holder not to (always) enforce its right, sufficiency requires that $\left(\pi_h^L + \pi_e^L\right)_{UE} < \pi_h^A + \pi_e^A \Leftrightarrow \left(\pi_e^L\right)_{UE} - \pi_e^A < \pi_h^A - \left(\pi_h^L\right)_{UE}$. This means that the highest licensing price the Infringer has the opportunity to offer is smaller than the minimum accepted by the Patent Holder ($\tilde{L} < \hat{L}$). As a result, the patent is not enforced except by the smallest Patent Holder's types. Otherwise, the infringement is accommodated.

Thus, the characteristic features of these equilibria under the Unjust Enrichment rule and when the patent is weak $\left(\theta < \theta'_{UE}\right)$, rely on whether the Infringer is better off in reaching a settlement at the pretrial stage with a subset of Patentees, or whether he prefers to induce some of them to accommodate infringement, while the others litigate and go at trial. This is because the screening effect coming from Patentee's legal costs may be of two types: either there is a separation between high (now settling for $\tilde{L}$) and low (going to trial) Patentee's types; or there is a separation between high (now accommodating) and low (still going to trial) Patentee's types. We emphasize now

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[13]Choi (proposition 5 p 153, 2007) for the case of a drastic innovation and zero litigation costs, finds that any probabilistic patent is enforced and infringed, whatever its strength. Schankerman and Scotchmer (2011) obtain the detterence of infringement for an ironclad patent, under a condition of dilution of industry profits (difference between industry profit with and without infringement) – remark that our condition is different (difference between industry profit under infringement and litigation, and industry profit under infringement and accomodation).

the role of the Infringer's legal costs (see the appendix for the formal analysis). Considering equilibrium of Part i), it comes that the influence of the Infringer's legal costs is similar to the one observed before: when the Infringer's legal cost is smaller than the expected gains of the negotiation ($\tilde{c} = c_e - (1 - F(\tilde{c}_h))\,(c_e + \tilde{c}_h) \leq 0$, with $\tilde{c}_h$ the new marginal Plaintiff being indifferent between accommodating and litigating the case), infringement is never deterred whatever the patent strength; in contrast when the Infringer's legal cost is larger than the expected gains of the negotiation ($\tilde{c} > 0$), the Infringer is deterred if the strength of the patent in a sense is small $\left(\theta < \theta'_{UE}\right)$ but not too small (see in appendix). On the other hand considering now equilibrium of Part ii), assumption 5 is sufficient to warrant that infringement is never deterred.

To sum up, three main results of this section are worth mentioning. First, the Lost Profit rule yields a probability of trials at least as large as the Unjust Enrichment rule. According to proposition 1 and 2, both are associated with the same probability of trials $F(c_h^*)$; however, according to proposition 3, the Lost Profit rule gives more trials since $F(c_h^*) > F(\tilde{c}_h)$. Second, under the Unjust Enrichment rule, the enforcement of the patent may be impossible at least for some Patent Holder types (the largest values for c_h); these ones will be better off in accommodating the infringement of the patent rather than litigating it, given the impossibility to reach a settlement. Third, under Lost Profit, only weak patents are potentially enforceable, in the sense that the Patentee is allowed to argue that infringement has been harmful to him.

3   Damage rules and patent protection : profits *vs* social welfare

Propositions 1 to 3 show that the two damages rules may have very different selection effects on patent cases, both regarding the Infringer's incentives and the Patentee's incentives. In this section, we investigate some of the consequences of these differences. We will have a more specific focus on the Patent Holder's protection afforded by each rule, and to the issue of social welfare.

3.1   Market profits

Regarding the issue of the Patent Holder profits protection, note that the comparison between $u_h(c_h)_{LP}$ and $u_h(c_h)_{UE}$ makes sense only for $\theta \in\,]0, \theta_{LP}]$: this is because there is no damage under the Lost profit rule if $\theta > \theta_{LP}$, i.e. $D_{LP} = 0 (< D_{UE})$.

It can be seen that for any $\theta \in (0, \theta_{LP})$, the difference:

$$u_h(c_h)_{LP} - u_h(c_h)_{UE} = \left(\pi_h^L + \theta\left[\pi_h^N - \pi_h^L\right]\right)_{LP} - \left(\pi_h^L + \theta\left[\pi_e^L - \pi_e^N\right]\right)_{UE}$$

can be rearranged as the sum of two effects. On the one hand, the difference between the value of expected damages $\theta\left[\pi_h^N - \pi_h^L\right]_{LP} - \theta\left[\pi_e^L - \pi_e^N\right]_{UE}$; this is a direct effect, in terms of compensation *per se* allowed to the Patent Holder in case of trial, and its sign is ambiguous. On the other hand, there is an indirect (feedback) effect since the choice of the damage rule affects the Patentee's markets profits, and thus the difference $\left(\pi_h^L\right)_{LP} - \left(\pi_h^L\right)_{UE}$; this reflects the market discipline imposed by each rule on competitors.

In the specific case where $k_h = k_e$, it can be verified that $\left(\pi_h^L\right)_{LP} = \left(\pi_e^L\right)_{UE}$ and $\left(\pi_h^L\right)_{UE} = \left(\pi_e^L\right)_{LP}$, which implies that $u_h(c_h)_{LP} > u_h(c_h)_{UE}$, and to the converse $u_e(c_e)_{LP} < u_e(c_e)_{UE}$. In the general case where $k_h \neq k_e$, we sum up our results in the next proposition.

Proposition 4 *A) If $\theta \leq \hat{\theta}$ where $\hat{\theta} < \theta_{LP}$, then the Lost Profit rule affords the Patentee with a better protection than the Unjust Enrichment one. If $\theta > \hat{\theta}$, the comparison is ambiguous.*

B) If $\theta \geq \hat{\theta}$, then the Infringer earns higher profits under the Unjust Enrichment rule than under the Lost Profit one. If $\theta < \hat{\theta}$, the comparison is ambiguous.

Proof. A) Using the equilibrium values of profits and rearranging, we find that $D_{LP} > D_{UE}$ if $(3-\theta)^2 > 9\left(\frac{(a-2k_h+k_e)^2+(a-2k_e+k_h)^2}{(a-2k_h+\bar{k})^2+(a-2\bar{k}+k_h)^2}\right)$ or equivalently if $\theta < \hat{\theta} \equiv 3\left(1 - \sqrt{\frac{(a-2k_h+k_e)^2+(a-2k_e+k_h)^2}{(a-2k_h+\bar{k})^2+(a-2\bar{k}+k_h)^2}}\right)$. Simple but tedious manipulations[14] show that $\hat{\theta} < \theta_{LP}$. To the converse, $D_{LP} < D_{UE}$ if $\theta > \hat{\theta}$. Hence, considering the case for a strong (weak) patent, *i.e.* for θ large (respectively, small) enough, the Unjust Enrichment rule grants higher (respectively smaller) damages than the Lost Profit rule.

