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Document de Travail
Working Paper
2020-25

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Should environment be a concern for competition policy when firms face environmental liability ?

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July 6, 2020

Abstract

This paper considers an oligopoly where firms produce a joint and indivisible environmental harm as a by-product of their output. We first analyze the effects on the equilibrium of alternative designs in environmental liability law, secondly, we discuss the rationale for "non-conventional" competition policies, i.e. more concerned with public interest such as the preservation of human health or environment. We study firms decisions of care and output under various liability regimes (strict liability vs negligence) associated with alternative damages apportionment rules (per capita vs market share rule), and in some cases with damages multipliers. We find that basing an environmental liability law on the combination of strict liability, the per capita rule, and an "optimal" damages multiplier, is consistent with a conservative competition policy, focused on consumers surplus, since, weakening firms' market power also increases aggregate expenditures in environment preservation and social welfare. In contrast, a shift to the market share rule, or to a negligence regime, may be consistent with a restriction of competition, since firms' entry may instead lead to a decrease in aggregate environmental expenditures and losses of social welfare. Nevertheless the fine tuning of the policy requires specific information from a Competition Authority, which we discuss as well.

Keywords: Strict liability; negligence; damages apportionment rules; market share liability; environmental liability; competition policy.

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1 Introduction

In the very recent period, competition policy has been advocated as a mean to reach environment preservation. Before Ursula von der Leyen the new President of the European Commission, announced as a priority the issue of environment sustainability (European Green Deal), the European Parliament in February 2019 asked for an adaptation of competition policies to take into account issues of public interest such as environmental sustainability, corporate social responsibility and so on.[1] The argument is that "competition law may be an obstacle to competition restraints that would nonetheless promote welfare, by ensuring more sustainable consumption and production for instance [...] When production involves a negative externality, such as a harmful by-product of output (e.g. pollution) or the decentralized use of a common resource, then a restraint of trade among competitors and the ensuing drop in output will necessarily limit the negative externality (see Cosnita-Langlais (2020) page 1)". In this paper, we contribute to this debate, analyzing possible designs for environmental liability laws, considering how firms' decisions are constrained by these alternative environmental liability laws, and finally discussing the consequences for (non-conventional) competition policies.

We consider the realistic situation where firms in an oligopoly produce a joint and indivisible environmental harm as the by-product of their output. Firms' investments for the preservation of the environment are similar to contributions to a public good, but aside from the precaution cost, the existence of environmental liability law compels firms to bear supplementary costs reflecting their liability burden, adding to productive expenditures in production inputs. The characteristics of this liability cost for environmental harm reflect the specific design of the law, which may be described as the combination of a liability regime (negligence vs strict liability) that requires or not a targeted precautionary activity to avoid any liability burden, a damages apportionment rule (per capita vs market share rule) that defines the way total damages for harm done to the environment are allocated among responsible firms, and a damages multiplier (loosely speaking, punitive damages) that expands the effective damages paid above the observed value of the environmental damages suffered. Here however, we consider a basic competition policy focused on consumers surplus and the expansion of aggregate output on the market thanks to firms' entry on the market.[2] Our concern is twofold: On the one hand, do environmental liability laws give efficient incentives to firms for the protection of environment, or at least, is there a specific legal design that dominates the others according to the Social Welfare criterion ? On the other hand, to what extent are environmental liability laws constraining competition policy, justifying non-conventional, more lenient actions with the purpose of increasing firms' expenditures in environment preservation ? The analytical framework we use to tackle these issues is based on a symmetric oligopoly *à la Cournot*, where

[1] https://www.competitionpolicyinternational.com/eu-parliament-demands-fundamental-overhaul-of-competition-policy/.

[2] One of us has analyzed the stability of cartels under the market share rule (Baumann, Cosnita-Langlais and Charreire, 2020).

firms produce a homogeneous good for a constant marginal cost. The cost of precautionary measures is modeled as a fixed cost with respect to production, and liability cost as a function of the expected environmental harm specific to the design of the liability law. The expected environmental harm is related both to expenditures in precaution and production, and increases with the industry output at a more-than-proportional rate. This "cumulative effect" (Daughety and Reinganum 2014) is relevant in contexts where environmental harms are the result of non linear effects, i.e. responses more than proportional to the exposition to dangerous/toxic substances, and/or because harms can only be observed after lagtimes and long latency periods, once a serious environmental deterioration is achieved.[3]

Regarding our first research question, we show that environmental liability laws generally fail in achieving efficient care expenditures. Firms follow suboptimal decision rules in the setting of precautionary measures as well as in the choice of output, and thus reach equilibrium levels that may be excessive as well as insufficient with regards to socially optimal ones. The introduction of optimal damages multipliers under strict liability allows firms to adopt efficient rules in care activity, but the outcome at equilibrium is nevertheless that care as well as output are lower than what is required in a Pareto efficiency. Under negligence (with a standard of care) firms face efficient incentives for care activities, but choose excessive levels of output and care. The main implication of this part of the paper is that environmental liability laws, alone, cannot attain the social optimum in a context where care and output decisions are interrelated.

With regards to our second research question, we analyze how the specific design of environmental liability laws is constraining the objective of competition policy. A law based on strict liability associated with the per capita rule, and an optimal damages multiplier, has an appealing feature for a Competition Authority. In implementing the law, Court actions do not impinge on its domain: a standard/conservative competition policy will succeed in increasing consumer surplus thanks to the expansion of aggregate output, and this will be accompanied by an increase in aggregare care expenditures. Moreover, such a policy is welfare improving, and should market structure be closer to perfect competition then the equilibrium would coincide with the social optimum. In contrast, we show that an environmental liability law based on the market share rule under strict liability, or negligence with a standard of care, does not verify this property. Such designs translate to firms a structure of liability cost that develops anti-competitive effects, as it increases with the number of firms on the market. Thus, firms' entry may lead to a cut in individual output and care decisions large enough to produce a decrease of the aggregate output and care levels, despite more firms compete on the market. Nonetheless, Social Welfare under such laws is maximized for a finite number of firms; in contrast, as the market structure becomes closer to perfect competition, the aggregate care and output levels fall short of their optimal values. In all, strict liability

[3] Daughety and Reinganum (2014) give several examples in the area of environment, food, and health. Friehe and Langlais (2017) deal explicitly with cases where the cumulative effect arises because of repeated accidents, and they analyze the issue of dynamic incentives under tort law.

with the per capita rule and optimal damages multipliers appears as an "ideal" environmental liability law, a kind of first best design, and is consistent with a standard competition policy dedicated to the improvement of competition on the market. This ideal policy mix requires no specific coordination between Courts and the Competition Authority. In turn, alternative designs for environmental liability law put stronger constraints on firms as well as a Competition Authority, that may justify non-conventional competition policies, limiting firms' entry, as a kind of second best solution. However, to reach the fine tuning of such orientation, a Competition Authority needs specific information on Courts' behavior as well as care technologies. To sum up, the key point for a Competition Authority is the knowledge of the impact of liability law on firms production costs, since this cost depends on the liability regime as well as the sharing rule of damages between firms.

Section 2 reviews the literature. Section 3 introduces the model and solves the social optimum. Section 4 analyzes the equilibrium of the industry under Cournot competition, when strict liability is associated with either the *per capita* or the *market share* rule. We study whether the combination of a damages multiplier and firms' entry have the potential to recover the social optimum. Section 5 considers the implications of a shift from strict liability to the negligence rule. Section 6 affords some robustness checks, and discuss the implications for competition policy. Section 7 concludes.

2 Literature review

The central issue of the paper being the design of environmental liability laws, it is worth starting with some institutional and legal considerations, and existing environmental laws. Comprehensive Environmental Response, Compensation, and Liability Act - CERCLA - in USA has been adopted by the Congress in 1980 (and amended by the Superfund Amendments and Reauthorization Act of 1986) as a tool allowing to clean up uncontrolled or abandoned hazardous-waste sites existing throughout the United States, as well as accidents, spills, and other emergency releases of pollutants and contaminants into the environment. All activities targeted by the Act are subject to strict liability, and in cases where the Superfund is activated, the government is allowed to use punitive damages; finally, liability is joint and several.[4] But CERCLA is silent on issues in relation with damages apportionment. The Environmental Liability Directive of the European Union was passed for similar reasons, stating in its preamble that: "There are currently many contaminated sites in the Community, posing significant health risks, and the loss of biodiversity has dramatically accelerated over the last decades. Failure to act could result

[4] According to CERCLA, the *liable party* is the owners or operators of sites where a hazardous substance has been released, as well as the generators and transporters of hazardous substances which have been released. In cases where the Environmental Protection Agency is forced to use the Superfund, *punitive damages* may be imposed up to three times the cleanup costs incurred from the owner or operator of the property or from the generator of the hazardous materials. When activating the Superfund, the Environmental Protection Agency designates a *Potentially Responsibile Party* (ideally, *the deep pocket*) to implement or finance the cleanup of a site on which hazardous materials have been found. Because *liability is joint and several*, PRP is sent scrambling to identify other PRPs to whom it can look for contribution (Smith 2012). Later on, the 1990 Oil Pollution Act has been prompted after the 1988 Exxon Valdez disaster (major oil spills in US waters).

in increased site contamination and greater loss of biodiversity in the future" (Environmental Liability Directive 2004/35/CE of the European Parliament and the Council, alinea (1) page 2). Alike CERCLA, the European Directive introduces a distinction between operations and activities that are subject to strict liability all being listed in its Annex III,[5] and those (not listed) that are subject to negligence. However, it does not make a definite choice regarding the rule governing damages sharing among multiple party causation; instead, it states that liability apportionment should be determined in accordance with national law.

Indeed, statute law (both in common law countries and civil law countries) does not provide such provision for apportioning damages among multiple tortfeasors, but case law provides traditional solutions. Courts decisions are founded on the seriousness of each defendant's misconduct to establish how the damages compensating the victims will be shared among the different injurers. Accordingly, two polar rules have emerged and are specifically of interest here, e.g. the *no liability rule* and the *per capita rule*.[6] Less often, Courts use the solution called the *market share rule*, according to which total damages are shared between tortfeasors competing in the same industry, in proportion to their market share. It first appeared in 1980 in the Californian case "Sindell v. Abbott Laboratories", and US Courts have limited its use up to now to toxic torts (exposure to a chemical) such as in the abestos litigation (*Becker v. Baron Bros*, 649 A.2d 613 (N.J. 1994).) or the MTBE (a gasoline additive) litigation (*In re Methyl Tertiary Butyl Ether*, 175 F. Supp. 2d 593; S.D.N.Y. 2001). In France, two cases come to the mind where the market share has been applied: the Orly Airport litigation for noise pollution (1988, Cass. 2e civ, No 86-12.543), and more recently the *Distilbène litigation* [7], a case close to Sindell's one. Some scholars advocate for an extensive use of the market share rule to environmental liability or competition law (Ferey and G'sell 2013, G'sell 2010).

As a paper focused on environmental liability, our paper of course has connections with the vivid literature of the 90s focused on extended liability as a solution to injurer's insolvency and the judgment-proof problem (see for example Beard 1990, Boyd and Ingberman 1994,1997, Boyer and Laffont 1997, Innes 1999, Pitchford 1995, Ringleb and Wiggins 1990, Shavell 1986). Van Veld (2007) considers the effects of liability on firms' size, and discusses the welfare impacts following the restructuring of judgment-proof firms. In the present set up with a symmetric oligopoly, the judgment proofness is of weakest interest since either all firms are judgment-proof, or no firm is. In contrast, our paper considers the interplay

[5] Indeed, Annex III lists oprations and activities that are otherwise convered by a regulation of the European Parliament or directive of the European Council.

[6] Basic causation requirements imply that the contribution of each defendant, among the pool of identified tortfeasors, should be proportional to its contribution to the harm of the victims. Thus, if no fault is established for any defendant, then no one is liable (no liability rule); if all the defendants committed a fault with the same intensity, then the damages are equally shared between them (per capita rule). In France, for example, Article L. 162-18 of Code de l'environnement, states that: *Lorsqu'un dommage à l'environnement a plusieurs causes, le coût des mesures de prévention ou de réparation est réparti par l'autorité visée au 2 de l'article L. 165-2 entre les exploitants, à concurrence de la participation de leur activité au dommage ou à la menace imminente de dommage.*

[7] See Tribunal de Grande Instance of Nanterre, April 10, 2014 n 12/12349 and n 12/13064. Both cases concern the diethylstilbestrol (DES), a product delivered to pregnant women and which caused years later injuries to the children exposed *in utero*.

between liability and market mechanisms, taking into account the strategic interactions between firms.

