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Bertrand Chopard
Edwige Marion
Ludivine Roussey

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Does the Appeals Process Lower the Occurrence of Legal Errors?

Bertrand Chopard[*], Edwige Marion[†]and Ludivine Roussey[‡§]

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Abstract

This paper challenges the commonly held idea that the appeals process lowers the occurrence of legal errors. We show that, even if incorporating the right to bring an appeal in criminal adjudication directly offers the opportunity to correct mistakes made at trial, the final impact on the appeals process on the accuracy of judicial decisions is ambiguous because of its effect through trial court decision-makers' effort and crime deterrence. We find out that according to (i) whether trial court decision-makers are rather reputation-concerned or socially-motivated, (ii) the distribution of the population of potential offenders and (iii) the marginal effect of decision-makers' effort on the probability of wrongful conviction compared to wrongful acquittal, implementing an appeals process may spur decision-makers to reduce their effort and thus be detrimental to the accuracy of legal decisions. We show that this result is true whether an appeal can be brought both after an acquittal or a conviction at trial, or the right to bring an appeal is limited by double jeopardy.

Keywords: appeals, legal errors, crime deterrence.

J.E.L. Classification: K14, K4.

1 Introduction

Appeals allow for parties in lawsuits to seek reconsideration of their arguments. There is a commonly held idea that appeals mechanically improve the accuracy of judicial decisions by offering the opportunity to correct legal errors at trial. Posner (1972) first highlighted this argument in his *Economic Analysis of Law* and Shavell (1995) formalized it. According to the latter, the appeals process may be understood as a means of error correction from the moment that litigants possess private information about whether an error has been made at trial and that appeal courts can verify these errors with higher precision than trial courts. Under these quite reasonable assumptions, a litigant will not bring an appeal if there is no mistake at first instance. Consequently, new errors cannot be made on appeal. On the contrary, appeals can (imperfectly) correct errors made at first instance. Shavell goes further: not only does the appeals process lower the occurence of legal errors but it does so at a lower cost than the alternative of enhancing the quality of the trial process. These arguments justify the existence of appeals process in most adjudication systems. For example the right to bring an appeal has been introduced in France in 2001 for serious crimes in order to correct errors potentially

made by Cours d'Assises. Since then, an appeal can be brought either after convictions or acquittals, by the defendant or the prosecutor. It is worthwhile noticing that, in some countries, the right to bring an appeal is more limited. In the United States, double jeopardy thus forbids the prosecution to appeal from an acquittal verdict.

The purpose of this paper is to reconsider the commonly held idea that appeals necessarily lower the occurence of legal errors. This idea is probably true if the appeals process does not impact the behavior of trial court decision-makers, but it is likely to be challenged otherwise. We thus explore the possible changes in the behavior of trial court decision-makers (judges, juries...) after the introduction of an appeals process. We show that if the appeals process can *directly* reduce the occurrence of legal errors, it may also *indirectly* either increase or decrease the occurrence of legal errors, according to the incentives given to the trial court decision-maker. Intuitively, the appeals process may change the behavior of trial court decision-makers in two opposite ways. On the one hand, it plays as a sanction mechanism for trial court decision-makers who fear reversal (*reputation-concerned* decision-makers). As such, it may spur them to devote more effort to obtain accurate decisions than they would in the absence of an appeals process. On the other hand, since the appeals process mechanically reduces the occurence of legal errors, it may substitute to the effort of trial court decision-makers who are concerned with the quality of final verdicts (*socially-motivated* decision-makers). In this case, it dissuades them from making efforts because the same final quality of judicial decisions can be attained with less effort thanks to the appeals process. Finally, we point out that this behavioral trade-off is not neutral to the final quality of court decisions and that it may qualify the widespread idea that appeals always improve the quality of judicial systems. The originality of our paper is not only to consider a representative trial court decision-maker that can be *both* career-concerned *and* socially-motivated but also to take into account the interaction between the trial court decision-maker's behavior and crime deterrence. To the best of our knowledge, how these aspects influence the occurence of legal errors when the right to bring an appeal is introduced in a criminal procedure has never been investigated before. And yet, this is of great importance because this leads to reconsider the idea that the appeals process is always a good way of improving the accuracy of judicial decisions. This paper suggests that whether this idea is true or not especially depends on the interaction between the characteristics of trial court decision-makers and potential offenders.

Our paper is closely related to the literature on the appeals process. We already mentioned Shavell's seminal paper on the appeals process as a means of error correction (1995). In this paper, the basic model supposes that the likelyhood of error at trial depends only on the resources devoted by the State to the trial process. In an extension to the basic model, Shavell interestingly envisages the case in which trial court judges' efforts may influence the chance of error at trial. However, he only considers that trial court judges are reputation-concerned, so that the appeals process necessarily spurs them to expend effort in order to make more accurate decisions and to avoid reversal. Shavell (2006) more precisely develops the idea that the threat of appeal leads judges to make decisions that more closely resemble the socially optimal decision. Levy (2005) also studies the effect of the appeals process on judges' incentives. He shows that in the presence of an appeals process, careerist judges may seek to provoke appeal, so as to signal their abilities and improve their reputation. Iossa and Palumbo (2007) analyze the degree of opportunism of the decision-maker when his decision may be subject to an appellate review. They find out different results depending on whether information on the dispute is provided by the parties themselves or by an independent investigator. More precisely, they show that information provision by the parties generates more efficient monitoring through appeals and less opportunism by the deicison-maker than information provision by the investigator. Daughety and Reinganum (2000) do not focus on the first-instance decision-maker's behavior but on parties incentives to bring an appeal when

the appeals court can draw inference from the fact that an appeal is brought. Finally, a forthcoming paper by Oytana studies the raison d'être of the appeals process when judges make their decisions on the basis of information gathered by biased experts. In this context, Oytana shows that the appeals process limits the adverse impact of biased experts in terms of legal errors.

The present paper is organized as follows. Section 2 presents the model of legal errors correction. In section 3, we extend the model to study the direct and the indirect effects of the appeals process on the occurrence of legal errors. In section 4, we discuss the double jeopardy clause. Section 5 concludes.

2 The model

We consider the case for a risk neutral decision-maker (judge, juries, prosecutors) who has enough influence to reduce the probability of judicial errors at the first order trial. He chooses some effort level, $e > 0$, at convex cost $\psi(e)$ in terms of disutility of effort, time, or any other resources. By increasing his effort, e, he may reduce the occurrence of legal errors. We denote $p(e)$ the likelihood of relaxing a guilty defendant, with $p'(e) < 0$ and $p''(e) >$, and $\beta(e)$, the likelihood of wrongly convicting an innocent individual, with $\beta'(e) < 0$ and $\beta''(e) > 0$. We assumed that $p(e) > \beta(e)$. This assumption reflects the fact that a high standard of evidence is usually required in criminal procedure, thus limiting wrongful conviction (RiZzolli and Saraceno, 2013). The decision-maker is paid a gross wage w. In addition, he may suffer a private loss associated with his own mistakes αh, where h represents the social harm due to judicial errors and α refers to the decision-maker's sensitivity to social harm. Finally, we consider a continuum of risk neutral individuals each defined by a private benefit from harmful act, as in Polinsky and Shavell (1984). We denote the latter b and take it to be distributed on a finite support $V \equiv (0, B)$, with cumulative distribution function $F(.)$. The corresponding density $f(.)$ is continuously differentiable and positive in the interval $(0, B)$, and equal to zero outside this interval. Given e, an individual will commit an harmful act if and only if his benefit, b, exceeds the expected sanction, $t[1 - p(e) - \beta(e)]S \equiv \tilde{b}_{NA}(e)$, where the value $\tilde{b}_{NA}(e)$ will be referred to as the borderline type when it is impossible to bring an appeal (NA means "no appeal"). Thus, there is a given probability (the product of the probability of detection, t, with the probability of conviction, $1 - p(e) - \beta(e)$) that an act will be subject to a sanction, S. A higher probability of relaxing a guilty individual increases the payoff of committing an act whereas a higher probability of wrongly convicting an innocent person will decrease the payoff of abiding by the law (Garoupa and Rizzolli, 2012). Following Png (1986), we also assume that the effects of judicial errors on crime deterrence are symmetric, meaning that the two probabilities of legal errors have the same impact on the potential offender, that is on the returns from honesty and from harmful act (Lando (2006), Rizzolli and Stanca (2012)).[1]

In this setting, the decision-maker chooses e to solve:

$$\max_e \left\{ u_{NA}(e) = w - \psi(e) - (1 - F(\tilde{b}_{NA}(e)))p(e)\alpha h - F(\tilde{b}_{NA}(e))\beta(e)\alpha h \right\}$$

where the last two terms of $u_{NA}(e)$ represent respectively the expected cost of wrongful acquittal weighted by the probability to be faced with a guilty individual (or the probability of crime), and the expected cost of wrongful conviction weighted by the probability to be faced with an innocent indivudual. Note that neither the social harm of legal errors (h) nor the decision-maker's sensitivity to social harm of errors (α) differ from one type of legal error to the other one.

