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External debt and real exchange rates' adjustment in the euro area: New evidence from a nonlinear NATREX model

Cécile Couharde* Serge Rey† Audrey Sallenave ‡

Abstract: In this paper, we revisit medium- to long-run real exchange rate determination within the euro area, focusing on the role of external debt. Accordingly, we rely on the NATREX approach which provides an explicit framework of the external debt-real exchange rates nexus. In particular, given the indebtedness levels reached by the euro area economies, we investigate potential nonlinearity in real exchange rates dynamics, according to the level of the external debt. Our results evidence that during the monetary union, gross and net external debt positions of the euro area countries have exerted pressures on real exchange rate dynamics within the area. Moreover, we find that, beyond a threshold reached by the external debt, euro area countries are found to be in a vulnerable position, leading to an unavoidable adjustment process. Nevertheless, the adjustment process, while effective, is found to be low and occurs slowly.

JEL Classification: C23; F31; O47

Keywords: Euro area; External debt; NATREX approach; Panel Smooth Transition Regression models; Real exchange rates.

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1. Introduction

Since the advent of the single currency, the financial integration process has strongly increased in the euro area and has resulted in the build-up of current account imbalances within the euro area (see among others, Barnes *et al.*, 2010; Lane, 2013). In particular, there has been a wide divergence between surpluses in the core countries and deficits in the peripheral countries that have started with monetary union in 1999. Since the global crisis, a small correction of these divergences has taken place, but a clear structural difference remains between between the core and peripheral countries of the euro zone. It has been argued that these imbalances have been unsustainable (European Commission, 2012; Jaumotte and Sodsriwiboon, 2010; Darvas, 2012), resulting in increased exchange rate's misalignments within the euro area, as evidenced by several empirical papers based on equilibrium exchange rates approaches. Among these studies, some have resorted to the Fundamental Equilibrium Exchange Rate (FEER) approach and have shown the role played by countries' structural current account surpluses (deficits) in possible currency misalignments (Jeong *et al.*, 2010; Cline and Williamson, 2011). Coudert *et al.* (2013) have also addressed this issue by using a Behavioral Equilibrium Exchange Rate (BEER) approach. In these papers, the correction of currency misalignments takes place through adjustments in real exchange rates and/or improvements of country's international investment position. However the conditions under which this correction can occur are not explicitly modelled.

Unlike these approaches, The NATural Real EXchange rate (NATREX) approach (Stein, 1990) provides an explicit framework in which real exchange rate reversion is related to the external debt burden through the following mechanism. In short-medium run, a surge in gross external debt, say because of nonoptimal policies, may become potentially dynamically unstable as the increased debt raises the current account deficit, which then increases the debt further. Nevertheless, when the external debt-to-GDP ratio reaches a threshold, the expected growth rates of net worth and consumption decline which allows an adjustment process (Stein, 1994). The higher will be this effect, the faster will be the convergence to a steady state. This mechanism seems particularly appealing to the euro area as in a single currency area adjustments in the real exchange rate cannot be - by definition - obtained through adjustments in the nominal exchange rate.

Against this background, our paper adopts the NATREX approach and revisits the nexus between the external debt and real exchange rates within the euro area, by investigating the two following issues. First, does the level of the external debt explain the global pattern of real exchange rates in the euro area? Second, is there a threshold for the external debt at which real exchange rates within the euro area converge towards their long-run equilibrium value?

To handle with these issues, we investigate and test a potential threshold effect in the relationship between the external debt and real exchange rates dynamics on a sample of 11 euro area countries over the 2003Q3-2012Q3 period. We then contribute to the literature by relying on a Panel Smooth Transition Regression (PSTR) specification to analyse the dynamic behaviour of real exchange rates. First, in comparison to the majority of empirical studies which focus on the medium-long equilibrium, we analyse dynamics of real exchange rates and more specifically their convergence process towards the long-term equilibrium. Second, the use of a threshold panel methodology allows us to analyse the nonlinear impact of external debt on real exchange rates dynamics, in contrast to empirical studies on the NATREX approach which usually assume a linear relationship between the external debt level and real exchange rates. It is obvious that this assumption is not suited to estimate adequately the dynamics of real exchange rates as it is described in the NATREX approach.

Our results evidence that real exchange rates within the euro area become increasingly sensible to interest rate differential, as the level of the external debt increases. Moreover, beyond a threshold reached by the external debt-to-GDP ratio level, real exchange rates of the euro area countries tend to converge more quickly towards their long-run equilibrium level. Nevertheless, this adjustment process, while effective, is found to be low and occurs slowly.

The rest of this paper is organized as follows. The next section presents the NATREX theoretical model, highlighting the potential sources of nonlinearities in real exchange rates dynamics. Section 3 describes the methodology and data. Section 4 discusses the estimation results. Finally, Section 5 concludes.

2. The NATREX model

Like most models of equilibrium real exchange rates, the NATREX gives to savings and investment a key role in the dynamics of real exchange rate through adjustments in current account (Stein and Allen, 1995; Stein, 2006). The NATREX is the equilibrium real exchange rate that satisfies both the equilibrium of goods market and the balance of payments, where the output is at its potential level and in the absence of speculative capital movements, cyclical factors and changes in foreign exchange reserves (Allen, 1995, 6). Under the assumption of neutrality of money, only real variables called "core" will affect investment and savings, and therefore the equilibrium real exchange rate. In fact more than one model, one should speak of a class of models NATREX (Federici and Gandolfo, 2002) which can be adapted to the characteristics of economies: country size relative to its major partners, degree of substitutability of goods and financial assets, etc. The approach can be summarized as follows.

NATREX relies on a theoretical framework based on rigorous methods of intertemporal optimization under uncertainty, to describe the behaviour of individual agents. Although Edwards' model (Edwards, 1989; model of equilibrium real exchange rate in small open economies) and Obstfeld and Rogoff's model (Obstfeld and Rogoff, 1995; model of the nominal exchange rate) have, among others, resorted to such approaches, the latest versions of NATREX (Stein, 2006) use more sophisticated methods of stochastic optimal control/dynamic programming in an environment of uncertainty that makes the future unpredictable and thereby show that the optimal private consumption is proportional to the net wealth.

Unlike alternative equilibrium exchange rates models, the NATREX distinguishes explicitly between the medium-run equilibrium (medium-run NATREX) and the long-run equilibrium (long-run NATREX). Let consider the general case and the following three horizons: the short, medium and long runs. In the short run, the real exchange rate (R) depends on exogenous fundamentals (noted Z), endogenous fundamentals (noted D) and cyclical and speculative factors (noted U): $R = R(Z, D, U)$. The real exchange rate observed at date t is thus not always equal to its equilibrium value (NATREX), but can be decomposed into a sum of three terms:

$$R_t(D_t, U_t : Z_t) = \left[R_t(D_t, U_t : Z_t) - R_t^{MR}(D_t : Z_t)\right] + \left[R_t^{MR}(D_t : Z_t) - R_t^{LR}(Z_t)\right] + R_t^{LR}(Z_t) \quad (1)$$

The first term on the right side represents the deviation of the short-run real exchange rate, affected by speculative factors, to the medium-run NATREX $R_t^{MR}(D_t : Z_t)$. The second term describes the difference between the medium-run NATREX and the long-run NATREX, $R_t^{LR}(Z_t)$. This long-run equilibrium is reached when the effects of cyclical factors have vanished and when the endogenous fundamentals (physical capital stock and external debt) have converged to their steady values. It then only depends on exogenous fundamental variables (the relative productivity of the economy as a whole, the ratio of social consumption/GDP or social time preference).

2.1. Microeconomic foundations

The function of consumption/savings of each country is derived from maximizing the expectation of an intertemporal utility function over an infinite horizon (Stein and Paladino, 2001). Using methods of stochastic optimal control/dynamic programming, Stein (1994) shows that the optimal private consumption is proportional to the net wealth. If we define the net wealth as the difference between the capital stock (K) and the net foreign assets position (F), we obtain for private consumption $C = \beta(K(t) - F(t))$, where β is the discount rate. Public consumption (G) is the product between a tax rate δ_g and the GDP (Y), $G(t) = \delta_g(t) \times Y(t)$. Social consumption ($C_s$) is the sum of private and government consumption: $C_s = C + G$. Social savings (S) is defined as the difference between the GNP and social consumption $S = Y - rF - C_s = S(K, F : Z_c)$ with Z_c the exogenous fundamentals: the social time preference $\delta = C_s/Y$, and r the real interest rate. When the country is a net debtor, the slowdown in wealth and private consumption generates a stabilizing effect in the long term which prevents foreign debt from spiralling out of control.

The investment depends positively on the Keynes-Tobin q-ratio.[1] Assuming that the produced goods are sold at world prices and that to produce the same goods the country imports raw materials, it is possible to show that the q-ratio depends on the real exchange rate and exogenous fundamentals (Z_q): terms of trade, real wages and the marginal productivity of labour and raw materials. The investment function can be written as $I = I_q = J\left(R; Z_q\right)$ with $I_{Z_q} > 0$ and $I_R < 0$.

The trade balance surplus is the domestic excess supply of tradable goods. If TB represents net exports (trade balance), it will be positively related to productivity in the exportable sector (Z_b) and to the foreign countries' real GDP, and negatively to the real exchange rate and to the home country's real GDP. We consider that the production functions for tradable in domestic and foreign countries are modelled in an AK fashion. So TB can be written as $TB = TB\left(R, K, K^*, Z_b\right)$ where K^* represents the capital stock abroad. The current account (CA) is the sum of the trade balance and the net flow of interest payments to foreigners (rF): $CA = CA\left(R, Z_b, r, K, K^*, F\right)$.

