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Abstract :

Cross-border investments in public debt securities are a key driver of sovereign yields. Everything else being equal, higher external demand lowers the cost of public financing through debt. However, from a financial stability perspective, it seems important to assess the resilience of sovereign debt against non-resident divestments, as cross-border flows tend to be volatile.

Our paper adresses this issue for the French public debt. It relies on estimating the increase in yields that is necessary for the domestic financial sector to reallocate enough of its securities portfolio to compensate for hypothetical non-resident divestments.

Our results indicate that if the share of non-residents in non-central bank holdings of French public debt had decreased regularly by an extra 1.25 pp compared to actual evolution, over the year from 2016-Q3 to 2017-Q3 (hence amounting to a 5 pp decrease in total), the sovereign yield would have been by around 38bp higher in 2017-Q3. Given the amount and the maturity of gross debt issuances over this period, and using the German bund yield curves for discounting, the corresponding extra cost for public finances in 2017-Q3 would have been around 6Bn €.

JEL classification: F34, G11, H63

Keywords: government bond yields, external debt, investor base, home bias, portfolio choice

1 Introduction

With increasing sovereign debt held by foreigners, countries' vulnerability to shifts in international markets gathers a significant political attention. The demand from international investors is often considered as less stable than the demand from domestic ones, and it is sometimes suggested that domestic financing of public deficit is “safer” than international financing. For example, during the financial crisis, annual portfolio investments in debt securities have dropped by nearly 1 700 Bn $ over one year, indicating that in this period the global supply of bonds has been increasingly absorbed by domestic investors.[1] Nonetheless, domestic financing would necessarily be more costly. Everything else being equal, decreased non-resident demand implies lower price for domestic debt, *i.e.* higher yields and higher borrowing costs.

Non-resident investments are indeed a key driver of sovereign yields. They have notably been put forward to explain the excessive divergence in sovereign spreads in advanced economies and in the Eurozone since the global crisis (as cross-border investment flows distorted by “safe haven” considerations may have benefited “core” economies and penalized “peripheral” ones to levels not explained by economic fundamentals). They have also been considered as a possible explanation for the “Greenspan's conundrum”, i.e. the continued decrease in Treasury yields observed in 2004-2005 despite tightening monetary conditions.

This dependence on external financing has important consequences for financial stability. In this paper, we adress this issue by measuring the extra borrowing cost that would result from non-residents investing away from French public debt.

We thus relate to the literature that links sovereign bond yields to foreign holdings of public debt. This literature has pointed out the positive impact of investor base internationalisation on lowering borrowing costs. For example, Warnock and Warnock (2009) estimate that between May 2004 and May 2005, foreign inflows in US debt have decreased the 10-year Treasury yield by 80 bp. In a cross-country analysis, Andritzky (2012) evaluates that for G20 countries a 10 pp increase in the share of non-resident holdings is associated with 25 to 40 bp decrease in yields level, and a 9 bp increase in the standard deviation of yields. Arslanap and Poghosyan (2016) estimate that a 1 pp increase in the share of government debt held by foreign investors explains a 6-10 bp reduction in long-term sovereign yields. For Beltran et al. (2012) a decrease by $100 billion of foreign official inflows into US Treasuries within one month would cause an increase of the 5-year Treasury rates by 40-60 bp in the short-run.

1 Cornand et al. (2016) also point out that home bias has tended to increase in countries in stress during the recent crisis on sovereign debts in the Eurozone.

In our opinion, one of the issue in this literature is that it has relied mainly on the empirical relationship between the composition of investor base and the level of sovereign yields.[2] But variations in foreign holdings, or in foreign inflows, do not tell much about the underlying dynamics, as they can reflect variations in both foreign and domestic demand. As an illustration of this difficulty, Asonuma et al. (2015) find that higher home bias also decreases borrowing costs, even if one expects foreign holdings and home bias to be negatively correlated.[3]

We believe that the proper approach to the issue of public debt's vulnerability to external demand shocks is to look at the *slope* of domestic demand. This reasoning is the main motivation and contribution of our paper. To this extent, we also relate to Arslanap and Tsuda (2012), who measure the vulnerability of a country's public debt according to the fraction of the banking sector's assets that should be reallocated under a given stress scenario. Where we go further is that we estimate the cost of such a reallocation in a market paradigm, based on estimating the parameters of domestic investors portfolio allocation decisions.

In our opinion, this approach is not only more relevant and straigthforward given our purpose, but it is also more convenient from an empirical perspective. Notably, it allows us to disregard the issue of the other determinants of sovereign yields, to the extent that variations in these determinants are embedded in spreads and affect portfolio composition through spreads. Quantitative Easing policies for example, while their effect on spreads depends on the parameters of portfolio rebalancing, are not expected to influence these parameters.

Turning to methodological purposes, we consider that the sovereign spread is the equilibrium price that clears the market for sovereign bonds. In this market, the demand is composed of three main consumers: the central bank, non-residents, and the domestic financial sector. Of these three consumers, only the demand of the domestic financial sector is price-dependent. The demand from the central bank is policy-dependent - and actually treated as a “negative supply” - and the demand from the non-residents is identified to the negative exogenous shock that

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2 There are exceptions of papers with structural approaches. Andritzky (2012) for example follows a portfolio balance approach assuming the demand for financial assets can be represented by a mean-variance investor. He finds that an exogenous purchase of 10% of outstanding government securities would lower US bonds returns by 16 bp. Kaminska et al. (2011) use the “preferred-habitat” model and estimate that foreign purchases of US Treasuries in July 2003-July 2004 have decreased the 10Y-sovereign rate by 100 bp.

3 Home bias is measured as the share of banks assets in domestic public debt. It is not perfectly correlated with the share of foreigners in holdings of public debt, but there should be a negative correlation for given banking sector size and domestic non-banks holdings.

is precisely the source of the vulnerability under consideration. Assuming further that issuances of public debt are price-inelastic, at least in the short-run, the sovereign spread is then determined by the demand function of the domestic financial sector, and this demand is considered to depend on the allocation choice made within its securities portfolio.

Our findings show that if the share of non-residents in non-central bank holdings of French public debt had decreased by an extra 5pp over the year from 2016-Q3 to 2017-Q3, the sovereign yield would have been by 38bp higher in 2017-Q3. Given the amount and the maturity of gross debt issuances over this period, and using the German bund yield curves for discounting, the corresponding extra cost for public finances in 2017-Q3 would have been around 6Bn €. From a policy perspective, it therefore stresses the importance of maintaining the internationalisation of French public debt investor base.

The rest of the paper is organized as follows. Section 2 describes the data and illustrates some descriptive statistics. Section 3 deals with the estimation of the elasticity of domestic investors' demand for French public debt. Section 4 builds on the results from section 3 to compute the effect of decreased non-resident investments on sovereign spread. Section 5 points the limits and provides some robustness checks. Section 6 concludes.