The second effect $\left(\pi_h^L\right)_{LP} - \left(\pi_h^L\right)_{UE}$ is unambiguously positive since we have shown that: $\pi_h^N > \left(\pi_h^L\right)_{LP} > \pi_h^A > \left(\pi_h^L\right)_{UE}$ (given $\theta < \theta_{LP}$). Hence, regarding this indirect effect on market profits, the Lost Profit rule gives a higher profit to the Patent Holder than the Unjust Enrichment rule.

Collecting both effects, we obtain that $\theta \leq \hat{\theta}$ implies:

$$\left(\pi_h^L + \theta\left[\pi_h^N - \pi_h^L\right]\right)_{LP} \geq \left(\pi_h^L + \theta\left[\pi_e^L - \pi_e^N\right]\right)_{UE}$$

otherwise, the result is ambiguous.

B) Note that the consequences for the Infringer's gross profits are also easy to describe; given that $u_e(c_e)_{LP} - u_e(c_e)_{UE} = \left(\pi_e^L\right)_{LP} - \left(\pi_h^L\right)_{UE} - \theta(D_{LP} - D_{UE})$, with $\pi_e^N < \left(\pi_e^L\right)_{LP} < \pi_e^A < \left(\pi_e^L\right)_{UE}$, the result is direct by symmetry. ∎

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[14] Available on request.

A main consequence of proposition 4 is worth to note, since it is related to the analysis of R&D incentives. As discussed by Choi (2009)[15] the incentives to invest in R&D in non tournament models of innovation (a single investor; cf Reinganum (1989)), are driven by the Patentee's expected profits; in contrast, in tournament models (multiple investors), they depend on the difference between the Patentee's expected profit and the potential competitor's one. As a result, our analysis implies that in tournament models, the Lost Profit rule gives more *R&D* investments than the Unjust Enrichment one only for weak patents, i.e. only if $\theta < \hat{\theta}(< \theta_{LP})$; in a sense, this can be understood as a pervasive effect of the Lost Profit rule, since it preserves the incentives to innovate in the range of patents having a high risk of infringement. Otherwise, the effect is ambiguous. Finally, in tournament models, no rule is clearly better than the other, whatever the strength of the patent.

3.2 Social welfare

Social welfare will be defined here as the standard Marshallian surplus (sum of the consumers' surplus and the firms' gross profits, including firms' legal costs). When the Unjust Enrichment rule is used, several kinds of equilibria may occur; in order to distinguish them, let us denote as:

- $SW_{1,LP}$ the social welfare associated with proposition 1;
- $SW_{2,UE}$ the social welfare associated with proposition 2;
- $SW_{3i,UE}$ the social welfare associated with proposition 3i);
- $SW_{3ii,UE}$ the social welfare associated with proposition 3ii).

Now defining as:

$$SW_i^L = \int_0^{(y_h^L + y_e^L)_i} P(x)dx - k_h(y_h^L)_i - k_e(y_e^L)_i \text{ for } i = LP, UE$$

$$SW^A = \int_0^{(y_h^A + y_e^A)} P(x)dx - k_h y_h^A - k_e y_e^A$$

it can be verified that:

$$SW_{1,LP} - SW_{2,UE} = SW_{LP}^L - SW_{UE}^L$$

$$SW_{1,LP} - SW_{3i,UE} = SW_{LP}^L - SW_{UE}^L - \int_{\tilde{c}_h}^{c_h^*}(c_h + c_e)dF(c_h)$$

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[15]Let us remind that Choi (2009) discusses the case for a drastic innovation.

$$SW_{1,LP} - SW_{3ii,UE} \quad = SW_{LP}^L - SW_{UE}^L - \int_{\tilde{c}_h}^{c_h^*}(c_h + c_e)dF(c_h) - (1 - F(\tilde{c}_h))\left(SW^A - SW_{UE}^L\right)$$

Remark that the comparisons of welfare makes sense only for values of θ properly chosen; this means that comparing proposition 1 and proposition P2 (or P3) is possible only when the considered value for θ supports both equilibria (see the appendix). Notwithstanding these qualifications, we can show that the next results hold:

Proposition 5 *A) (proposition 1 vs proposition 2) Social welfare is higher under the Lost Profit (Unjust Enrichment) rule if $k_h < k_e$ (resp. $k_h > k_e$).*

B) (proposition 1 vs proposition 3) i) When $\left(\pi_h^L + \pi_e^L\right)_{UE} > \pi_h^A + \pi_e^A$, social welfare is higher under the Unjust Enrichment rule if $k_h > k_e$. The result is ambiguous if $k_h < k_e$. ii) When $\left(\pi_h^L + \pi_e^L\right)_{UE} < \pi_h^A + \pi_e^A$, the result is ambiguous whatever the sign of $k_h - k_e$.

Proof. A) We have to sign $SW_{1,LP} - SW_{2,UE}$. Given that both damage rules are associated with the same value for the marginal Plaintiff (Patentee), and thus with the same expected legal costs, this implies that $SW_{1,LP} - SW_{2,UE} = SW_{LP}^L - SW_{UE}^L$. Thus, the proof is the same as in Choi (proposition 3, 2009)[16].

B) The relevant welfare comparison is based either on $SW_{1,LP} - SW_{3i,UE}$ or $SW_{1,LP} - SW_{3ii,UE}$, meaning that we have to take into account that the value for the marginal Patentee is no longer the same between the equilibria compared here, and/or that the Patentee may accommodate at equilibrium. Then additional effects must be considered due to legal costs and existing differences in the probability of a trial. For this purpose, let us distinguish two different cases.

i) First, consider the case where $\left(\pi_h^L + \pi_e^L\right)_{UE} > \pi_h^A + \pi_e^A$. Then $SW_{1,LP} - SW_{3i,UE}$ depends, beyond $SW_{LP}^L - SW_{UE}^L$ (which behaves as before), on the difference in legal costs. As there are more trials under the Lost profit rule than under the Unjust Enrichment one (since $c_h^* > \tilde{c}_h$), the expected legal costs under the Lost Profit rule are higher than the ones under the Unjust Enrichment rule: $\int_{\tilde{c}_h}^{c_h^*}(c_h + c_e)dF(c_h) > 0$. As a consequence, if $k_e < k_h$, the social welfare is larger under the Unjust Enrichment rule; but if the condition $k_e > k_h$ holds, the comparison is ambiguous.

ii) Second, assume now that $\left(\pi_h^L + \pi_e^L\right)_{UE} < \pi_h^A + \pi_e^A$. In this case, the sign of $SW_{1,LP} - SW_{3ii,UE}$ is also governed by a third effect, reflecting that the Patentee may sometimes use his outside option under the Unjust Enrichment rule (i.e. there is a positive probability that he opts

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[16]The result does not depend on whether the innovation is drastic or not.