The market share rule, mainly used in personal injury cases, has motivated a debate among US scholars focusing its consistency with basic causation requirements, as well as its moral/ethical roots (see Dillbary 2011, Priest 2010). The debate has known a revival in France with the 2014 *Distilbène litigation*. Several French scholars argue in favor of traditional solutions adopted for damages apportionment (Molfessis 2015, Quézel-Ambrunaz 2010). Others have defended the market share rule on the grounds that market shares may be a proxy for the likelihood of individual liability at the stage of damages apportionment, in contexts of joint liability characterized by hard uncertainty and ambiguous causation (Ferey and G'sell 2013, G'sell 2010), such as when the set of all potential offenders is identified without any doubt, but it is impossible to establish the origin of the harmful product (evidence are missing, or destroyed). In contrast, our paper studies the case for joint liability from an *ex ante* perspective, affording a comparative analysis of different apportionment rules (per capita vs market share rules) and their impact on the incentives to undertake precaution in richer strategic environments (oligopoly market). Moreover, we discuss different policy implications, including the choice of a liability regime (strict liability vs negligence).

The interactions between liability laws and other kinds of public interventions has motivated an important literature in *Law & Economics,* to begin with the mix between *ex ante* regulation and *ex post* liability in different informational contexts (Bhole and Wagner (2008), Friehe and Langlais (2015), Innes (2004), Kolstad, Ulen and Johnson (1990), Schmitz (2000), Shavell (1984a,b)). Studies focused on the mix between competition policy and liability law are scarce. Marino (1991) studies product liability for joint but divisible harms, and analyses the effects of firms' entry in a oligopoly under strict liability and the market share rule. Daughety and Reinganum (2014) consider the case of product liability law for divisible but cumulative harms, and analyze firms' entry under strict liability with a "modified" market share rule vs negligence, while Friehe (2014) addresses the issue of tacit collusion under liability laws. Baumann, Cosnita-Langlais and Charreire (2020) study also the stability of cartels under liability laws in case of indivisible environmental harms, but do not introduce precautionary expenditures. Our paper considers instead the case for environmental liability law, and discuss the interplay between competition policy and different designs for environmental liability laws.

Our work is a contribution to the debate regarding the definition of new areas for competition policy, i.e. whether competition authorities should take into account public interests above competition objectives. Existing works focused on the issue of environmental protection assume that firms' contributions to environmental protection are voluntary (Hashimzade and Myles (2017), Schinkel and Toth (2019), Spiegel and Schinkel (2017), Treuren and Schinkel (2018)). In contrast, our analysis states that firms' decisions are constrained by environmental liability, and suggests that this raises coordination issues between Courts (focused on the incentives to precautionary expenditures) and Competition Authorities (focused on consumers surplus), in order to improve environment protection measures.

Lastly, our paper is obviously connected to the vast literature on product liability law. Tracing back to the pioneering works by Marino (1988,1991) and Polinsky and Rogerson (1983), a recent stream of the *Law & Economics* literature has analyzed the performances of product liability in alternative competitive environments.[8] To sum up, this literature arrives at a quite optimistic conclusion regarding product liability in oligopolistic markets when consumers' expected harm is modeled as a linear function of firms' individual output[9]: under any liability regime (strict liability, negligence, as well as no liability) firms are induced to choose the first best optimal level of care. Hence, there would be "no role for the influence of market structure or strategic interaction on liability policy" (Daughety and Reinganum 2013) in this case. This equivalence of liability rules, and the efficiency result fall down when the expected harm to victims depends on the level of output in a more complex way (Marino 1988, Daughety and Reinganum 2014).[10] Daughety and Reinganum (2014) show that for cumulative harms to consumers, strict liability associated with a "modified" market share rule (see below) and an optimal damages multiplier, dominates the negligence rule (that degenerates into a no liability rule). In contrast our paper shows that for a joint, indivisible and cumulative environmental harm, strict liability with the per capita rule and an optimal damages multiplier, dominates strict liability with the market share rule, as well as the negligence rule – whether negligence is based on a due care level, or a standard of care.

3 The framework

3.1 Assumptions

We consider a symmetric oligopoly *à la Cournot* where $n > 2$ firms compete for a homogeneous product. Both consumers and firms are risk neutral. The quantity of goods produced by firm i is denoted $q_i (i = 1, ..., n)$, and $Q = \sum_{i=1}^{n} q_i$ represents the industry output. The market demand is $P(Q) = a - bQ$, $(a > 0, b > 0)$. We assume that a firm i $(\forall = 1, ..., n)$ operates at a cost given by $C(q_i, x_i) = x_i$, where x_i represents the level of care of firm i; this means that the marginal cost corresponding to the use of all productive inputs (aside from care expenditures) is constant and null, and that care expenditures do not depend on firm's activity level. Finally following Tietenberg (1989), the product may cause a joint and

[8] To have a focus on oligopoly markets: quantity competition (Baumann and Friehe 2015, Daughety and Reinganum 2013, 2014, 2017), or price competition (Cournot with product differentiation: Baumann, Friehe and Rasch 2018; spatial competition: Baumann, Friehe and Rasch 2016, Chen and Hua 2017).

[9] Polinsky and Rogerson (1983) assume that the harm conditional to an accident is constant, and that the probability of accident is proportional to care. Marino (1991,1988) and Daughety and Reinganum (2014) reach similar conclusions when the probability of harm is linear in the output.

[10] Marino (1988) considers "scale effects" on the probability of accident, when it depends on the level of output, and shows that comparative performances of the liability regimes depends on the one hand on the nature of these scale effects (the probability of harm may increase or decreases with the output), and on the other, on the intensity of competition (pure competition vs Cournot oligopoly). Daughety and Reinganum (2014) deal with a case of a cumulative harm, e.g. when the expected harm is proportional to the square of the output; they find that for both a monopoly and an oligopoly (assuming that firms' specific risks are independent) strict liability maintains efficient incentives to take precaution, in contrast to no liability and negligence; however, strict liability induces an underprovision both of care and output at equilibrium as a result of the distortions due to the imperfect competition. Daughety and Reinganum show that the superiority of strict liability holds when firms specific risks are interdependent, to the extent that strict liability is associated with the use of an optimal damages multiplier in order to maintain efficient incentives to take precaution.

indivisible harm to society (third party victims, not consuming the good),[11] such that the expected harm is defined as $H(Q,X) = h(X).Q^2$, where $h(X)$ such that $h(0) > 0$ and $h(X) < 1$ for any $X \geq 0$, is the joint probability of harm and $X = \sum_{i=1}^{n} x_i$ is the aggregate care expenditure.

Finally, most of the central results of the paper are obtained thanks to a set of very simple assumptions, namely:

H1a: $b > 2.h(0)$; **H1b:** $h'(X) < 0 < h''(X)$ for any $X \geq 0$; **H1c:** $H(Q,X)$ is convex in (Q,X).

H1a is a very basic assumption regarding two parameters of the paper − requiring that the price sensibility of market demand is not too large compared to the baseline probability of accident. H1b is usual in the literature on liability rules, saying that the probability function is decreasing and convex with care expenditures. H1c has sound economic motivations, since it establishes that the expected external harm to society is supposed to be a convex function, which is a very natural and reasonable assumption for a cost function. Remark that H1a and H1b together imply that the next condition is satisfied:

$$\text{For any } X > 0 \text{ and } \alpha \leq 2 : b > \alpha\,(h(0) - h(X)) > -h'(X).\alpha X \tag{C1}$$

Several basic static comparative results require (C1) to hold. H1c in turn implies:

$$\text{For any } (Q,X) > 0 : \left[h(X)h''(X) - 2\,(h'(X))^2\right] > 0 \tag{C2}$$

Second order conditions are satisfied (most of the time) under (C2), as well as the stability of Nash equilibria (see below). When necessary, we will introduce additional restrictions in order to qualify our results.

Remark 1: Care expenditures are captured here as a fixed cost, independent of the units of output produced − care is durable, with the terminology of Nussim and Tabbach (2009). This makes sense when such expenditures correspond to the acquisition of safety technologies which are very specific assets adapted to a firm's business, and as such represent inputs that are essential for the firm's activity but that cannot be relocated to other businesses. This encompasses a large set of safety devices: it can be (smoke) alarms and detectors (for the leakage of toxic substances), a videosurveillance device, or the installation of a sufficient number of circuit-brakers. In the same vein, it can be also the investment in a containment system (to prevent the leakage of dangerous products outside of the plant, and limit the release in the environment), the capacity of which does not reflect the normal activity on the plant

[11] An alternative interpretation relies on the fact that once the case is litigated, it will be impossible for Courts – at a sustainable economic and social cost – to disentangle the individual responsibilities of each firm (evidence are missing, or destroyed; see the Distilbène litigation), at least too costly to reach this goal (given the number of injurers and the complexity of the phenomenon). For example, in the case of the Orly Airport litigation for noise pollution (1988, Cass. 2e civ, No 86-12.543), the individual contribution of each planes and/or company cannot be easily assessed, at least at a reasonable economic or social cost - neither instantaneously, nor across periods.

on a day-to-day basis, but an abnormal event, such as the biggest accident at the plant. Alternatively, care has a durable nature when it corresponds to R&D investments dedicated to the design of "green products" and entails a substitution between polluting goods and products with lower environmental impacts. In contrast, precaution is said to be non-durable when it is related to the usual activity at the plant and the quantity supplied. It can be establishing check-lists and adopting regular check-up procedures for equipment, together with their regular cleaning to avoid and detect potential failures and repair in time; as well as the observance of resting periods for the employees in order to limit their fatigue and the occurrence of human failures. These kinds of safety measures imply expenditures that are roughly speaking proportional to the number of working hours for employees as well as the intensity with which equipments are used, and thus proportional to the final output of the plant. Care can also be understood as having a non-durable nature when we consider various activities dedicated to the collect of end-of-life products, and/or efforts focused on recycling industrial wastes and components of end of life products.[12] In a companion paper (Charreire and Langlais 2020), we compare Joint Liability vs Joint and Several Liability for a non-durable care, under strict liability; we show that the main results of the current paper (propositions 1 to 5) still hold.

Remark 2: Indivisibility in environmental harms mean two different things for practical purposes. It may be the result of simultaneous actions by firms the activities of which are concentrated on the same geographical area; we can think of noise pollution in the suburban neighborhood of Orly airport (see below, a famous litigation case), or river pollution with acids from tannery activities and dyeing plants in Indian cities, or Great Lakes pollution by mercury from the automobile industry in North America. However, indivisibility do not require the contemporaneous actions of firms to exist. In famous litigation cases, the existence of indivisibilities has been motivated *de facto* because it was impossible for Courts to separate individual responsibilities at a reasonable economic and social cost (see "Sindell v. Abbott Laboratories", or the MTBE litigation in USA; in France, more recently the *Distilbène litigation*).

3.2 The benchmark : Social Welfare maximization

Social Welfare is the sum of consumers' total utility minus the sum of firms' operating costs (cost of care), minus the expected harm: $SW = \int_0^Q P(z)dz - \sum_{i=1}^n c(x_i) - H(Q, X)$. We will directly use firms' symmetry in order to reduce the dimension of the optimization problem, allowing the benevolent planner to focus on a symmetric optimum where $q_1 = ... = q_n = q$ and $x_1 = ... = x_n = x$, with the aggregate output and care levels given by respectively $Q = nq$ and $X = nx$. As a result, Social Welfare may be written as a function of (q, x):

[12] Considering the different precautionary measures undertaken at a plant, one may argue obviously that some will have a durable nature and the others will be of a non-durable nature. We contrast the two "pure" cases for pedagogical reasons, whereas the literature about product liability assumes usually that care is non-durable.

$$SW = anq - \frac{b}{2}(nq)^2 - n.x - h(nx)(nq)^2 \tag{1}$$

The first-order conditions for an interior solution (q^{sw}, x^{sw}) are written:

$$a - bnq = 2h(nx)nq \tag{2}$$

$$-h'(nx).(nq)^2 = 1 \tag{3}$$

meaning that the optimal output level (see condition (2)) must be pushed up to the point where average market proceeds (inverse of market demand, LHS) are equal to the marginal cost associated with expected harm (RHS); similarly (condition (3)), optimal care expenditures are such that the marginal cost of care (RHS) is equal to the social marginal benefits associated with the decrease in the expected harm (LHS). Second order conditions are verified by convexity of the expected harm function (see Appendix 1); they also imply that output and care are strategic complements, and that the Nash equilibrium satisfying (2)-(3) is unique and stable.