2 Data and descriptive statistics

2.1 Data

Our analysis relies on data on security holdings that come from the Banque de France's database PROTIDE. It contains the security holdings of resident investors, as well as the security holdings of non-resident investors in French securities. It is compiled at a quarterly frequency since 2008, from direct reporting by the end-investors or indirect reporting via the custodians of securities. Positions are reported at market-value, aggregated by investors' institutional sector, and detailed at the security-by-security level. Direct investments are excluded, as well as investments in non-quoted shares.

From this database, we have extracted the portfolio of the domestic financial sector (banks, insurances, monetary and investment fund or other investment companies), and we have consolidated it by removing holdings of securities issued by the domestic financial sector. Our data covers the period from 2008-Q1 to 2017-Q3.

The PROTIDE database on security holdings also contains basic information regarding security characteristics that allow us to identify notably the instrument type and the issuer's domicile country and institutional sector. These characteristics come from the Eurosystem's referential on securities “CSDB” (Centralized Security DataBase), or from Banque de France's own referential. We have used them to aggregate security-by-security positions in 10 asset classes: French public debt securities (0) ; French corporate debt securities (1) ; “Non-peripheral” Eurozone debt securities (2) ; “Peripheral” Eurozone debt securities (3) ; Non-Eurozone debt securities (4) ; French quoted shares (5) ; “Non-peripheral” Eurozone quoted shares (6) ; “Peripheral” Eurozone quoted shares (7) ; Non-Eurozone quoted shares (8) ; Investment fund shares (9).

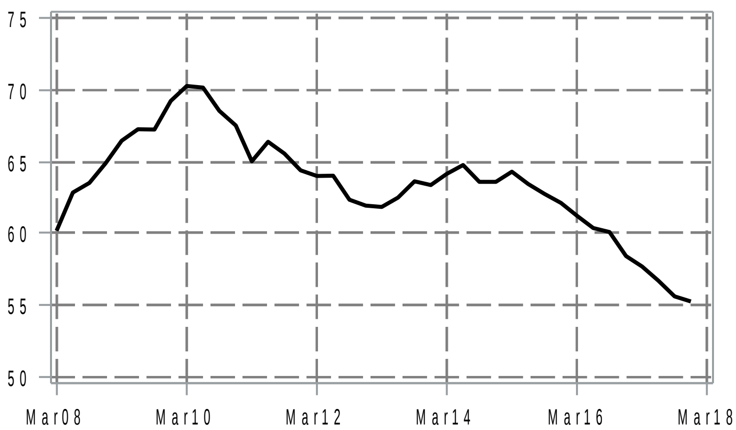
For these asset classes, we have added descriptive variables that either come from Bloomberg (stock indices, sovereign rates), or from other series compiled by the Banque de France (exchange rates, yields on French non-financial corporations debt securities).

2.2 Descriptive statistics on domestic and non-resident holdings of French public debt

End-2017, foreign investors were holding 55 % of French public debt securities (Figure 1). This share has decreased steadily from 64 % since March 2015, as a result of the Public Sector Purchase Program (PSPP) under which the Banque de France has bought large amounts of French public debt. What is not held by foreign investors is almost entirely held by the domestic financial sector: this justifies our decision to exclude domestic investors that do not belong to the financial sector from our analysis[4].

4They account for around 1 % of total holdings.

<u>Figure 1 : Share of non-resident investors in holdings of French public debt</u>

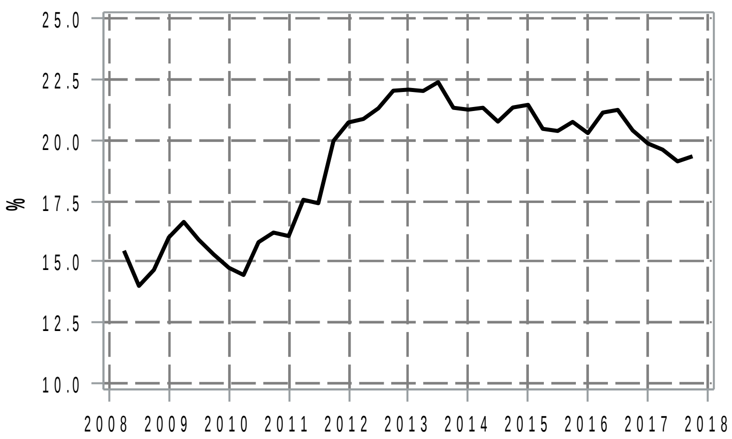


Source: Banque de France

Mid-2010, foreign investors were holding more than 70 % of French public debt. But over 9 months, France experienced a drop in foreign investments that resulted in a 5 pp decrease in the share of foreigners in the investor base for public debt. This decrease was mainly absorbed by domestic insurance companies, whose share rose by 4.5 pp. However, contrary to what would be expected, these reallocations did not increase the French-German spread. Instead, it decreased over the period. The explanation is that at the same time, other asset classes (mainly bonds from the Eurozone periphery countries) were found less attractive by French investors.

As shown by Figure 2, the share of French public debt in the (consolidated) portfolio of domestic financial investors has increased over the past 10 years from 15 % to around 20 %, reaching a maximum at 22.5 % mid-2013. This increase responded mainly to the decrease in holdings of debt securities from peripheral Eurozone countries.

<u>Figure 2 : Share of French public debt securities in the consolidated portfolio of the French financial sector</u>



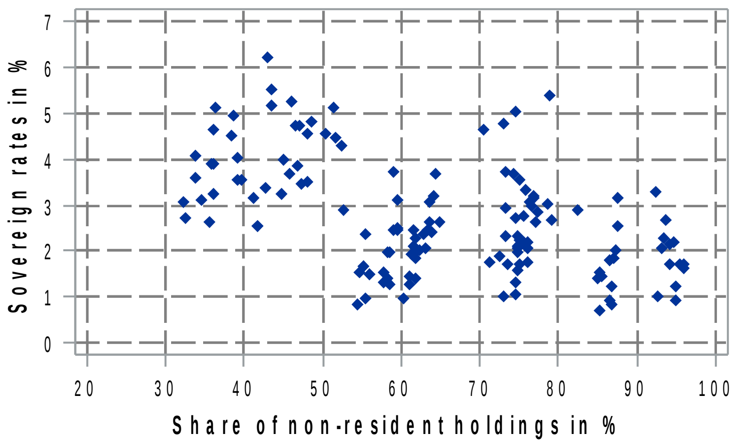
Source: Banque de France

The level of foreign holdings of public debt securities in France is quite high. It is associated with relatively low interest rates on public securities. This is consistent with the general case - that has been made clear in the literature (see section 1) - about the association between low interest rates and high foreign holdings.