for accommodating the entry of the Infringer) rather than enforcing his right. Thus, we have to establish[17] the conditions under which it is socially desirable that under the Unjust Enrichment rule at least some Patentee's types accommodate rather than sue. Solving explicitly, we find that:

$$SW_{UE}^L - SW^A = \frac{\theta \left(a - 2k_e + k_h\right)}{18b \left(3 - \theta\right)^2} \left[- \left(6 - \theta\right)\left(a - 2k_e + k_h\right) + 12\left(3 - \theta\right)\left(k_h - k_e\right)\right]$$

The bracketed term is negative when $k_h < k_e$, which implies that $SW^A - SW_i^L > 0$ (and thus the third effect runs the opposite to the two first ones). But the sign is indeterminate when $k_h > k_e$, so is it for the sign of $SW^A - SW_i^L$. ■

In part A), $SW_{1,LP} - SW_{2,UE} = SW_{LP}^L - SW_{UE}^L$ depends on two effects going in opposite senses, respectively the integral term and the production costs. On the one hand, the Patent Holder – whatever the sign of $k_h - k_e$ – produces a larger share of the total output under the Lost Profit rule $((y_h^L)_{LP} > (y_h^L)_{UE})$, whereas the reverse is true for the potential infringer $((y_e^L)_{LP} < (y_e^L)_{UE})$; this effect in turn affects production costs accordingly. Thus, when $k_h > k_e$, an inefficient allocation of market shares is obtained under the Lost Profit since the Patentee produces more than the Infringer despite higher production costs; but, when $k_h < k_e$, there is no inefficiency associated with the Lost profit rule. On the other hand, it can be verified that the market output under the Lost Profit rule $(y_h^L + y_e^L)_{LP}$ is larger (smaller) than that under the Unjust Enrichment rule $(y_h^L + y_e^L)_{UE}$ if $k_h > k_e$ (respectively if $k_h < k_e$), which yields that the integral term under Lost Profit is larger (respectively smaller). It can be shown[18] that when $k_h > k_e$ the inefficiency effect associated with Lost Profit dominates (is dominated by) the first (respectively, second) one such that $SW_{LP}^L - SW_{UE}^L < 0$; but when $k_h > k_e$, it is direct that $SW_{LP}^L - SW_{UE}^L > 0$.

To sum up, our analysis shows that the comparison of social welfare is sensible to the patent strength θ, to the accommodation of infringement, and to the sign of the difference in marginal costs $k_h - k_e$. Comparing equilibria where any Patentee enforces his right and never accommodates (specifically when $\theta_{UE}' < \theta_{LP} < \theta_{UE}''$), it appears that the Lost Profit rule is better from a social point of view as long as the Patent Holder is at least as efficient as the Infringer ($k_h \leq k_e$). Otherwise, the Unjust Enrichment is better from a social point of view. But considering a weak patent ($\theta < \theta_{UE}'$), the advantages of one rule over the other do not necessarily hold for a given condition on marginal costs. As long as infringement is not accommodated and the Infringer becomes more efficient than the Patent Holder ($k_h > k_e$), the Unjust Enrichment rule should be preferred from a social point of view whatever the patent strength. In other cases, including the possibility of accommodation of infringement, we find that no rule is better than the other.

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[17]Note that the sign of $SW_{LP}^L - SW_{UE}^L$ is obtained under the same conditions as in case i).

[18]See also Choi (2009) for a drastic innovation.

4 Conclusion

Our paper adds to the existing literature regarding the analysis of the impacts of damage rules in patent litigations. For the case of non drastic innovations, we introduce a comprehensive model of litigations with pretrial negotiations, which allows to analyze the decisions to enforce as well as to infringe a patent, and captures the feedback influences of amicable settlements on the market stage. We show that when the value of patents is probabilistic (the verdict at trial in uncertain) and the occurrence of a trial is not certain (negotiation may occur), the choice of the damage rule has a complex effects on the Infringer's decision to infringe or not a patent, on the Patentee's decision to sue rather than accommodate infringement, and then on the decision to settle rather than litigate in front of Courts the case. As a result, while the Unjust Enrichment rule never yields more trials than the Lost Profit one, we obtain that generally speaking, no rule is better than the other regarding the preservation of Patentee's profits and incentives to invest in *R&D*, or regarding social welfare, whatever the difference between marginal costs of production.

Our paper also sheds some new light on the controversy regarding the existence of infringement at equilibrium under the Lost Profit rule (Shankerman and Scotchmer, 2001; Anton and Yao, 2006; Choi, 2009), and allows to enlarge the view in several ways – specifically regarding some weakness of the Unjust Enrichment rule which have been overlooked up to now. Schankerman and Scotchmer (2001) consider ironclad patents, and find that infringement occurs under the Lost Profit only when it dissipates the industry profits (i.e. when the sum of firms' profits is less under infringement than without infringement). In Anton and Yao (2006), infringement of a probabilistic patent correspondig to a non drastic innovation always exists at equilibrium under the Lost Profit since the imitator has the opportunity to engage in passive infringement where he produces the non infringement output at a lower cost, but such that he is never liable (since there is no harm at equilibrium). Finally, Choi (2009) shows for drastic innovations and probabilistic IPR, that infringement always occurs under both rules. In contrast, our paper puts the emphasize the role of the timing of decisions as well as the importance of the set of strategies held by competing firms (enlarging the one considered by Anton and Yao, or Choi). On the one hand, central to our analysis is the assumption of a sequential move between the Infringer and the Patentee, where their opt-out option is firm specific (non infringement vs accocomodation) – as a result, dissipation of industry' profits does not necessarily means the same thing for both. On the other hand, also important to our results is the introduction of a pretrial stage and its consequences for firms' decisions such as enforcing as well infringing a patent.

Infringement may not always exist here at equilibrium because of the patent strength and the

Infringer's legal costs, although not in a trivial way. We have assumed that the Infringer's legal cost is bounded above, in order to rule out cases of deterrence which are not essential here – such as when patent infringement is deterred despite it can be expected that infringement would be accomodated given the very small probability to lose at trial. Thus our findings show more specifically that the incentives to infringe the patent are generally driven by the difference between the Infringer's legal cost and the expected value of the negotiation gains (the size of which depends on the *marginal* Plaintiff's legal cost, up to the Infringer's one), under both damages rules. To sum up, when the Infringer's legal cost is small as compared to the expected value of the negotiation gains, then the Infringer always infringe a patent (weak enough); otherwise, the Infringer may be deterred from doing so. But regarding the incentives to enforce a patent, we have shown that generally the strength of a patent , and the size of the Patentee's legal cost influence the screening of types between a settlement and a trial under both rules; we have found that any patent is enforced under the Lost Profit rule (provided it is weak enough to be associated with a harm when infringed). To the opposite, under the Unjust Enrichment rule high legal costs may induce accommodation by some Patentee's type, rather than litigating the case; this is more likely to occur when the patent is weak enough and when infringement is associated with the dissipation of industry' profits (but in a sense different than in Schankerman and Scotchmer: the sum of firms' profits is less under infringement than under accomodation).