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[1] For instance, Rizzolli and Stanca (2012) explain how wrongful conviction may become more detrimental to deterrence than wrongful acquittal if we introduce risk-aversion and loss aversion in the standard model of public enforcement.

Differentiating $u_{NA}(e)$ with respect to e leads to lemma 1 below (all proofs are reported in appendix).

Lemma 1 *There exists a unique effort level at equilibrium e_{NA}^* defined by:*

$$\psi'(e_{NA}^*) = \alpha h \left(\begin{array}{c} -p'(e_{NA}^*)[1 - F(\tilde{b}_{NA}(e_{NA}^*))] - \beta'(e_{NA}^*)F(\tilde{b}_{NA}(e_{NA}^*)) - \\ f(\tilde{b}_{NA}(e_{NA}^*))tS[p'(e_{NA}^*) + \beta'(e_{NA}^*)][p(e_{NA}^*) - \beta(e_{NA}^*)] \end{array} \right). \tag{1}$$

The first order condition can be rewritten:

$$\underbrace{\psi'(e)}_{C} = \underbrace{-\left(p'(e)(1 - F(\tilde{b}_{NA}(e))) + \beta'(e)F(\tilde{b}_{NA}(e))\right)\alpha h}_{B_p^{NA}} + \underbrace{\frac{\partial F(\tilde{b}_{NA}(e))}{\partial e}(p(e) - \beta(e))\,\alpha h}_{B_d^{NA}}. \tag{2}$$

The term C represents the decision-maker's disutility of raising e by one unit. Together, terms B_p^{NA} and B_d^{NA} are the decision-maker's total marginal benefit of raising e by one unit under two effects: the reduction of the probability of legal errors and crime deterrence.

We first explore the term B_p^{NA}. It represents the marginal benefit due to the fact that, by increasing his effort by one unit, the decision-maker reduces the *probability* of relaxing a guilty defendant by $-p'(e)$ units, and the *probability* of wrongly convicting an innocent individual by $-\beta'(e)$ units. Thus, the decision-maker reduces his expected private loss in case of a legal error by $-\left(p'(e)(1 - F(\tilde{b}_{NA}(e))) + \beta'(e)F(\tilde{b}_{NA}(e))\right)\alpha h$ units because losses associated to both types of legal errors are weighted by the probabilities that the defendant is either guilty or innocent.

Second, term B_d^{NA} captures the *deterrence* effect. By increasing his effort by one unit, the decision-maker deters crime. Then, it simultaneously reduces the likelihood to be faced with a guilty individual by $-\frac{\partial(1 - F(\tilde{b}_{NA}(e)))}{\partial e} = \frac{\partial F(\tilde{b}_{NA}(e))}{\partial e}$ units, and increases the probability to be faced with an innocent individual by $\frac{\partial(F(\tilde{b}_{NA}(e)))}{\partial e}$ units. As it is more likely to wrongly relax a guilty individual than to wrongly convict an innocent individual ($p(e) > \beta(e)$), such an increase of e by one unit is profitable. More precisely, it reduces the decision-maker's expected loss associated with legal errors by $-\left(\frac{\partial(1 - F(\tilde{b}_{NA}(e)))}{\partial e}p(e)\alpha h + \frac{\partial F(\tilde{b}_{NA}(e))}{\partial e}\beta(e)\alpha h\right)$ units.

The second order condition for a maximum is:

$$\frac{d^2 u_{NA}(e)}{de^2} = f'(\tilde{b}_{NA}(e))(-t^2 S^2)(p'(e) + \beta'(e))^2(p(e) - \beta(e))\alpha h + 2f(\tilde{b}_{NA}(e))tS(p'(e) + \beta'(e))(p'(e) - \beta'(e))\alpha h \leq 0. \tag{3}$$

Following Kaplow (1990), it appears that the first term in (3) is the product of the decision-maker's net marginal benefit per individual deterred and the rate at which the population density changes, which is of indeterminate sign. The word "net" means that an increase in e has a deterrence effect. It leads to a reduction in the risk to be faced with a guilty individual, and thus to wrongly relax him with probability $p(e)$. But it leads also to an increase in the probability to be faced with an innocent individual, and thus to wrongly convict him with probability $\beta(e) < p(e)$ by assumption. The second term in (3) reflects that for each additional individual deterred, the decision-maker simultaneously looses the private benefit of a one-unit increase in his effort to reduce the risk of wrongly relaxing a guilty individual ($-p'(e)\alpha h$) and wins the marginal benefit of a one-unit increase in his effort to reduce the risk of wrongly convicting an innocent one ($-\beta'(e)\alpha h$). As long as the decision-maker's effort is deterrent ($p'(e) + \beta'(e) < 0$), fewer individuals then remain undeterred, and the second term in (3) is either positive or negative according to the sign of $p'(e) - \beta'(e)$ which is indeterminate.

To sum up, whether the SOC will hold depends on the sign and the magnitude of the term $p'(e) - \beta'(e)$, and the shape of distribution $f(.)$ weighted by the gap between the two probabilities of legal errors, $p(e) - \beta(e)$. In particular, if $p'(e) - \beta'(e)$ is relatively low and negative, equation (3) fails unless $f(.)$ is increasing at a

sufficient rate in the relevant range. To illustrate consider the case of an uniform distribution: the first term in (3) is zero, and the effort satisfying the FOC will be a maximum if and only if $p'(e) \geq \beta'(e)$. In words, the decision-maker's effort should rather reduce at the margin the risk of wrongly convicting an innocent individual instead of reducing the risk of wrongly relaxing a guilty one for the SOC to hold. Finally, to ensure the existence of a maximum, we can rewrite equation (3) as follows:

Assumption 1:

$$\varepsilon(b) < \left(\frac{2(p'(e) - \beta'(e))}{(p'(e) + \beta'(e))} \right) \left(\frac{1 - (p(e) + \beta(e))}{(p(e) - \beta(e))} \right) \equiv \overline{\varepsilon}_{NA}$$

where $\varepsilon(b) = \frac{f'(b)}{f(b)} b$ is the elasticity of the density.

The static comparative results are summarized in Proposition 1 below:

Proposition 1 *An increase in either the sanction, S, or the probability of detection, t, implies a decrease in the decision-maker's effort if and only if*

$$\varepsilon(\tilde{b}_{NA}(e)) < \frac{(p'(e) - \beta'(e)) + 2(\beta'(e)\beta(e) - p'(e)p(e))}{(p'(e) + \beta'(e))(p(e) - \beta(e))}. \tag{4}$$

An increase in either the social harm due to judicial errors, h, or the decision-maker's sensitivity to social harm of errors, α, implies an increase in decision-maker's effort.

An increase in the decision-maker's private loss associated with a legal error, αh, contributes to rise both B_p^{NA} and B_d^{NA}, thus encouraging him to increase his effort because legal errors are more costly. The effect of the penalty, S, and the probability of detection, t, on the optimal level of effort is ambiguous. A variation of these parameters may affect B_p^{NA} and B_d^{NA} in a different way.

First, an increase in the penalty or the probability of detection reduces the probability of crime (by increasing $\tilde{b}_{NA}(e)$). Since the probability of crime becomes lower, B_p^{NA} may either increase or decrease according to the sign and the magnitude of the difference $p'(e) - \beta'(e)$. On the one side, the decision-maker has a lower incentive to reduce the probability to wrongly relax a guilty individual simply because there is a lower probability to be faced with such an individual. On the other side, the opposite effect is at stake when we consider the risk of wrongly convicting an innocent person. In this sense, the penalty and the probability of detection can be considered as either substitutes or complements for the decision-maker's effort.