2.2. Dynamic adjustment in the NATREX model

One important contribution of the NATREX approach is to distinguish between medium-run and long-run equilibrium real exchange rates and to explain the adjustment process of the real exchange rate towards its long-run equilibrium. The medium-run NATREX, i.e. the medium-run equilibrium real exchange rate, ensures that the current account valued in terms of the internal equilibrium will be equal to the desired social savings minus

1. The q-ratio can be defined as the ratio between the expectation of the discounted value of all future cash flows generated by the capital increase and the value of investment (acquisition value of capital). For more details, see Rey (2009), Hoarau (2013).

investment desired.[2] In the long run, capital stock and foreign debt have converged to their steady state. So, the long-run equilibrium will require, in addition to medium-run equilibrium, two additional conditions: (i) the capital/potential output ratio must be constant. This implies that the capital stock (K) will increase at the same rate as output (Y): $dk/dt = 0$, where $k = K/Y$; (ii) the net external position/potential output ratio is constant, that is to say that the external debt (F) expressed in terms of GDP is stabilized: $df/dt = 0$, where $f = F/Y$. The steady long-term NATREX will only depend on exogenous fundamentals, relative social consumptions (in terms of GDP) that measure the time preferences (δ, δ^*), and the relative productivities of the entire economy (ρ, ρ^*): $R^{LT} = R^{LT}(Z)$ with $Z = (\delta, \delta^*, \rho, \rho^*)$.

The transition to the longer-run equilibrium is obtained by also considering the dynamics of endogenous variations in capital and foreign debt. The dynamics of the system can be described from the dynamics of external debt, capital stock and the real exchange rate. The rate of change of the external debt is the current account deficit, i.e. investment (j) less savings (s): $df/dt = -ca = j - s = L(k, f; Z)$. Except r and r^*, the lower case for the variables denotes the ratio of the variable to GDP.

In the absence of capital depreciation, the change in the capital stock is $dk/dt = j - g \times k = J(k, f; Z)$, where g is the growth rate. To obtain the dynamic equation of the real exchange rate we consider the conditions for medium-term macroeconomic equilibrium in the domestic and foreign countries.

From equilibrium equation of domestic country we deduce:

$$R = H(r, k, k^*, f; Z) \tag{2}$$

2. Internal equilibrium holds when the rate of capacity utilization is at its stationary average. This means that there are no deflationary pressures related to under-utilization of capacity, or inflationary pressures due to an overheating economy. In the absence of speculative capital flows based on current expectations, and changes in foreign exchange reserves, external equilibrium requires the equalization of the domestic and the foreign long-term real interest rates. The achievement of internal and external equilibrium yields the medium-run NATREX.

and from equation of foreign country:[3]

$$R = h\left(r^{*}, k^{*}, k, f; Z\right) \tag{3}$$

The dynamic of real interest differential is given by:

$$d\left(r - r^{*}\right)/dt = -\gamma\left(r - r^{*}\right) \tag{4}$$

We differentiate the Equations 2 and 3, and we extract k^{*} from an equation and we replace it in the other equation. Combining with 4, we deduce the dynamic equation of the real exchange rate:

$$dR/dt = a_1 dk/dt + a_2 df/dt + a_3\left(r - r^{*}\right) + a_4 dZ/dt \tag{5}$$

where Z stands for the long-run, i.e. exogenous, fundamentals of the real exchange rate: relative time preferences and relative productivities.

The convergence process of the real exchange rate, R, to its long-run NATREX value, R^{LR}, implies that changes in the real exchange rate, ΔR, along its trajectory to its longer-run equilibrium, can be estimated through the following equation:

$$\Delta R_t = -\alpha_0\left[R_{t-1} - R_{t-1}^{LR}\left(Z_{t-1}\right)\right] + \beta_0 \Delta k_t + \beta_1 \Delta f_t + \beta_2\left(r_t - r_t^{*}\right) + u_t \tag{6}$$

where $\Delta x_t \equiv x_t - x_{t-1}$. The term in square brackets is the difference between the actual real exchange rate, R, and the longer-term equilibrium real exchange rate, R^{LT}, and represents, by definition, the misalignment of the real exchange rate. This misalignment can occur either because the actual real exchange rate has changed –for example as a result of inflation differentials– and/or because the long-run equilibrium exchange rate has changed as a result of changes in the exogenous fundamentals.

In the long run, the capital stock and the foreign debt reach their steady-state values and the domestic interest rate is equal to the foreign one. Hence the equilibrium long-run

3. In a two-country world, we have $F + F^{*}/N = 0 \Rightarrow F^{*} = -FN$; with N, the nominal exchange rate.

real exchange rate, R^{LT}, is only influenced by long-run real fundamentals Z: relative consumption preferences and relative productivities. In the long run, the standard response of the real exchange rate to exogenous thrift and productivity shocks within the NATREX model, as suggested by Stein (1994), is the following: in the long run, a positive shock to consumption preferences will lead to a real depreciation while a positive productivity shock will appreciate the equilibrium exchange rate.

2.3. Accounting for nonlinearities

Nevertheless, the convergence process of the real exchange rate from its medium to its long-run value, in the NATREX approach, is not monotonous as it depends critically on the adjustment speed of the two stock variables: the capital stock and net foreign assets. This particularity of the NATREX model has two important consequences.

First, the adjustment of the real exchange rate to its long-run equilibrium level may be non linear, depending on the behaviour of underlying economic fundamentals. Therefore, this non linearity in real exchange rate dynamics does not necessarily come from rigidity prices; but rather it could be due to the dynamics of wealth's accumulation (decumulation) –through current account surpluses (deficits)– and of investment in the capital stock which may both depend on the level of these two stock variables. Accordingly, responses of the real exchange rate to its determinants may vary according to a threshold value reached by those stock variables.

Second, the long-run effect of exogenous fundamentals on the real exchange rate can be ambiguous and depends on the characteristics of the economies under review. Indeed, suppose an increase in time preference, i.e. a decrease of social savings. This reduction will lead to a deterioration of the current account and will be followed by capital inflows leading to a real exchange rate appreciation. The deterioration in the current account raises the foreign debt ratio while a lower investment ratio due to the appreciated currency decreases the capital stock ratio. This trend is maintained until the foreign debt achieves the critical size that, once reached, consumption will slowly start declining due to the associated wealth effects. The system is then stabilized, as the adjustment of consumption scales back the trade balance deficit and thus the current account. If those wealth effects or consumption's influence on the trade balance are low, a depreciation of the real exchange rate with respect to the medium-run equilibrium will be necessary to improve the current account ratio to a level, which stabilizes the foreign debt ratio in the long-run equilibrium.

So, in short-medium run, the real exchange rate will tend to deviate significantly from its long-run equilibrium with the increased level of foreign debt, while in the medium-long run, it will begin to converge to its long-run level due to an adjustment process coming from the following forces. First, the increase in the external debt will reduce net wealth and absorption. Second, the depreciation of the real exchange rate will improve the trade balance. In addition, the real depreciation will increase the Tobin q-ratio, investment and economic growth. Thus real exchange rate dynamics, as decribed by Equation 6, is expected to vary according a threshold valued reached by the level of the foreign debt.

3. Methodology and data

We consider the 10 founder members of the euro area and Greece over the period 2003Q3-2012Q3. Included countries are then Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain. The relatively short time span is due to the availability on a quarterly basis of external debt data. It has however the advantage to cover more accurately the process of European Monetary Union (EMU) implementation and is thus less prone to structural changes.

3.1. Estimating the long-run relationship

To account for the long-run relationship between the real exchange rate and its determinants, we follow the NATREX approach in which the real effective exchange rate is expressed as a function of relative social consumption and relative productivity.

Among the number of alternative estimation methods to estimate long-run relationship in panel data, we choose to implement the conditional pooled mean group (CPMG) panel model[4] because of its appealing features. Indeed, as the PMG estimator of Pesaran *et al.* (1997), it does not impose untenable exogeneity restrictions on the series being considered, restricts the long-run coefficients to be homogenous over the cross-sections, but allows for heterogeneity in intercepts, short-run coefficients (including the speed of adjustment) and error variances. It can be argued that country heterogeneity is particularly relevant in short-run relationships, given that countries can be affected by several constraints in short-time horizons, albeit to different degrees. On the other hand, we can expect that long-run relationships between variables are homogeneous across countries. Moreover, the CPMG is valid in the presence of cross-sectional dependencies. This hypothesis is likely to hold for our sample as the EMU has strengthened interdependence between euro area countries.

The CPMG estimator is based on an Autoregressive Distributive Lag (ARDL) model. We consider here the following ARDL(p, q) model consistent with the determination of the long-run real exchange rate in the NATREX approach:

$$R_{it} = \omega_i + \Sigma_{j=1}^{p} \rho_{ij} R_{it-j} + \Sigma_{j=0}^{q} \zeta'_{ij} Z_{it-j} + u_{it} \tag{7}$$

where $i = 1, 2, .., N$ indexes countries, $t = 1, 2, .., T$ indexes time periods, R_{it} denotes the real exchange rate (the dependent variable with coefficients ρ_{ij} on its lagged values), ω_i represents the country-specific intercept term (fixed effect) and Z_{it} and ζ'_{ij} represent $(m \times 1)$ vectors of long-run explanatory variables (relative productivity, relative time preference) and coefficients, respectively.