To conclude, let us underline that several results may be very specific to the timing of the model introduced here. The specific influence of the order of moves will be assessed in future research, since the incentives to enforce as well infringe the patent are closely related to the specific opt-out option of firms. We have also highlighted the importance of two parameters of the litigation technology: the intrinsic strength of the patent, and the distribution of legal costs, which are exogenously given here. Given that the strength of the patent may summarize the probability of detecting infringement and the probability of conviction, it would be worth to relate them to the behavior of Patentee, as well as the behavior of Courts, and thus the distribution of both public and private legal costs. This is also left for future research.

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5   Appendix

5.1   Proof of Proposition 1

The complete proof requires that some intermediate results have to be established.

- **Stage 4**

The pretrial bargaining game in stage 4 is solved as follows. The reason why the Infringer has to litigate (reaches stage 4) is that at least some (types of) Patent Holders preferred to chose *Litigate* rather than *Accommodate* at stage 3. Let us assume that there exists a cut-off value $\hat{c}_h$ with $\underline{c}_h < \hat{c}_h \leq \bar{c}_h$ such that Patent Holders with a $c_h < \hat{c}_h$ chooses *Litigate*, in contrast to Patent Holders with a $c_h \geq \hat{c}_h$ who chooses *Accommodate*. The intuition is that a Patent holder having large litigation costs should be prone to accommodate.

Conditional on the fact that at stage 4 the Infringer expects to face Patent Holders with a $c_h < \hat{c}_h$, the settlement offer L may separate the Patent Holders between those who accept L - any Patent Holder who litigates and for whom $u_h(L) \geq u_h(c_h)$ will accept - and those who reject L - any Patent Holder who litigates and for whom $u_h(L) < u_h(c_h)$ will reject the offer. Let us denote as c_L the marginal holder's type who is indifferent between litigate and settle; its type is given by: $u_h(c_L) = u_h(L) \Leftrightarrow \theta \pi_h^N + (1 - \theta) \pi_h^L - c_L = \pi_h^L + L$ or rearranging :

$$c_L = \theta \left[\pi_h^N - \pi_h^L \right] - L \tag{1}$$

Thus, the Infringer faces with probability $\left(1 - \frac{F(c_L)}{F(\hat{c}_h)}\right)$ a Patent Holder who will be prone to chose *Accept* (the settlement demand L); and with probability $\frac{F(c_L)}{F(\hat{c}_h)}$, he faces a Patent Holder who will prefer to chose *Reject* (go to trial).

As a result, the best licensing price L^* for the patent infringer can now be defined as the solution of the maximization of his ex ante total benefit for the case:

$$
\begin{aligned}
U_e\left(L; c_L\right) &= \left(1 - \frac{F(c_L)}{F(\hat{c}_h)}\right) u_e(L) + \frac{F(c_L)}{F(\hat{c}_h)} u_e(c_e) \\
&= \pi_e^L - \left(1 - \frac{F(c_L)}{F(\hat{c}_h)}\right) L - \frac{F(c_L)}{F(\hat{c}_h)} \left(\theta\left[\pi_h^N - \pi_h^L\right] + c_e\right)
\end{aligned} \tag{2}
$$

Lemma 6 *Under assumption 1, the solution to the maximization of (3) under (2), is unique and corresponds to the licensing offer L^* and the cut-off value for the Patent Holder's type $c_{L^*} = c_h^*$ which are implicitly obtained by solving the system:*

$$
L^* = \theta\left[\pi_h^N - \pi_h^L\right] - c_h^* \tag{3}
$$

$$
\left(\frac{F(\hat{c}_h) - F}{f}\right)_{|c_h^*} = c_e + c_h^* \tag{4}
$$

Proof. The derivative of $U_e\left(L; c_L\right)$ in L is :

$$
\frac{\partial}{\partial L} U_e\left(L; c_L\right) = -\left(1 - \frac{F(c_L)}{F(\hat{c}_h)}\right) + \frac{f(c_L)}{F(\hat{c}_h)}\left[u_e(L) - u_e(c_e)\right] \tag{5}
$$

The first term is the marginal cost of the offer L. Indeed, the Infringer will get an increase in its cost of making an offer with a probability of $1 - \frac{F(c_L)}{F(\hat{c}_h)}$. The second term is the marginal benefit of the licensing offer, which is the result of the impact of the offer:

- on the probability of an amicable settlement: $\frac{d}{dL}\left(1 - \frac{F(c_L)}{F(\hat{c}_h)}\right) = \frac{f(c_L)}{F(\hat{c}_h)}$,
- on the gains of the negotiation evaluated for the marginal Patent Holder: $u_e(L) - u_e(c_e) = c_e + c_L > 0$.

Substituting for $u_e(L)$, $u_e(c_e)$ and $L = u_h(c_L) - \pi_h^L$, and rearranging the terms gives the first order condition for an interior solution (L^*, c_h^*):

$$
0 = -\left(1 - \frac{F(c_h^*)}{F(\hat{c}_h)}\right) + \frac{f(c_h^*)}{F(\hat{c}_h)}\left[c_e + c_h^*\right] \tag{6}
$$

which leads to condition (3). Under assumption 1, it is easy to verify that the second order condition holds (implying more generally that $U_e\left(L; c_L\right)$ is concave). Given that the RHS in (3) is increasing in c_h, it is obvious that both the existence and uniqueness result from assumption 1. ∎

Since under assumption 4, we have $\theta \left[\pi_h^N - \pi_h^L \right] > c_h^*$, the equilibrium of the entry game is associated with a frequency of trial $\frac{F(c_h^*)}{F(\hat{c}_h)}$ and a frequency of settlement $1 - \frac{F(c_h^*)}{F(\hat{c}_h)}$, with $L^* > 0$, where c_h^* is the solution to (5).

• **stage 3**

Let us show first that by sequential rationality, we have: $\hat{c}_h = \bar{c}_h$. As previously shown, Patent Holders with a $c_h < \hat{c}_h$ choose *Litigate* (and, either reach a settlement, or end up in trial), implying that they obtain at least $u_h(L^*) \equiv u_h(c_h^*)$. Litigating is the best strategy for any $c_h < \hat{c}_h$ if:

$$u_h(L^*) = \pi_h^L + \theta \left(\pi_h^N - \pi_h^L \right) - c_h^* > \pi_h^A \tag{7}$$

But if this inequality holds, it prevents that accommodating for Patent Holders with a $c_h \geq \hat{c}_h$ be sequentially rational: given that in the bargaining process, any type $c_h \in]c_h^*, \hat{c}_h]$ is pooled together with c_h^*, observe that any $c_h \geq \hat{c}_h$ would be better off in litigating and settling his case, rather that accommodating. Thus, by sequential rationality, we must have: $\hat{c}_h = \bar{c}_h$. We show now that the inequality (7) always holds, considering the market conditions.