This point is also empirically confirmed in Eurozone data on security holdings (Figure 3). A simple OLS estimation for Eurozone countries shows that an increase by 10 pp in the share of foreigners in non-Eurosystem holdings of public debt securities is associated with a decrease by 28 bp in the 10-year sovereign rate (highly significant). In a panel regression framework (from 2014-Q2 to 2017-Q2), controlling for both time and country fixed effects, the association is even stronger with a coefficient of -41 bp per 10 pp.

Figure 3 : Sovereign rates and non-resident holdings of central government securities

Non-resident holdings are expressed as the share of total holdings excluding holdings by the Eurosystem



Sources : ECB databases on securities (SHSDB and CSDB)

3 Estimating the demand of the French financial sector for the French public debt

The purpose of this section is to estimate the elasticity of domestic financial investors' demand for French public debt. We model portfolio composition as a softmax composition of assets “scores”, and assume that these scores can be written as linear functions of observables. These linear equations can be derived from pairwise comparison of asset classes weights in the portfolio, and are estimated using standard regression techniques, adressing in particular the issue of error-term covariance and lag-dependency.

This estimation will be the base for measuring the extent to which the premium on public debt must increase in order for domestic demand to cover for the withdrawal of foreign investors.

3.1 The setup

We assume that the share of the asset class **i** in the portfolio of investors can be written as a softmax function:

$$y_{it} = \frac{e^{\delta_{it}}}{\sum_k e^{\delta_{kt}}} \quad (1)$$

Where $\mathbf{y_{it}}$ is the share of asset class **i** in the portfolio of investors at time **t**, and $\boldsymbol{\delta_{it}}$ is the "score" of the asset class **i**. This score reflects the "attractiveness" of the asset class **i**, but it also carries out other effects that may influence the weight of a given asset class in the portfolio, such as valorization effects or lag-dependency. We assume that $\boldsymbol{\delta_{it}}$ can be written as a linear function of the asset class characteristics:

$$\delta_{it} = \rho \ln(y_{it-1}) + \alpha_i + X_{it}\beta_i + X_{\kappa(i)t}\beta_{\kappa(i)} + X_{r(i)t}\beta_{r(i)} + \eta_{it} \quad (2)$$

Where the **X**s are the explanatory variables of the model ; the **β**s are the coefficients of the model ; **κ(.)** and **r(.)** are functions that associate an asset class to respectively its instrument type (debt securities, quoted share, investment fund shares) and its counterpart region (domestic, Eurozone, outside Eurozone) ; **α**s are fixed effects ; and **η**s are error terms.

This specification allows a flexible description of the investors' demand for financial assets. It is "agnostic" in the sense that it does not rely on a presupposed maximizing behavior of investors. For example, the home bias naturally fits in this specification, not only as a fixed effect carried out by $\boldsymbol{\alpha_i}$, but also as an heterogeneous way for valorizing the asset characteristic $\mathbf{X_i}$ (through $\boldsymbol{\beta_i}$).

Taking ratios and logs allows us to linearize the equation (assuming the absence of zero holdings):

$$\begin{aligned} \ln(y_{it}/y_{jt}) &= \rho \ln(y_{it-1}/y_{jt-1}) + \alpha_i - \alpha_j + X_{it}\beta_i - X_{jt}\beta_j \\ &+ \underbrace{X_{\kappa(i)t}\beta_{\kappa(i)} - X_{\kappa(j)t}\beta_{\kappa(j)}}_{=0 \text{ if } \kappa(i)=\kappa(j)} + \underbrace{X_{r(i)t}\beta_{r(i)} - X_{r(j)t}\beta_{r(j)}}_{=0 \text{ if } r(i)=r(j)} + \epsilon_{ijt} \end{aligned} \quad (3)$$

With $\epsilon_{ijt} = \eta_{it} - \eta_{jt}$. It is quite clear that we must not expect **ε**s to be independent, neither in the cross-section dimension nor in the time dimension. Instead, we allow correlation between contemporaneous **ε** to be different from 0, such that for a given period, the covariance matrix of the error term is written as **Σ**. Moreover, we allow error terms to be autocorrelated of order 1, with the coefficient of autocorrelation being independent of the asset class. Hence, the covariance matrix of the vector of stacked error terms $\boldsymbol{\varepsilon}_{ijt}$ can be written as $\boldsymbol{\Gamma} \otimes \boldsymbol{\Sigma}$, where $\boldsymbol{\Gamma}_{t1,t2} = \boldsymbol{\gamma}^{|t1-t2|}$.

3.2 The definition of asset classes and explanatory variables

We apply the above specification on the consolidated portfolio of domestic financial investors broken down in 10 asset classes according to asset types (debt, quoted shares, investment fund shares), counterpart regions (for debt securities and quoted shares only: domestic, core Eurozone, non-core Eurozone, outside Eurozone), and counterpart sector (for domestic securities only: general government or non-financial corporation).

Explanatory variables include sovereign spreads, German yields on government securities, exchange rates, stock indices and VIX. The way they enter the model is made explicit in Table 1. The French sovereign spread is our only variable of interest: all other variables can be considered as control variables, that are taken to reflect variations in attractiveness, market values, and euro countervalues.

The coefficient associated to the French sovereign spread reflects the way the French financial sector modifies its position in domestic public debt depending on variations in spreads. It is not to be interpreted as a structural preference for the “return” component of public debt securities: such a preference would be expected to be strictly positive, whereas our coefficient is expected to be positive only if the French investor is less “risk-averse” than the global investor when investing in French public debt.[5]

To see this point, one may consider a stylized case, with global demand for an asset **i** written as a function of returns and risks $D_i(r_i, \sigma_i)$ where $\mathbf{r_i}$ stands for returns and $\boldsymbol{\sigma}_\mathbf{i}$ stands for risks (such that $\mathbf{D_i}$ is increasing in its first argument and decreasing in its second). Market-clearing conditions for this asset implicitly define a function $\sigma_i = \sigma_i(r_i)$ that satisfies:

5 We believe that a more structural decomposition of the “score” of French public debt would be uncertain. For example, adding risk-related variables, or proxies such as CDS, generates instability in the estimation, as spreads and CDS are closely related. Therefore, we consider it more robust to rely on spreads only, with the understanding that the associated coefficient is not to be interprated as a structural preference for returns.

$$\frac{\partial \sigma_i}{\partial r_i} = -\frac{\partial D_i / \partial r_i}{\partial D_i / \partial \sigma_i} \tag{4}$$