The convexity of the expected harm function also implies that (q^{sw}, x^{sw}) decreases with the number of firms (obvious, the proof is omitted). In contrast, the aggregate output and care at the social optimum do not depend on the number of firms: substituting with $Q = nq$ and $X = nx$, it is easy to see that (2)-(3) give $Q^{sw} = \frac{a}{b+2h(X^{sw})}$ and $-h'(X^{sw})(Q^{sw})^2 = 1$ which do not depend on n. As a consequence, the optimal expected harm $h'(X^{sw})(Q^{sw})^2$ and Social Welfare at optimum $SW(Q^{sw}, X^{sw})$ do not depend on n. Hence, an increase in n has also no effect on Social Welfare (as long as the cost of care is the unique source of fixed costs for firms, which is consistent with our interpretation that care is durable).

4 Oligopoly equilibrium under strict liability

We assume in this section that the liability regime is based on strict liability according to which Courts do not rely on a negligence test, and only need to establish causation to prove the responsibility of firms. We discuss the case for negligence in section 5. In our context of joint and indivisible harms to the environment, the legal doctrine that allows the Plaintiff to sue several defendants in a single trial is Joint Liability. Given Joint Liability, the main issue when Defendants are strictly liable is the way total damages will be apportioned between them.

Remark 3: Other legal doctrines of large application exist, for cases where multiparties are involved in the harm a victim is suffering from (Kornhauser and Revesz 1989). One is Several Liability, according to which the Plaintiff is entitled with the right to sue each one of the firms in separate trials. Several

Liability is not relevant here because in cases with indivisibility, establishing facts on an individual basis and separate causations for each defendant is not possible – or will leave the Plaintiff with the impossibility to obtain the payment of damages and be compensated for the harms borne. An alternative doctrine is Joint and Several Liability which makes each defendant responsible *in solidum*, and provides the Plaintiff with the opportunity to obtain the payment of total damages from any one of the defendants. We discuss the case with Joint and Several Liability in a companion paper.

Remark 4: We do not provide specific details regarding procedural rules, nor with settlement issues (existence of pre-trial negotiations), and generally speaking we ignore the existence of litigation costs, as well as strategic aspects of settlements and litigations and the associated impacts on litigation costs. Similarly, we are not specific with Plaintiff type/identity. It may be a (public) environmental agency – in this case, the trial will be heard by an administrative jurisdiction/Court; or it may be a private association involved in the protection of environment, or generally speaking a third-party victim (the Plaintiff is an individual who has neither economic nor contractual relationships with the Defendants) – hence, the case will be settled in front of a civil Court. Finally, we assume here that in the implementation of the law, Courts choose the way total damages are shared between multidefendants. This corresponds to reality for some national jurisdictions (see the Distilbène litigation in France), whereas for other jurisdictions or under alternative doctrines, the sharing of total damages is a decision of the Plaintiff (see the implementation of CERCLA by Environmental Agencies in USA) who may strategically consider the solvability of the different defendants to secure the recovering of damages. Indeed, this is neutral here since we assume complete information, we rule out the insolvability problem, and do not consider strategic aspects of litigations. The rationale is that the paper is rather focused on liability costs and the way some specific designs of liability rules may shape this liability cost, with the ensuing consequences for firms.

4.1 Joint liability and damages sharing

Let us denote $L_i^{JL}(q_i, x_i) = s_i \times \alpha \times H(X, Q)$ the amount of compensation accruing to firm i; s_i is firm's i liability share, and $\alpha > 0$ is a damages multiplier, exogenously set to 4.2, that Courts may use to inflate total damages. We will investigate the effects of two different damages sharing rules.

The per capita rule. Courts may divide total expected damages equally between all firms pertaining to the industry, i.e. $s_i = \frac{1}{n}$, $\forall i = 1, ..., n$. (C2) guarantees the convexity of the individual liability cost function $L_i^{pc}(q_i, x_i) = \frac{\alpha}{n} H(Q, X)$ with respect to (q_i, x_i). In this case, each firm $i (\forall i = 1, ..., n)$ chooses a level of output and a level of care in order to maximize its profit:

$$\Pi^{pc}(q_i, x_i) = (a - bQ)q_i - x_i - \left(\frac{\alpha}{n}\right) h(X) Q^2 \tag{4}$$

Using the first order conditions (see Appendix 1; second order conditions are met under (C2)), the symmetric Cournot-Nash equilibrium where $q_1 = ... = q_n = q^{pc}$ and $x_1 = ... = x_n = x^{pc}$ solves the system:

$$a - b(1+n)q = 2\alpha h(nx)q \tag{5}$$

$$-h'(nx).\alpha n q^2 = 1 \tag{6}$$

meaning that the output level (see condition (5)) must be pushed up to the point where marginal market proceeds are equal to the marginal cost of liability (RHS); similarly (condition (6)), care expenditures are such that the marginal cost of care (RHS) is equal to their marginal benefits, defined as the decrease in the expected liability cost (LHS). Remark more specifically that the LHS in (5) exhibits the standard distortion due to imperfect competition according to which the marginal market proceeds are smaller than the market demand price (RHS in (5)). In Appendix 1, we verify that output and care levels defined by (5)-(6) are strategic complements, and that the associated Nash equilibrium is stable and unique (under (C2)).

The market share rule. Courts may assess individual contributions according to individual market shares, i.e. $s_i = \frac{q_i}{Q}$, $\forall i = 1, ..., n$. Let us assume that the individual liability cost function $L_i^{ms}(q_i, x_i) = \alpha \left(\dfrac{q_i}{Q} \right) H(Q, X)$ is convex in (q_i, x_i), which implies that $2q_i Q.h(X).h''(X) - (Q + q_i)^2 (h'(X))^2 > 0$. This requirement is stronger than (C2), but as shown in Appendix 1, it allows that second order conditions for profit maximization under the market share rule are satisfied. Firm i chooses now a level of output and care that maximize the profit:

$$\Pi^{ms}(q_i, x_i) = (a - bQ)q_i - x_i - \alpha h(X) q_i Q \tag{7}$$

Once more, using the first order conditions and focusing on the symmetric Cournot-Nash equilibrium, the solution $q_1 = ... = q_n = q^{ms}$ and $x_1 = ... = x_n = x^{ms}$ solves the system:

$$a - b(1+n)q = (1+n)\alpha h(nx)q \tag{8}$$

$$-h'(nx).\alpha n q^2 = 1 \tag{9}$$

with a meaning equivalent to (5)-(6). The LHS in (8) also exhibits the standard distortion due to imperfect competition according to which the marginal market proceeds are smaller than the market demand price. In Appendix 1, we verify that output and care levels defined by (8)-(9) are strategic complements, and that the associated Nash equilibrium is stable and unique (under (C2)).

Before going into the analysis of the performances of both rules, and the comparison with the social optimum, remark how the different sharing rules shape the relationship between output decisions and the individual cost of liability, and the way it translates to the marginal cost of production. More precisely, it turns out that the marginal cost of production/liability under the market share is proportional to the number of Defendants (firms). Throughout the paper, this difference with the per capita rule will drive the results. Remark also that despite the damages multiplier is exogenously set, we assume until the next section that it cannot include values that are too large (see below). This qualification will be relaxed later on, when we will discuss the case for optimal damages multipliers. We have the following result:

Proposition 1 *Assume $1 \leq \alpha < n$. i) Strict liability under the per capita rule yields a level of output and a level of care larger than under the market share rule ($x^{pc} > x^{ms}$ and $q^{pc} > q^{ms}$). ii) Under strict liability with the per capita rule, the equilibrium output and care levels may be larger as well as smaller than their socially optimal levels. iii) Under strict liability with the market share rule, the equilibrium output and care levels may be larger as well as smaller than their socially optimal levels if $\alpha \leq \frac{2n}{1+n}$; if $\alpha > \frac{2n}{1+n}$, the equilibrium output and care levels are smaller than their socially optimal levels.*

Proof: Let us denote $q^{sw}(x)$ the output level that solves (2), for any given value of care; while $x^{sw}(q)$ is the care level that solves (3), for any given value of the output. Similarly, let us denote $q^{pc}(x)$ the output level that solves (5), for any given value of care; while $x^{pc}(q)$ is the care level that solves (6), for any given value of the output; and let $q^{ms}(x)$ be the output level that solves (8), for any given value of care; while $x^{ms}(q)$ is the care level that solves (9), for any given value of the output. i) According to conditions (6) and (9): $\forall q > 0$, $x^{pc}(q) = x^{ms}(q)$. In contrast, according to (5) and (8): $2qh(nx) < (1+n)qh(nx)$ implying that: $\forall x > 0$, $q^{pc}(x) > q^{ms}(x)$. Hence: $x^{pc} > x^{ms}$ and $q^{pc} > q^{ms}$ since care and output are strategic complements under both rules. ii) and iii) From (3)-(6)-(9), it comes that: $-h'(nx).\alpha nq^2 < -h'(nx).(nq)^2$; hence: $\forall q > 0$, $x^{sw}(q) > x^{pc}(q) = x^{ms}(q)$. In turn, (2)-(5)-(8) show there are two opposite effects on output. On the one hand (LHS): $a - (1+n)bq < a - nbq$; this reduces the incentives to produce under strict liability at any level of care, compared to the social optimum. On the other hand (RHS): if $1 \leq \alpha \leq \frac{2n}{1+n}$, then $2nh(nx)q > (1+n)\alpha h(nx)q > 2\alpha h(nx)q$; this increases now the incentives to produce at any level of care, compared with the social optimum. Thus, $q^{sw}(x)$ is not generally comparable with $q^{pc}(x)$ or $q^{ms}(x)$. But if $\alpha > \frac{2n}{1+n}$, then $2nh(nx)q < (1+n)\alpha h(nx)q$ and thus $q^{pc}(x) > q^{ms}(x)$, implying that $x^{sw} > x^{ms}$ and $q^{sw} > q^{ms}$. Hence the result. ■

For both damages apportionment rules, firms have the same best response in terms of care to any feasible activity level. However, firms' private marginal benefits associated with care under strict liability are smaller than their socially optimal value, implying that firms have inefficient incentives to take care at any output level under strict liability.

In contrast, for any given feasible level of care, firms face a marginal cost for liability under the per capita rule that is smaller than under the market share rule (for the same marginal market proceeds); thus at any level of care, firms' best response function in terms of output is larger under the per capita than under the market share rule. The intuition is simple to get: under both liability sharing rules, firms obtain obviously the same market share at equilibrium, $1/n$. However, output decisions are driven by the ex ante individual liability cost which is rule-specific. Under the market share rule, the cost of liability for a firm is related to its individual market power, the larger its market share, the larger its liability burden. Thus, aside from the standard negative competition externality, the market share rule adds a liability cost externality that drives down the output decisions of firms at any level of care. As a result, the market share rule provides firms with more incentives to reduce output and care than the per capita rule.

This said, strict liability (whatever the damage sharing rule) leads to an inefficient rule for output choice since the furniture of the output is driven by two opposite effects. On the one hand, firms' marginal market proceeds under strict liability (for both rules) fall short of their socially optimal value: this is nothing else but the standard output distortion of output that reflects firms' market power under imperfect competition. On the other hand, the marginal cost of liability accruing to a firm under the per capita rule falls short of its optimal value; this is also true under the market share rule if the damages multiplier is small enough ($\alpha \leq \frac{2n}{1+n}$). As a result, strict liability (whatever the damage sharing rule) also exerts inefficient incentives to produce at any level of care, but this is the result of two countervailing influences and the net effect is ambiguous: firms may produce either too much or not enough at any care level. Hence the comparison between the equilibrium values for output and cares under strict liability (for any damages sharing rule) and their optimal values is also generally ambiguous.[13] When the damages multiplier trespasses a threshold ($\alpha > \frac{2n}{1+n}$), the marginal cost of liability under the market share rule increases above its social value, thus shifting the equilibrium below the social optimum.

Let us now consider the impact of firms' entry on the equilibrium under strict liability; we still assume that the damages multiplier is small enough.