Second, raising the penalty or the probability of detection contributes also to either increase or decrease B_d^{NA}. Indeed, notice that:

$$B_d^{NA} = \frac{\partial F(\tilde{b}_{NA}(e))}{\partial e} (p(e) - \beta(e)) \alpha h = (p(e) - \beta(e)) \alpha h f(\tilde{b}_{NA}(e)) \frac{d\tilde{b}_{NA}(e)}{de} > 0.$$

It appears that the penalty and the probability of detection modify the magnitude of the positive effect of the decision-maker's effort on the decrease in the probability of crime through the two terms $\frac{d\tilde{b}_{NA}(e)}{de}$ and $f(\tilde{b}_{NA}(e))$. First, $\frac{d\tilde{b}_{NA}(e)}{de}$ is increasing with the penalty and the probability of detection. Thus, an increase in one of these two parameters strengthens the positive effect of the decision-maker's effort on crime deterrence. Second, $f(\tilde{b}_{NA}(e))$ may either increase or decrease with the penalty or the probability of detection according to the rate at which the population density changes. Finally, remark that both these effects are weighted by the gap between the two probabilities of legal errors, $p(e) - \beta(e)$. Therefore, the lower is this gap, the lower is the effect of the penalty and the probability of detection on the extent to which the decision-maker's effort is profitable through criminal decision making.

Put together, condition (4) resumes all these effects and indicates whether the penalty and the probability of detection can be viewed as complements or substitutes for the decision-maker's effort to reduce the occurrence of legal errors. Below, we study the interaction between the decision-maker's choice of effort in reducing the risk of legal errors and the correction of legal errors by appeals courts.

3 Appeals process as a means of legal errors correction

Following Shavell (1995), it is assumed that an appeal is brought if and only if an error has been made at trial. It supposes that litigants know whether an error has been made at trial and that the appeals court makes judicial decisions with higher precision than the trial court[2]. As Shavell, we abstract from the fact that appeals courts could draw inferences from the fact that litigants bring appeals and consequently systematically reverse trial court decisions. Indeed, using such inferences is first not what happens in practice: appeals courts cannot use this knowledge to make their decisions ; they fully reconsider cases. Second it is not socially desirable that appeals courts use inferences from the fact that appeals are brought. As explained by Shavell (1995) "*if appeals courts were to reverse all decisions, on the basis of their inference that all appellants are the victims of error, then disappointed litigants who are not the victims of error clearly would have an incentive to bring appeals, for they could obtain sure reversal*". Consequently, the utility of the appeals process in error correction would be diminished. For these reasons, we assume that the probability of reversal by the appeals court, $q \in (0,1)$ is exogenous and common knowledge. For sketch of simplicity, this probability does not depend on the type of legal error.

Under this new setting, the threshold value of the benefit of crime equals:

$$\tilde{b}_A(e) = tS[1 - (1-q)(\beta(e) + p(e))].$$

Notice that $\tilde{b}_A(e) > \tilde{b}_{NA}(e)$, which means that the probability of crime will decrease if an appeals court corrects legal errors if any.

We now consider that the decision-maker suffers a reputational cost or a disutility z if he is reversed by the appeals court. The decision-maker is thus now said to be socially-motivated when he is more sensitive to social harm than to his reputation ($\alpha h > z$). Otherwise, we will refer to him as a reputation-concerned decision-maker ($\alpha h < z$).

The decision-maker chooses e to solve:

$$\max_e \left\{ u_A(e) = w - \psi(e) - (1 - F(\tilde{b}_A(e)))p(e)((1-q)\alpha h + qz) - F(\tilde{b}_A(e))\beta(e)((1-q)\alpha h + qz) \right\}$$

under the condition below in order to control for the problem's concavity

Assumption 2:

$$\varepsilon(b) < \left(\frac{2(p'(e) - \beta'(e))}{(p'(e) + \beta'(e))} \right) \left(\frac{1 - (p(e) + \beta(e))(1-q)}{(p(e) - \beta(e))(1-q)} \right) \equiv \bar{\varepsilon}_A.$$

Differentiating $u_A(e)$ with respect to e leads to lemma 2:

Lemma 2 *There exists a unique effort level at equilibrium e_A^* defined by:*

$$
\begin{aligned}
\psi'(e_A^*) =\ & f(\tilde{b}_A(e_A^*)))(-tS)(1-q)(\beta'(e_A^*) + p'(e_A^*))\left(p(e_A^*) - \beta(e_A^*)\right)((1-q)\alpha h + qz) - \\
& \left((1 - F(\tilde{b}_A(e_A^*)))p'(e_A^*) + F(\tilde{b}_A(e_A^*))\beta'(e_A^*)\right)((1-q)\alpha h + qz)
\end{aligned}
\tag{5}
$$

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[2] Regarding the relevance of the second hypothesis, Shavell (2006) argues that the society invests special efforts in selecting appeals court decision-makers to ensure that their preferences are more aligned with society's than trial court decision-makers'. For example, appeals courts decide in larger panels. One can also think that appeals court judges are more experienced.

The first-order condition may be rewritten as follows:

$$\underbrace{\psi'(e)}_{C} = \underbrace{-\Big(p'(e)(1 - F(\tilde{b}_A(e))) + \beta'(e)F(\tilde{b}_A(e))\Big)((1-q)\alpha h + qz)}_{B_p^A} + \underbrace{\frac{\partial F(\tilde{b}_A(e))}{\partial e}(p(e) - \beta(e))((1-q)\alpha h + qz)}_{B_d^A}.$$

(6)

where B_p^A and B_d^A are the decision-maker's marginal benefits of raising e by one unit.

We now explore the conditions under which the appeals process may encourage or discourage the decision-maker's to increase his effort according to the parameters of the model. To begin with, notice that the appeals process leads to a decrease (resp. an increase) in the decision-maker's effort if $\Delta B_p = B_p^A - B_p^{NA} < 0$ (resp. $\Delta B_p > 0$) and $\Delta B_d = B_d^A - B_d^{NA} < 0$ (resp. $\Delta B_d > 0$), that is if the appeals process reduces (resp. increases) the marginal benefits associated with the reduction of the *probability* of a legal error and crime *deterrence*. Below, we explore separately these two conditions.

First of all, we rewrite ΔB_d as follows:

$$\Delta B_d = (p(e) - \beta(e))\frac{\partial F(\tilde{b}_A(e))}{\partial e}((1-q)\alpha h + qz) - (p(e) - \beta(e))\frac{\partial F(\tilde{b}_{NA}(e))}{\partial e}\alpha h.$$

For given probabilities of legal errors, appeals process has an influence on the decision-maker's choice through two channels: (i) his private loss associated with a legal error (which moves from αh to $(1-q)\alpha h + qz$), and (ii) his capacity to deter crime at the margin (captured by respectively $\frac{\partial F(\tilde{b}_A(e))}{\partial e}$ and $\frac{\partial F(\tilde{b}_{NA}(e))}{\partial e}$).

The intuition for point (i) runs as follows. If the decision-maker is *reputation-concerned*, then implementing an appeals process creates an incentive for him to increase his effort due to his disutility of being overturned. Indeed, if $\alpha h < z$, the appeals process increases the decision-maker's private loss associated with legal errors because $(1-q)\alpha h + qz > \alpha h$. At the opposite, if the decision-maker is *socially-motivated* ($\alpha h > z$), then implementing an appeals process spurs him to decrease his effort because he now relies on the appeals court to correct his mistakes. The appeals process thus diminishes the decision-maker's private loss associated with legal errors ($(1-q)\alpha h + qz < \alpha h$).

To better understand point (ii), we use lemma 3 below:

Lemma 3 $\frac{\partial F(\tilde{b}_A(e))}{\partial e} > \frac{\partial F(\tilde{b}_{NA}(e))}{\partial e} \Leftrightarrow 0 < \frac{1-(1-q)[p(e)+\beta(e)]}{(1-q)(p(e)+\beta(e))} < \varepsilon(\tilde{b}_A(e)) < \overline{\varepsilon}_A.$

Here, the rate at which the population density changes (captured by the term $\varepsilon(\tilde{b}_A(e))$) determines how a one-unit increase in the decison-maker's effort will either increase or decrease the marginal impact of his effort on the probability to be faced with either a guilty or an innocent defendant (and by the way, on the risk to commit one specific type of legal error). To illustrate, consider anew the case of an uniform distribution. Thus, $\varepsilon(\tilde{b}_A(e)) = 0$ and $\frac{\partial F(\tilde{b}_A(e))}{\partial e} < \frac{\partial F(\tilde{b}_{NA}(e))}{\partial e}$: the appeals process discourages the decision-maker to exert more effort, everything else equal (in particular, the decision-maker's type, see point (i) above) because one more effort is less deterrent with the appeals process.