A given investor in this framework would have a demand for asset **i** given by $d_i(r_i, \sigma_i) = d_i(r_i, \sigma_i(r_i))$, and it would vary with respect to $\mathbf{r_i}$ according to:

$$\frac{d}{d r_i} d_i(r_i, \sigma(r_i)) = \frac{\partial d_i}{\partial r_i} + \frac{\partial d_i}{\partial \sigma_i} \frac{\partial \sigma_i}{\partial r_i} = \frac{\partial d_i}{\partial r_i} - \frac{\partial D_i}{\partial r_i} \frac{\partial d_i / \partial \sigma_i}{\partial D_i / \partial \sigma_i} \tag{5}$$

Hence, changes in $\mathbf{d_i}$ relative to changes in $\mathbf{r_i}$ would be positive if the given investor has a more pronounced taste for returns and/or a less pronounced aversion for risks than the representative investor, i.e. if:

$$\frac{\partial d_i / \partial r_i}{\partial D_i / \partial r_i} > \frac{\partial d_i / \partial \sigma_i}{\partial D_i / \partial \sigma_i} \tag{6}$$

<u>Table 1 : Description of asset classes and explanatory variables</u>

Asset type	**Counterpart country**	**Counterpart sector**	**Id Number**	$X_{\kappa(i)t}$	$X_{r(i)t}$		X_{it}	
					in reg 1	*in reg 2*	*in reg 1*	*in reg 2*
Debt securities	Domestic (France)	General administration	0	German sovereign rate (β_{0-4})	-	-	FR sov spread (β_0)	FR sov spread (β_0)
		Corporate sector	1				French NFC average spread (β_1)	French NFC average spread (β_1)
	"non-peripheral" Eurozone countries		2				DE sov. Rate (β_2)	PCA1 of BE, NL and AT sov spreads (β_2)
	"peripheral" Eurozone countries		3				IT sov spread (β_3)	PCA1 of IT, ES and PT sov spreads (β_3)
	Outside the Eurozone		4		USD/EUR exchange rate ($\beta_{4,8}$)	PCA1 of USD/EUR GBP/EUR JPY/EUR exchange rates ($\beta_{4,8}$)	US sov spread (β4)	PCA1 of US, GB and JP sov spreads (β_4)
Quoted shares	Domestic (France)		5	VIX (β_{5-8})	-	-	FR CAC 40 (β_5)	FR CAC 40 (β_5)
	"non-peripheral" Eurozone countries		6				DE DAX (β_6)	PCA1 of DE DAX, BE BEL20, NL AEX, AT ATX (β_6)
	"peripheral" Eurozone countries		7				IT FTSE MIB (β_7)	PCA1 of IT FTSE MIB, ES IBEX35, PT PSI20 (β_7)
	Outside the Eurozone		8		USD/EUR exchange rate ($\beta_{4,8}$)	PCA1 of USD/EUR GBP/EUR JPY/EUR exchange rates ($\beta_{4,8}$)	US SP500 (β_8)	PCA1 of US SP500, GB FTSE100, JP Nikkei225 (β_8)
Investment fund shares			9	German sovereign rate and VIX (β_9)	USD/EUR exchange rate (β_9)	PCA1 of USD/EUR GBP/EUR JPY/EUR exchange rates (β_9)	-	-

- *"Reg1" and "reg2" refer to alternative specifications in the empirical analysis.*
- *Explanatory variables referring to stock indices have to be understood as referring to the growth rate of the given index, over the t-3 months to t+3 months window.*
- *Yields and spreads in explanatory variables are measured for a 10-years maturity.*
- *"PCA1" refers to the first component of the principal component analysis of the given variables.*
- *NFC = Non-financial corporations ; sov = sovereign.*

3.3 Results

The model is estimated with FGLS, in which endogeneity concerns from the dynamic bias and the French-German spread are managed with instrumentation. For the dynamic bias, the lag-dependent variable is instrumented by its predicted value from its regression on the first and second lags of exogeneous variables. The French-German spread is instrumented by its predicted value from its regression on the German sovereign rate, the Italian spread, the difference between French and German CDS and the lagged value of the French-German spread. More details are provided in annex.

In estimating the model, it is also necessary for parsimony to select a subset of explanatory variables, in particular when it comes to describing characteristics for an asset class referring to regions such as "peripheral Eurozone" countries. In order to do so, we apply two strategies. In the first regression, we have selected the country variable corresponding to the "main" economy in the region. In the second regression, we have used principal component analysis to reduce the number of variables. For example, for the description of sovereign spreads in countries in the periphery of the Eurozone, we have used the Italian sovereign spread in regression 1, and the first component on Italian, Spanish and Portuguese sovereign spreads in regression 2. Details are provided in the table of results (Table 2).

The results of this estimation are satisfying. A significant positive correlation is found between investment position in domestic public debt and the French-German spread. The domestic financial sector invests more in domestic public debt when its returns are higher. As was pointed out in section 3.2, this result suggests heterogeneous risk/returns preferences between domestic and foreign investors, with the formers being less risk-averse. This can be interpreted as a consequence of the home bias.

The German sovereign rate has a negative, but non-significant, relation with investment positions in debt securities. This is explained by the fact that the coefficients on debt interest rates also reflect valuation effects. Since holdings are measured at market value, an increase in the interest rate is mechanically associated with a decrease in the value of the investment position in debt securities.

These valuation effects should also affect investment positions in quoted shares that have more volatile prices. But although we found positive relationships between stock indices and investment positions, they are not significant in general. One reason may be that a part of the covariance is captured by the VIX variable that is correlated with the growth rate of stock indices.

Table 2: estimations results

	Variable	Assets classes for which the coefficient applies	Regression 1		Regression 2	
			Coefficients	Standard errors	Coefficients	Standard errors
Non asset-specific variables	Lag dependent variable	All variables 0-9	0.573 ***	0.1279	0.727 ***	0.1103
	DE sov rate	debt securities ($\beta_{0\text{-}4}$)	-0.019	0.0177	-0.024	0.016
		IFS (β_9)	-0.008	0.0206	-0.028	0.0182
	VIX	quoted shares ($\beta_{5\text{-}8}$)	-0.009 ***	0.0024	-0.007 ***	0.0022
		IFS (β_9)	-0.007 ***	0.0016	-0.006 ***	0.0017
	Exchange rates a)	debt securities and quoted shares from outside the eurozone ($\beta_{4,8}$)	-0.155	0.0999	-0.02 **	0.0098
		IFS (β_9)	-0.393 **	0.1618	-0.016	0.0145
Asset-specific variables	**FR-DE sov spread**	**French Gov debt securities (β_0)**	**0.133 ****	**0.0646**	**0.128 ****	**0.0621**
	French NFC average spread with DE sov rate	French corporate debt securities (β_1)	0.007	0.0238	0.021	0.0254
	DE sov rates and core eurozone sov spreads b)	Debt securities from non-peripheral eurozone (β_2)	0.011	0.008	0.013 **	0.0057
	Peripheral eurozone sov spreads c)	Debt securities from peripheral eurozone (β_3)	-0.029 *	0.0157	-0.008	0.009
	Outside eurozone sov spread d)	Debt securities from outside eurozone (β_4)	-0.018	0.0196	-0.014	0.0094
	FR stock index (CAC 40)	French quoted shares (β_5)	0.103	0.1036	0.15	0.1021
	Core eurozone stock indices e)	Quoted shares from non-peripheral eurozone (β_6)	0.242 **	0.1071	0.022 ***	0.0077
	Peripheral eurozone stock indices f)	Quoted shares from peripheral eurozone (β_7)	0.11	0.164	0.018	0.0179
	Outside eurozone stock indices g)	Quoted shares from outside eurozone (β_8)	0.251	0.1922	0.027 **	0.0121
	Fixed effects	included	Y		Y	
		Number of observations	1665		1665	
		R^2	0.79		0.84	