To allow for cross-sectional correlation of the error terms, u_{it} is specified as

$$u_{it} = \gamma'_i c f_t + \varepsilon_{it} \tag{8}$$

4. For more details, see Cavalcanti *et al.* (2012).

The source of error term dependencies across countries is captured by the common factors cf_t; the impacts of these factors on each country are governed by γ_i coefficients. The error component ε_{it} is assumed to be distributed independently across i and t with zero mean and variance $\sigma_i^2 > 0$.

To control for the common factors modelled as unobservable, the ARDL model (7) is augmented with the cross-sectional averages of the model's observable variables following the correlated effects augmentation of Pesaran (2006). Combining Equations 7 and 8 and cross-sectional averages of the dependent variable and of the regressors gives the following:

$$\overline{R}_t = \overline{\omega} + \Sigma_{j=1}^{p}\overline{\rho}_j\overline{R}_{t-j} + \Sigma_{j=0}^{q}\overline{\xi'}_j\overline{Z}_{t-j} + \overline{\gamma}cf_t + \overline{\varepsilon}_t \tag{9}$$

where variables denoted by a bar stand for the cross-sectional averages of the corresponding variables in year t.

Since the error component ε_{it} by assumption is independently distributed across i and t, $\overline{\varepsilon_t}$ tends to zero in root mean square error as N becomes large. The common factors can then be captured through a linear combination of the cross-sectional averages of the dependent variable and the regressors:

$$\gamma_i'cf_t = v_i\overline{\gamma'}cf_t = \eta_i\overline{R}_t + \overline{\xi'}_i\overline{Z}_t + \Sigma_{j=0}^{p-1}\nu_i\Delta\overline{R}_{t-j} + \Sigma_{j=0}^{q-1}\zeta_{ij}\Delta\overline{Z}_{t-j} - v_i\overline{\omega} \tag{10}$$

with reparametrizations $\eta_i = v_i\left(1 - \Sigma_{j=1}^{p}\overline{\rho}_j\right)$, $\xi_i = v_i\left(\Sigma_{j=0}^{p}\overline{\zeta}_j\right)$, $\nu_{ij} = v_i\left(\Sigma_{j+1}^{p}\overline{\rho}_{j+1}\right)$ and $\zeta_{ij} = v_i\left(\Sigma_{j+1}^{p}\overline{\xi}_{j+1}\right)$.

Using Equations 8 and 10, the error correction representation of the panel ARDL model (7) can be written as

$$\begin{aligned}\Delta R_{it} = {} & \mu_i + \alpha_i R_{it-1} + \beta_i'Z_{it} + \Sigma_{j=1}^{p-1}\phi_{ij}\Delta R_{it-j} \\ & + \Sigma_{j=0}^{q-1}\delta_{ij}'\Delta Z_{it-j} + \eta_i\overline{R}_t + \xi_t'\overline{Z}_t + \Sigma_{j=0}^{p-1}\nu_{ij}\Delta\overline{R}_{t-j} + \Sigma_{j=0}^{q-1}\zeta_{ij}'\Delta\overline{Z}_{t-j} + \varepsilon_{it}\end{aligned} \tag{11}$$

with $\mu_i = \omega_i - v_i\overline{\omega}$, $\alpha_i = -\left(1 - \Sigma_{j=1}^{p}\rho_{ij}\right)$, $\beta_i = \Sigma_{j=0}^{q}\xi_{ij}$, $\phi_{ij} = -\Sigma_{j+1}^{p}\rho_{ij+1}$ and $\delta_{ij} = -\Sigma_{j+1}^{q}\xi_{ij+1}$.

From Equation 11 the long-run relationship between the real effective exchange and its fundamentals is given by

$$R_{it} = -\left(\frac{\beta_t'}{\alpha_i}\right) Z_{it} - \left(\frac{\mu_i}{\alpha_i}\right) - \left(\frac{\eta_i \overline{R}_t + \xi_t' \overline{Z}_t}{\alpha_i}\right) = \theta_i' Z_{it} - \left(\frac{\mu_i + \chi_i' g_t}{\alpha_i}\right) = \theta_i' Z_{it} + \eta_{it} \quad (12)$$

where $g_t = \left(\overline{R}_t, \overline{Z}_t\right)$ represents the level parts of the common factors and $\chi_i' = (\eta_i, \xi_t')'$ contains the loadings on these common factors.

Finally, the CPMG estimator imposes the long-run coefficients to be the same across countries, i.e. $\theta_i = \theta$ for $i = 1, 2, ..., N$. The null of long-run homogeneity $H_0 : \theta_i = \theta$ can be tested using the Hausman statistics for the coefficient on each of the explanatory variables and for all of them jointly. The long-run coefficients between R_i and Z_i , given by $\theta_i = \theta$, and the speed of adjustment towards the long-run relation for country i, given by α_i, constitute the key coefficients of economic interest.

3.2. The nonlinear NATREX model

As described by Equation 6, the dynamic process of real exchanges rates underlying the NATREX model can be adequately estimated by a vector error-correction econometric model (VECM): the long-run NATREX corresponding to the hypothesized cointegrating equation, short-run dynamics reflecting the short-run adjustment of real exchanges rates towards their long-run equilibrium level and the error correction term measuring the speed at which prior deviations of real exchange rates from their long-run equilibrium are corrected. Nevertheless, in the classical VECM, real exchange rate dynamics is linear. But, as mentioned before, real exchange rate dynamics may be non linear, depending on the adjustment speed of the capital stock and net foreign assets to their steady levels. In order to take into account those potential nonlinearities, we propose to estimate a regime-specific model allowing for threshold or switching effects in real exchange rates' dynamics. More specifically, we rely on the PSTR methodology proposed by González *et al.* (2005). This specification seems relevant as it allows the observations to be divided in different regimes, with estimated coefficients that vary depending on the considered regime. The change in the estimated value of coefficients is smooth and gradual, since

PSTR models are regime-switching processes in which the transition from one state to the other is smooth rather than discrete.

In the simplest case with two extreme regimes and a single transition function, the introduction of nonlinearity in Equation 6 leads to the following PSTR error-correction model:

$$\Delta R_{it} = \mu_i + \alpha_0 mis_{it} + \beta_0 D_{it} + (\alpha_1 mis_{it} + \beta_1 D_{it}) \times h\left(debt_{it}; \gamma, c\right) + \varepsilon_{it} \tag{13}$$

with:

$$mis_{it} = R_{it} - \left(\hat{\alpha}_i + \hat{\beta} Z_{it}\right) \tag{14}$$

R_{it} is the logarithm of the real exchange rate of country i at time t; mis_{it} stands for the misalignment, i.e. the difference between the observed real exchange rate and its estimated long-run equilibrium value. This latter value is expressed as a function of the long-run fundamental variables (Z_{it}) that impact the long-run NATREX. $\hat{\alpha}_i$ and $\hat{\beta}$ respectively denote the estimated long-run fixed effects and fundamentals' coefficients from the cointegrating relationship between the real effective exchange rate and its long-run determinants.

D_{it} is the vector of the short-medium-run determinants of the real exchange rate. These short-medium-run determinants are also selected on the basis of the NATREX model described above, i.e the rate of change of the external debt-to-GDP ratio, the investment ratio and the real interest rate differential. $debt_{it}$ is the external debt-to-GDP ratio of country i at time t, considered here as the transition variable.

The transition function $h\left(debt_{it}; \gamma, c\right)$ is a continuous and bounded function of the threshold variable defined by the external debt-to-GDP ratio, $debt_{it}$. Gonzalez *et al.* (2005) consider the following transition function:

$$h\left(debt_{it}; \gamma, c\right) = \left[1 + exp\left(-\gamma \Pi_{k=1}^{m}\left(debt_{it} - c_k\right)\right)\right]^{-1} \tag{15}$$

where γ ($\gamma > 0$) stands for the slope of the transition function and c_k ($k = 1, 2, .., m$) are the threshold parameters satisfying $c_1 \leq c_2 \leq ... \leq c_m$. According to Gonzalez *et al.* (2005), the transition function can be of order one (logistic function) or order two (quadratic function) in order to capture the non linearities derived from the regime switching. For $m = 1$, the PSTR model implies that the two extreme regimes are associated with high and low values of the transition variable. For $m = 2$, the PSTR model becomes a three-regime threshold model where the intermediate regime follows a different pattern, while the two other extreme share the same dynamics. The logistic function appears to be the most suitable for describing the impact of the level of the external debt on real exchange rate dynamics as depicted by the NATREX model. The reason is that this specification allows real exchange rate dynamics to vary, depending on the threshold reached by the external debt. More precisely, the coefficient of the misalignment and short-run determinants elasticities of the real exchange rate are given by the parameters (α_0, β_0) when the transition function $h\left(debt_{it}; \gamma, c\right)$ tends to 0, while they correspond respectively to the sum of the parameters $(\alpha_0 + \alpha_1)$ and $(\beta_0 + \beta_1)$ when the transition function $h\left(debt_{it}; \gamma, c\right)$ tends to 1. Between these two extremes, the coefficient of the misalignment and short-run determinants elasticities of the real exchange rate are defined as a weighted average of the parameters (α_0, α_1) and (β_0, β_1).

The model (13) can be rewritten as

$$\Delta R_{it} = \mu_i + \psi_0 W_{it} + \psi_1' W_{it} \times h\left(debt_{it}; \gamma, c\right) + \varepsilon_{it} \tag{16}$$

where $\psi_j' = \left(\alpha_j, \beta_j\right)'$ for $j = 0, 1$; $W_{it} = \left(mis_{it-1}, D_{it}\right)'$.