Now, we study ΔB_p or the way that the appeals process affects the decision-maker's marginal benefit resulting from the fact that, by increasing his effort, he also reduces the probabilities of legal errors. By definition:

$$\Delta B_p = -\Big(p'(e)(1 - F(\tilde{b}_A(e))) + \beta'(e)F(\tilde{b}_A(e))\Big)((1-q)\alpha h + qz) - \Big(-\Big(p'(e)(1 - F(\tilde{b}_{NA}(e))) + \beta'(e)F(\tilde{b}_{NA}(e))\Big)\alpha h\Big).$$

Thus, ΔB_p depends on the way that the appeals process affects (i) the decision-maker's private loss associated with legal errors, and (ii) the expected marginal effect of the decision-maker's effort on the probabilities of legal errors, that is the expression $\Big(p'(e)(1 - F(\tilde{b}_i(e))) + \beta'(e)F(\tilde{b}_i(e))\Big)$ for $i = \{A, NA\}$.

The explanation of point (i) is identical to the one above and depends on the decision-maker's type: either *socially-motivated* or *reputation-concerned*. Further, notice for point (ii) that the following is true. As $\tilde{b}_A(e) > \tilde{b}_{NA}(e)$, we have $\Delta B_p < 0 \Leftrightarrow -p'(e) > -\beta'(e) > 0$. In words, recall that a one-unit increase in the decision-maker's effort allows for a reduction of legal errors on both innocent individuals (with probability $F(\tilde{b}_i(e))$ for $i = \{A, NA\}$) and guilty ones (with probability $1 - F(\tilde{b}_i(e))$ for $i = \{A, NA\}$). As the appeals process directly deters crime (that is for a given e), fewers individuals remain undeterred. Thus, due to the appeals process, the decision-maker's expected benefit of a reduction in the probability of wrongly relaxing a guilty individual becomes *lower* whereas the expected benefit of a reduction in the probability of wrongly convicting an innocent individual becomes *higher*. As a consequence, we show that the appeals process will encourage the decision-maker to exert *less* effort ($\Delta B_p < 0$) if and only if his effort reduces more the probability of wrongly relaxing a guilty individual than the probability of wrongly convicting an innocent one, that is if $-p'(e) > -\beta'(e) > 0$.

Put together, we show that the appeals process has an *indirect* impact on the probabilities of legal errors through three channels: the type of the decision-maker, either *socially-motivated* or *reputation concerned* ($\alpha h \gtrless z$), the magnitude of the marginal impact of the decision-maker's effort on the probabilities of legal errors ($-p'(e) \gtrless -\beta'(e)$), the sign and the magnitude of the rate at which the population density changes ($\varepsilon(\tilde{b}_A(e)) \gtrless \frac{1-(1-q)[p(e)+\beta(e)]}{(1-q)(p(e)+\beta(e))}$, see Lemma 3). We summarize these effects in Table 1 below:

Table 1: appeals process and variations in the decision-maker's marginal benefits.

$\varepsilon(\tilde{b}_A(e)) > \frac{1-(1-q)[p(e)+\beta(e)]}{(1-q)(p(e)+\beta(e))}$	$\alpha h > z$	$\alpha h < z$
$-p'(e) > -\beta'(e)$	$\Delta B_d \gtrless 0$ $\Delta B_p < 0$	$\Delta B_d > 0$ $\Delta B_p \gtrless 0$
$-p'(e) < -\beta'(e)$	$\Delta B_d \gtrless 0$ $\Delta B_p \gtrless 0$	$\Delta B_d > 0$ $\Delta B_p > 0$
$\varepsilon(\tilde{b}_A(e)) < \frac{1-(1-q)[p(e)+\beta(e)]}{(1-q)(p(e)+\beta(e))}$	$\alpha h > z$	$\alpha h < z$
$-p'(e) > -\beta'(e)$	$\Delta B_d < 0$ $\Delta B_p < 0$	$\Delta B_d \gtrless 0$ $\Delta B_p \gtrless 0$
$-p'(e) < -\beta'(e)$	$\Delta B_d < 0$ $\Delta B_p \gtrless 0$	$\Delta B_d \gtrless 0$ $\Delta B_p > 0$

We show in Table 1 that the effect of the appeals process on the decision-maker's incentives is largely unclear. Intuitively, if the decision-maker is *reputation-concerned* ($\alpha h < z$), concerned only by the risk of wrongly relaxing a guilty individual ($\beta(e) = 0$), and unsensitive to the way judicial errors may have a deterrence effect, then we can conclude that the appeals process spurs him to exert more effort to obtain more accurate decisions and avoid a reversal on appeal (Shavell, 1995, 2006). We show here that adding on the fact that decision-makers may be rather *socially-motivated*, that there is a room for two types of legal errors, and that decision-makers can also internalize the impact of both their own effort and the appeals process on the deterrence, qualifies the previous result.

More precisely, we show that a *reputation-concerned* decision-maker will still have unambiguously an incentive to increase his effort *only* if $\varepsilon(\tilde{b}_A(e)) > \frac{1-(1-q)[p(e)+\beta(e)]}{(1-q)(p(e)+\beta(e))}$ and $-p'(e) < -\beta'(e)$: see row 3 column 3 in Table 1. A *socially-motivated* decision-maker will have unambiguously an incentive to decrease his effort if $\varepsilon(\tilde{b}_A(e)) < \frac{1-(1-q)[p(e)+\beta(e)]}{(1-q)(p(e)+\beta(e))}$ and $-p'(e) > -\beta'(e)$: see row 6 column 3 in Table 1. Thus, neither the decision-maker's type by itself, nor the sensitivity of the probabilities of legal errors to the decision-maker's effort by itself are sufficient to predict the way that the appeals process may have an indirect influence on the emergence

of legal errors. Finally, remark that an appeals process will all the more encourage the decision-maker to reduce the risk of a legal error (*i*) that the decision-maker is rather more *reputation-concerned* ($\alpha h < z$), (*ii*) that the elasticity of the density function of the private benefit of crime for the threshold individual, $\varepsilon(\tilde{b}_A(e))$, is high, and (*iii*) that the decision-maker is rather concerned by the reduction of the risk of wrongly convicting an innocent individual (in comparison to the risk of wrongly relaxing a guilty individual, $-p'(e) < -\beta'(e)$).

Next, we summarized the results of the comparative statics analysis in the proposition below. The explanation is similar to the one of Proposition 1.

Proposition 2 *An increase in either the sanction, S, or the probability of detection, t, implies a decrease in the decision-maker's effort if and only if*

$$\varepsilon(\tilde{b}_A(e)) < \frac{(p'(e) - \beta'(e)) + 2(1-q)(\beta'(e)\beta(e) - p'(e)p(e))}{(p'(e) + \beta'(e))(p(e) - \beta(e))(1-q)}.$$

An increase in either the social harm due to a judicial errors, h, or the decision-maker's sensitivity to social harm of errors, α, implies an increase in the decision-maker's effort at equilibrium.

To supplement the previous analysis, we compare the expected social cost of legal errors without any appeals process (namely $ESCE_{NA}(e^*_{NA})$ at equilibrium) with the expected social cost of legal errors when an appeals process prevails ($ESCE_A(e^*_A)$ at equilibrium), where:

$$ESCE_{NA}(e^*_{NA}) = \left([1 - F(\widetilde{b}_{NA}(e^*_{NA}))]p(e^*_{NA}) + F(\widetilde{b}_{NA}(e^*_{NA}))\beta(e^*_{NA}) \right) h$$

and

$$ESCE_A(e^*_A) = \left([1 - F(\widetilde{b}_A(e^*_A))]p(e^*_A) + F(\widetilde{b}_A(e^*_A))\beta(e^*_A) \right)(1-q)h.$$

If $q = 0$ then $e^*_A = e^*_{NA}$ and $ESCE_{NA}(e^*_{NA}) = ESCE_A(e^*_A)$. If $q > 0$, we have shown that the appeals process may either encourage or discourage the decision-maker to increase his effort, and thus reduce the occurence of a legal error. Thus, even if the appeals process *directly* reduces the risk to commit a egal error (see the term $1 - q$ in $ESCE_A(e^*_A)$), the effect of the appeals process on the expected social cost of errors remains unclear because of its *indirect* effet via e^*_A.

4   The double jeopardy

We now take into account the doctrine known in the US as *double jeopardy*. This constitutional clause refers to the impossibility of appealing acquittals by public prosecutors in criminal cases[3]. In our setting, this means that the appeals court may now only finally relax a previously wrongly convicted person. Thus, if double jeopardy exists, it modifies both the definition of the threshold value of the benefit of crime and the decision-maker's utility. The borderline type then equals:

$$\tilde{b}_A^{DJ}(e) = tS\left(1 - p(e) - (1-q)\beta(e)\right)$$

where $\tilde{b}_A(e) > \tilde{b}_A^{DJ}(e)$. The appeals process is less deterrent because the risk of finally wrongly relaxing a guilty person is higher (in comparison to the case where appeals process corrects both types of legal errors, as in section 3).