a) In regression 1 : USD/EUR exchange rate. In regression 2 : PCA component 1 for USD/EUR, GBP/EUR, JPY/EUR

b) In regression 1 : DE sov rate. In regression 2 : PCA component 1 BE, NL, AT sov spreads

c) In regression 1 : IT sov spread. In regression 2 : PCA component 1 for IT, ES and PT sov spreads

d) In regression 1 : US sov spread. In regression 2 : PCA component 1 US, GB, JP sov spreads

e) In regression 1 : DE stock index (DAX). In regression 2 : PCA components 1 DE (DAX), BE (BEL20), NL (AEX), AT (ATX) stock indices

f) In regression 1 : IT stock index (FTSE MIB). In regression 2 : PCA component 1 IT (FTSE MIB), ES (IBEX 35), PT (PSI20) stock indices

g) In regression 1 : US stock index (SP500). In regression 2 : PCA component 1 US (SP500), GB (FTSE100), JP (Nikkei 225) stock indices

*Significance at 1% is indicated with ***, 5% with ** and 10% with *.*
R^2 are computed against the model with fixed effects only.

4 Counterfactual analysis: Estimating the response of French sovereign spreads to variations in non-resident investments in French public debt

4.1 Using market-clearing condition to move from the effect of spread on domestic investments to the impact of non-resident investments on spread

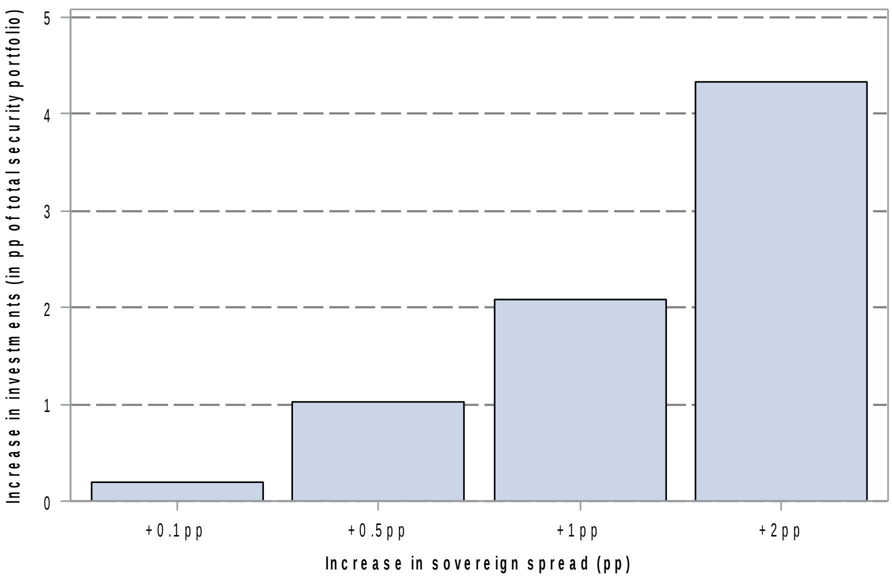
The econometric estimates from section 3 allow us to measure how domestic investments in public debt are impacted by changes in the French-German sovereign spread. Indeed, we have:

$$\frac{\partial \ln y_{fpd,t}}{\partial s_t} = \beta_s \left(1 - y_{fpd,t}\right) \qquad (7)$$

where **s** = French-German spread, and **fpd** = French public debt. This notation will be used in this section for more clarity: with respect to the notations in Table 1 in the previous section, we have $\mathbf{y_{fpd,t} = y_{0,t}}$ and $\boldsymbol{\beta_s = \beta_0}$.

For illustration, Figure 4 below plots the impact of a variation in the French sovereign spread on the share of the portfolio of the French financial sector invested in domestic public debt. Starting with an initial allocation of 19 % of total portfolio, an increase by 1 pp in the sovereign spread results in allocating 2 extra pp of this portfolio to French public debt.

Figure 4 : Impact of French sovereign spread on the investment position in domestic public debt of the domestic financial sector *(β_S = 0.13)*



From this point, to compute the impact of non-resident investments on the French spread requires considering the market-clearing condition that the equilibrium spread must satisfy. This market-clearing condition writes:

$$AO_t = I_{nr,t} + I_{dfs,t} + I_{oth,t} \tag{8}$$

Where:

- **AO** = amount outstanding of French Public Debt not held by the central bank (at market value)
- $\mathbf{I_a}$ = investment position of **a** in French Public Debt
- **nr** = non-resident ; **dfs** = domestic financial sector ; **oth** = other domestic sectors
- $\mathbf{I_{dfs} = y_{fpd}.TPF}$ where **TPF** is the total security portfolio of the domestic financial sector.

It also requires the following assumptions (discussed in section 5):

- $\mathbf{I_{oth}}$ is independent of **s** (investments of the domestic non-financial sector in French public debt are actually residual) such that an exogenous change in $\mathbf{I_{nr}}$ must be compensated by spread-driven changes in $\mathbf{I_{dfs}}$ and **AO**.

- The relationship between **I_{dfs}** and **s** is entirely captured by the portfolio allocation decision (i.e. by **y_{fpd})** meaning that **s** does not impact the total amount of securities in the portfolio of the French financial sector (**TPF**).[6] This relationship is approximated at order 1, using $\partial \ln y_{fpd,t} / \partial s_t$.

- The relationship between **AO** and **s** consists only in valuation effect (i.e. the decrease in prices that brings the increase in yields), such that face-valued outstanding amount of French public debt is not affected by changes in interest rates. These valuation effects ($\partial AO / \partial s$) are approximated with an average price-to-yield sensitivity computed from a security-by-security analysis, for a set of securities representing 74% of total amount outstanding.