3.3. Data

As we are interested by analysing the adjustment of real exchanges rates towards their long-run equilibrium level within the euro area, we consider as the dependant variable, the real effective exchange rate. The quarterly data are from the Bank for International Settlements. Real effective exchange rates are calculated as the weighted averages of bilateral exchange rates adjusted by relative consumer prices. They correspond to narrow indices, i.e. indices calculated over a sample of 27 economies.[5] The weighting pattern is time varying, and the most recent weights are based on trade in 2008-2010. The series are expressed in logarithms and expressed so that a rise (resp. decrease) denotes a currency appreciation (depreciation).

The measure of time preference is defined as the ratio of social and private consumption to the gross domestic product. Data of both social and private consumption are extracted from the OECD database.

Due to availability of quarterly data, relative sectoral productivity is proxied by the consumer-price-to-producer-price ratio. Unlike the consumer price index (CPI), the producer price index (PPI) only covers tradable goods. Thus the consumer-price-to-producer-price ratio explicitly differentiates between the tradable and nontradable sectors. CPI and PPI are, respectively, expressed in logarithms and taken from the OECD database and datastream.

Time preference and relative sectoral productivity are calculated for each euro area country and for a weighted average of its trading partners. The measure of time preference and relative sectoral productivity for country i at time t relative to its trading partners (j) are then given by

$$pref_{it} = \frac{\frac{C_{it}+G_{it}}{GDP_{it}}}{\Sigma_{j\neq i} w_{ij} \left(\frac{C_{jt}+G_{jt}}{GDP_{jt}}\right)} \tag{17}$$

where C_{it} and G_{it} denote respectively private and public consumption of country i at time t.

$$prod_{it} = ln\left(\frac{CPI_{it}}{PPI_{it}}\right) - \Sigma_{j\neq i} w_{ij} ln\left(\frac{CPI_{jt}}{PPI_{jt}}\right) \tag{18}$$

where CPI_{it} and PPI_{it} stand respectively for CPI and PPI of country i at time t.

For consistency, we retain the same time-varying weights as those used in the real effective exchange rate calculations.

Investment-to-GDP ratio corresponds to gross fixed capital formation in percentage of

5. Austria, Australia, Belgium, Canada, Denmark, Euro area, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Korea, Mexico, the Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, Taiwan (China), the United Kingdom, the United States.

GDP; data are extracted from the OECD's quarterly national accounts data set. We have collected long-run interest rates and CPI and used them to calculate real interest rate differential of each euro area country against the US long-term interest real rate (considered here as the world real interest rate). The source is the online database of the OECD.

Finally, external debt corresponds to external gross debt which measures the outstanding amount of actual current, and not contingent, liabilities that require payment(s) of principal and/or interest by the debtor at some point(s) in the future and that are owed to nonresidents by residents of an economy.[6] The gross external debt, by capturing an economy's external exposure to international debt markets, gives then an idea to expected transfers to creditors which, if too excessive, may create difficulties with interest payments when real interest rates rise and/or income generation slows down, leading to the insolvency of some economic agents. Data of gross external debt are extracted from the World Bank database, Quarterly External Debt Statistic.

4. Estimation results

4.1. Estimating the long-run relationship

We begin by examining the stationarity properties of the various variables entering our panel ARDL real exchange rate model (11). We first test the presence of a similar pattern across euro area countries by using the cross-sectional dependence (CD) statistic of Pesaran (2004). Results, reported in table A.1 in the Appendix, reject the null hypothesis of cross-section independence between countries for the real effective exchange rate and relative productivity. Second, the ARDL model variables should be either integrated of order zero or one, $I(0)$ or $I(1)$. To test for the order of integration, we only consider unit root tests that have the best properties in finite samples, i.e., that remain relatively strong with a limited number of observations: the first generation test of Madalla and Wu (1999) and the second generation tests of Pesaran (2007) that allow for cross-section dependence. Results reported in Tables A.2 and A.3 in the Appendix provide strong evidence that our

6. World Bank, External Debt Statistics, Guide for Compilers and Users, September 2012.

three variables of interest are $I(1)$ variables. We can thus turn to estimation results for the panel ARDL model (11).

Table 1 reports CPMG estimation results. In addition to these CPMG results we also report the conditional mean group estimates (CMG) estimates, which are averages of the individual country coefficients. Under long-run slope homogeneity, CPMG estimates are consistent and efficient, while the CMG approach provides consistent and efficient estimates of the averages of long-run coefficients if heterogeneity is present. We test for long-run homogeneity using the Hausman statistic for the coefficients on each of the explanatory variables and for all of them jointly based on the null of equivalence between the CPMG and CMG estimations. If we reject the null hypothesis (i.e. a probability value lower than 0.05), the homogeneity assumption on long-run coefficients across countries is invalid. According to the Hausman statistics, the long-run homogeneity restriction is not rejected for individual parameters and jointly in all regressions. Thus, we focus on the results obtained using the CPMG estimator, which is more appropriate. The CPMG results indicate that the error-correction coefficient is statistically significant and negative, and therefore the null hypothesis of no long-run relationship is rejected. In the long run, the real exchange rate is, as expected, negatively related to relative time preference and positively related to relative productivity. Moreover, the CPMG estimates of these two explanatory variables are not statistically significant in the short run, which means that real exchange rates are linked to relative time preference and relative productivity only in the long run. Overall, there results show that the NATREX model is appropriate for describing the long-run real exchange rates of our sample of euro area countries over the period under study.

Finally, using the CPMG estimates, we calculate currency misalignments which are derived from the difference between observed real effective exchange rate and their equilibrium level (Equation 14). Following this definition, a negative sign indicates an undervaluation of the real effective exchange rate whereas a positive sign indicates an overvaluation of the currency. Figures A.1 and A.2 in the Appendix display respectively the evolution of real effective exchange rates (observed and equilibrium) and of misalign-

Table 1. *Common correlation effect Pooled Mean Group (CPMG) and Mean Group (CMG)*

	CPMG	**CMG**
Error-correction term	-0.140^{*}	-0.509^{***}
	(0.074)	(0.078)
Long-run coefficients		
$prod$	0.435^{***}	0.110
	(0.101)	(0.127)
$pref$	-0.246^{**}	-0.081
	(0.095)	(0.108)
$\overline{R}$	0.825^{***}	1.008^{***}
	(0.086)	(0.143)
$\overline{prod}$	-1.314^{***}	-0.341
	(0.260)	(0.363)
$\overline{pref}$	1.685^{***}	0.220
	(0.268)	(0.630)
Short-run coefficients		
$\Delta prod$	0.097	0.024
	(0.070)	(0.084)
$\Delta pref$	0.001	0.008
	(0.057)	(0.053)
$\overline{\Delta R}$	0.882^{***}	0.499^{***}
	(0.113)	(0.090)
$\overline{\Delta prod}$	0.041	0.176
	(0.149)	(0.219)
$\overline{\Delta pref}$	-0.201	-0.184
	(0.132)	(0.205)
Constant	-0.251^{*}	-0.084
	(0.133)	(1.091)
Hausman test	$\chi^2\,(5) = 8.03$	
	$Prob > \chi^2 = 0.15$	
Number of countries	11	
Number of observations	396	

*Notes: All estimations include a constant country-specific term. Standard errors are presented below the corresponding coefficients in brackets. Symbols ***, ** and * denote significance at 1%, 5% and 10% respectively. The bars over the variables indicate the cross-sectional averages of these variables. The dependent variable is the growth rate of the real effective exchange rate. The Schwarz Bayesian Criterion (SBC) has been used to select the lag orders for each group in which the maximum lag is set to two. Null hypothesis of the Hausman test indicates no systematic difference in coefficients.*

ments over the sample period and for the 11 considered countries of the euro zone.

Figures A.2 illustrate the increase in currency misalignments that has occurred since the launch of the monetary union. For most countries, this increase has been associated to an appreciation of their real effective exchange rates driven by a real appreciation of the euro towards third currencies[7] and/or higher inflation compared to all partners. Real exchange rates of three peripheral countries (Greece, Ireland, Spain) exhibited the highest level of real overvaluation before the financial crisis. Corrective mechanisms have come into play with the financial crisis as a convergence of real exchange rates towards their equilibrium level can be observed in core countries (Austria, Belgium, the Netherlands, Finland) and some of the peripheral countries (Italy, Portugal). Nevertheless, these mechanisms did not wipe off overvaluations in Spain and Greece.[8] In those countries, the deterioration of long-term funadmentals has been ongoing and has depreciated the equilibrium exchange rate (Figures A.1).

4.2. Short-run dynamics of real exchange rates: does the level of the external debt matter?

Finally, we assess the potential non linear effect of the external debt on real exchange rates dynamics by estimating the PSTR error correction model, as described by Equation 13. Accordingly, we follow the methodology proposed by Gonzalez *et al.* (2005) to apply PSTR models.