Under the **Lost Profits rule,** with a linear demand $P = a - b(y_h + y_e)$, the equilibrium is obtain, first solving for the first best response function of both firms. In the post-entry game where the Infringer chooses not to infringe, $\left(y_h^N, y_e^N \right)$ is the solution to the system:

$$
\begin{aligned}
y_h^N &= y_h(y_e^N) = \arg \max_{y_h} \left\{ \pi_h^N = (P - k_h) y_h \text{ s.t. } P = a - b(y_h + y_e^N) \right\} \\
y_e^N &= y_e(y_h^N) = \arg \max_{y_e} \left\{ \pi_e^N = (P - \bar{k}) y_e \text{ s.t. } P = a - b(y_e + y_h^N) \right\}
\end{aligned}
$$

In the post-entry game where the Infringer chooses to infringe but the Patent Holder accommodates, $\left(y_h^A, y_e^A \right)$ is the solution to the system:

$$
\begin{aligned}
y_h^A &= y_h(y_e^A) = \arg \max_{y_h} \left\{ \pi_h^A = (P - k_h) y_h \text{ s.t. } P = a - b(y_h + y_e^A) \right\} \\
y_e^A &= y_e(y_h^A) = \arg \max_{y_e} \left\{ \pi_e^A = (P - k_e) y_e \text{ s.t. } P = a - b(y_h^A + y_e) \right\}
\end{aligned}
$$

When the Patent Holder chooses to litigate, the Infringer's expected total benefit associated with the decision to infringe is :

$$
\begin{aligned}
U_e \left(L^*; c_h^* \right) &= \left(1 - F\left(c_h^* \right) \right) \left(\pi_e^L - L^* \right) + F\left(c_h^* \right) \left(\pi_e^L - \theta \left[\pi_h^N - \pi_h^L \right] - c_e \right) \\
&= \pi_e^L - \left(1 - F\left(c_h^* \right) \right) L^* - F\left(c_h^* \right) \left(\theta \left[\pi_h^N - \pi_h^L \right] + c_e \right)
\end{aligned}
$$

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Substituting with L^* gives:

$$U_e\left(L^*; c_h^*\right) = \pi_e^L - \theta\left[\pi_h^N - \pi_h^L\right] + (1 - F\left(c_h^*\right)) c_h^* - F\left(c_h^*\right) c_e$$

Thus, the CN equilibrium $\left(y_h^L, y_e^L\right)$ is the solution to the system:

$$y_h^L = y_h(y_e^L) = \arg\max_{y_h}\left\{u_h(c_h) = \theta\pi_h^N + (1-\theta)(P - k_h)y_h - c_h \text{ s.t. } P = a - b(y_h + y_e^L)\right\}$$

$$y_e^L = y_e(y_h^L) = \arg\max_{y_e}\left\{\begin{array}{c} U_e\left(L^*; c_h^*\right) = (P - k_e)y_e - \theta\left[\pi_h^N - (P - k_h)y_h\right] + (1 - F\left(c_h^*\right)) c_h^* - F\left(c_h^*\right) c_e \\ \text{s.t. } P = a - b(y_h^L + y_e) \end{array}\right\}$$

Under assumptions 2 and 3, the characteristic features of the three market equilibria are described in the next schedule:

No Infringement	$y_h^N = \frac{a - 2k_h + \bar{k}}{3b}$ $y_e^N = \frac{a - 2\bar{k} + k_h}{3b}$ $P^N = \frac{a + k_h + \bar{k}}{3}$	$\pi_h^N = b\left(y_h^N\right)^2$ $\pi_e^N = b\left(y_e^N\right)^2$
Accommodation	$y_h^A = \frac{a - 2k_h + k_e}{3b}$ $y_e^A = \frac{a - 2k_e + k_h}{3b}$ $P^A = \frac{a + k_h + k_e}{3}$	$\pi_h^A = b\left(y_h^A\right)^2$ $\pi_e^A = b\left(y_e^A\right)^2$
Litigation	$y_h^L = \frac{a - 2k_h + k_e}{b(3 - \theta)}$ $y_e^L = \frac{a - 2k_e + k_h - \theta(a - k_h)}{b(3 - \theta)}$ $P^L = \frac{a + k_h + k_e - \theta k_h}{3 - \theta}$	$\pi_h^L = b\left(y_h^L\right)^2$ $\pi_e^L = b\left(y_e^L\right)^2$

According to the conditions imposed on marginal costs, we have:

$$y_h^N > y_h^A \text{ and } y_h^L > y_h^A$$

$$y_e^N < y_e^A \text{ and } y_e^L < y_e^A$$

but we need that : $\bar{k} - k_e > \frac{\theta}{3}(a - 2k_h + \bar{k}) \Leftrightarrow \theta < \theta_{LP} \equiv 3\left(\frac{\bar{k} - k_e}{a - 2k_h + \bar{k}}\right)$ to have $y_h^N > y_h^L$ and $y_e^N < y_e^L$ - i.e. to have that Lost Profit are positive, since, given the ranking of output levels, we have the ranking of equilibrium profits:

$$\pi_h^N > \pi_h^L > \pi_h^A$$

$$\pi_e^N < \pi_e^L < \pi_e^A$$

we also have under this inequality : $P^L < P^N$.

Thus, the inequality (7) always holds, and we have: $\hat{c}_h = \bar{c}_h$. Then, the belief consistency requirement implies that $\frac{F(c_h^*)}{F(\hat{c}_h)} = F(c_h^*)$. Finally, the equilibrium in the subgame corresponding to the one described in lemma 5 satisfies the conditions:

$$L^* = \theta\left[\pi_h^N - \pi_h^L\right] - c_h^*$$

$$\left(\frac{1-F}{f}\right)_{|c_h^*} = c_e + c_h^*$$

- **stage 2**

The Infringer chooses the entry with infringement if $U_e\left(L^*; c_h^*\right) \geq \pi_e^N$ or :

$$\left(\pi_e^L - \pi_e^N\right) - \theta\left[\pi_h^N - \pi_h^L\right] \geq F\left(c_h^*\right)c_e - \left(1 - F\left(c_h^*\right)\right)c_h^* \tag{8}$$

Note that under Lost Profit, the condition $\theta < \theta_{LP}$ implies $\pi_h^N - \pi_h^L > 0$ and $\pi_e^L - \pi_e^N > 0$. Thus, both the RHS and the LHS in (8) have an ambiguous sign. It can be verified that:

- $c^* = F\left(c_h^*\right)c_e - \left(1 - F\left(c_h^*\right)\right)c_h^*$ is an increasing function in c_e since:

$$\frac{\partial c^*}{\partial c_e} = F\left(c_h^*\right) - \left(\left(1 - F\left(c_h^*\right)\right) - f(c_h^*)\left[c_e + c_h^*\right]\right)\frac{\partial c_h^*}{\partial c_e} = F\left(c_h^*\right) > 0$$

- for $\theta = 0$, we have $\pi_e^L = \pi_e^A$ and thus $U_e\left(L^*; c_h^*\right) - \pi_e^N = \left(\pi_e^A - \pi_e^N\right) - c^*$. Thus assumption 5 is sufficient to have $U_e\left(L^*; c_h^*\right) - \pi_e^N > 0$ at $\theta = 0$: assumption 5 prevents that the size of the legal costs deters the Infringer to infringe the patent in the neighborhood of $\theta = 0$.