Proposition 2 ***(firms' entry under strict liability with a exogenous damages multiplier)*** *i) Under both rules, firms' entry yields a decrease in individual output and care levels. ii) Under the per capita rule, the aggregate output increases with the number of firms, while the aggregate care decreases with the number of firms. iii) Under the market share rule, the aggregate output increase (decrease) with the number of firms when b is large (small) enough, while the aggregate care always decreases with the number of firms.*

Proof. See Appendix 2. i) We show that under the capita rule, the result holds for $\alpha \leq n-1$; although the market share rule does not require such a restriction. ii) In contrast to the social optimum,

[13] It can be understood as the result of the comparison between the slope of two curves: the slope of the marginal market proceeds (mainly driven by the value of b), and the slope of the marginal cost of liability; the proof is omitted.

it is obvious that the equilibrium levels of aggregate output and care under both rules depend on the number of firms. Using (5)-(6), the aggregate output and care levels under the per capita rule satisfy:

$$a - b\left(1 + \frac{1}{n}\right)Q = \frac{2\alpha}{n}h(X)Q \tag{5bis}$$

$$-h'(X).\frac{\alpha}{n}Q^2 = 1 \tag{6bis}$$

It is easy to see that firms' entry entails opposite effects on aggregate output and care expenditures. The LHS in (5bis) is increasing with n, while the RHS is decreasing with n: hence the larger n, the larger the aggregate output $Q^{pc}(X)$ at any level of X. In turn, the LHS in (6bis) is decreasing with n, while the RHS does not depend on n: hence the larger n, the smaller the aggregate care $X^{pc}(Q)$ at any level of Q. However, the conditions $b > 2h(0)$ and $1 < \alpha \leq n - 1$ are sufficient to sign the effect at equilibrium.

iii) Similar effects arise under the market share rule. Using (8)-(9), the aggregate output and care levels satisfy now:

$$a - b\left(1 + \frac{1}{n}\right)Q = \alpha\left(1 + \frac{1}{n}\right)h(X)Q \tag{8bis}$$

$$-h'(X).\frac{\alpha}{n}Q^2 = 1 \tag{9bis}$$

The LHS in (8bis) is increasing with n, while the RHS is decreasing with n: hence the larger n, the larger the aggregate output $Q^{ms}(X)$ at any level of X. In turn, since (9bis) is identical to (6bis), the larger n, the smaller the aggregate care $X^{ms}(Q)$ at any level of Q. Once again, the net effect at equilibrium is ambiguous. Explicit comparative statics show that H1a is not enough, and a sufficient condition for the aggregate output to increase (decrease) is that b be large (small) enough with respect to a new theshold value defined in Appendix 2, larger than $2h(0)$. ■

Proposition 2 establishes that following firms' entry in oligopoly, the effect on output and care at firm level under both rules is the usual/intuitive one. Less market power yields less production at firms level, and given the strategic complementarity between care and output, this also implies less individual care.

The impact on the aggregate care level, identical under both rules, is particularly interesting. The intuition of the result is related to the situation where firms share a joint harm they have produced, and the public good characteristics of care activity: as the number of firms increases, the equilibrium individual share $(1/n)$ in total harm also decreases under both rules, hence diminishing the expected benefits of care more than what would be justified by the decrease in the output level. As a result, firms' entry has such a large a negative impact on individual care levels, that the aggregate care level also decreases: the additional number of firms that invest in care does not compensate the decrease in the individual care (there are more firms, but poorly investing in care).

In contrast, the impact on the aggregate output is rule-specific. Under the per capita rule, we obtain the usual effect according to which the aggregate supply expands following firms' entry (the individual output decreases, but this effect is compensated by more firms producing the output). Inspection of the RHS (5bis) shows that the feedback influence of care on output decision is neglictable as n becomes large under the per capita rule. Thus the effect on the LHS is dominating. In contrast, the effect is ambiguous with the market share rule: inspection of the RHS in (8bis) suggests that the explanation is in the importance of the feedback influence of care on output (the RHS is proportional to $\left(1+\frac{1}{n}\right)h(X)$ and thus tends to 1 as n becomes large) compared to the impact on marginal market proceeds (the LHS is also proportional to $\left(1+\frac{1}{n}\right).b$), the net effect depending on the price sensibility of the market demand (see Appendix 2 for an explicit proof). Depending on the relative size of the marginal cost of production and marginal market proceeds, the decrease in the individual output may be so large that it results in a decrease of the output at the industry level – the entry of new firms does not compensate the large cut in individual production levels.

Obviously, these distortions in care and output decisions imply a loss of welfare. Specifically as regards with the discussion below, the differentiated responses of aggregate output and care suggest that there may exist a trade-off between consumers surplus and environmental harm. Let us assess how Social Welfare evolves as the number of firms increases, starting with the per capita rule. Evaluating (1) at (Q^{pc}, X^{pc}) and differentiating in n, we have (using (5bis)-(6bis)):

$$\frac{dSW}{dn}(Q^{pc}, X^{pc}) = \left[b - 2h(X^{pc})(n-\alpha)\right].\frac{Q^{pc}}{n}.\frac{dQ^{pc}}{dn} - (n-\alpha)\frac{(Q^{pc})^2}{n}\left(h'(X^{pc}).\frac{dX^{pc}}{dn}\right)$$

where $h'(X^{pc}).\frac{dX^{pc}}{dn} > 0$. Still considering a damages multiplier small enough ($1 \leq \alpha \leq n$), it comes that the impact of firms' entry on Social Welfare is generally ambiguous. When the slope of the (inverse) market demand is small enough ($b < 2h(X^{pc})(n-\alpha)$), indeed Social Welfare decreases – the intuition is that aggregate output is too large, and aggregate care expenditures are too low, implying losses of welfare related to an excessive expected environmental harm that is not compensated by consumers surplus. However, as the damage multiplier increases (such that $b > 2h(X^{pc})(n-\alpha)$), there is a balance between the welfare gains provided by aggregate output expansion and the losses that result from the increase in expected harm (decrease in aggregate care expenditures).

Turning to the market rule now, evaluating (1) at (Q^{ms}, X^{ms}) and differentiating in n yields (using (8bis)-(9bis)):

$$\frac{dSW}{dn}(Q^{ms}, X^{ms}) = \left[b - 2h(X^{ms})\left(n - \frac{\alpha}{2}(1+n)\right)\right].\frac{Q^{ms}}{n}.\frac{dQ^{ms}}{dn} - (n-\alpha)\frac{(Q^{ms})^2}{n}\left(h'(X^{ms}).\frac{dX^{ms}}{dn}\right)$$

with once more $h'(X^{ms}).\frac{dX^{ms}}{dn} > 0$. The same comments are relevant here, with one noticeable difference. Although the impact on Social Welfare resulting from the decrease in aggregate care expenditures is still conditioned by the sign of $n - \alpha$, the impact of the increase in the aggregate output is now depending on the sign of $\frac{2n}{1+n} - \alpha$. As seen before (in the proof of proposition 1), when $\alpha > \frac{2n}{1+n}$ the marginal cost of liability is above its socially optimal value; thus the equilibrium corresponds to output and care levels that are below their optimal values. Hence in this situation, increasing n provides welfare gains thanks to the increase in aggregate output, despite the decrease in aggregate expenditures – the intuition is that in this situation the market equilibrium is associated with output distortions that large, that an increase in the number of firms will provide a gain for consumers, whatever the price sensibility of market demand, which will do more than compensate the loss associated with environment deprivation. This effect disappears when $\alpha < \frac{2n}{1+n}$, and we obtain effects on Social Welfare similar to the per capita rule.

These considerations suggest that there may exist a finite number of firms that maximizes Social Welfare, at least as long as the damage multiplier is low enough, under strict liability. However, in any case, the distortion in care decisions is an issue. We now discuss the potential for improving firms care decisions, and the consequences that arise.

4.2 Damages multipliers and Social Welfare under joint liability

We have found for both rules that firms choose an inefficient rule of care (this is not an efficient response at any level of output). Thus, firms' incentives in care activities may be corrected, with the means of a specific value for the damages multiplier $\alpha > 1$, at the disposal of Courts. We focus here on this optimal value of this damages multiplier.

Proposition 3 ***(impact of an optimal damages multiplier)*** *i) For both rules, the optimal damages multiplier is $\alpha^* = n$. ii) In a regime of strict liability (the damages sharing rule being either the per capita or the market share rule) with an optimal damage multiplier, the industry provides insufficient levels of output and care, compared with the social optimum ($q^{sw} > q^{pc} > q^{ms}$ and $x^{sw} > x^{pc} > x^{ms}$).*

Proof: i) Comparing the LHS in (6) or (9) and (3), it comes that $\alpha^* = n \Rightarrow -h'(nx).\alpha^* nq^2 = -h'(nx).(nq)^2$, in which case $x^{sw}(q) = x^{pc}(q) = x^{ms}(q)$, $\forall q > 0$. ii) According to the RHS of (2) and (5), when $\alpha^* = n \Rightarrow 2\alpha^* h(nx)q = 2nh(nx)q$ implying that $q^{sw}(x) > q^{pc}(x)$, $\forall x > 0$. Given i), we obtain that: $q^{sw} > q^{pc}$ and $x^{sw} > x^{pc}$. In turn considering the RHS in (8) and (2), $\alpha^* = n \Rightarrow (1+n)\alpha^* h(nx)q > 2nh(nx)q$, implying that $q^{sw}(x) > q^{ms}(x)$, $\forall x > 0$; as a result given i): $q^{sw} > q^{ms}$ and $x^{sw} > x^{ms}$. ■

Part i) of Proposition 3 suggests that an optimal damages multiplier may be quite easy to assess for Courts, since it corresponds to industry size $\alpha^* = n$. Part ii) illustrates that when the per capita rule is combined with a damages multiplier optimally chosen, firms face a marginal cost of liability equal to its social value. Hence an optimal damages multiplier solves the issue of care incentives (firms face efficient incentives to take care at any level of output), and also part of the distortion on production. Of course, this is not sufficient to reach optimal levels of care and output at equilibrium, since the distortion coming from imperfect competition still holds. A single instrument such as a damages multiplier is obviously not enough to solve two distortions; nevertheless, under the capita rule, the optimal damages multiplier allows to improve both the incentives to take care and the incentives to produce. This property does not hold under the market share rule. Under the market share rule, the optimal damages multiplier will exert perverse incentives on output choice now, since it puts an excessive liability cost on firms. This aggravates the problem of output underprovision created by the distortion due to imperfect competition.

The issue we consider now is whether fostering firms' entry will allow to reach the social optimum given the constraint of the environmental liability law considered here. As it will appear, we may need a very simple and intuitive assumption in order to qualify some effects:

H2: $-h'(nx).\,(nq)^2$ is decreasing with n.

H2 means that the marginal benefit of care must be decreasing with the number of firms; it implies that the next condition holds in this case:

$$\text{For any } x, n:\; 2h'(nx) + nxh''(nx) > 0 \tag{C3}$$

The next proposition collects the results for the per capita rule.

Proposition 4 *Assume environmental liability law relies on the combination (strict liability, the per capita rule, an optimal damages multiplier); then: i) The individual output decreases with the number of firms; individual care decreases with the number of firms under (C3). ii) The aggregate output and care levels increase with the number of firms. iii) As $n \to \infty$, the equilibrium industry $(Q^{pc}_{\infty}, X^{pc}_{\infty})$ converges to the social optimum (Q^{sw}, X^{sw}). iv) fostering firms' entry is always welfare improving.*

Proof. i) See Appendix 2. Regarding the ambiguous impact on care, it is shown that (C3) is a sufficient condition for x^{pc} to decrease with n; thus, should the inequality (C3) not hold, it may be that x^{pc} still decreases with n. ii) To illustrate, let us write (5bis)-(6bis) substituting with $\alpha^* = n$; it comes:

$$a - b\left(1 + \frac{1}{n}\right)Q = 2h(X)Q \tag{5bis, α^*}$$

$$-h'(X)Q^2 = 1 \tag{6bis, α^*}$$

Condition (6bis,α^*) does not depend on n. In turn, (5bis) shows that an increase in n shifts upward the marginal market proceeds (LHS). Thus, an increase in n yields an increase in Q^{pc} and X^{pc}. iii) As $n \to \infty$, the LHS in (5bis,α^*) is equal to the demand price; thus as $n \to \infty$, the equilibrium industry $(Q^{pc}_\infty, X^{pc}_\infty)$ converges to the social optimum (Q^{sw}, X^{sw}). iv) Evaluating (1) at (Q^{pc}, X^{pc}) and differentiating in n, we have (using (5bis,α^*)-(6bis,α^*) now):

$$\frac{dSW}{dn}(Q^{pc}, X^{pc}) = \frac{b}{n}.Q^{pc}.\frac{dQ^{pc}}{dn} > 0$$

since $\frac{dQ^{pc}}{dn} > 0$. ■

The optimal damages multiplier correct firms incentives regarding care decisions; as the public good effects is removed, firms are bound to follow efficient rule of care at any level of output, on the pathway that leads to the social optimum. Part i) and ii) of Proposition 4 illustrates a nice property of an optimal damages multiplier under the per capita rule. Specifically, the aggregate output and the aggregate care both increase now as more firms enter the market. Part iii) of Proposition 4 shows that under the per capita rule, fostering firms' entry on the market increases Social Welfare, and when competition between firms turns out to be perfect, the first best optimum is recovered. In this perspective the per capita rule appears as a quite flexible damages apportioning arrangement: once the incentives to invest in care and the incentives to produce are aligned with the socially optimal ones, there is a clear-cut separation between Courts' action and Competition Authorities policies. The decisions of the former do not impinge on the domain of the latter, and as long as Courts commit to use optimal damages multipliers, Competition Authorities have room to pursue their traditional objectives moving the equilibrium closer to the social optimum.