The decision-maker chooses e to solve:

$$\max_e \left\{ u_A^{DJ}(e) = w - \psi(e) - (1 - F(\tilde{b}_A^{DJ}(e)))p(e)\alpha h - F(\tilde{b}_A^{DJ}(e))\beta(e)((1-q)\alpha h + qz) \right\}.$$

---

[3]See Rizzolli (2010) for a discussion on the history and spirit of double jeopardy.

under the condition below in order to control for the problem's concavity.

Assumption 3:

$$(p(e)\alpha h - \beta(e)((1-q)\alpha h + qz)) \left(\varepsilon(b) \left(p'(e) + (1-q)\beta'(e) \right)^2 - \left(p''(e) + (1-q)\beta''(e) \right) (1 - p(e) - (1-q)\beta(e)) \right)$$
$$< \quad 2 \left(p'(e) + (1-q)\beta'(e) \right) (1 - p(e) - (1-q)\beta(e)) \left(p'(e)\alpha h - \beta'(e)((1-q)\alpha h + qz) \right).$$

Differentiating $u_A^{DJ}(e)$ with respect to e leads to lemma 4 below:

Lemma 4 *There exists a unique effort level at equilibrium $e_A^{DJ\,*}$ defined by:*

$$\psi'(e_A^{DJ*}) = f(\tilde{b}_A^{DJ}(e_A^{DJ*}))(-tS)((1-q)\beta'(e_A^{DJ*}) + p'(e_A^{DJ*}))(p(e_A^{DJ*})\alpha h - \beta(e_A^{DJ*})((1-q)\alpha h + qz)) -$$
$$\left((1 - F(\tilde{b}_A^{DJ}(e_A^{DJ*})))p'(e_A^{DJ*})\alpha h + F(\tilde{b}_A^{DJ}(e_A^{DJ*}))\beta'(e_A^{DJ*})((1-q)\alpha h + qz) \right). \qquad (7)$$

The first order condition can be rewritten:

$$\underbrace{\psi'(e)}_{C} = \underbrace{- \left(p'(e)(1 - F(\tilde{b}_A^{DJ}(e))) + \beta'(e)F(\tilde{b}_A^{DJ}(e)) \right) ((1-q)\alpha h + qz) + q(z - \alpha h)p'(e)(1 - F(\tilde{b}_A^{DJ}(e))) +}_{(B_p^A)^{DJ}}$$

$$\underbrace{\frac{\partial F(\tilde{b}_A^{DJ}(e))}{\partial e} (p(e) - \beta(e))((1-q)\alpha h + qz) - q(z - \alpha h)(p(e) - \beta(e))\frac{\partial F(\tilde{b}_A^{DJ}(e))}{\partial e}}_{(B_d^A)^{DJ}}. \qquad (8)$$

The term C represents the decision-maker's disutility of raising e by one unit. Terms $(B_p^A)^{DJ}$ and $(B_d^A)^{DJ}$ are the decision-maker's total marginal benefits of raising e by one unit, associated with the reduction of the probability of legal errors and crime deterrence. In comparison to the benchmark detailed in section 3, double jeopardy changes the decision-maker's marginal benefits at two levels.

By comparing the two FOC (6) and (8), we first show that the double jeopardy adds the term $q(z - \alpha h)p'(e)(1 - F(\tilde{b}_A^{DJ}(e)))$ in $(B_p^A)^{DJ}$ and the term $-q(z - \alpha h)(p(e) - \beta(e))\frac{\partial F(\tilde{b}_A^{DJ}(e))}{\partial e}$ in $(B_d^A)^{DJ}$. In words, it means that the *reputation-concerned* decision-maker's marginal benefits, $(B_p^A)^{DJ}$ and $(B_d^A)^{DJ}$, may be lower because the decision-maker cannot be reversed (and does not suffer a disutility z) if he does wrongly relax a guilty person. At the opposite, the *socially-motivated* decision-maker's marginal benefits, $(B_p^A)^{DJ}$ and $(B_d^A)^{DJ}$, may be greater because the decision-maker cannot rely on appeals courts to finally convict guilty defendants.

Second, we show that fewer individuals remain undeterred with double jeopardy: $\tilde{b}_A(e) > \tilde{b}_A^{DJ}(e)$. The comparison between $(B_p^A)^{DJ}$ and B_p^A, and the one between $(B_d^A)^{DJ}$ and B_d^A thus become much more complex. Not surprisingly, they depend on parameters $p'(e)$, $\beta'(e)$ and the rate at which the population density changes.

Finally, we explore the conditions under which the appeals process may still encourage or discourage the decision-maker's to increase his effort according to the parameters of the model. To begin with, notice that the appeals process anew leads to a decrease (resp. an increase) in the decision-maker's effort if $\Delta B_p = (B_p^A)^{DJ} - B_p^{NA} < 0$ (resp. $\Delta B_p > 0$) and $\Delta B_d = (B_d^A)^{DJ} - B_d^{NA} < 0$ (resp. $\Delta B_d > 0$).

After some manipulations, we have:

$$\Delta B_d = (p(e) - \beta(e))\alpha h \left(\frac{\partial F(\tilde{b}_A^{DJ}(e))}{\partial e} - \frac{\partial F(\tilde{b}_{NA}(e))}{\partial e} \right)$$

and

$$\Delta B_p = \left(p'(e) - \beta'(e) \right) \left(F(\tilde{b}_A^{DJ}(e)) - F(\tilde{b}_{NA}(e)) \right)\alpha h - \beta'(e)F(\tilde{b}_A^{DJ}(e))q(z - \alpha h)$$

where $F(\tilde{b}_A^{DJ}(e)) - F(\tilde{b}_{NA}(e)) > 0$ and

Lemma 5 $\frac{\partial F(\tilde{b}_A^{DJ}(e))}{\partial e} > \frac{\partial F(\tilde{b}_{NA}(e))}{\partial e} \Leftrightarrow 0 < \frac{\beta'(e)}{\beta(e)} \frac{1-p(e)-(1-q)\beta(e)}{p'(e)+(1-q)\beta'(e)} < \varepsilon(\tilde{b}_A^{DJ}(e))$.

Thus, the appeals process still has an *indirect* impact on the probabilities of legal errors through the type of the decision-maker, either *socially-motivated* or *reputation concerned* $(\alpha h \gtrless z)$, the magnitude of the marginal impact of the decision-maker's effort on the probabilities of legal errors $(-p'(e) \gtrless -\beta'(e))$, the sign and the magnitude of the rate at which the population density changes $(\varepsilon(\tilde{b}_A^{DJ}(e)) \gtrless \frac{\beta'(e)}{\beta(e)} \frac{1-p(e)-(1-q)\beta(e)}{p'(e)+(1-q)\beta'(e)})$. We summarize these effects in Table 2 below:

Table 2: variations in the decision-maker's marginal benefits.

$\varepsilon(\tilde{b}_A^{DJ}(e)) > \frac{\beta'(e)}{\beta(e)} \frac{1-p(e)-(1-q)\beta(e)}{p'(e)+(1-q)\beta'(e)}$	$\alpha h > z$	$\alpha h < z$
$-p'(e) > -\beta'(e)$	$\Delta B_d > 0$ $\Delta B_p < 0$	$\Delta B_d > 0$ $\Delta B_p \gtrless 0$
$-p'(e) < -\beta'(e)$	$\Delta B_d > 0$ $\Delta B_p \gtrless 0$	$\Delta B_d > 0$ $\Delta B_p > 0$
$\varepsilon(\tilde{b}_A^{DJ}(e)) < \frac{\beta'(e)}{\beta(e)} \frac{1-p(e)-(1-q)\beta(e)}{p'(e)+(1-q)\beta'(e)}$	$\alpha h > z$	$\alpha h < z$
$-p'(e) > -\beta'(e)$	$\Delta B_d < 0$ $\Delta B_p < 0$	$\Delta B_d < 0$ $\Delta B_p \gtrless 0$
$-p'(e) < -\beta'(e)$	$\Delta B_d < 0$ $\Delta B_p \gtrless 0$	$\Delta B_d < 0$ $\Delta B_p > 0$

Note: we derive from Lemma 5 that $\Delta B_d > 0 \Leftrightarrow \varepsilon(\tilde{b}_A^{DJ}(e)) > \frac{\beta'(e)}{\beta(e)} \frac{1-p(e)-(1-q)\beta(e)}{p'(e)+(1-q)\beta'(e)}$.