Given these considerations, the impact of an exogenous change in **I_{nr}** to **I^*_{nr}**, on French sovereign spread is:

$$(s_t^* - s_t) = \frac{I^*_{nr,t} - I_{nr,t}}{\frac{\partial AO_t}{\partial s_t} - TPF_t \beta_s y_{fpd,t} (1 - y_{fpd,t})} \quad (9)$$

For example, given our estimated **β_s** = 0.13, and an estimated value $\partial AO / \partial s = -0.07 \times AO$ in 2017-Q3 ; and given observed **AO** = 2 212 Bn € , **TPF** = 3 185 Bn €, and **y_{fpd}** = 19 % in 2017-Q3, we obtain that a decrease by 10 Bn € in **I_{nr}** changes the equilibrium sovereign spread by +5 bp.

4.2 Running a counterfactual analysis over 1 year

To understand the dynamic response of spreads for a given trajectory in non-resident investments, we compute the equilibrium path of spreads over several counterfactual trajectories in non-resident investments over the period 2016-Q3 to 2017-Q3.

The scenarios we test are the followings:

6 In Section 5, we discuss the possibility that the total amount of securities in the domestic investors' portfolio decreases because of valuation effects applied to public debt securities

- Scenario A : non-residents' share in non-central bank holdings of French public debt decreases by an extra 0.5pp every quarter over the period ;
- Scenario B : non-residents' share in non-central bank holdings of French public debt decreases by an extra 1.25pp every quarter over the period ;
- Scenario C : non-residents' share in non-central bank holdings of French public debt decreases by an extra 2pp every quarter over the period.

The computation of a counterfactual spread in each of these scenarios is based on the same kind of assumptions and relations than those in Section 4.1. We assume that everything but holdings of the non-residents and the domestic financial sector, the market value of the outstanding debt, and the sovereign spread, remain identical in the counterfactual scenario. The only difference is that we must take into account the impact of changes in lagged value of the dependent variable $\mathbf{y_{fpd,t-1}}$ on the contemporaneous value $\mathbf{y_{fpd,t}}$ **.** Also, unlike previously, the impact of $\mathbf{s_t}$ and $\mathbf{y_{fpd,t-1}}$ on $\mathbf{y_{fpd,t}}$ is computed from exact relations, solved numerically, and not from first-order approximations. Stated generally, the algorithm writes for a given period:

- Get counterfactual value for $\mathbf{I^*_{nr}}$ from defined scenario
- Numerically find the counterfactual value **s*** that ensures:

$$AO^*_t = I^*_{nr,t} + I^*_{dfs,t} + I^*_{oth,t} \tag{9}$$

- Move to next period

and it is based on the following equalities (where we have denoted with a “ * ” the counterfactual values - the absence of a “ * ” indicating factual values – and where we introduce the variable **NRS** the non-residents' share in holdings of the French public debt not held by the central bank, defined by the counterfactual scenario):

$$I^*_{oth,t} = I_{oth,t}$$

$$I^*_{nr,t} = NRS^*_t AO^*_t$$

$$I^*_{dfs,t} = y^*_{fpd,t} TPF_t$$

$$AO^*_t = AO_t + \frac{\partial AO_t}{\partial s_t}(s^*_t - s_t)$$

$$y^*_{fpd,t} = \frac{e^{\delta^*_{fpd,t}}}{\sum_k e^{\delta^*_{kt}}} \tag{10}$$

$$\delta^*_{fpd,t} = \hat{\rho}\ln(y^*_{fpd,t-1}) + s^*_t \hat{\beta}_{fpd} + X_{\kappa(fpd)t}\hat{\beta}_{\kappa(fpd)} + X_{r(fpd)t}\hat{\beta}_{r(fpd)} + \hat{\eta}_{fpd,t}$$

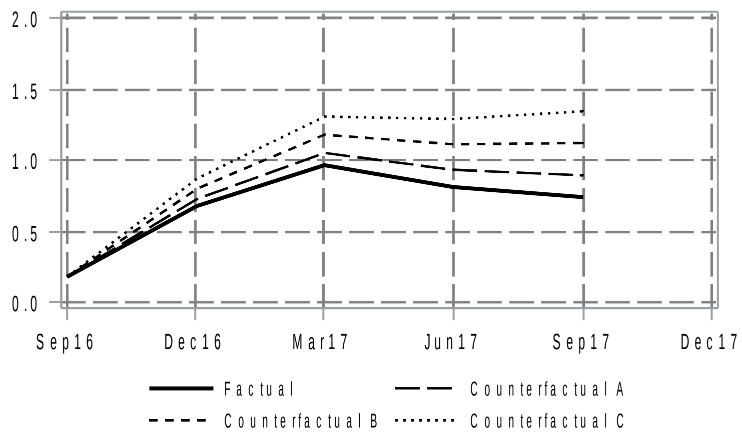
$$\delta^*_{k,t} = \hat{\rho}\ln(y^*_{k,t-1}) + X_{kt}\hat{\beta}_{fpd} + X_{\kappa(k)t}\hat{\beta}_{\kappa(k)} + X_{r(k)t}\hat{\beta}_{r(k)} + \hat{\eta}_{k,t}$$

for k ≠ fpd

For the values $\beta_s = 0.13$ and $\rho = 0.57$ (estimated coefficients in reg 1), running this counterfactual analysis reveals that for a 5 pp extra decrease in the share of non-resident holdings over one year (scenario B), the sovereign interest rate is increased by 38bp. In the worst scenario C, it is increased by 60bp, and in the low scenario A, it is increased by 15bp[7] (Figure 5).

7 Using coefficients estimated in Reg2 does not change the results very much : in scenario A, the sovereign yield is increased by 14 bp, in scenario B, it is increased by 36 bp, and in scenario C, it is increased by 58 bp.

Figure 5 : Trajectory of French sovereign interest rate, in observed and counterfactual scenarios



5 Limits and robustness checks

5.1 Limits

Our methodology is based on a “ceteris paribus” approach, in which all of the variations of the sovereign spread triggered by non-resident investments are explained by the need for portfolio reallocation within the domestic financial sector, and in which only the sovereign spread adjusts to allow market-clearing.

Although being appealing, this approach has some limits. To put it in the most general terms: changing one equilibrium has ripple effects on all other equilibria, and “holding all else constant” implies disregarding these ripple effects. The aim of this section is to discuss and weight these limits.