- for $\theta = \theta_{LP}$, we have $\pi_h^N = \pi_h^L$ and $\pi_e^L = \pi_e^N$, which implies that $U_e\left(L^*; c_h^*\right) - \pi_e^N = -c^* \lessgtr 0$.

On the other hand, note also that we have (making use of the envelop theorem):

$$\frac{\partial U_e}{\partial \theta}\left(L^*; c_h^*\right) = \frac{\partial \pi_e^L}{\partial y_h^L}\frac{\partial y_h^L}{\partial \theta} - \left[\pi_h^N - \pi_h^L\right]$$

where it can be verified that under the Lost Profit rule: $\frac{\partial \pi_e^L}{\partial y_h^L} < 0$ and $\frac{\partial y_h^L}{\partial \theta} > 0$ (given our assumptions 2 and 3; see also Choi (2009)). Hence:

- for $\theta < \theta_{LP}$, we obtain that $\frac{\partial U_e}{\partial \theta}\left(L^*; c_h^*\right) = \underbrace{\frac{\partial \pi_e^L}{\partial y_h^L}\frac{\partial y_h^L}{\partial \theta}}_{<0} - \underbrace{\left[\pi_h^N - \pi_h^L\right]}_{>0} < 0$.

- for $\theta = \theta_{LP}$, we obtain that $\frac{\partial U_e}{\partial \theta}\left(L^*; c_h^*\right) = \frac{\partial \pi_e^L}{\partial y_h^L}\frac{\partial y_h^L}{\partial \theta} < 0$.

To sum up, under the condition $\left(\pi_e^A - \pi_e^N\right) - c_e > 0$:

- if $c^* = 0$ then $\theta < \theta_{LP}$ implies $U_e\left(L^*; c_h^*\right) > \pi_e^N$.

- if $c^* < 0$ then $\theta \leq \theta_{LP}$ implies $U_e\left(L^*; c_h^*\right) > \pi_e^N$.

- if $c^* > 0$ then $\theta < \tilde{\theta} < \theta_{LP}$ (where $\tilde{\theta}$ satisfies $U_e\left(L^*; c_h^*\right) = \pi_e^N$) implies $U_e\left(L^*; c_h^*\right) > \pi_e^N$.

5.2 Proof of propositions 2 and 3

By the same argument as the one developed for proposition 1, it is easy to verify that at stage 4 (pretrial bargaining stage) lemma 5 may be substituted with lemma 6, where $D_{UE} = \pi_e^L - \pi_e^N$ and:

$$c_L = \theta \left[\pi_e^L - \pi_e^N \right] - L \tag{2'}$$

Lemma 7 *Under assumption 1, the solution to the maximization of (3) under (2'), is unique and corresponds to the licensing offer L^* and the cut-off value for the Patent Holder's type $c_{L^*} = c_h^*$ which are implicitly obtained by solving the system:*

$$L^{**} = \theta \left[\pi_e^L - \pi_e^N \right] - c_h^* \tag{9}$$

$$\left(\frac{F(\hat{c}_h) - F}{f} \right)_{|c_h^*} = c_e + c_h^* \tag{10}$$

We omit the proof since it is similar to the proof of proposition 1. Remark that conditions (4) and (10) are identical. The main differences comparing the Unjust Enrichment rule and the Lost Profit rule appears at stages 2 and 3:

- given that the best settlement offer is now $L^{**} = \theta \left[\pi_e^L - \pi_e^N \right] - c_h^*$, *Litigate* is the best action for any $c_h \in [\underline{c}_h, \bar{c}_h]$ only if:

$$u_h(L^{**}) = \pi_h^L + \theta \left[\pi_e^L - \pi_e^N \right] - c_h^* > \pi_h^A \tag{11}$$

We will show that this inequality may not always hold, at least for the highest values of c_h.

- let us consider the market stage. When the case is litigated, the Infringer's expected total benefit associated with the decision to infringe is :

$$
\begin{aligned}
U_e \left(L^{**}; c_h^* \right) &= \left(1 - \frac{F\left(c_h^* \right)}{F(\hat{c}_h)} \right) \left(\pi_e^L - L^{**} \right) + \frac{F\left(c_h^* \right)}{F(\hat{c}_h)} \left(\pi_e^L - \theta \left[\pi_e^L - \pi_e^N \right] - c_e \right) \\
&= \pi_e^L - \theta \left[\pi_e^L - \pi_e^N \right] + \left(1 - \frac{F\left(c_h^* \right)}{F(\hat{c}_h)} \right) c_h^* - \frac{F\left(c_h^* \right)}{F(\hat{c}_h)} c_e \\
&= \theta \pi_e^N + (1 - \theta) \pi_e^L + \left(1 - \frac{F\left(c_h^* \right)}{F(\hat{c}_h)} \right) c_h^* - \frac{F\left(c_h^* \right)}{F(\hat{c}_h)} c_e
\end{aligned}
$$

Thus, the Cournot-Nash equilibrium $\left(y_h^L, y_e^L \right)$ is the solution to:

$$y_h^L \;=\; y_h(y_e^L) = \arg\max_{y_h}\left\{u_h(c_h) = (P-k_h)y_h + \theta\left[(P-k_e)y_e - \pi_e^N\right] - c_h \;\text{ s.t. }\; P = a - b(y_h+y_e)\right\}$$

$$y_e^L \;=\; y_e(y_h^L) = \arg\max_{y_e}\left\{
\begin{array}{c}
U_e\left(L^{**};c_h^*\right) = \theta\pi_e^N + (1-\theta)\left(P-k_e\right)y_e + \left(1 - \frac{F(c_h^*)}{F(\hat{c}_h)}\right)c_h^* - \frac{F(c_h^*)}{F(\hat{c}_h)}c_e \\[2mm]
\text{s.t. }\; P = a - b(y_h+y_e)
\end{array}\right\}$$

with:

Litigation	$y_h^L = \frac{a-2k_h+k_e-\theta(a-k_e)}{b(3-\theta)}$ $y_e^L = \frac{a-2k_e+k_h}{b(3-\theta)}$ $P^L = \frac{a+k_h+k_e-\theta k_e}{3-\theta}$	$\pi_h^L = b\left(y_h^L\right)^2$ $\pi_e^L = b\left(y_e^L\right)^2$

and according to the conditions imposed on marginal costs, it is direct that[19] :

$$y_h^N \;>\; y_h^A > y_h^L \Rightarrow \pi_h^N > \pi_h^A > \pi_h^L$$

$$y_e^N \;<\; y_e^A < y_e^L \Rightarrow \pi_e^N < \pi_e^A < \pi_e^L$$

This implies that (11) may not hold since under assumption 4: $\theta\left[\pi_e^L - \pi_e^N\right] - c_h^* > 0$, but now with $\pi_h^A > \pi_h^L$. Thus, different kinds of equilibria may be obtained in the pretrial negotiation subgame:

- assume $\pi_h^L + \theta\left[\pi_e^L - \pi_e^N\right] - c_h^* > \pi_h^A$: in this case, once more it is easy to see that by sequential rationality $\hat{c}_h = \bar{c}_h$. Thus any Patent Holder $c_h \in [c_h^*, \bar{c}_h]$, where c_h^* is the same as under the Lost Profit rule, chooses (*Litigate,Accept*), while any Patent Holder $c_h \in [\underline{c}_h, c_h^*)$ chooses (*Litigate, Reject*). In this case, the belief consistency requirement implies that at stage 2, the Infringer obtains $U_e\left(L^{**};c_h^*\right)$ and *Infringe* is the best decision if $U_e\left(L^{**};c_h^*\right) \geq \pi_e^N$ or:

$$(1-\theta)\left(\pi_e^L - \pi_e^N\right) \geq F\left(c_h^*\right)c_e - \left(1 - F\left(c_h^*\right)\right)c_h^* \tag{12}$$

Let us investigate the values of θ that are consistent with such an equilibrium. First, it is easy to verify that for $\theta = 0$, we have $\pi_e^L = \pi_e^A$ and $\pi_h^L = \pi_h^A$ and thus $u_h\left(L^{**}\right) - \pi_e^A = -c_h^*$. Second, as θ increases, there are two opposites forces driving the sign of $u_h\left(L^{**}\right) - \pi_e^A$: on the one hand

---

[19]The condition required for Unjust Enrichment, i.e. to have $\pi_e^L - \pi_e^N > 0$, is:

$$k_e < \bar{k} + \frac{\theta}{6}(a - 2\bar{k} + k_h)$$

which is always satified.

$\pi_h^L - \pi_h^A - c_h^*$ has a negative impact on $u_h\left(L^{**}\right) - \pi_e^A$; on the other hand, $\theta\left[\pi_e^L - \pi_e^N\right]$ has a positive impact. Formally, making use of the envelop theorem, we have:

$$\frac{\partial u_h}{\partial \theta}\left(L^{**}\right) = \frac{\partial \pi_h^L}{\partial y_e^L}\frac{\partial y_e^L}{\partial \theta} + \left[\pi_e^L - \pi_e^N\right]$$

where it can be verified that under the Unjust Enrichment rule: $\frac{\partial \pi_h^L}{\partial y_e^L} < 0$ and $\frac{\partial y_e^L}{\partial \theta} > 0$ (given our assumptions 2 and 3; see also Choi (2009)). Hence, we obtain that:

$$\frac{\partial u_h}{\partial \theta}\left(L^{**}\right) = \underbrace{\frac{\partial \pi_h^L}{\partial y_e^L}\frac{\partial y_e^L}{\partial \theta}}_{<0} + \underbrace{\left[\pi_e^A - \pi_e^N\right]}_{>0}$$

It can be conjectured that in the neighborhood of $\theta = 0$, the first term dominates the second such that $\frac{\partial u_h}{\partial \theta}\left(L^{**}\right) < 0$; in contrast as θ becomes large enough, the second term is the dominant one, such that $\frac{\partial u_h}{\partial \theta}\left(L^{**}\right) > 0$. In this perspective, let us assume that there exists a $\theta'_{UE} < 1$ for which $u_h(L^{**}) = \pi_h^A \Leftrightarrow \theta'_{UE} \equiv \frac{\pi_h^A - \left(\pi_h^L\right)_{UE} + c_h^*}{\left(\pi_e^L\right)_{UE} - \pi_e^N}$. Then: if $\theta > \theta'_{UE}$, we have $u_h\left(L^{**}\right) > \pi_e^A$; i.e. the strength of the Patent Holder's case must be strong enough, in order that he chooses *Litigate* at equilibrium, whatever his type.

Turning to the Infringer, the best decision of the Infringer at the initial node is *Infringe* if: $U_e\left(L^{**}; c_h^*\right) > \pi_e^N$, which writes now as:

$$(1-\theta)\left[\left(\pi_e^L\right)_{UE} - \pi_e^N\right] - c^* > 0$$

Thus, remark that:

- for $\theta = 0$, $U_e\left(L^{**}; c_h^*\right) = \pi_e^A - \pi_e^N - c^*$; thus, assumption 5 is still sufficient to obtain $U_e\left(L^*; c_h^*\right) - \pi_e^A > 0$ at $\theta = 0$.

- for $\theta \in (0,1)$, $(1-\theta)\left[\left(\pi_e^L\right)_{UE} - \pi_e^N\right] > 0$.

- for $\theta = 1$, $U_e\left(L^{**}; c_h^*\right) = -c^*$.

Hence, when $c^* \le 0$, then $U_e\left(L^{**}; c_h^*\right) > \pi_e^N$ $\forall \theta \in [0,1]$; but when $c^* > 0$, let us define as $\theta''_{UE} < 1$, the patent strength for which $U_e\left(L^{**}; c_h^*\right) = \pi_e^N \Leftrightarrow \theta''_{UE} \equiv 1 - \frac{c^*}{\left(\pi_e^L\right)_{UE} - \pi_e^N}$: it must be that $\theta < \theta''_{UE}$, i.e. the strength of the Patent Holder's case must be weak enough, in order that *Infringe* is the best decision for the Infringer: $U_e\left(L^{**}; c_h^*\right) > \pi_e^N$. Thus, if $\theta'_{UE} < \theta''_{UE}$, the equilibrium is the one described in proposition 2.

- assume $\pi_h^L + \theta\left[\pi_e^L - \pi_e^N\right] - c_h^* < \pi_h^A \Leftrightarrow \theta < \theta'_{UE}$: in this case, if $\pi_h^L + \theta\left[\pi_e^L - \pi_e^N\right] - \underline{c}_h < \pi_h^A$, then any equilibrium may be built such that whatever the proposal $L \ge 0$ made by the Infringer at the pretrial stage, every Patent Holder types choose *Accommodate*, and the

Infringer chooses *Infringe*. In terms of outcomes, all these equilibria are associated with the output levels y_h^A, y_e^A. Thus, assume in contrast that a Patent Holder type denoted $\tilde{c}_h \in (\underline{c}_h, c_h^*)$ exists such that $\pi_h^L + \theta \left[\pi_e^L - \pi_e^N \right] - \tilde{c}_h = \pi_h^A \Leftrightarrow \tilde{c}_h = \left(\pi_h^L - \pi_h^A \right) + \theta \left[\pi_e^L - \pi_e^N \right]$ and consider the licensing price $\tilde{L}(> L^{**})$ required in order that $\pi_h^L + \tilde{L} = \pi_h^A \Leftrightarrow \tilde{L} = \pi_h^A - \pi_h^L = \theta \left[\pi_e^L - \pi_e^N \right] - \tilde{c}_h > 0$. Different equilibria may be build in this case.