By comparing Table 1 (without double jeopardy) and Table 2 (with double jeopardy), we first notice that the discussion on ΔB_p is similar. As a result, the double jeopardy does not significantly matter here. Second, it appears that when the double jeopardy exists, the sign of ΔB_d does not depend on the type of the decision-maker anymore (contrary to the case where appeals courts may correct both types of legal errors). The intuition of this result is the following. Recall that a one-unit increase in the decision-maker's effort has a deterrent effect $(\frac{\partial F(\tilde{b}_A^{DJ}(e))}{\partial e} > 0)$. Thus, there is a lower probability to be faced with an undeterred individual, *and* a higher probability to be faced with an innocent individual. Further, the magnitude of each variation is identical. As the likelihood of relaxing a guilty defendant is strictly superior to the probability of wrongly convicting an innocent individual, $p(e) > \beta(e)$, then such an increase in the decision-maker's effort is profitable. Now, by preventing appeals courts from finally convicting undeterred individuals, double jeopardy then partially clears the *reputation-concerned* decision-maker's incentive to increase his effort (due to his disutility of being overturned), and simultaneously partially clears the *socially-motivated* decision-maker's incentive to reduce his effort (due to the free riding effect). Finally, notice that this effect (and indirectly the type of the decision-maker) is at stake for both types of legal errors as explained above. As a result, the decision-maker's type, either *socially-motivated* or *reputation-concerned*, has not anymore an influence on the way that appeals process indirectly influences the occurrence of legal errors *via* the so called *deterrence* effect.

To sum up, even if the effect of the appeals process on the decision-maker's incentives is less ambiguous, the theoretical prediction detailed in section 3 still holds. Further, let us define the expected social cost of legal errors when an appeals process with double jeopardy prevails as follows:

$$ESCE_A^{DJ}(e_A^{DJ*}) = (1 - F(\tilde{b}_A(e_A^{DJ*})))p(e_A^{DJ*})h + F(\tilde{b}_A(e_A^{DJ*}))\beta(e_A^{DJ*})(1-q)h.$$

In comparison to the case where appeals courts may correct both types of legal errors, their expected social cost tends to be automatically higher because the *direct* effect of the appeals process on the correction of legal

errors is lower. But the *indirect* effect remains unclear, and its magnitude may be either superior or inferior to the one exposed in section 3.

5 Conclusion

In this paper we have focused on how the introduction of an appeals process may indirectly impact the occurrence of legal errors, via trial court decision-makers' efforts. So far, the dominant idea has been that the appeals process lowers the occurence of legal errors both directly, by offering the opportunity to reconsider cases and indirectly, by spurring trial court decision-makers to exert more effort to avoid reversal on appeal. By considering the fact that trial court decision-makers may be more concerned by errors than by reputation and that their behavior has a deterrent effect on potential offenders, we have pointed out that the appeals process does not necessarily improve the accuracy of judicial decisions. Although we have adopted a setting in which the appeals process indeed directly (mechanically) improves the accuracy of judicial decisions (because an appeals is brought if and only if an error has been made at trial), our paper shows that whether the appeals process will finally allow reducing legal errors also depends on a positive or negative indirect effect through the trial court decision-makers' efforts. We have highlighted that this indirect effect depends on the type of the decision-maker (reputation-concerned or socially-motivated), the distribution of the population of potential offenders and the marginal effect of the decision-maker's effort on the probability of wrongful conviction compared to the probability of wrongful acquittal. Our results are qualitatively the same whether an appeal can be brought after conviction and acquittal verdicts at trial, or appeals are limited by double jeopardy. On a social point of view, these results question the opportunity of incorporing an appeals process compared to the alternative of enhancing the quality of the trial process, as regards the occurrence of legal errors. Indeed, our results suggest that, according to the characteristics of decision-makers and potential offenders, investing in more skilled trial court decision-makers or increasing the number of decision-makers who hear each case, could be preferable to using an appeals process (even if it is a more costly solution) since implementing an appeals process may even increase the occurence of legal errors.

Proofs

Proof of Lemma 1. Differentiating $u_{NA}(e)$ with respect to e shows that $u'_{NA}(e)$ is equal to

$$-\psi'(e) - \alpha h p'(e)[1 - F(\tilde{b}_{NA}(e))] - \alpha h \beta'(e) F(\tilde{b}_{NA}(e)) - \alpha h f(\tilde{b}_{NA}(e)) tS[p'(e) + \beta'(e)][p(e) - \beta(e)].$$

Differentiating twice $u_{NA}(e)$ with respect to e gives

$$
\begin{aligned}
u''_{NA}(e) &= -\psi''(e) + f'(\tilde{b}_{NA}(e))(-t^2 S^2)(p'(e) + \beta'(e))^2(p(e) - \beta(e))\alpha h \\
&\quad + f(\tilde{b}_{NA}(e))(-tS)(p''(e) + \beta''(e))(p(e) - \beta(e))\alpha h + f(\tilde{b}_{NA}(e))(-tS)(p'(e) + \beta'(e))(p'(e) - \beta'(e))\alpha h \\
&\quad - \alpha h \left[\begin{array}{c} -f(\tilde{b}_{NA}(e))(-tS)(p'(e) + \beta'(e))p'(e) + (1 - F(\tilde{b}_{NA}(e))p''(e) + \\ f(\tilde{b}_{NA}(e))(-tS)(p'(e) + \beta'(e))\beta'(e) + F(\tilde{b}_{NA}(e))\beta''(e) \end{array} \right].
\end{aligned}
$$

We know that $\psi''(e) > 0$, $p'(e) < 0$, $p''(e) > 0$, $\beta'(e) < 0$, $\beta''(e) > 0$ and $p(e) - \beta(e) > 0$. This implies that a sufficient condition for having $u''_{NA}(e) < 0$ is

$$
\begin{aligned}
&f'(\tilde{b}_{NA}(e))(-t^2 S^2)(p'(e) + \beta'(e))^2(p(e) - \beta(e)) + f(\tilde{b}_{NA}(e)) tS(p'(e) + \beta'(e))(p'(e) - \beta'(e)) \\
&- f(\tilde{b}_{NA}(e))(-tS)(p'(e) + \beta'(e))(p'(e) - \beta'(e)) \\
&> 0
\end{aligned}
$$

or

$$\varepsilon(\tilde{b}_{NA}(e)) < \left(\frac{2(p'(e) - \beta'(e))}{(p'(e) + \beta'(e))}\right)\left(\frac{1 - (p(e) + \beta(e))}{(p(e) - \beta(e))}\right).$$

The inequality above is satisfied under Assumption 1. Finally, notice for the existence of an interior solution (the function u_{NA} being continuously differentiable twice) that

$$\lim_{e \to 0} u'_{NA}(e) = -\alpha h a(1 - F(\tilde{b}_{NA}(0))) - \alpha h c F(\tilde{b}_{NA}(0)) - \alpha h f(\tilde{b}_{NA}(0)) t S[a + c][p(0) - \beta(0)] > 0$$

with $a = \lim_{e \to 0} p'(e) < 0$ and $c = \lim_{e \to 0} \beta'(e) < 0$. Further, we have

$$\lim_{e \to +\infty} u'_{NA}(e) = -\infty$$

because $\lim_{e \to +\infty} \psi'(e) = +\infty$ and $\lim_{e \to +\infty} p'(e) = \lim_{e \to +\infty} \beta'(e) = 0$. $\blacksquare$

Proof of Proposition 1. Since

$$u'_{NA}(e, x) = -\psi'(e) - (\alpha h)\left(p'(e)[1 - F(\tilde{b}_{NA}(e))] + \beta'(e)F(\tilde{b}_{NA}(e))\right) - f(\tilde{b}_{NA}(e))tS\left(p'(e) + \beta'(e)\right)(p(e) - \beta(e))\alpha h = 0$$

for $x = \{S, \alpha, h, t\}$, then by applying the implicit function theorem, we have

$$\frac{\partial e}{\partial S} = -\frac{\frac{\partial u'_{NA}}{\partial S}}{\frac{\partial u'_{NA}}{\partial e}} \; ; \; \frac{\partial e}{\partial \alpha} = -\frac{\frac{\partial u'_{NA}}{\partial \alpha}}{\frac{\partial u'_{NA}}{\partial e}} \; ; \; \frac{\partial e}{\partial h} = -\frac{\frac{\partial u'_{NA}}{\partial h}}{\frac{\partial u'_{NA}}{\partial e}} \text{ and } \frac{\partial e}{\partial t} = -\frac{\frac{\partial u'_{NA}}{\partial t}}{\frac{\partial u'_{NA}}{\partial e}}$$

where the derivatives $\frac{\partial u'_{NA}}{\partial x}$ for $x = \{S, \alpha, h, t\}$ and the derivative $\frac{\partial u'_{NA}}{\partial e}$ are calculated respectively at the points (e, x) such that $u'_{NA}(e, x) = 0$ for $x = \{S, \alpha, h, t\}$. Thus

$$\frac{\partial e}{\partial S} = -\frac{\alpha h \left(\begin{array}{c} -f'(\tilde{b}_{NA}(e))t^2 S(1 - (p(e) + \beta(e)))(p'(e) + \beta'(e))(p(e) - \beta(e)) \\ -f(\tilde{b}_{NA}(e))t(p'(e) + \beta'(e))(p(e) - \beta(e)) + f(\tilde{b}_{NA}(e))t(1 - (p(e) + \beta(e)))(p'(e) - \beta'(e)) \end{array}\right)}{\frac{\partial u'_{NA}}{\partial e}}.$$