We first test for the null hypothesis of linearity. This test also permits to choose the adequate value of the parameter r (the number of regimes). However, as the PSTR model contains unidentified nuisance parameters, the test is nonstandard. An equivalent hypothesis is then tested in an auxiliary regression where the transition function is replaced

7. The euro had strongly appreciated against third currencies, by 41% from 2000 to 2009 ahead of the 7% drop in 2010.

8. For a detailed analysis of the competitiveness problem, see Sinn (2014, 105).

by its first-order Taylor expansion around $\gamma = 0$:

$$\Delta R_{it} = \mu_i + \psi_0' W_{it} + \Gamma_1' W_{it} debt_{it} + \Gamma_2' W_{it} debt_{it}^2 + ... + \Gamma_m' W_{it} debt_{it}^m + \varepsilon_{it} \quad (19)$$

where $\psi_0' = (\alpha_0, \beta_0)'$; $W_{it} = (mis_{it-1}, D_{it})'$ and the parameters Γ_k' are a multiple of the slope parameter γ. Thus testing the linearity against the PSTR model consists in testing $\Gamma_1' = \Gamma_2' = ... = \Gamma_m' = 0$ in the linear panel model described by Equation 19.

The process of choosing the value of r is sequential. Firstly, we compare the one regime model (homogeneous model, $r = 0$) to the two regimes model ($r = 1$). Then, if we accept the hypothesis of non homogeneity in the first step, we compare the two regimes model to the three regimes model ($r = 2$).[9] This test of linearity is run considering the gross external debt (in % of GDP) as the transition variable. It consists in applying the Lagrange Multiplier (LM) test developed by Gonzalez *et al.* (2005): $LM = TN\left(SSR_0 - SSR_1\right)/SSR_0$ where SSR_0 is the sum of squared residuals of the model with fixed effects and SSR_1 is the sum of squared residuals of the alternative equation (PSTR model with two regimes). We compute also an LRT (Likelihood Ratio Test) statistic defined as $LRT = -2\left[log\left(SSR_1\right) - log\left(SSR_0\right)\right]$.

The results of the LM and LRT tests are displayed in Table 2. As expected, the null hypothesis of linearity is rejected in favour of the PSTR alternative, and the significance level is far beyond the usual 1%. Moreover, the null hypothesis of $r = 1$ is accepted, supporting our assumption that short-run dynamics of real exchange rates is asymmetric, depending on the level of the external debt-to-GDP ratio. We now proceed to the estimation of our nonlinear PSTR specification. Results are reported in Table 3. The second and third columns give respectively the values of the parameters $\psi_0 = (\alpha_0, \beta_0)$ (regime 1) and $\psi_1 = (\alpha_1, \beta_1)$ (regime 2). As highlighted by Colletaz and Hurlin (2006), it is however difficult to directly interpret the values of those coefficients as they correspond to extreme situations. It is then more relevant to interpret the sign of these parameters which indicates an increase or a decrease of the elasticity with the value of the threshold variable.

9. By setting $r = 2$, we allow for three regimes where each one has its own slope and location parameters.

Table 2. *Nonlinearity tests*

Test	*LM* test	*LRT* test
$H_0 : r = 0$ versus $H_1 : r = 1$	21.59	23.16
	$(2.06e10^{-4})$	$(1.75e10^{-4})$
$H_0 : r = 1$ versus $H_1 : r = 2$	13.27	12.19
	(0.010)	(0.016)
$H_0 : r = 2$ versus $H_1 : r = 3$	10.74	11.65
	(0.030)	(0.021)

Notes: The LM and pseudo-LRT statistics have a chi-square distribution with K degrees of freedom, where K is the number of explanatory variables with only one location parameter ($m = 1$). Corresponding p-values are given in parentheses.

Our results confirm that the effect of the external debt-to-GDP ratio on real exchange rates' dynamics is clearly nonlinear, depending on its level. As shown in Table 3, the estimated threshold for the external debt ratio that triggers the regime switch is quite high: 223% of the GDP. But this level was reached by the vast majority of countries belonging to our sample at the end of the period. Indeed, the evolution of the external debt in % of GDP in the euro area (see figures B.1 reported in the Appendix) has been characterized by a noticeable increasing trend over the past decade. This increasing trend has been the result of a combination of several factors. According to Dias (2010), the observed increase between 2003 and 2006 was driven by stable economic conditions and lower long-term real interest rates. This situation was followed by an increasing borrowing undertaken by many governments of the euro area in response to the global financial crisis.

Some further findings can be highlighted from the estimated coefficients for euro area countries. First, the real exchange rate tends to return to its level consistent with fundamentals –i.e. its long-run equilibrium value– as the misalignment variable is significant with the expected negative sign. This mean-reverting behaviour is effective in the low-debt regime. It is slightly strengthened when the external debt-to-GDP ratio increases, as the negative coefficient α_1 adds up to the negative α_0. Thus, when the external debt-to-GDP is above the estimated threshold, the convergence process of real exchange rates towards their long-run equilibrium level is faster. This result is in accordance with the wealth effect exerted by the external debt on savings, as described by the NATREX model.

Table 3. *Estimation of the PSTR model*

Variable	Regime 1	Regime 2
Real exchange rate misalignment	-0.106^{***}	-0.042^{***}
	(-5.202)	(-4.086)
External debt-to-GDP ratio (rate of change)	0.068^{***}	-0.007
	(4.556)	(-0.296)
Investment-to-GDP ratio (log)	-0.008	0.058^{***}
	(-0.833)	(4.658)
Real interest rate differential	-0.074^{**}	0.337^{***}
	(-1.976)	(4.000)
γ	2.756	
Threshold (c)	223%	
N	396%	

Notes: Estimation of Equation 13:

$$\Delta R_{it} = \mu_i + \alpha_0 mis_{it} + \beta_0 D_{it} + (\alpha_1 mis_{it} + \beta_1 D_{it}) \times h\,(debt_{it}; \gamma, c) + \varepsilon_{it}$$

*where $debt_{it}$ stands for the gross external debt-to-GDP ratio and is considered as the transition variable. Coefficients α_0 (α_1) and β_0 (β_1) are reported in columns "Regime 1" ("Regime 2") that stand for the low (high) indebtedness regime. Standard errors corrected for heteroscedasticity are in parentheses. ** (resp. ***) stands for a significant coefficient at the 5% (resp. 1%) statistical level.*

When the external debt increases, the wealth of the nation decreases, so that the agents reduce their consumption. This results in a decrease in the current account deficit and a stabilization of the external debt. The higher will be this effect, the faster will be the convergence to a steady state.

The rate of change of the external debt-to-GDP ratio impacts significantly real exchange rate dynamics only in the low-indebtedness regime. When the external debt-to-GDP ratio is below 223%, the impact of additional external debt is significantly positive. However, once the external debt threshold is reached, this positive effect becomes no significant.

The investment ratio is also a significant explanatory variable, the significance of its effect, here again depending on the indebtedness situation of the euro area countries. In the low-indebtedness regime, the coefficient is found nonsignificant. The positive effect of the investment rate becomes however significant when the external debt-to-GDP ratio exceeds the given threshold.

Finally, the effect exerted by the interest rate differential is significant in the two regimes but the size of this effect also depends on the indebtedness situation of the euro area countries. In the low-indebtedness regime, the coefficient is negative but becomes positive with the increase of the external debt-to-GDP ratio. Indeed, higher external debt is likely to be associated by investors with higher risk premiums, which in turn may contribute to an appreciation of the real exchange rate driven by higher interest rate differential.

Overall our result show that when switching in the second regime (high indebtedness regime), real exchange rates become increasingly sensible to interest rate differential. They also confirm that when the external debt level has reached a certain threshold, an adjustment process comes into place. This adjustment process strengthens the convergence process of real exchange rates towards their long-run equilibrium value, as depicted by the NATREX's dynamics. However, the pace of this adjustment is rather slow, as showed by the low value of the slope of the transition function (γ). Moreover, the magnitude of the effect is small: switching from one regime to the other one results in a small increase in the coefficient of the real exchange rate misalignment.

4.3. What's about the net external debt?

Gross external debt per se only captures one side of an economy's external exposure to international debt markets. As depicted by Figures B.2 in the Appendix, some countries (as Belgium, the Netherlands) can have an important gross external debt while at the same time being creditors. On the other hand, the peripheral countries (Italy, Greece, Portugal and Spain) with lower levels of gross external debt have a negative net external position. In this sense, the net external debt position, obtained by subtracting the gross external debt assets from the liabilities, can provide additional insights into the sustainability of external debt and the adjustment process within the euro area. Acknowledging this fact, we re-run the estimations over our sample of euro area countries in order to check if the effect of the net external position on real exchange rate's dynamics is also non linear.[10]

10. Net external positions correspond to net international investment positions. Data are taken from the World Bank database, External Debt Statistics.