- **case 1:** Consider that the Infringer proposes $\tilde{L}$ at stage 4; this implies that any Patent Holder $c_h \in [\tilde{c}_h, \bar{c}_h]$ chooses *Litigate* and accepts $\tilde{L}$, while any Patent Holder $c_h \in [\underline{c}_h, \tilde{c}_h)$ chooses *Litigate* and rejects $\tilde{L}$. At stage 2: the Infringer chooses the entry with infringement if $U_e(\tilde{L}; \tilde{c}_h) > \pi_e^N$ or:

$$F(\tilde{c}_h) \left(\pi_e^L - \theta \left(\pi_e^L - \pi_e^N \right) - c_e \right) + (1 - F(\tilde{c}_h)) (\pi_e^L - \tilde{L}) \geq \pi_e^N$$

$$\Updownarrow$$

$$(1 - \theta) \left(\pi_e^L - \pi_e^N \right) - F(\tilde{c}_h) c_e + (1 - F(\tilde{c}_h)) \tilde{c}_h \geq 0$$

Given assumption 5, and $\pi_e^L - \pi_e^N > 0$, then once more either $\tilde{c} = F(\tilde{c}_h) c_e - (1 - F(\tilde{c}_h)) \tilde{c}_h = c_e - (1 - F(\tilde{c}_h)) (c_e + \tilde{c}_h) \leq 0$, and then $U_e \left(\tilde{L}; \tilde{c}_h \right) > \pi_e^N \, \forall \theta \in (0, \theta_{UE}']$; or $\tilde{c} > 0$ and $U_e(\tilde{L}; \tilde{c}_h) > \pi_e^N$ only for $\theta \in (0, \tilde{\theta}_{UE}]$ where $\tilde{\theta}_{UE} < \theta_{UE}'$ is such that $U_e(\tilde{L}; \tilde{c}_h) = \pi_e^N$.

Note that the Infringer has no incentive to increase L over $\tilde{L}$ (in order to induce the separation of Patent Holder's types between *Accept* and *Reject* with a cut-off value smaller than $\tilde{c}_h$). By definition, $U_e(L; c_L)$ reaches its maximum for $(L^{**}; c_h^*)$: by the second order condition (concavity), $U_e(L; c_L)$ is thus decreasing (respectively increasing) for any $c_h < (>) c_h^*$, or equivalently, $U_e(L; c_L)$ is decreasing (respectively increasing) for any $L > (<) L^*$. Thus for any $L > \tilde{L}$ (associated with $c_h < \tilde{c}_h$) we have: $U_e(L; c_L) < U_e(\tilde{L}; \tilde{c}_h)$.

This is the case described in proposition 3i); however, to complete the proof that this is an equilibrium we must show that it resists also to a reduction of L under $\tilde{L}$. Let us analyze this point:

- **case 2:** Consider now that the Infringer proposes any $\hat{L} \in [0, \tilde{L})$ at stage 4; since $\hat{L} < \tilde{L}$ then $u_h(\hat{L}) < \pi_h^A$, and thus (since a $\tilde{c}_h \in [\underline{c}_h, c_h^*)$ exists) any Patent Holder $c_h \in [\tilde{c}_h, \bar{c}_h]$ chooses now *Accommodate*, while any Patent Holder $c_h \in [\underline{c}_h, \tilde{c}_h)$ chooses *(Litigate, Reject)* and obtains a judgment. At stage 2, the Infringer chooses the entry with infringement if $U_e(\hat{L}; \tilde{c}_h) > \pi_e^N$ or:

$$F(\tilde{c}_h) \left(\pi_e^L - \theta \left(\pi_e^L - \pi_e^N \right) - c_e \right) + (1 - F(\tilde{c}_h)) \pi_e^A \quad > \quad \pi_e^N$$

$$\Updownarrow$$

$$F(\tilde{c}_h) (1 - \theta) \left(\pi_e^L - \pi_e^N \right) + (1 - F(\tilde{c}_h)) \left(\pi_e^A - \pi_e^N \right) \quad > \quad F(\tilde{c}_h) c_e$$

Under assumption 5 we have: $\pi_e^A - \pi_e^N - F(\tilde{c}_h) c_e > 0$. For $\theta = 0$, we have $\pi_e^L = \pi_e^A$, en thus $U_e(\hat{L}; \tilde{c}_h) - \pi_e^N = \pi_e^A - \pi_e^N - F(\tilde{c}_h) c_e > 0$; for $\theta = 1$ then $U_e(\hat{L}; \tilde{c}_h) - \pi_e^N = (1 - F(\tilde{c}_h)) \left(\pi_e^A - \pi_e^N \right) - F(\tilde{c}_h) c_e > 0$. Moreover for any $\theta \in (0, 1)$:

$$\frac{\partial U_e}{\partial \theta}(\hat{L}; \tilde{c}_h) = (1 - \theta) \frac{\partial \pi_e^L}{\partial y_h^L} \frac{\partial y_h^L}{\partial \theta} > 0$$

since $\frac{\partial \pi_e^L}{\partial y_h^L} < 0$ and $\frac{\partial y_h^L}{\partial \theta} < 0$. As a result, the inequality $U_e(\hat{L}; \tilde{c}_h) > \pi_e^N$ holds once more for any $\theta \in (0, \theta'_{UE})$. This is the case described now in proposition 3ii).

Thus, in the case $\theta < \theta'_{UE}$ and if there is a $c_h \in [\underline{c}_h, \bar{c}_h]$ for which $\pi_h^L + \theta \left[\pi_e^L - \pi_e^N \right] - c_h = \pi_h^A$, then:

- when $U_e(\hat{L}; \tilde{c}_h) > U_e(\tilde{L}; \tilde{c}_h)$, which requires that: $\pi_e^A > \pi_e^L - \tilde{L} \Leftrightarrow \pi_h^A + \pi_e^A > \pi_e^L + \pi_h^L$, the equilibrium has the features described in proposition 3ii).

- in contrast, when the opposite inequality holds $\pi_h^A + \pi_e^A < \pi_e^L + \pi_h^L$, the equilibrium is the one described in proposition 3i).

5.3 Consistency of welfare comparisons

The next table illustrates the difficulties associated with the comparisons of welfare, through a simple numerical simulation.

Calibration: $a = 70; b = 1; \bar{k} = 20; c_h^* = 0 = c^*$

	$k_h = 15; k_e = 18$	$k_h = 15 = k_e$	$k_h = 4 = k_e$	$k_h = 15; k_e = 8$	$k_h = 8; k_e = 18$
θ_{LP}	0.1	0.25	0.58	0.6	0.1
θ'_{UE}	0.99	0.79	0.57	0.35	> 1
θ''_{UE}	0.9	1	1	1	1

Note that this is only illustrative, since we do not provide a comprehensive analysis of sensibility, and moreover we neglect the influence of legal costs. However, it shows that the result may be very sensitive to the difference between marginal production costs. In this set of calculus, we find that

the comparison of welfare is consistent (given that $\theta'_{UE} < \theta_{LP} < \theta''_{UE}$) only for $k_h = 15, k_e = 8$ (fourth column).