We know that $\alpha h > 0$ and $\frac{\partial u'_{NA}}{\partial e} < 0$ (by the second order condition). This implies that the sign of $\frac{\partial e}{\partial S}$ equals the sign of the bracketed term in the numerator of the expression above. After some manipulations, we find that:

$$\left(\begin{array}{c} -f'(\tilde{b}_{NA}(e))t^2 S(1 - (p(e) + \beta(e)))(p'(e) + \beta'(e))(p(e) - \beta(e)) - f(\tilde{b}_{NA}(e))t(p'(e) + \beta'(e))(p(e) - \beta(e)) + \\ f(\tilde{b}_{NA}(e))t(1 - (p(e) + \beta(e)))(p'(e) - \beta'(e)) \end{array}\right) > 0$$

is equivalent to:

$$\varepsilon(\tilde{b}_{NA}(e)) > \frac{(p'(e) - \beta'(e)) + 2(\beta'(e)\beta(e) - p'(e)p(e))}{(p'(e) + \beta'(e))(p(e) - \beta(e))}.$$

By a very similar token we can show that, under the same condition regarding $\varepsilon(\tilde{b}_{NA}(e))$, we have $\frac{\partial e}{\partial t} > 0$. Finally, we have:

$$\frac{\partial e}{\partial \alpha} = -\frac{-h\left(p'(e)[1 - F(\tilde{b}_{NA}(e))] + \beta'(e)F(\tilde{b}_{NA}(e))\right) - hf(\tilde{b}_{NA}(e))tS\left(p'(e) + \beta'(e)\right)(p(e) - \beta(e))}{\frac{\partial u'_{NA}}{\partial e}} > 0$$

and

$$\frac{\partial e}{\partial h} = -\frac{-\alpha\left(p'(e)[1 - F(\tilde{b}_{NA}(e))] + \beta'(e)F(\tilde{b}_{NA}(e))\right) - \alpha f(\tilde{b}_{NA}(e))tS\left(p'(e) + \beta'(e)\right)(p(e) - \beta(e))}{\frac{\partial u'_{NA}}{\partial e}} > 0.$$

$\blacksquare$

Proof of Lemma 2. Differentiating $u_A(e)$ with respect to e shows that $u'_A(e)$ is equal to

$$-\psi'(e)-((1-q)\alpha h+qz)\left(p'(e)[1-F(\tilde{b}_A(e))]+\beta'(e)F(\tilde{b}_A(e))\right)-(1-q)f(\tilde{b}_A(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)]((1-q)\alpha h+qz).$$

Differentiating twice $u_A(e)$ with respect to e gives

$$
\begin{aligned}
u''_A(e) =\ & -\psi''(e)+f'(\tilde{b}_3(e))(-t^2S^2)(1-q)^2(p'(e)+\beta'(e))^2(p(e)-\beta(e))((1-q)\alpha h+qz)+\\
& f(\tilde{b}_A(e))(-tS)(1-q)(p''(e)+\beta''(e))(p(e)-\beta(e))((1-q)\alpha h+qz)+\\
& f(\tilde{b}_A(e))(-tS)(1-q)(p'(e)+\beta'(e))(p'(e)-\beta'(e))((1-q)\alpha h+qz)-\\
& ((1-q)\alpha h+qz)\left[\begin{array}{c}-f(\tilde{b}_A(e))(-tS)(1-q)(p'(e)+\beta'(e))p'(e)+(1-F(\tilde{b}_A(e))p''(e)+\\ f(\tilde{b}_A(e))(-tS)(1-q)(p'(e)+\beta'(e))\beta'(e))+F(\tilde{b}_A(e))\beta''(e)\end{array}\right].
\end{aligned}
$$

We know that $\psi''(e)>0$, $p'(e)<0$, $p''(e)>0$, $\beta'(e)<0$, $\beta''(e)>0$ and $p(e)-\beta(e)>0$. This implies that a sufficient condition for having $u''_A(e)<0$ is

$$
\begin{aligned}
& f'(\tilde{b}_A(e))(-t^2S^2)(1-q)^2(p'(e)+\beta'(e))^2(p(e)-\beta(e))+f(\tilde{b}_A(e))tS(1-q)(p'(e)+\beta'(e))(p'(e)-\beta'(e))\\
& -f(\tilde{b}_A(e))(-tS)(1-q)(p'(e)+\beta'(e))(p'(e)-\beta'(e))\\
> &\ 0.
\end{aligned}
$$

The inequality above is equivalent to

$$\varepsilon(\tilde{b}_A(e))<\left(\frac{2(p'(e)-\beta'(e))}{(p'(e)+\beta'(e))}\right)\left(\frac{1-(p(e)+\beta(e))(1-q)}{(p(e)-\beta(e))(1-q)}\right)$$

which holds under Assumption 2. Finally, notice for the existence of an interior solution (the function u_A being continuously differentiable twice) that

$$\lim_{e\to0}u'_A(e)=-((1-q)\alpha h+qz)a(1-F(\tilde{b}_A(0)))-((1-q)\alpha h+qz)cF(\tilde{b}_A(0))-((1-q)\alpha h+qz)f(\tilde{b}_A(0))tS[a+c][p(0)-\beta(0)]>0$$

because $a=\lim_{e\to0}p'(e)<0$, $c=\lim_{e\to0}\beta'(e)<0$. Finally, we have $\lim_{e\to+\infty}u'_A(e)=-\infty$ because $\lim_{e\to+\infty}\psi'(e)=+\infty$ and $\lim_{e\to+\infty}p'(e)=\lim_{e\to+\infty}\beta'(e)=0$. $\blacksquare$

Proof of Lemma 3. Note that

$$\frac{\partial(F(\tilde{b}_A(e)))}{\partial e}=-f(\tilde{b}_A(e))tS(1-q)\left(p'(e)+\beta'(e)\right)>0.$$

It follows that

$$\frac{\partial\left(\frac{\partial F(\tilde{b}_A(e))}{\partial e}\right)}{\partial q}=-tS\left(p'(e)+\beta'(e)\right)\left[f'(\tilde{b}_A(e))tS(1-q)\left(p(e)+\beta(e)\right)-f(\tilde{b}_A(e))\right].$$

As $-tS\left(p'(e)+\beta'(e)\right)>0$ we have $\frac{\partial(F(\tilde{b}_A(e)))}{\partial e}>\frac{\partial(F(\tilde{b}_{NA}(e)))}{\partial e}$ if and only if

$$f'(\tilde{b}_A(e))tS(1-q)\left(p(e)+\beta(e)\right)-f(\tilde{b}_A(e))>0$$

or

$$\varepsilon(\tilde{b}_A(e))>\frac{1-(1-q)(p(e)+\beta(e))}{(1-q)(p(e)+\beta(e))}.$$