According to the nonlinearity tests displayed in table 4, we find again strong evidence of a non linear relationship between the net external debt and real exchange rates dynamics. Moreover, the results of specification tests of no remaining nonlinearity also indicate that two distinct regimes are at work. In order words, as before, real exchange rates dynamics is found to be asymmetric, depending on the indebtedness situation of the euro area countries. We report the results of the PSTR estimation when taking the net external

Table 4. *Nonlinearity tests (net external debt as transition variable)*

Test	*LM* test	*LRT* test
$H_0 : r = 0$ versus $H_1 : r = 1$	21.29	21.89
	$(2.77e10^{-4})$	$(2.11e10^{-4})$
$H_0 : r = 1$ versus $H_1 : r = 2$	14.16	14.42
	(0.068)	(0.061)

Notes: The LM and pseudo-LRT statistics have a chi-square distribution with K degrees of freedom, where K is the number of explanatory variables with only one location parameter ($m = 1$). Corresponding p-values are given in parentheses.

debt-to-GDP ratio as the transition variable in table 5. Results –displayed in this table– confirm our previous findings which then appear to be at stake when considering the net external debt. Indeed, looking at our main variables of interest, our results first confirm a nonlinear impact exerted by the level of the net external debt on the short-run dynamics of real exchange rates. The estimated threshold value (4.7% of GDP) delineates the two following regimes. A first one is characterized by values of the net external debt below 4.7% of GDP which correspond to negative external positions. A second one, prevailing for values of the net external debt higher than 4.7% of GDP, corresponds to positive external positions. Negative external positions lead to an adjustment process of real exchange rates towards their equilibrium levels, while this process does not hold anymore for positive external positions, suggesting here again that an adjustment process come into place with the level of indebtedness. Moreover, the effect of short-medium-run determinants of real exchange rates is sensitive to the net external position. Indeed, the appreciation of the real exchange rate is driven, in the short run, by the increase of the net external debt in the indebtedness regime, while being driven by the investment ratio in the other

Table 5. *Estimation of the PSTR model*

Variable	Regime 1	Regime 2
Real exchange rate misalignment	-0.100^{***}	0.011
	(0.020)	(0.014)
External debt-to-GDP ratio (rate of change)	0.028^{***}	-0.012
	(0.006)	(0.0168)
Investment-to-GDP ratio (log)	0.044^{*}	0.338^{**}
	(0.023)	(0.115)
Real interest rate differential	0.023^{**}	0.003
	(0.010)	(0.004)
γ	83.2	
Threshold (c)	4.7%	
N	396%	

Notes: Estimation of Equation 13:

$$\Delta R_{it} = \mu_i + \alpha_0 mis_{it} + \beta_0 D_{it} + (\alpha_1 mis_{it} + \beta_1 D_{it}) \times h\,(debt_{it}; \gamma, c) + \varepsilon_{it}$$

*where $debt_{it}$ stands for the net external debt-to-GDP ratio and is considered as the transition variable. Coefficients α_0 (α_1) and β_0 (β_1) are reported in columns "Regime 1" ("Regime 2") that stand for the low (high) indebtedness regime. Standard errors corrected for heteroscedasticity are in parentheses. * (resp. **, ***) stands for a significant coefficient at the 10% (resp. 5%, 1%) statistical level.*

regime. The effects of the interest rate differential on the real exchange rate is in line with those shown for the gross external debt, with a significant impact only for the indebtedness regime. Finally, the slope parameter of the transition function (γ) is higher than the previous one, suggesting that governments and/or international markets are more sensible to country's risk exposure identified by net external positions.

5. Conclusion

This paper revisits the link between the external debt and real exchange rates dynamics within the euro area. More specifically, as described by the NATREX approach, we check the existence of potential adjustment effects exerted by the external debt, and more precisely we account for nonlinearities of real exchange rates' dynamics depending on the external indebtedness level of the euro area countries. Accordingly, we use a threshold panel methodology, in order to test a potential threshold effect in the relationship between the external debt and real exchange rates dynamics on a sample of 11 euro area countries over the 2003Q3-2012Q3 period.

As regards the long run, our results support the prediction of the NATREX approach. Indeed, in the long run, real exchange rates of euro area countries tend to depreciate in the case of a higher time preference and to appreciate with an increase in relative productivity.

As regards the short run, we show that since the launch of the euro, increasing gross external debt positions of most euro area economies have exerted pressures on real exchange rate dynamics within the area. Moreover, we find that, beyond a threshold reached by the external debt-to-GDP ratio, euro area countries can reach a vulnerable position that implies an unavoidable adjustment process. This adjustment is evident in a faster convergence of real exchange rates towards their long-run equilibrium. However, our results show that, while effective, this adjustment process is slow and low. Moreover, real exchange rates are found to be more sensible to the interest differential with the increase in indebtedness. Those results are robust to the use of the net external debt instead of the gross external debt. The slow and low adjustment process of real exchange rates towards their long-run value may explain the persistence of currency misalignments observed in the euro area and more generally the build-up of an unstable situation which reached its peak with the sovereign debt crisis in peripheral countries.

Finally, our findings have some policy implications. In particular, they raise the importance of closely monitoring developments in external debt: large-scale macroeconomic imbalances and persistent currency misalignments within the euro area could have been avoided if some headline indicators of external debt had been defined as early as the beginning of the monetary union.

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Appendix

Table A.1. *Cross-sectional dependence (CD) test.*

	CD test	ρ	$\lvert\rho\rvert$
R	32.45^{***}	0.719	0.719
$prod$	-2.16^{**}	-0.048	0.513
$pref$	1.13	0.012	0.557

Notes: Pesaran's (2004) cross-sectional dependence (CD) test follows a standard normal distribution. Under the null hypothesis, the cross-sectional dependence test is no dependence between cross section units. *** *indicates significance at the 1% level and* ** *at the 5% level.*

Table A.2. *Maddala and Wu (1999) panel unit root tests*

Variable	lags	P_{MW}	p-value	Variable	lags	P_{MW}	p-value
			Without trend				
R	0	17.02	0.762	ΔR	0	313.76	0.000
	1	18.78	0.659		1	127.56	0.000
	2	11.88	0.960		2	100.13	0.000
	3	17.94	0.709		3	47.66	0.001
$prod$	0	23.56	0.371	$\Delta prod$	0	238.17	0.000
	1	22.31	0.442		1	212.60	0.000
	2	13.27	0.925		2	126.49	0.000
	3	13.43	0.920		3	67.81	0.000
$pref$	0	23.27	0.386	$\Delta pref$	0	298.64	0.000
	1	25.20	0.287		1	184.50	0.000
	2	26.96	0.213		2	81.75	0.000
	3	35.34	0.036		3	76.88	0.000
			With trend				
R	0	0.92	1.000	ΔR	0	288.22	0.000
	1	2.37	1.000		1	106.65	0.000
	2	1.34	1.000		2	106.83	0.000
	3	0.88	1.000		3	43.48	0.004
$prod$	0	33.32	0.057	$\Delta prod$	0	193.11	0.000
	1	29.17	0.140		1	169.01	0.000
	2	13.70	0.912		2	95.69	0.000
	3	14.12	0.897		3	42.95	0.005
$pref$	0	26.68	0.223	$\Delta pref$	0	252.54	0.000
	1	25.82	0.259		1	150.45	0.000
	2	21.86	0.468		2	60.96	0.000
	3	35.46	0.035		3	61.86	0.000

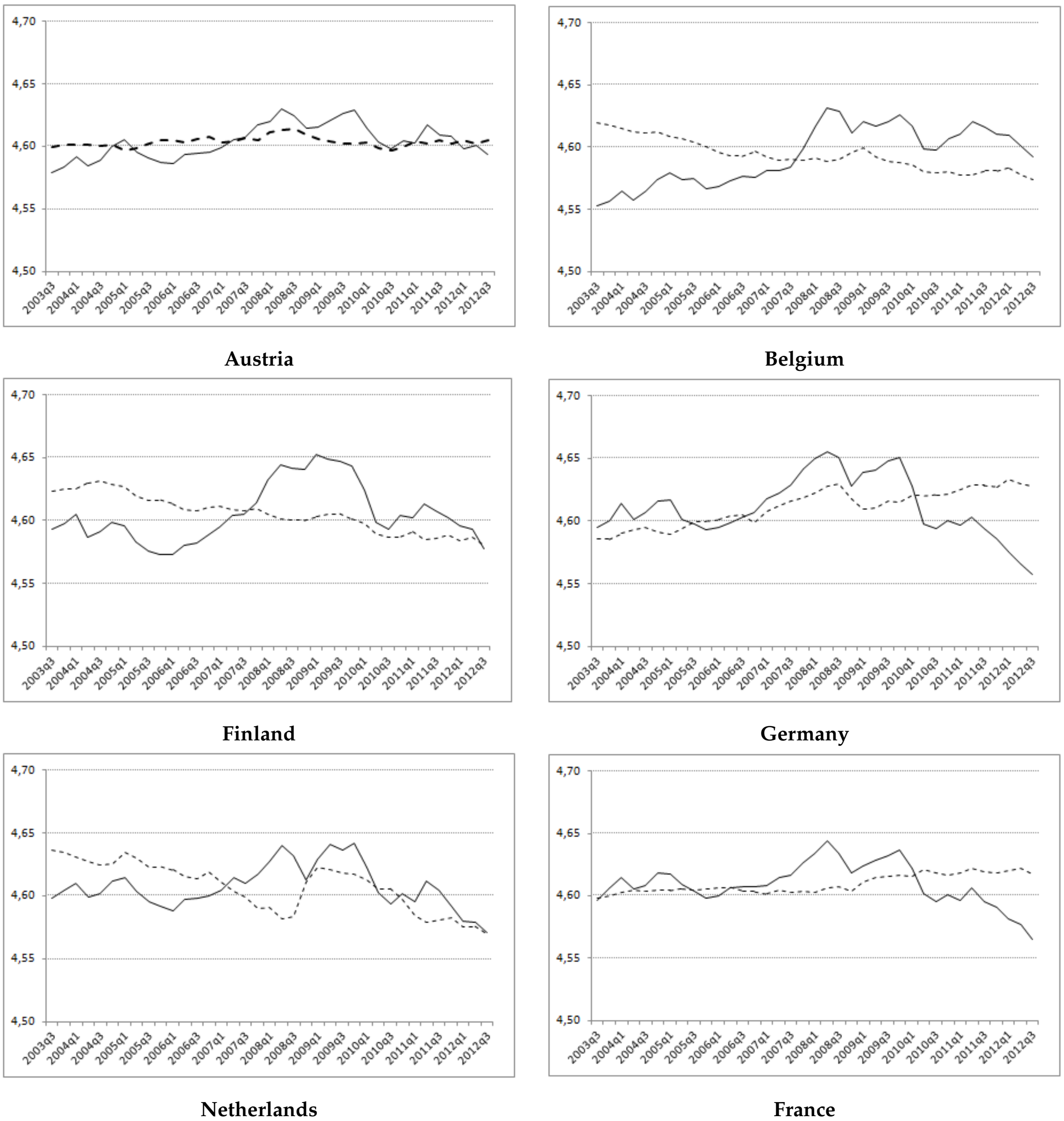
Notes: The statistic, P_{MW}, proposed by Maddala and Wu (1999) has a chi-square distribution with 2N degrees of freedom, when T tends to infinity and N is fixed. The test is based on the null hypothesis of unit root.