In turn, the next proposition collects the results for the market share rule:

Proposition 5 *Assume environmental liability law is designed according to the combination (strict liability, the market share rule, an optimal damages multiplier); then: i) The individual output decreases with the number of firms; (C3) is sufficient for that individual care decreases with the number of firms. ii) If b is large (small) enough, then the aggregate output and care levels increase (decrease) with the number of firms. iii) As $n \to \infty$, the industry vanishes, i.e. $(Q^{ms}_\infty = 0, X^{ms}_\infty = 0)$. iv) If $\frac{dQ^{ms}}{dn} > 0$, fostering firms' entry is Social Welfare improving. If $\frac{dQ^{ms}}{dn} < 0$, fostering firms' entry reduces Social Welfare. v) Social Welfare is maximized for a finite number of firms, $n^{ms} = \sqrt{\frac{b}{h(X^{ms})}}$.*

Proof. i) See Appendix 2. Once more we find that (C3) is a sufficient condition for x^{pc} to decrease with n. ii) To illustrate, let us substitute $\alpha^* = n$ in conditions (8bis)-(9bis):

$$a - b\left(1 + \frac{1}{n}\right)Q = (1+n)h(X)Q \tag{8bis,α^*}$$

$$-h'(X).Q^2 = 1 \tag{9bis,α^*}$$

Condition (9bis,α^*) is similar to (6bis,α^*). According to (8bis,α^*), the marginal cost of liability increases with n, which drives downward the output; to the opposite, the marginal market proceeds also increases with n, driving upward the output. Thus, $Q^{ms}(X)$ may increase or decrease at any level of X, depending on whether the impact of n on the market proceeds is larger or smaller than on the cost of liability. In Appendix 2, we show that $b > n^2 h(X)$ implies $\frac{dQ^{ms}}{dn} > 0$ and $\frac{dX^{ms}}{dn} > 0$; to the converse, $b < n^2 h(X)$ implies $\frac{dQ^{ms}}{dn} < 0$ and $\frac{dX^{ms}}{dn} < 0$. iii) According to the RHS in (8bis), the marginal cost of liability goes to infinity as $n \to \infty$. Thus the aggregate output and care levels become smaller and smaller with n ($Q^{ms}_\infty \to 0$ and $X^{ms}_\infty \to 0$). iv) Evaluating (1) at (Q^{ms}, X^{ms}) and differentiating in n (using (8bis,α^*)-(9bis,α^*)), we obtain:

$$\frac{dSW}{dn}(Q^{ms}, X^{ms}) = \left(\frac{b}{n} + (n-1)h(X^{ms})\right).Q^{ms}.\frac{dQ^{ms}}{dn}$$

The result iv) is straightforward. v) In Appendix 2, we show that Social Welfare is maximized neither with perfect competition ($n \to \infty$) nor with a monopoly ($n = 1$); there exists a finite number of firms $n^{ms} > 1$, for which $SW(Q^{ms}, X^{ms})$ is maximized, satisfying the condition:

$$\frac{dSW}{dn}(Q^{ms}, X^{ms}) = 0 \Rightarrow \frac{dQ^{ms}}{dn} = 0 \tag{10}$$

Given that $\left(\frac{b}{n} + (n-1)h(X^{ms})\right) > 0$, it must be that $\frac{dQ^{ms}}{dn} = 0 \Leftrightarrow b - n^2 h(X) = 0$; solving yields: $n^{ms} = \sqrt{\frac{b}{h(X^{ms})}}$. ■

Proposition 5 (Part i) and ii)) shows that if at firms level, the behavior of the individual output and care levels under the market share rule are qualitatively very similar to those obtained for the per capita rule, at the industry/aggregate level, the implied adjustments following firms' entry are very different (rule-specific). In short, optimal damages multipliers do not remove the public good effect for sure. Proposition ii) highlights that under the market share rule, the net effect at the aggregate level is driven in a complex way by the relative size of the slopes of market demand and the marginal cost of liability. When n increases (see (8bis,α^*)), both the aggregate marginal proceeds (LHS) and aggregate marginal cost of liability (RHS) increase. If the price-elasticity of market demand is low (b is large) enough, the first effect (LHS) dominates, entailing an increase in the aggregate output and care levels – the rationale is that despite the increase in the individual marginal liability cost at a firm level, the cut in the individual output level is of limited scale, such that the aggregate output increases with firms' entry. In contrast as the price-elasticity of market demand becomes low (b is large) enough, the second effect (RHS) is large compared to the first one – the associated cut in the output level at a firm level is that large that it is not be compensated at the aggregate level by the entry of new firms; as a result, there is a contraction in the market supply, associated with a decrease in aggregate care expenditures. On the other hand, Proposition 5 Parts iii) and i) illustrate that the implications for Social Welfare analysis are also very contrasted between both rules; part iii) may be also understood as follows. Starting with a given number of firms, and fostering firms' entry may allow in a first stage to improve both the output and care level – this is the "traditional" effect: as n increases, both individual output and care expenditures decrease at firm level, but given that more firms operate on the market, the aggregate output and care expenditures both increase with n. However, as n becomes great enough, fostering firms' entry a little further may have countervailing effects, in the sense that this will induce a decrease of individual output and care expenditures at firm level that large, that the aggregate output and Social Welfare also decrease. This explains Part v): restricting firms' entry is socially worth in this case.

More generally, the implication of proposition 5 is that the market share rule appears as liability rule less flexible, since Courts' action impinge on the domain of Competition policy. The use of the market share rule yields an additional distortion in output decisions at firms' level (the higher the market share, the higher the (marginal) cost of liability), and thus it is not granted that fostering firms' entry under the market share rule be socially welfare improving. Moreover recovering perfect competition under the market share would entail a severe contraction in the aggregate output supply and aggregate care expenditures, and Social Welfare would be reduced compared to the oligopoly.

At that point, two issues are worth of consideration.

Remark 5 (No liability regime): Before tuning to the analysis of environmental liability regimes based on a negligence test, let us consider the impact of a no liability regime. In this case, each firm i chooses a level of output and a level of care in order to maximize its profit: $\Pi^{nl}(q_i, x_i) = P(Q).q_i - x_i$. Thus, firms do not invest in care $x^{nl} = 0$, and we obtain the standard symmetric Cournot-Nash equilibrium where firms choose a level of output equal to $q^{nl} = \frac{a}{(1+n)b}$, associated with an aggregate output $Q^{nl} = \frac{na}{(1+n)b}$. It is straightforward that compared with the equilibrium with strict liability (whatever the damages rule), the no liability regime yields the highest level of output $q^{nl} > q^{pc} > q^{ms}$ and the lowest level of care expenditures $x^{nl} = 0 < x^{pc} < x^{ms}$. However, the comparison with the social optimum yields an ambiguous result; it can be verified that $b > (<)2nh(x^{sw}) \Rightarrow q^{sw} > (<)q^{nl}$. Moreover, we obtain $\frac{dSW}{dn}(Q^{nl}, 0)) = \left[\frac{b}{n} - 2h(0)\right].Q^{nl}.\frac{dQ^{nl}}{dn}$ where $\frac{dQ^{nl}}{dn} > 0$. On the one hand: $\lim_{n\to\infty} Q^{nl} = \frac{a}{b}$ and $\lim_{n\to\infty} \frac{dQ^{nl}}{dn} = 0$, implying that $\lim_{n\to\infty} \frac{dSW}{dn}(Q^{nl}, 0)) \to 0_-$; on the other hand, $Q^{nl}(n = 1) = \frac{a}{2b}$ and $\left(\frac{dQ^{nl}}{dn}\right)_{n=1} = \frac{a}{4b}$, implying that $\left(\frac{dSW}{dn}(Q^{nl}, 0))\right)_{n=1} > 0$ under H1a. As a result the number of firms that maximizes Social Welfare under no liability is $n^{nl} = \frac{b}{2h(0)}$.

Remark 6: Our analysis of the market share rule (propositions 2, 5 and 8) reaches conclusions that are close to Marino (1989) although with a very different framework. Marino's paper considers the case of joint but divisible and non cumulative harms (the equivalent here with our notations is $H = (\sum_{i=1}^{n} h(x_i)q_i)$), and finds that under the market share rule (without an optimal damages multiplier) firms undertake inefficient decisions of care (both in terms of response to any output level, and in terms of equilibrium level); moreover, he shows that the equilibrium care level decreases with firms' entry, and that Social Welfare is maximized for a finite number of firms. In contrast, recent papers afford formal arguments according to which the market share rule allows to reach an efficient goal. The papers by Dehez and Ferey (2013) and Ferey and Dehez (2016) rely on a cooperative game-theoretic frameworks to analyze apportionment rules that satisfy both efficiency and fairness criterions; however, they consider the case for a exogenous harm, and this way eliminate the issue of care incentives and the interdependence with endogenous decisions of production, as well as the backward influence of market interactions. Daughety and Reinganum (2014)[14] discuss the case where the joint harm is supposed to be cumulative but divisible (the equivalent with our notations is $H = \left(\sum_{i=1}^{n} h(x_i)q_i\right)^2$). This enable them to introduce a "modified market share" rule in a strict liability regime, whose equivalent with our notations here is $\frac{h(x_i)q_i}{\sum_{i=1}^{n} h(x_i)q_i}$; this rule is relevant in cases where the aggregate harm is seen as the result of the combination of existing divisible individual harms having non linear (cumulative) effects. Daughety and Reinganum show that the oligopoly equilibrium obtained under this modified market share rule, associated with an optimal damages multiplier, has several properties that are close to those described for the per capita rule in our framework (see proposition 4), with the noticeable exception that the equilibrium level of individual care in their set up increases with the number of firms (whereas, it is decreasing in our set up). Remark that this modified market share rule is not relevant for our case of indivisible environmental harms,[15] and going back to the "simple" market share rule $\frac{q_i}{Q}$ makes sense in this context. However, we find instead that the use of the basic per capita rule for apportioning indivisible environmental damages between firms has nice properties for care incentives and production decisions, since it introduces no distortion in competition above those due to strategic market interactions. In contrast, the market share rule will exert perverse incentives on firms regarding their choice of output, since the liability cost increases with their market power, thus aggravating the issue of output underprovision due to imperfect competition. Interestingly enough this result mirrors an old debate about the comparison between the per capita rule and the market share rule in personal injury case (see Kornhauser and Revesz 1989), although we reach the exact opposite conclusion, as far as imperfect quantity competition is concerned.

[14] Although Daughety and Reinganum (2024) paper addresses the issue of product liability, their results regarding strict liability are still relevant for environment liability. In turn, what is specific to a product liability context is their comparative analysis of no liability, negligence vs strict liability – as well as our comparative analysis of these different liability regimes is specific to the case with indivisible environmental harms we focus on; see above Remark 1.

[15] Remark also to have a complete picture that in the situation investigated by Marino (1989), the "modified market share" rule indeed provides firms with efficient incentives in care decision, without requiring any damage multiplier to achieve this result. The proof is obvious: with joint but divisible and non cumulative harms, the individual liability cost borne by each firm under strict liability with this modified market share is equal to individual harm:

$$L_i(q_i, x_i) = \left(\frac{h(x_i)q_i}{\sum_{i=1}^{n} h(x_i)q_i}\right) \times \left(\sum_{i=1}^{n} h(x_i)q_i\right) = h(x_i)q_i$$

Thus, each firm faces the social cost it imposes to the society, and the efficient goal is attained.

5 Oligopoly equilibrium under negligence

CERCLA as well as The Environmental Liability Directive of the European Union are focused on specific polluting or dangerous activities and/or operations, thus being explicitly recognized as subject to strict liability. By default, those not listed are subject to negligence. Having provided the analysis of strict liability, we now investigate here the case with the negligence regime.