Going back to the assumptions we made in section 4.1, we believe that two of them are of little consequence. The assumption that the behavior of domestic investors from the non-financial sector would not be affected by variations of the sovereign spread has a small impact, since the non-financial sector represents only a residual fraction of holdings of the French public debt (around 1%). The assumption that the face-valued amount outstanding of public debt is unchanged by the sovereign spread is more important, as we may consider that issuances of debt should decline as financing costs increase. However, the growth of public debt is a sticky process with various determinants, and more importantly, it is also non-relevant to our purpose, in that we are considering the vulnerability of French public debt for a pre-determined indebtedness trajectory.

In section 4.1, we also assumed that the total amount of the security portfolio of the domestic financial sector remained unaffected by the changes in the sovereign spread. This assumption is more critical: we may just as well believe that increased spread may decrease the total portfolio (because of valuation effect applied to the position in public debt securities) or increase it (because of investment flows). Given the purpose of this work, it is more conservative to make the former assumption: this is performed as robustness check in section 5.2.

Last but not least, we have also assumed that other financial variables were unaffected by the counterfactual scenario. This is disputable, in particular for the exchange rate and the yields on domestic securities issued by non-financial corporations. Concerning the exchange rate, it implies for example that the overall demand for euro denominated securities remains stable in the counterfactual. Concerning the spread on securities issued by non-financial corporations, it would mean that the decrease in the demand from domestic financial sector is compensated by either an increase in the demand of other investors, or – more probably in our opinion – a decrease in issuances. Such mechanisms make sense considering the non-significance of the coefficient associated with the spread on French NFC debt securities in Table 2 : if domestic demand is not very sensitive to prices, then "something else" must adjust to allow market clearing. Moreover, in recent years, yields have been negatively correlated with growth in the French corporate bond market volumes. In section 5.2, we look at the counterfactual scenarios when the domestic financial sector is forced to keep a constant amount invested in the French securities issued by non-financial corporations (both debt and stocks).

5.2 Robustness checks

We perform various tests to check the robustness of our estimations.

Concerning the estimation of the parameter β_s in the portofolio allocation decision, we test alternative specifications:

- We change the instruments for the French-German spread in Reg1, by dropping its lag value as a predictor of its current value. With this specification, the coefficient β_s increases to 0.17 (vs 0.13 in Reg1) but is less significant (at 10% level).
- We run Reg1 dropping asset classes referring to stocks. With this specification, the coefficient β_s drops at 0.10 (vs 0.13 in Reg1) and is significant only at 15% due to the decrease in the number of observations.
- We run Reg1 keeping only observations prior to the Public Sector Purchase Program (March 2015). With this specification, the coefficient β_s remains stable at 0.14 (vs. 0.13 in Reg1) but is not significant due to the decrease in the number of observations.

Concerning the counterfactual scenarios:

- We relax the assumption that the total amount of the security portfolio of the domestic financial sector remains unaffected by the changes in the sovereign spread. Alternatively, we assume that this amount decreases due to valuation effects on public debt securities. This does not change dramatically the results provided in section 4. In the median scenario B, this assumption adds an extra 2 bp (40 bp instead of 38 bp) to the variations in yields triggered by non-residents' divestments.
- We run the counterfactual, keeping the investments in French NFC securities and stocks constant. In this case, the sovereign rate in scenario B is increased by 48 bp instead of 38 bp. This difference is explained by the fact that forcing reallocations toward domestic public debts to occur only against foreign securities is quite restrictive, and therefore requires higher degree of compensation for domestic investors.

6 Conclusion

The goal of this paper is to assess the effect of non-resident investments on the French sovereign spread. It relies on estimating the increase in yields that is necessary for the domestic financial sector to reallocate enough of its securities portfolio to compensate for non-resident divestments.

Given our estimates of the impact of yields on the financial sector portfolio composition, we conclude that if the share of non-residents in non-central bank holdings of French public debt had decreased by 5pp over the year from 2016-Q3 to 2017-Q3, the sovereign yield would have been by around 38bp higher in 2017-Q3. Given the amount and the maturity of gross debt issuances over this period, and using the German bund yield curves for discounting, the corresponding extra cost for public finances in 2017-Q3 would have been around 6Bn €.

This estimate is higher than Andritzky's (2012), who obtains that a 5pp decrease in the share of non-resident holdings is associated with 20bp increase in the yields on sovereign bonds. Therefore, our estimation points to a greater vulnerability of French public debt toward non-resident investments. It highlights the importance of maintaining a stable and diversified foreign investor base for the French public debt.

The question that arises from this result would be to identify the characteristics that make public debt attractive to foreign investors. By construction, we have considered the demand from foreign investors to be exogenous, disregarding its determinants. Understanding these determinants would be the main step forward to move from a "blind" to a meaningful understanding of a "shock on foreign demand". In particular, this further work should allow to measure the probability of a shift in foreign demand, as well as the response of foreign investors to an increase in domestic sovereign yields.

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Annex: Details on the model's estimation

The estimation of the econometric model in section 3 relies on the following considerations:

- The model can be written in a more standard way as $Y_t = \rho Y_{t-1} + Z_t \beta + V_t$, or, stacking indices "t", as $\widetilde{Y} = \widetilde{Z}\widetilde{\beta} + \widetilde{V}$.

  Once this re-arranging has been obtained, least squares estimates can be derived from simple matrix algebra.

- The covariance matrix of $\widetilde{V}$ is not diagonal, but is assumed to be of the form **Γ ⊗ Σ,** as a result of the correlation of error-terms in series and in cross-sections. To be efficient, least squares estimates should therefore account for the covariance matrix, justifying the use of FGLS regression.

- The FGLS regression is affected by endogeneity issues, due to the lag-dependent variables and reverse causality between domestic demand error term and the French spread, that can be handled by instrumentation.

We provide more details on these three points in turn.

1. Re-arranging the model

To begin with, we recall that the model in section 3 is written as:

$$\ln(y_{it}/y_{jt}) = \rho \ln(y_{it-1}/y_{jt-1}) + \alpha_i - \alpha_j + X_{it}\beta_i - X_{jt}\beta_j + \underbrace{X_{\kappa(i)t}\beta_{\kappa(i)} - X_{\kappa(j)t}\beta_{\kappa(j)}}_{=0 \text{ if } \kappa(i)=\kappa(j)}$$
$$+ \underbrace{X_{r(i)t}\beta_{r(i)} - X_{r(j)t}\beta_{r(j)}}_{=0 \text{ if } r(i)=r(j)} + \epsilon_{ijt}$$

where $i \in \{1, \ldots, N\}$, j > i (to avoid "duplicates" observations) and $t \in \{1, \ldots, T\}$, and where κ(.) and r(.) are surjective functions from {1, … ,N} (set of asset classes) to respectively {1, … , K} (set of instrument types) and {1, … R} (set of counterpart regions).