Table A.3. *Pesaran (2007) Panel Unit Root test (CIPS)*

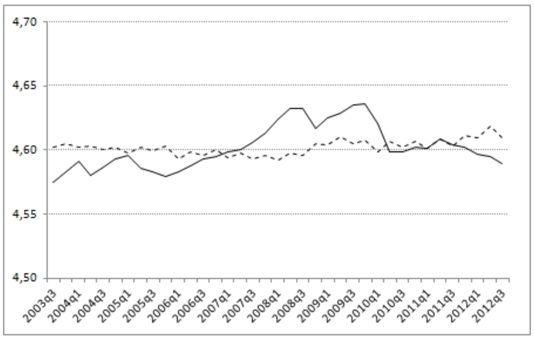
Variable	lags	$CIPS$	p-value	Variable	lags	$CIPS$	p-value
			Without trend				
R	0	0.96	0.833	ΔR	0	-13.27	0.000
	1	1.66	0.952		1	-4.85	0.000
	2	0.90	0.817		2	-5.06	0.000
	3	1.55	0.940		3	0.72	0.765
$prod$	0	-1.91	0.028	$\Delta prod$	0	-11.37	0.000
	1	0.70	0.761		1	-6.69	0.000
	2	1.73	0.958		2	-5.77	0.000
	3	2.68	0.996		3	-2.28	0.011
$pref$	0	-1.60	0.054	$\Delta pref$	0	-11.11	0.000
	1	-2.50	0.006		1	-8.36	0.000
	2	-1.52	0.064		2	-3.81	0.000
	3	-2.07	0.019		3	-3.03	0.001
			With trend				
R	0	-0.81	0.209	ΔR	0	-12.46	0.000
	1	3.77	1.000		1	-3.36	0.000
	2	0.43	0.667		2	-3.70	0.000
	3	3.63	1.000		3	2.43	0.992
$prod$	0	-0.37	0.355	$\Delta prod$	0	-10.71	0.000
	1	1.57	0.942		1	-5.33	0.000
	2	2.79	0.997		2	-5.06	0.000
	3	4.08	1.000		3	-1.11	0.133
$pref$	0	-0.49	0.312	$\Delta pref$	0	-10.07	0.000
	1	-1.25	0.105		1	-7.57	0.000
	2	0.12	0.551		2	-2.58	0.005
	3	-0.54	0.293		3	-2.45	0.007

Notes: Pesaran's test consists in adding to the standard Im-Pesaran-Shin (IPS) test, the mean and the lagged mean of the observed series which is sufficient to filter asymptotically the effects of unobserved common component when the number of countries tends to infinity.

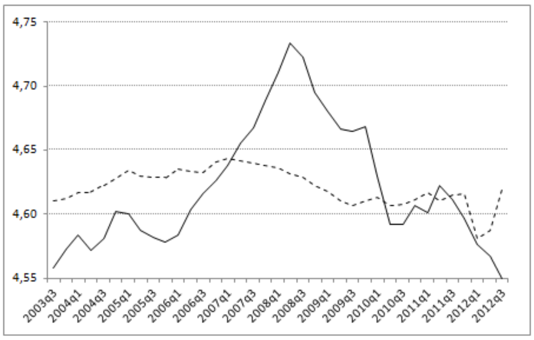
Figure A.1. *Real effective exchange rates (observed and equilibrium level)*



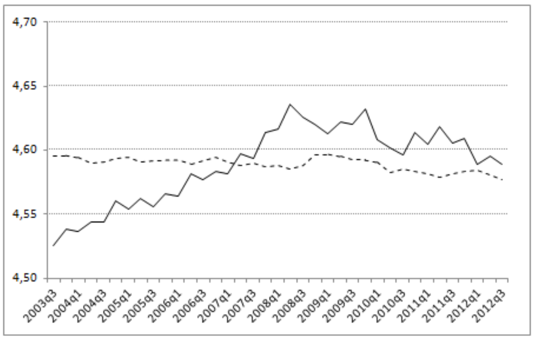
Note: An increase (resp. decrease) of the real effective exchange rate indicates an appreciation (resp. depreciation). The solid (dotted) black line represents the observed (equilibrium) real effective exchange rate.