Assume Courts rely on a negligence test considering a flexible standard of care:[16] a firm will be considered as not negligent to the extent that its care expenditures proove to be an efficient behavioral response, in the sense of firm's best care expenditure it may have chosen considering any relevant foreseeable instances, that Courts consider as being adapted to the situation – in which case the firm will again avoid any liability cost (i.e. $s_i = 0$). It is natural to consider in that case that Courts promote a standard of care $\hat{x} = x_i(q_i, q_{-i}, x_{-i})$ defined as:

$$-h'(x_i + x_{-i}).(q_i + q_{-i})^2 = 1 \tag{11}$$

for any positive (q_i, q_{-i}, x_{-i}). The rationale for Courts is that an individual firm will not be seen as liable as long as its contribution to the safety have been designed in order to minimize the expected cost of the accident, given its own output and its competitors decisions (including output and care), i.e. $x_i(q_i, q_{-i}, x_{-i}) = \min_{x_i}(h(x_i + x_{-i}).(q_i + q_{-i})^2 + x_i)$ whatever (q_i, q_{-i}, x_{-i}).

Let us show that in this regime of negligence, there exists a Nash equilibrium where all firms comply with the standard of care.

Assume that one firm is negligent, while its $n-1$ competitors do comply with the standard required. The profit of the non compliant firm is: $\hat{\Pi}^{neg}(q_i, x_i) = (a - bQ)q_i - x_i - Q^2 h(X)$, since it bears the full external cost. As a result, it faces the liability cost of strict liability with the *per capita* rule when $\alpha^* = n$, implying that the firm chooses a level of care that satisfy (11) – this amounts to say that the deviant firm has inconsistent belief and should have not considered to be liable because Courts shouldn't have conclude for its liability.

As a result, the equilibrium now is such that any firm abides the standard of care $x_i(q_i, q_{-i}, x_{-i})$, and the individual output is maximizing the profit:

$$\hat{\Pi}^{nl}(q_i, x_i(q_i, q_{-i}, x_{-i})) = (a - bQ)q_i - x_i(q_i, q_{-i}, x_{-i}) \tag{12}$$

under $x_i(q_i, q_{-i}, x_{-i})$ defined by the constraint (11). The first and second order conditions are given in Appendix 3, such that the symmetric Cournot-Nash equilibrium where $q_1 = ... = q_n = q^{neg}$ and

[16] A classical distinctions in the *Law & Economics* literature (Kaplow 1992, Sullivan 1992) is made between a *rule* and a *standard* of care. When Courts use a rule of care, a firm will be considered as not negligent once its provides at least a predetermined fixed level of care ($\hat{x}$, i.e. the due care level). A natural candidate for such a due care level, usually considered in the literature, is the socially optimal level of care $\hat{x} = x^{sw}$ (see Charreire and Langlais 2020).

$x_1 = ... = x_n = x^{neg}$ solves the system:

$$a - b(1+n)q = 2\left(\frac{(h'(nx))^2}{h''(nx)}\right) nq \quad (13)$$

$$-h'(nx).\,(nq)^2 = 1 \quad (3)$$

The RHS in condition (13) shows that the marginal cost of liability accruing to each firm has now a more complex expression compared with (2); it still depends on the expected harm, although in a more elaborate way (captured through the two first derivatives of $h(X)$). This implies that negligence associated with a standard of care outperforms strict liability as we show now:

Proposition 6 ***(negligence with a standard of care vs strict liability, and social optimum)*** *i) Negligence with a standard of care leads to a level of output and a level of care larger than under strict liability associated with an optimal multiplier $\alpha^* = n$ (either with the per capita or the market share: $q^{neg} > q^{pc} > q^{ms}$ and $x^{neg} > x^{pc} > x^{ms}$). ii) If b is large (small) enough, then negligence with a standard of care yields equilibrium levels of output and care smaller (respectively, larger) than their optimal values (b large enough $\Rightarrow q^{neg} < q^{sw}$ and $x^{neg} < x^{sw}$; b small enough $\Rightarrow q^{neg} > q^{sw}$ and $x^{neg} > x^{sw}$).*

Proof. i) Under (C2) we have $h(nx) > 2\frac{(h'(nx))^2}{h''(nx)}$ and thus $h(nx) > \frac{(h'(nx))^2}{h''(nx)}$; as a result (comparing the RHS in (5), (8) when $\alpha^* = n$ with (13)): $q^{ms}(x) < q^{pc}(x) < q^{neg}(x)\ \forall x$. On the other hand, $x^{ms}(q) = x^{pc}(q) = x^{neg}(q)\ \forall q$ if $\alpha^* = n$. Hence: $q^{ms} < q^{pc} < q^{neg}$ and $x^{ms} < x^{pc} < x^{neg}$. $q^{neg}(x)\ \forall x$. ii) See also Appendix 4. By construction, $x^{sw}(q) = x^{neg}(q)$ for any feasible $q > 0$. From the comparison of the LSH in (2) and (13), it comes that $a - (1+n)bq < a - nbq$; in contrast, the comparison of the RHS shows that: $2nqh(nx) > 2nq\frac{(h'(nx))^2}{h''(nx)}$. Hence the comparison of $q^{sw}(x)$ and $q^{neg}(x)$ at any $x > 0$ is ambiguous. So it is at equilibrium. In Appendix 4, it is shown that: i) if $b > 2n\left(h(nx) - \frac{(h'(nx))^2}{h''(nx)}\right)$, it comes $q^{sw}(x) > q^{neg}(x)$ for any feasible $x > 0$; thus we obtain $x^{sw} > x^{neg}$ and $q^{sw} > q^{neg}$; but ii) if $b < 2n\left(h(nx) - \frac{(h'(nx))^2}{h''(nx)}\right)$, then $q^{sw}(x) < q^{neg}(x)$ for any feasible $x > 0$; and thus we have $x^{sw} < x^{neg}$ and $q^{sw} < q^{neg}$. ■

Proposition 6 Part i) reflects that the marginal cost of liability under negligence with a standard of care is smaller than under strict liability associated with the per capita rule, at any level of care. The intuition is that the incentives constraint (11) afford firms with a strategic advantage: anticipating its care activity as being an efficient response to the output decision, this materializes through a smaller (marginal) cost of liability, at any level of output. As a consequence, the equilibrium output and care levels under negligence are larger than under strict liability (whatever the damages rule).

Part ii) shows that under negligence with a standard of care, firms face once more two opposite incentives regarding the furniture of the output. On the one hand, firms' marginal market benefits under negligence with a standard of care are smaller than their socially optimal value (this is the standard distortion due to imperfect competition) – this reduces the incentives to produce compared to the social optimum, at any level of care. On the other hand, the marginal cost of liability accruing to firms under negligence with a standard of care is lower than its socially optimal value (according to the convexity of the expected harm function) – this increases now the incentives to produce under strict liability, compared to the optimum, at any level of care. The net effect at equilibrium is ambiguous, and depends on the relative size of the slopes of the two marginal market proceeds (mainly, the value of b), and the slopes of the two marginal cost of liability (convexity of the expected harm function).

We show now that the equilibrium attained in this regime is inefficient from a social point of view, with either too much or in contrast not enough of both output and care.

Regarding the impact of firms' entry, the consequences are described in the next proposition.

Proposition 7 *Assume environmental liability law is designed according to negligence with a standard of care; then: i) The individual output and care may increase as well as decrease with the number of firms. ii) The aggregate output and care levels increase with the number of firms. iii) As $n \to \infty$, the equilibrium industry converges to levels $(Q_\infty^{neg}, X_\infty^{neg})$ above the social optimum (Q^{sw}, X^{sw}). iv) If b is large (small) enough, fostering firms' entry increases (decreases) Social Welfare. v) Social Welfare is maximized for a finite number of firms, $n^{neg} = \frac{b}{2}\left(\frac{h''(X^{neg})}{h(X^{neg}).h''(X^{neg})-(h'(X^{neg}))^2}\right)$.*

Proof. i) See Appendix 5. Specifically we show that if b is large (small) enough, then the individual output decreases (increases) with n. The effect on individual care requires more qualifications: we show that if b is large enough and (C3) holds, then the individual care decreases with n; in contrast, if b is small enough and (C3) does not hold ($2h'(nx) + nxh''(nx) < 0$ for any n, x), then the individual care increases with n. ii) Using (13)-(3), the aggregate output and aggregate care levels satisfy:

$$a - b\left(1 + \frac{1}{n}\right)Q = 2\left(\frac{(h'(X))^2}{h''(X)}\right)Q \tag{13bis}$$

$$-h'(X).Q^2 = 1 \tag{3bis}$$

We will denote as $Q^{neg}(X)$ the aggregate output level that solves (13bis), for any given value of aggregate care; while $X^{ms}(Q)$ is the aggregate care level that solves (3bis), for any given value of the aggregate output. According to (3bis), the aggregate care expenditures $X^{neg}(Q)$ doe not depend on n, but increase with Q. According to (13bis), the marginal market proceeds increases with n, hence $Q^{neg}(X)$ increases with n at any X. Hence ii). iii) In the limit case where $n \to \infty$, then according to the LHS in (13bis), the marginal market proceeds tends to the market price; given that the marginal cost of liability still satisfies $\frac{(h'(X))^2}{h''(X)} < h(X)\ \forall X > 0$, the aggregate output and care levels do not reach their socially optimal values $(Q^{sw}_{\infty}, X^{sw}_{\infty})$ as $n \to \infty$, but in contrast take some positive values larger than $(Q^{sw}_{\infty}, X^{sw}_{\infty})$ that solve (13bis)-(3bis), or: $Q^{neg}_{\infty} = a\left[b + 2\left(\frac{(h'(X^{neg}_{\infty}))^2}{h''(X^{neg}_{\infty})}\right)\right]^{-1}$ and $-h'(X^{neg}_{\infty}).(Q^{neg}_{\infty})^2 = 1$. iv) Evaluating (1) at (Q^{neg}, X^{neg}) and differentiating in n (using (17bis)-(3bis)), we obtain, denoting $V^{neg} = \frac{(h'(X^{neg}))^2}{h''(X^{neg})}$:

$$\frac{dSW}{dn}(Q^{neg}, X^{neg}) = \left(\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right)\right).Q^{neg}.\frac{dQ^{neg}}{dn}$$

with $\frac{dQ^{neg}}{dn} > 0$. Hence either a) $b - 2n\left(h(X^{neg}) - V^{neg}\right) > 0$ and thus $\frac{dSW}{dn}(Q^{neg}, X^{neg}) > 0$; or b) $b - 2n\left(h(X^{neg}) - V^{neg}\right) < 0$ and thus $\frac{dSW}{dn}(Q^{neg}, X^{neg}) < 0$. v) In Appendix 5, we show that Social Welfare is maximized neither with perfect competition ($n \to \infty$) nor with a monopoly ($n = 1$); there exists a finite number of firms, $n^{neg} > 1$, that maximizes $SW(Q^{neg}, X^{neg})$, which is the solution to:

$$\frac{dSW}{dn}(Q^{neg}, X^{neg}) = 0 \Rightarrow \frac{b}{n^{neg}} - 2\left(h(X^{neg}) - V^{neg}\right) = 0 \tag{14}$$

and solving yields $n^{neg} = \frac{b}{2(h(X^{neg}) - V^{neg})} = \frac{b}{2}\left(\frac{h''(X^{neg})}{h(X^{neg}).h''(X^{neg}) - (h'(X^{neg}))^2}\right)$. ■

Proposition 7 Part i) reflects that negligence with a standard of care affords firms a strategic advantage with regard to their output and care decisions, and this leaves them with a degree of freedom that materialize through the adaptation to market conditions and changes in their market power. On the one hand, depending on the price sensibility of market demand, firms may choose to increase or decrease their output following firms' entry; on the other hand, depending on whether the marginal benefits of care activities decrease or increase, firms may decrease or increase their investments in care. Although the public good effect is removed by the incentives constraint (11), this liability regime introduces a direct link between equilibrium care expenditures at firms level, and the variation of the marginal benefits of care with n. Part ii) illustrates that once the public good effect is neutralized thanks to the incentives constraint (11), firms' entry yields the normal, intuitive effect on the aggregate output and agggregate care. Part iii) may be understood as follows: as n becomes great enough, the marginal market proceeds increase, while the average marginal liability cost for the industry (RHS in (13bis) is constant: thus the aggregate output and aggregate care expenditures reach level above their optimal values. Part iv) and v) follow: an excessive aggregate output may entail loss of welfare; this restricting firms' entry may be socially worth.

6 Robustness check and discussion

Before going back on the debate regarding the orientations of competition policy and environmental goals, let us comment on some robustness check around the specification of the cost of care.