We want to show that this model can be written as:

$$Y_t = \rho Y_{t-1} + Z_t\beta + V_t$$

To this purpose, we begin by defining $\mathbf{Y_t}$ as the vector $\mathbf{[ln(y_{it} / y_{jt})]_{i,j>i}}$, of length $\mathbf{L=N(N-1)/2}$; and $\mathbf{V_t}$ as the vector $\mathbf{[\varepsilon_{ijt}]_{i,j>i}}$. Moreover, we define $\mathbf{l(i,j) = l}$ the function that gives the coordinate $\mathbf{l}$ of the observation $\mathbf{ln(y_{it} / y_{jt})}$ in $\mathbf{Y_t}$. Said otherwise, we have:

$$[Y_t]_{l(i,j)} = \ln(y_{it}/y_{jt})$$

$$[V_t]_{l(i,j)} = \epsilon_{ijt}$$

Then, we define $\mathbf{A}$ as the matrix $\mathbf{(I_{1.} \dots I_{N.}) - (I_{.1} \dots I_{.N})}$, where $\mathbf{I_{ij}}$ is the L-dimension vector that is 1 at the coordinate $\mathbf{l(i,j)}$ and 0 at all other coordinates, and where $I_{i.} = \sum_j I_{ij}$ and $I_{.j} = \sum_i I_{ij}$. We also define the vector $\boldsymbol{\alpha} = (\boldsymbol{\alpha_1}, \dots, \boldsymbol{\alpha_N})'$ such that:

$$[A\alpha]_{l(i,j)} = \alpha_i - \alpha_j$$

We define the matrix $\mathbf{Z_t^{assets}}$ as the matrix of asset-specific variables. The l^{th} row of $\mathbf{Z_t^{assets}}$ is given by $[Z_t^{assets}]_{l(i,j),\cdot} = (\varphi_{l(i,j)}(1) X_{1t}, \dots, \varphi_{l(i,j)}(N) X_{Nt})$ with $\varphi_{l(i,j)}(x)$ equals to 1 for x=i, -1 for x=j, and 0 otherwise. We also set $\beta^{assets} = (\beta_1', \dots, \beta_N')'$, such that:

$$[Z_t^{assets}\beta^{assets}]_{l(i,j)} = X_{it}\beta_i - X_{jt}\beta_j$$

We define the matrix $\mathbf{Z_t^{instru}}$ as the matrix of instrument type effects. The l^{th} row of $\mathbf{Z_t^{instru}}$ is given by $[Z_t^{instru}]_{l(i,j),\cdot} = (\psi_{l(i,j)}(k_1) X_{k_1 t}, \dots, \psi_{l(i,j)}(k_K) X_{k_K t})$ with $\psi_{l(i,j)}(x)$ equals 1 if x=κ(i) and κ(i)≠κ(j), -1 if x=κ(j) and κ(i)≠κ(j), 0 otherwise. We also set $\beta^{instru} = (\beta_{k1}', \dots, \beta_{kK}')'$, such that:

$$[Z_t^{instru}\beta^{instru}]_{l(i,j)} = \underbrace{X_{\kappa(i)t}\beta_{\kappa(i)} - X_{\kappa(j)t}}_{=0 \text{ if } \kappa(i)=\kappa(j)}\beta_{\kappa(j)t}$$

And we define identically $\mathbf{Z_t^{region}}$ and β^{region} such that:

$$[Z_t^{region}\beta^{region}]_{l(i,j)} = \underbrace{X_{r(i)t}\beta_{r(i)} - X_{r(j)t}}_{=0 \text{ if } r(i)=r(j)}\beta_{rt}$$

Eventually, the original model can be re-written as:

$$Y_t = \rho Y_{t-1} + A\alpha + Z_t^{assets}\beta^{assets} + Z_t^{instru}\beta^{instru} + Z_t^{region}\beta^{region} + V_t$$
$$Y_t = \rho Y_{t-1} + [A \quad Z_t^{assets} \quad Z_t^{assets} \quad Z_t^{region}][\alpha' \quad \beta^{assets}{}' \quad \beta^{instru}{}' \quad \beta^{region}{}']' + V_t$$

$$\boldsymbol{Y_t = \rho Y_{t-1} + Z_t \beta + V_t}$$

2. About the error term

We make no specific assumptions about the variance of V_t : $cov(V_t) = \Sigma$. We allow moreover V_t to be autocorrelated of order 1 : $V_t = \gamma V_{t-1} + \zeta_t$ where ζ_ts are iid and independent of V_s for s<t.

Hence, the covariance between the mth coordinate in V_{t1} and the nth coordinate in V_{t2} is given by (for $t_2<t_1$):

$$cov(V_{t_1}^m, V_{t_2}^n) = cov\left(\gamma^{t_1-t_2} V_{t_2}^m + \sum_{\tau=0}^{t_1-t_2-1} \gamma^\tau \zeta_{t_1-\tau}^m, V_{t_2}^n\right) = \gamma^{t_1-t_2} cov(V_{t_2}^m, V_{t_2}^n) = \gamma^{t_2-t_1}\Sigma^{(m,n)}$$

And the covariance matrix of the vector of stacked **V_t**s, **$[V_t]_t$,** is given by **$\Gamma \otimes \Sigma$**, where **$\Gamma^{t1,t2} = \gamma^{|t1-t2|}$**.

3. Estimating the model with FGLS and instrumentation of the dynamic bias

Using FGLS estimation is a standard practice in models with autocorrelated disturbances. It seems all the more adequate given that our model allows for another form of covariance in contemporaneous error terms (Σ).

In FGLS, a first step intends to provide an estimate of the parameters of the covariance matrix (γ and Σ) using OLS. However, in the presence of a lagged dependent value in the model, this first step does not give consistent estimates of the covariance matrix. This point is notably put forward by Hakanata (1974) and Wallis (1967). We follow the spirit of their solutions by performing the first step of FGLS using IV regression where Y_{t-1} is instrumented with $\hat{Y}_{t-1}$, given that $\hat{Y} = P_{Z,Z_{-1}} Y$, P_Z being the orthogonal projector on the linear subspace generated by Z. Moreover, we instrument the French-German spread by its predicted value from its regression on the German sovereign rate, the Italian spread and the lagged value of the French-German spread to remove the possibility of reverse causality between domestic demand error term and the French spread.

From the estimated residuals obtained in this regression, we estimate the covariance matrix of the residuals with:

$$\hat{\Sigma} = \frac{1}{T - dim(\beta)} \sum_t V_t V_t'$$

$$\hat{\rho} = \frac{1}{L} \sum_l \frac{\sum_t V_t^l V_{t-1}^l}{\sum_t (V_{t-1}^l)^2}$$

We finally use the estimated covariance matrix of the residuals for the regression of Y on the explanatory variables, instrumenting again the lag dependant variable and the French-German spread.