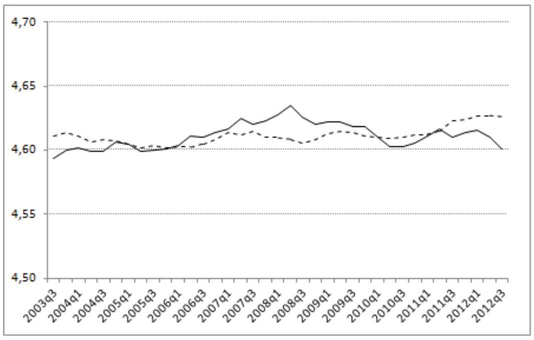
Italy



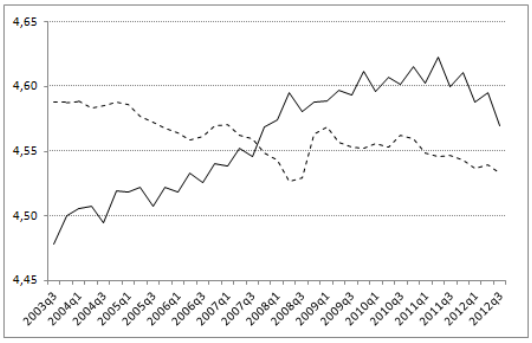
Ireland



Spain



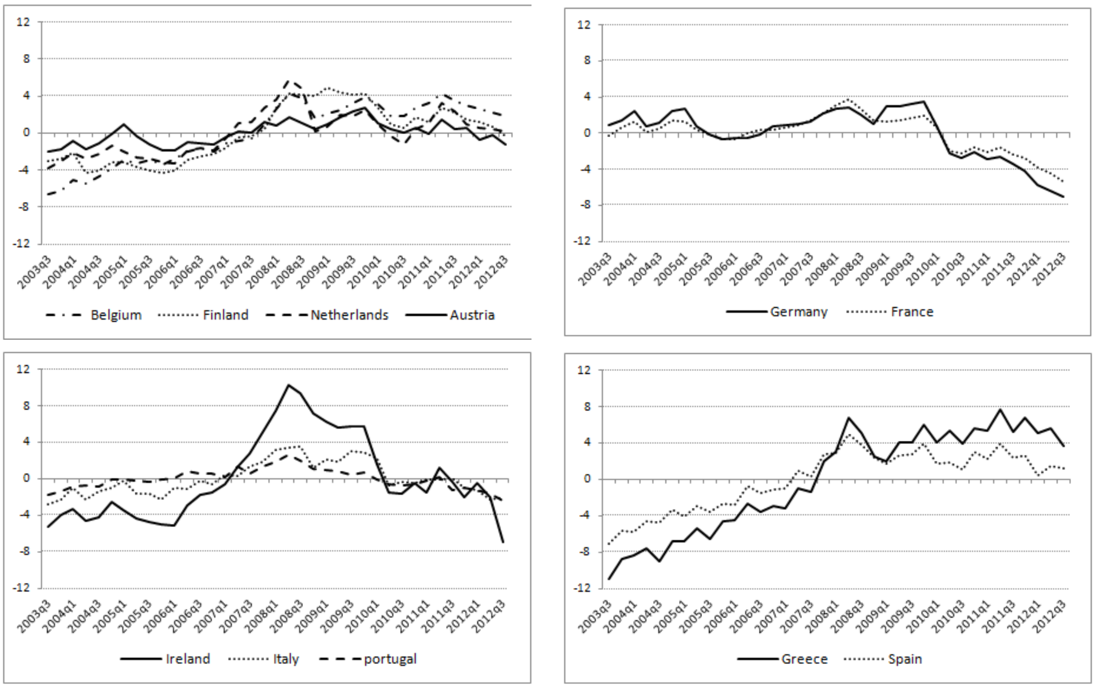
Portugal



Greece

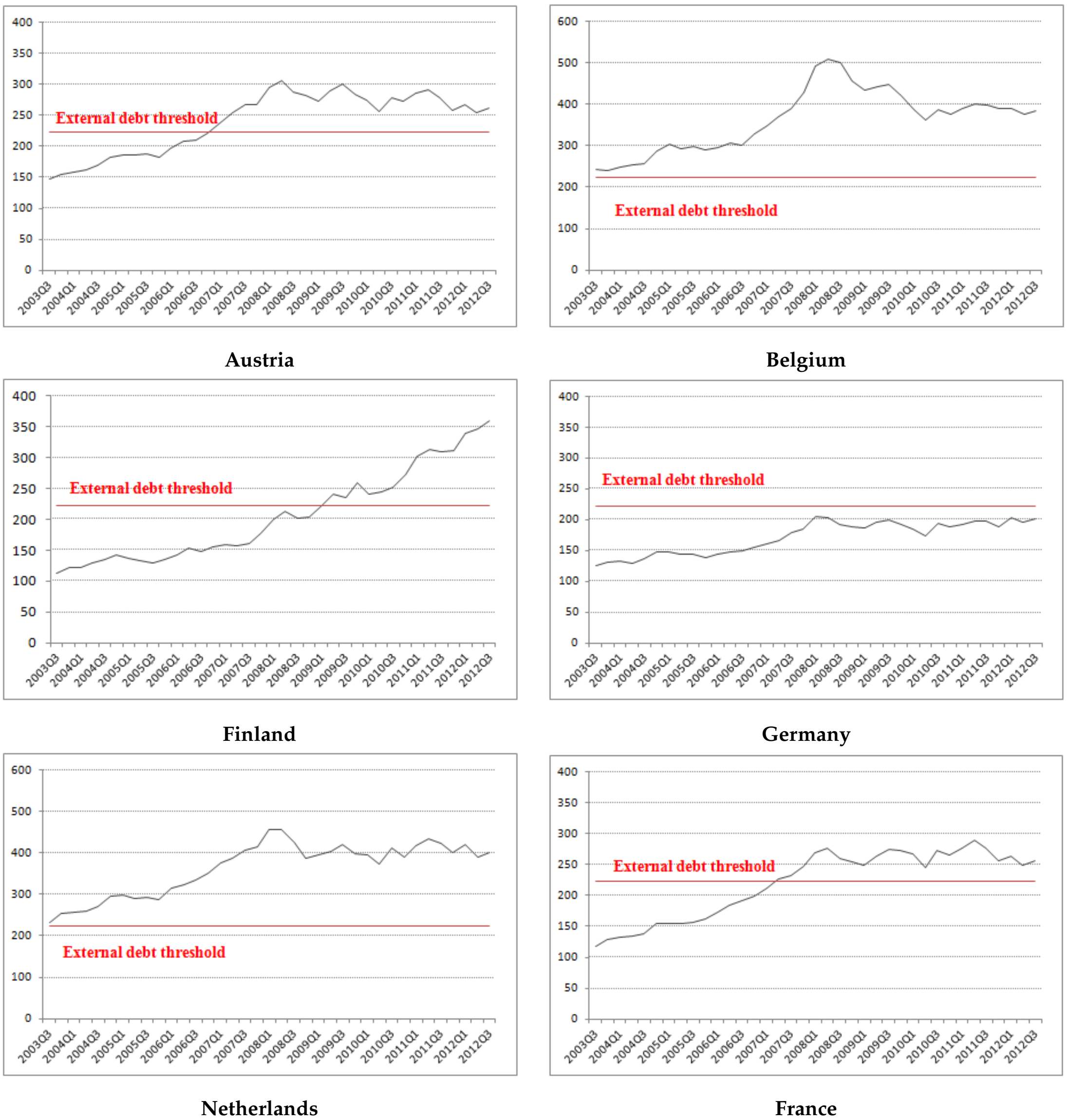
Note: An increase (resp. decrease) of the real effective exchange rate indicates an appreciation (resp. depreciation). The solid (dotted) black line represents the observed (equilibrium) real effective exchange rate.

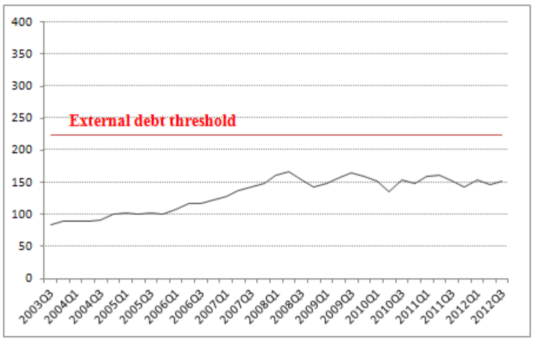
Figure A.2. *Currency misalignments (in %)*



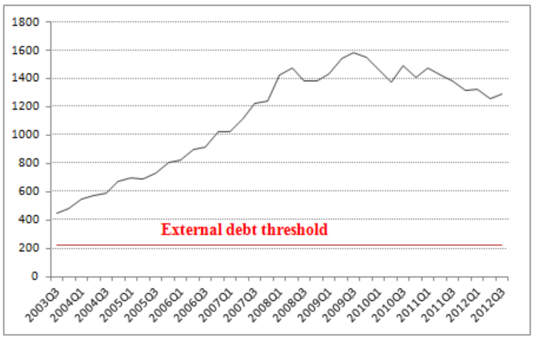
Note: A positive (resp. negative) value corresponds to a real overvaluation (resp. real undervaluation).

Figure B.1. *Gross external debt in % of GDP*

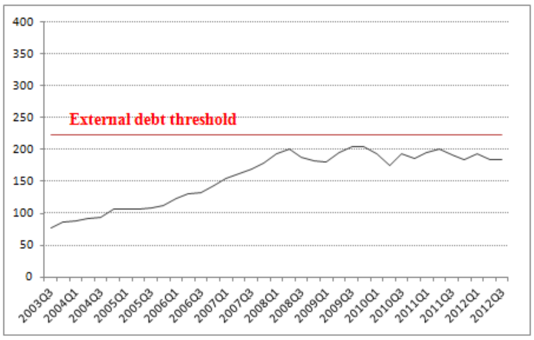




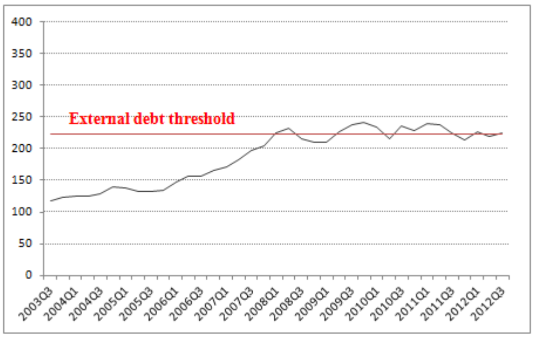
Italy



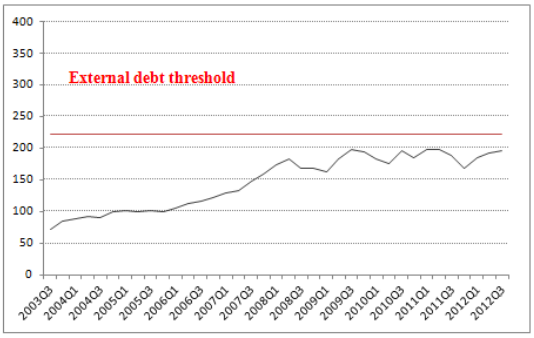
Ireland



Spain



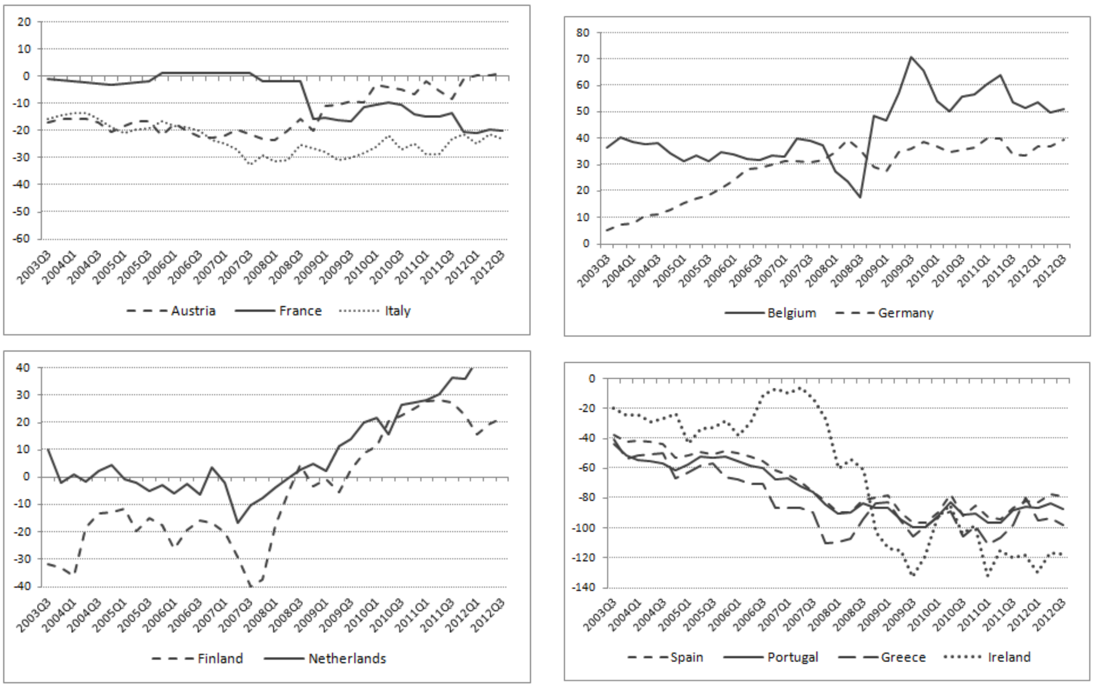
Portugal



Greece

Source: World Bank, Quarterly External Debt Statistics.

Figure B.2. *Net external positions in % of GDP*



Source: World Bank.