6.1 A more general cost of durable care

Assume now that the marginal cost of care is no longer constant; more generally, we assume $c(0) > 0$, $c'(x) > 0$, $c''(x) > 0$, and $c'''(x) \geq 0 \; \forall \; x$. Remark that as a result, the next condition always holds:

$$\text{For any } x > 0 : -c\,(x) + c'\,(x)\,.x > 0 \tag{C4}$$

It is immediate that in all regimes we have formerly discussed, the individual care is set according to a new condition where the (constant) marginal care is substituted now with $c'(x)$. The very first implication is that the aggregate output and care at the social optimum are parametrized by the number of firms (see Appendix 1). The reason is that the optimal aggregate care is now characterized by the condition:

$$(-h'(X))Q^2 = c'\left(\frac{X}{n}\right) \tag{3ter}$$

Thus as n increases, then the incentives to invest in care increases at the level of the industry (since $c'\left(\frac{X}{n}\right)$ decreases) which creates a feedback influence of care decision on firms output decisions, all else equal. Hence: as n increases, then both Q^{sw} and X^{sw} increase. Moreover, as $n \to \infty$ then the socially optimal aggregate output and care reach some upper limit values defined by $Q^{sw}_{\infty} = \frac{a}{b+2h(X^{sw}_{\infty})}$ and $-h'(X^{sw}_{\infty}) = \frac{c'(0)}{\left(\frac{a}{b+2h(X^{sw}_{\infty})}\right)^2}$. Finally, it is easy to verify that Social Welfare also increases with the number of firms at optimum, since evaluating (1) at (Q^{sw}, X^{sw}) and differentiating with respect to n yields (according to the envelop theorem) now:

$$\frac{dSW}{dn}(Q^{sw}, X^{sw}) = -c\left(\frac{X^{sw}}{n}\right) + c'\left(\frac{X^{sw}}{n}\right).\frac{X^{sw}}{n} > 0$$

under (C4). As far as the expenditures in care are the unique source of fixed costs for a firm (which is consistent with our interpretation that care is durable), this implies that it is socially optimal to have an infinite number of firms engaged in the activity (choosing both a level of output and care that are infinitively small).

The second major implication is related to the impact of firms' entry under strict liability with the market share (with $\alpha^* = n$).[17] Proposition 5 is now substituted with:

Proposition 8 *Assume environmental liability law is designed according to the combination (strict liability, the market share rule, an optimal damages multiplier). When the marginal cost of care is increasing: i) If b is large enough, the aggregate output and care levels increase with the number of firms. If b is small enough, the aggregate output and care levels may increase or decrease with the number of firms. ii)*

[17] It is immediate that propositions 1, 2, 3, and also proposition 4 – the properties of the per capita rule with an optimal damages multiplier – still hold. See Appendix 2.

As $n \to \infty$, the industry vanishes, i.e. $(Q^{ms}_{\infty} = 0, X^{ms}_{\infty} = 0)$. iii) If $\frac{dQ^{ms}}{dn} > 0$, fostering firms' entry is Social Welfare improving. If $\frac{dQ^{ms}}{dn} < 0$, fostering firms' entry may reduce or increase Social Welfare. iv) Social Welfare is maximzed for a finite number of firms, $n^{ms} > 1$ (see condition (15) below).

Proof. See Appendix 2. i) More specifically, the ambiguity when b is small relies on the fact that the aggregate care satisfies now condition (3b): thus as n increases, the marginal cost of care decreases while the marginal cost of liability increases; we show that when $c''(x)$ has large values, it is more likely that the aggregate output and care increase with n. iii) Same argument as for proposition 5. iv) When $c''(x) > 0$ at any x, then evaluating (1) at (Q^{ms}, X^{ms}) and differentiating in n yields now:

$$\frac{dSW}{dn}(Q^{ms}, X^{ms}) = \left(\frac{b}{n} + (n-1)h(X^{ms})\right).Q^{ms}.\frac{dQ^{ms}}{dn} + \left(-c\left(\frac{X^{ms}}{n}\right) + c'\left(\frac{X^{ms}}{n}\right).\frac{X^{ms}}{n}\right)$$

where $\left(-c\left(\frac{X^{ms}}{n}\right) + c'\left(\frac{X^{ms}}{n}\right).\frac{X^{ms}}{n}\right) > 0$ under (C4) for any finite n and X. iv) is thus straightforward. v) In Appendix 2, we show that Social Welfare is maximized neither with perfect competition ($n \to \infty$) nor with monopoly ($n = 1$). The finite number of firm that maximizes $SW(Q^{ms}, X^{ms})$, $n^{ms} > 1$, is now solving the condition:

$$-c\left(\frac{X^{ms}}{n^{ms}}\right) + c'\left(\frac{X^{ms}}{n^{ms}}\right).\frac{X^{ms}}{n^{ms}} = -\frac{dQ^{ms}}{dn}.\left(\frac{b}{n^{ms}} + (n^{ms}-1)h(X^{ms})\right).Q^{ms} \tag{15}$$

which implies that at n^{ms}, it must be that $\frac{dQ^{ms}}{dn} < 0$. Obviously, there may exist several values of n that satisfy $\frac{dSW}{dn}(Q^{ms}, X^{ms}) = 0$ given that this condition is highly non linear, all being local maximum. ■

The third important consequence is related to the regime of negligence with a standard of care. Proposition 9 is replaced with the next proposition, where $\hat{V}^{neg} = \left(\frac{\left(h'(X^{neg})\right)^2.(Q^{neg})^2}{h''(X^{neg}).(Q^{neg})^2 + c''\left(\frac{X^{neg}}{n}\right)}\right)$:

Proposition 9 *Assume environmental liability law is designed according to negligence with a standard of care. i) Assume that the marginal cost of care is linear ($c''(x) > 0$ but $c'''(x) = 0$ at any $x > 0$); then the aggregate output and care levels increase with the number of firms. ii) Assume instead that the marginal cost of care is convex in x ($c''(x) > 0$ and $c'''(x) > 0$ at any $x > 0$). If b is large enough, then the aggregate output and care levels both increase with the number of firms; whereas if b is low enough, then the aggregate output and care levels may increase as well as decrease with the number of firms. iii) If $\left(b - 2n\left(h(X^{neg}) - \hat{V}^{neg}\right)\right).\frac{dQ^{neg}}{dn} > 0$, then fostering firms' entry is always Social Welfare improving; but if $\left(b - 2n\left(h(X^{neg}) - \hat{V}^{neg}\right)\right).\frac{dQ^{neg}}{dn} < 0$, then fostering firms' entry may reduce Social Welfare. iv) As $n \to \infty$, the equilibrium industry $(Q^{neg}_{\infty}, X^{neg}_{\infty})$ converges to levels above the social optimum $(Q^{sw}_{\infty}, X^{sw}_{\infty})$. v) Social Welfare is maximized by a finite number of firms, $n^{neg} \geq 1$.*

Proof. i) and ii) In Appendix 5, it is shown that the aggregate output and care are now satisfying:

$$a-b\left(1+\frac{1}{n}\right)Q=2Q.V(Q,X) \tag{13ter}$$

$$-h'(X).Q^2=c'\left(\frac{X}{n}\right) \tag{3ter}$$

with $V(Q,X)=\left(\frac{\left(h'(X)\right)^2.Q^2}{h''(X).Q^2+c''\left(\frac{X}{n}\right)}\right)$. In order to illustrate the difference between i) and ii) note that when $c'''(x)=0$ at any x, then $\frac{dV}{dn}=0$ and (13ter)-(3ter) evolves with variations of n as in (13bis)-(3bis). In contrast when $c'''(x)>0$ at any x, then $\frac{dV}{dn}>0$ such that the RHS in (13ter) increases with n – this goes the opposite sense compared with the LHS of (13ter) which increases; hence the ambiguity regarding the net effect on the equilibrium, and the additional qualifications required. Appendix 5 gives the detailed analysis. When $c'''(x)=0$, then we are back to proposition 5 iii); while if $c'''(x)>0$, then a b large enough and/or a $c''(x)$ large enough are sufficient to obtain that the aggregate output and care increases with n. iii) Evaluating (1) at (Q^{neg},X^{neg}) and differentiating in n, we obtain:

$$\frac{dSW}{dn}(Q^{neg},X^{neg})=\frac{dQ^{neg}}{dn}.\left(\frac{b}{n}-2\left(h(X^{neg})-\hat{V}^{neg}\right)\right).Q^{neg}+\left(-c\left(\frac{X^{neg}}{n}\right)+c'\left(\frac{X^{neg}}{n}\right).\frac{X^{neg}}{n}\right)$$

where $\hat{V}^{neg}=\left(\frac{\left(h'(X^{neg})\right)^2.(Q^{neg})^2}{h''(X^{neg}).(Q^{neg})^2+c''\left(\frac{X^{neg}}{n}\right)}\right)>0$, and $\left(-c\left(\frac{X^{neg}}{n}\right)+c'\left(\frac{X^{neg}}{n}\right).\frac{X^{neg}}{n}\right)>0$ under (C4) for any finite n and X. Hence iii) is obvious. v) Appendix 5 establishes that Social Welfare is not maximized under perfect competition ($n\to\infty$); when $c'''(x)=0\ \forall x$, it is also shown that monopoly ($n=1$) reduces Social Welfare. Hence, when $c'''(x)=0\ \forall x$, there exists a finite number of firms, $n^{neg}>1$, that maximizes $SW(Q^{neg},X^{neg})$, defined as the solution to:

$$-c\left(\frac{X^{neg}}{n^{neg}}\right)+c'\left(\frac{X^{neg}}{n^{neg}}\right).\frac{X^{neg}}{n^{neg}}=-\frac{dQ^{neg}}{dn}.\left(\frac{b}{n^{neg}}-2\left(h(X^{neg})-\hat{V}^{neg}\right)\right).Q^{neg} \tag{16}$$

Note that this implies that the condition $\frac{b}{n^{neg}}-2\left(h(X^{neg})-\hat{V}^{neg}\right)<0$ is also verified (since $c'''(x)=0$ $\forall x\Rightarrow\frac{dQ^{neg}}{dn}>0$). In contrast, assuming $c'''(x)>0\ \forall x$ and $\left(\frac{dQ^{neg}}{dn}\right)_{n=1}>0$, implies that the number of firms that maximizes Social Welfare is finite, with $n^{neg}>1$, and satisfies (16). Otherwise soon as $\left(\frac{dQ^{neg}}{dn}\right)_{n=1}<0$ is verified, the market structure that maximizes Social Welfare may be either the monopoly ($n^{neg}=1$) or the oligopoly ($n^{neg}>1$ still given by (16)). Obviously, there may exist several values of n that satisfy (16) given that this condition is non linear, all being local maximum. ■

Proposition 9 illustrates that under the negligence rule with a standard of care, the net effect of firms' entry on the aggregate output and care is also driven in a complex way by the relative size of the slope of market demand (b), the slope of the marginal cost of care ($c''(x)$), and the slope of the marginal cost of liability (through $h''(x)$). As a consequence, starting from a given equilibrium (associated with a specific number of firms), and fostering firms' entry may allow in a first stage to increase the aggregate output and care closer to their socially optimal levels; however, as n becomes great enough, fostering firms' entry a little further may have countervailing effects, and reduce the aggregate output and care expenditures moving away from optimal levels. Proposition 11 suggests that the properties of the (marginal) cost of care plays an important role (more crucial than the properties of the probability function).

6.2 Environmental law and competition policy: tentative of interpretation

Our results illustrate that environmental harms are characterized by invisibility, strict liability fails to induce an efficient rule of care from firms, unless when Courts consider optimal damages multipliers. When Courts do so, then the specific sharing rule they use for apportioning total harms is not innocuous: if the per capita rule does not distort firms output decision, in contrast, the market share rule adds to the standard competitive externality (that drives the marginal market proceeds downwards) because it makes firms bearing an excessive marginal cost of liability, which aggravates the underprovision of output.

In turn negligence associated with a standard of care proves to be an effective rule in obtaining that firms abide with an efficient rule of care, but provides firms with a strategic advantage, they use in setting their output and care according to market conditions above or below the socially optimal ones. In all, any environmental liability law alone fails in reaching the social optimum: strict liability has a tendency to reach an equilibrium with too little care and output, while negligence may provide both outcomes at equilibrium, i.e. too much or not enough of both care and output. Thus, there is room for other kinds of public interventions. The point, we now turn to, is to discuss the constraints environmental laws are putting on a Competition Authority, and when competition policy may be oriented with respect to environmental goals.

Without going into details (see above), let us simply remind here that the results of the paper reflect nothing but the way the specific design of the environmental law shapes the liability cost borne by a firm, and the way this individual liability cost will vary with the number of firms (because of damages sharing rules, and/or because of return to scales in care activity). The debate in the public arena, although not explicitly articulated, seems to focus on protective measures at firms/individual level. The paper shows it is worth distinguishing between behaviors at firms/individual level, behaviors at industry/aggregate level, and finally the general move towards the social optimum.