$\blacksquare$

Proof of Proposition 2. Since

$$
\begin{aligned}
u'_A(e,x) =\ & -\psi'(e)-((1-q)\alpha h+qz)\left(p'(e)[1-F(\tilde{b}_A(e))]+\beta'(e)F(\tilde{b}_A(e))\right)\\
& -(1-q)f(\tilde{b}_A(e))tS\left(p'(e)+\beta'(e)\right)\left(p(e)-\beta(e)\right)((1-q)\alpha h+qz)\\
=\ & 0
\end{aligned}
$$

for $x = \{S, \alpha, h, t\}$, then by applying the implicit function theorem, we have

$$\frac{\partial e}{\partial S} = -\frac{\frac{\partial u'_A}{\partial S}}{\frac{\partial u'_A}{\partial e}} \ ; \ \frac{\partial e}{\partial \alpha} = -\frac{\frac{\partial u'_A}{\partial \alpha}}{\frac{\partial u'_A}{\partial e}} \ ; \ \frac{\partial e}{\partial h} = -\frac{\frac{\partial u'_A}{\partial h}}{\frac{\partial u'_A}{\partial e}} \text{ and } \frac{\partial e}{\partial t} = -\frac{\frac{\partial u'_A}{\partial t}}{\frac{\partial u'_A}{\partial e}}$$

where the derivatives $\frac{\partial u'_A}{\partial x}$ for $x = \{S, \alpha, h, t\}$ and the derivative $\frac{\partial u'_A}{\partial e}$ are calculated respectively at the points (e, x) such that $u'_A(e, x) = 0$ for $x = \{S, \alpha, h, t\}$. Thus

$$\frac{\partial e}{\partial S} = -\frac{((1-q)\alpha h + qz)\begin{pmatrix} -f'(\widetilde{b}_A(e))t^2 S(1-(1-q)(p(e)+\beta(e)))(1-q)(p'(e)+\beta'(e))(p(e)-\beta(e)) \\ -f(\widetilde{b}_A(e))t(1-q)(p'(e)+\beta'(e))(p(e)-\beta(e)) \\ +f(\widetilde{b}_A(e))t(1-(1-q)(p(e)+\beta(e)))(p'(e)-\beta'(e)) \end{pmatrix}}{\frac{\partial u'_A}{\partial e}}.$$

We know that $(1-q)\alpha h + qz > 0$ and $\frac{\partial u'_A}{\partial e} < 0$ (by the second order condition). This implies that the sign of $\frac{\partial e}{\partial S}$ equals the sign of the bracketed term in the numerator of the expression above. After some manipulations, we find that:

$$\begin{pmatrix} -f'(\widetilde{b}_A(e))t^2 S(1-(1-q)(p(e)+\beta(e)))(1-q)(p'(e)+\beta'(e))(p(e)-\beta(e))- \\ f(\widetilde{b}_A(e))t(1-q)(p'(e)+\beta'(e))(p(e)-\beta(e)) + f(\widetilde{b}_A(e))t(1-(1-q)(p(e)+\beta(e)))(p'(e)-\beta'(e)) \end{pmatrix} > 0$$

is equivalent to:

$$\varepsilon(\tilde{b}_A(e)) > \frac{(p'(e)-\beta'(e)) + 2(1-q)(\beta'(e)\beta(e)-p'(e)p(e))}{(p'(e)+\beta'(e))(p(e)-\beta(e))(1-q)}.$$

By a very similar token, we can show that under the same condition, we have $\frac{\partial e}{\partial t} > 0$. Finally, we also have:

$$\frac{\partial e}{\partial \alpha} = -\frac{-(1-q)h\left(p'(e)[1-F(\tilde{b}_A(e))]+\beta'(e)F(\tilde{b}_A(e))\right) - (1-q)^2 hf(\tilde{b}_A(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)]}{\frac{\partial u'_A}{\partial e}} > 0$$

and

$$\frac{\partial e}{\partial h} = -\frac{-(1-q)\alpha\left(p'(e)[1-F(\tilde{b}_A(e))]+\beta'(e)F(\tilde{b}_A(e))\right) - (1-q)^2 \alpha f(\tilde{b}_A(e))tS[p'(e)+\beta'(e)][p(e)-\beta(e)]}{\frac{\partial u'_A}{\partial e}} > 0.$$

∎

Proof of Lemma 4. Differentiating $u_A^{DJ}(e)$ with respect to e shows that the derivative of $u_A^{DJ}(e)$ is equal to

$$\begin{aligned} & -\psi'(e) + f(\tilde{b}_A^{DJ}(e))(-tS)((1-q)\beta'(e)+p'(e))(p(e)\alpha h - \beta(e)\left((1-q)\alpha h + qz\right)) \\ & - \left((1-F(\tilde{b}_A^{DJ}(e)))p'(e)\alpha h + F(\tilde{b}_A^{DJ}(e))\beta'(e)\left((1-q)\alpha h + qz\right)\right) \\ = & \ 0 \end{aligned}$$

Differentiating twice $u_A^{DJ}(e)$ with respect to e gives

$$\begin{aligned} u_A^{DJ''}(e) = & -\psi''(e) - f'(\tilde{b}_A(e))\frac{d\tilde{b}_A^{DJ}(e)}{de}tS(p'(e)+(1-q)\beta'(e))(p(e)\alpha h - \beta(e)((1-q)\alpha h + qz)) \\ & -f(\tilde{b}_A^{DJ}(e))tS(p''(e)+(1-q)\beta''(e))(p(e)\alpha h - \beta(e)((1-q)\alpha h + qz)) \\ & -f(\tilde{b}_A^{DJ}(e))tS(p'(e)+(1-q)\beta'(e))(p'(e)\alpha h - \beta'(e)((1-q)\alpha h + qz)) \\ & +f(\tilde{b}_A^{DJ}(e))\frac{d\tilde{b}_A^{DJ}(e)}{de}p'(e)\alpha h - f(\tilde{b}_A^{DJ}(e))\frac{d\tilde{b}_A^{DJ}(e)}{de}\beta'(e)((1-q)\alpha h + qz) \\ & -(1-F(\tilde{b}_A^{DJ}(e)))p''(e)\alpha h - F(\tilde{b}_A^{DJ}(e))\beta''(e)((1-q)\alpha h + qz). \end{aligned}$$

We know that $\psi''(e) > 0$, $p'(e) < 0$, $p''(e) > 0$, $\beta'(e) < 0$, $\beta''(e) > 0$, $p(e) - \beta(e) > 0$ and $\frac{d\tilde{b}_A^{DJ}(e)}{de} = -tS(p'(e) + (1-q)\beta'(e)) > 0$. This implies that a sufficient condition for having $u_A^{DJ''}(e) < 0$ is

$$f'(\tilde{b}_A^{DJ}(e))t^2S^2(p'(e) + (1-q)\beta'(e))^2\,(p(e)\alpha h - \beta(e)((1-q)\alpha h + qz))$$
$$-f(\tilde{b}_A^{DJ}(e))tS(p''(e) + (1-q)\beta''(e))\,(p(e)\alpha h - \beta(e)((1-q)\alpha h + qz))$$
$$-2f(\tilde{b}_A^{DJ}(e))tS(p'(e) + (1-q)\beta'(e))\,(p'(e)\alpha h - \beta'(e)((1-q)\alpha h + qz))$$
$$< \quad 0.$$

The inequality above is equivalent to

$$(p(e)\alpha h - \beta(e)((1-q)\alpha h + qz))\left(\varepsilon(b)\left(p'(e) + (1-q)\beta'(e)\right)^2 - \left(p''(e) + (1-q)\beta''(e)\right)\left(1 - p(e) - (1-q)\beta(e)\right)\right)$$
$$< \quad 2\left(p'(e) + (1-q)\beta'(e)\right)\left(1 - p(e) - (1-q)\beta(e)\right)\left(p'(e)\alpha h - \beta'(e)((1-q)\alpha h + qz)\right)$$

which holds under Assumption 3. Finally, notice for the existence of an interior solution that we have $\lim\limits_{e \to 0} u_A^{DJ'}(e) > 0$ and $\lim\limits_{e \to +\infty} u_A^{DJ'}(e) = -\infty$. ∎

Proof of Lemma 5. Note that

$$\frac{\partial(F(\tilde{b}_A^{DJ}(e)))}{\partial e} = -f(\tilde{b}_A^{DJ}(e))tS\left(p'(e) + (1-q)\beta'(e)\right) > 0.$$

It follows that

$$\frac{\partial\left(\frac{\partial F(\tilde{b}_A^{DJ}(e))}{\partial e}\right)}{\partial q} = -tS\left(f'(\tilde{b}_A^{DJ}(e))\beta(e)tS(p'(e) + (1-q)\beta'(e)) - f(\tilde{b}_A^{DJ}(e))\beta'(e)\right).$$

Thus, we have $\frac{\partial(F(\tilde{b}_A^{DJ}(e)))}{\partial e} > \frac{\partial(F(\tilde{b}_{NA}^{DJ}(e)))}{\partial e}$ if and only if

$$\varepsilon(\tilde{b}_A^{DJ}(e)) > \frac{\beta'(e)}{\beta(e)}\frac{1 - p(e) - (1-q)\beta(e)}{p'(e) + (1-q)\beta'(e)} > 0.$$

∎

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