In the different scenarios we have analyzed, the individual output and individual care levels are both decreasing with the number of firms on the market, these results being obtained under very simple assumptions (H1a, b and c). The additional assumption H2 is important to consider only when optimal damages multipliers are introduced; moreover, relaxing H2 in this case does not necessarily reverts the comparative statics for the individual care level.[18] Given the strategic complementarity between output

[18] To be a little more specific: relaxing H2: 1) may not necessarily change $\frac{dx^{pc}}{dn} < 0$ or $\frac{dx^{ms}}{dn} < 0$ for a positive sign, as

and care in our model, this is hardly a surprise: both individual output and care levels decrease with firms' entry. Does it support the view according to which less competition (the preservation of their market power) allows insider firms at individual level to invest more in the preservation of environment? Indeed in the present set up, this does reflect anything else but that firms adapt their care expenditures to their size and to the cost of liability – given that less competition would increase market power and a larger individual output, individual care expenditures would be also larger to avoid an excessive cost of liability.

Most important, our analysis suggests at the same time that the aggregate level of care may have erratic reactions following (restrictions to) firms entry. It depends on the specific design of the environmental liability law, but also on the properties of the technology of care (including the cost of care, and the environmental impact of care activities and productions). Saying differently, the environmental law put some constraints on a Competition Authority, and for achieving better decisions this later needs generally two different sets of information. On the one hand, it requires the understanding of the legal and the institutional design regarding to the way Courts implement the environmental liability law, which is a practical issue; as we remind before, CERCLA and the European Directive operate at first glance a clear separation between activities that are subject to strict liability and those driven by negligence. However, the interventions of a Competition Authority are designed to improve competition on markets in the sense of business. Thus identifying whether the product on the market is subject to strict liability rather than negligence may be uneasy (when it combines activities that are targeted by strict liability and others by negligence). Moreover, although the distinction between strict liability and negligence is a matter of statute law, it is not the case with damages sharing rule; it may be a matter of precedents (Courts may apportion damages between multidefendants according to one rule or the other, following some general guidelines, or conventional use, or the evolution of the doctrinal debate), or it may a right entitled to Plaintiffs. On the other hand, the Competition Authority needs to know the cost structures and benefits curve associated with safety activities in different business and markets at scrutiny, and the assessment of existing returns to scale – this is an empirical issue, but also a technical point since a high level of engineering expertise is required. For the purpose of this discussion, tables 1 to 3 summarize our central results.

long as the convexity of the expected harm is large enough (i.e. the term in (C2) is large enough); 2) and change $\frac{dx^{neg}}{dn} < 0$ in $\frac{dx^{neg}}{dn} > 0$ only when b is low enough.

	$\alpha \leq n-1$	$\alpha^* = n$
$c''(x) = 0$	$\frac{dQ^{pc}}{dn} > 0$ $\frac{dX^{pc}}{dn} < 0$	$\frac{dQ^{pc}}{dn} > 0$ $\frac{dX^{pc}}{dn} > 0$
$c''(x) > 0$	$\frac{dQ^{pc}}{dn} > 0$ $\frac{dX^{pc}}{dn} \gtrless 0$ ($c''(x)$ large $\Rightarrow \frac{dX^{pc}}{dn} > 0$)	$\frac{dQ^{pc}}{dn} > 0$ $\frac{dX^{pc}}{dn} > 0$

TABLE 1 – Aggregate equilibrium, firms' entry, and the per capita rule

	$\alpha \leq n-1$	$\alpha^* = n$
$c''(x) = 0$	b high (low) $\Rightarrow \frac{dQ^{ms}}{dn} > (<)0$ $\frac{dX^{ms}}{dn} < 0$	b high (low) $\Rightarrow \frac{dQ^{ms}}{dn} > (<)0$ b high (low) $\Rightarrow \frac{dX^{ms}}{dn} > (<)0$
$c''(x) > 0$	$\frac{dQ^{ms}}{dn} \gtrless 0$ ($c''(x)$ large $\Rightarrow \frac{dQ^{ms}}{dn} > 0$) $\frac{dX^{ms}}{dn} \gtrless 0$ ($c''(x)$ large $\Rightarrow \frac{dX^{ms}}{dn} > 0$)	$\frac{dQ^{ms}}{dn} \gtrless 0$ (b or $c''(x)$ large $\Rightarrow \frac{dQ^{ms}}{dn} > 0$) $\frac{dX^{ms}}{dn} \gtrless 0$ (b or $c''(x)$ large $\Rightarrow \frac{dX^{ms}}{dn} > 0$)

TABLE 2 – Aggregate equilibrium, firms' entry, and the market share rule

$c''(x) \geq 0, c'''(x) = 0$	$\frac{dQ^{neg}}{dn} > 0$ $\frac{dX^{neg}}{dn} > 0$
$c''(x) > 0, c'''(x) > 0$	b or $c''(x)$ large $\Rightarrow \frac{dQ^{neg}}{dn} > 0$ b or $c''(x)$ large $\Rightarrow \frac{dX^{neg}}{dn} > 0)$

TABLE 3 – Aggregate equilibrium, firms' entry and negligence with a standard of care

Skipping to the most clearcut result, we find that when the marginal cost of care is increasing (and associated with a large degree of convexity), it is very likely that aggregate care expenditures increase with firms' entry (thus, decrease with restriction to firms' entry). This holds under strict liability – whatever the damage sharing rule, and whether an optimal damages multiplier is used or not – as well as under negligence (with a standard of care). In turn when the marginal cost of care is constant (or with low returns to scale, i.e. $c''(x) \to 0$), the impact of (restrictions to) firms' entry on aggregate care expenditures may depend in a complex manner on the specific design of the liability law, as well as on the elasticity of market demand. Once again going to the essence, aggregate care increase for sure with firms' entry: 1) under strict liability associated with the per capita rule and an optimal damages multiplier, or 2) under the negligence rule with a standard of care. Apart of such designs/cases, things are less predictable with strict liability, and as a consequence, the fine tuning of competition policy requires more information, some being quite commonplace and standard for a Competition Authority (value of elasticity of demand, number of firms). But others that are less usual, such as those regarding the way Courts implement the law (use of sharing rules). Our results suggest that the opportunity to use damages multipliers is of great importance here; on the one hand, damages multiplier design the incentives to take care at individual level; on the other hand, their impact on aggregate care expenditures may dramatically change with the regime liability to which they are associated. Under strict liability, aggregate care expenditures always increase with entry restrictions when suboptimal (no) damages multiplier are used; but they decrease (may increase or decrease) with entry restrictions if an optimal damages multiplier is combined with the per capita rule (market share rule). It is worth to remind that if CERCLA considers the use of "punitive damages", the use of "punitive damages" is forbidden up to now in the European context outside of competition law. The point is that suboptimal (no) damages multipliers lead to distortions in care and output decisions at individual level, and restrictions to firms' entry produce rule-specific distortions in aggregate care levels, and thus welfare losses.

The final challenge is whether exist liability rules flexible enough, in combination with competition policy, in order to improve Social Welfare? Our results suggest as a first best policy mix achieving higher aggregate output and care expenditures, as well as higher Social Welfare, the association of an ("ideal") environmental liability law based on three pillars – a regime of strict liability, a damages sharing arrangement given by the per capita rule, an optimal damages multiplier – on the one hand, and on the other hand, any tools at the disposal of a competition policy focused on the maximization of consumers surplus (here, an increase in the aggregate output thanks to firms' entry). Each of these tools is in the hands of an independent body (Courts enforce environmental liability law; the Competition Authority implements the competition law), but this policy mix requires a low degree of coordination: as long as each body commits to its targeted objective, firms undertake efficient decisions at the individual level both (in terms of care and in terms of output), aggregate expenditures for environment preservation increase with the degree of competition on the market (towards the socially optimal level), and firms' entry is Social Welfare improving.

To the converse, an unusual lenient competition policy (preserving firms markets power, through restrictions to firms' entry) may be justified on the grounds that the existing design of the environmental liability law departs from the ideal one, and is constraining the Competition Authority action. The rationale is that any design for environmental liability laws (other than our "ideal" law) will fail to increase aggregate care expenditures and improve Social Welfare when exists an excessive number of firms on the market – thus restricting market access may be a kind of second best policy for a Competition Authority. But as explained above, this needs a close scrutiny at the situation, case by case, in order to reach a fine tuning – otherwise, the aggregate care expenditures may decrease with restrictions to firms' entry. When Courts depart from the per capita rule to adopt the market share rule, the knowledge of the price sensibility of market demand as, well as the specific characteristics of the technology of safety (cost structure for care, up to c''; and productivity of care h'') is determinant to establish that restricting firms' entry should improve aggregate care and Social Welfare. Instead, when the assessment of liability is set according to a negligence test, firms will always adhere to a (flexible) standard of care, but reaching the fine tuning of firms' entry is even more demanding for a Competition Authority: this means collecting even more specific information on the technology of safety (up to c'''), not only on the price-elasticity of market demand.

Last but not the least, our analysis shows that the our so-called "ideal" environmental liability laws enables a traditional competition policy (focused on the improvement of competition in the industry) to move the equilibrium closer to the social optimum (as $n \to \infty$); in contrast for any other liability regime, there exists a finite number of firms that maximizes Social Welfare. The consequence is that adopting restrictions to firms' entry cannot be seen as a second best solution given the institutional design of the environmental liability law, unless the current number of firms is above this threshold. This is an empirical/practical issue, but maybe an uneasy job to establish.[19]

7 Concluding remarks

Our analysis rests on a simple framework (linear demand, durable care), but relaxing such assumptions will not change the general story (see for example Charreire and Langlais (2020)). We are also very focused here on a rough instrument (firms' entry) for a Competition Authority; additional works in different competitive environments will be useful to complete the picture. The case for cartels and various forms of collusion will be of specific interest. In this perspective Baumann, Charreire and Cosnita-Langlais (2020) develop the analysis of cartels stability under an environmental liability law based on the market share rule.

Remark also that the different points addressed here, as well as the conclusions of the paper focused on the environmental liability law, extend very generally to any situation with "third-party" victims (that is, having no economic nor contractual relationships with the injurers), e.g. injurers competing on the same legal market, producing a good with potential harmful consequences for human health, society and so on (which are not the consumers of the good produced by firms).[20] Most of the conclusions will also easily extend to consumers harms as the result of competition distorsions. Nonetheless, this needs more scrutiny. Friehe, Langlais and Schulte (2019) provide the analysis of harms to both consumers and third-parties victims for a monopolistic market, and find significant departures related to the comparison

[19] As matter of comparison, see the analysis of the consequence of fusions in European markets for mobile communications (Genakos, Valletti, and Verboten 2015), or the debate about the introduction of a fourth operator on the French market (Thesmar and Landier 2012).

[20] Authorized by Title III of the Superfund Amendments and Reauthorization Act (SARA), the Emergency Planning & Community Right-to-Know Act (EPCRA) was enacted by Congress as the national legislation on community safety. This law is designed to help local communities protect public health, safety, and the environment from chemical hazards.

of strict liability and negligence for example.

Finally, remark that (optimal) damages multipliers appear as an important pillar of the first best as well as second best policies in our paper. These so-called "punitive damages" are still a matter of debate in Europe. With punitive damages, effective damages paid by each firm represent a multiple of effective harms to the environment that fall down public pockets, or provide fundings for a Compensation Fund for victims of environmental harms. Such high levels for effective damages raise, and at the same time may help in solving, the judgment-proofness problem or the disappearing defendants issues. These two important problems are beyond the scope of the present paper which consider symmetric firms. The judgment-proofness issue requires a more detailed analysis of liability sharing in an asymmetric environment, whether this is understood as considering an asymmetric oligopoly,[21] or vertically differentiated markets; the disappearing defendant problem calls for a dynamic approach. This will be the topic of future researches.

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[21] In Charreire and Langlais (2017) we provide the comparative analysis of the market share and the per capita rules for an asymmetric oligopoly. We show that the market share rule induces more distortions in output decisions than the per capita rule. On the one hand, high-cost firms obtain a strategic competitive advantage when aggregate damages are shared according to the market share rule compared with the capita rule, allowing them to reduce their output less than low-cost firms. On the other hand, the market share rule yields a distribution of equilibrium markets shares that is less spread than the per capita rule, away from the socially optimal one.

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APPENDIX 1

Second order conditions and stability of Nash equilibrium, for social welfare, and profit maximization under Joint Liability with strict liability

In this appendix, we consider the general specification for the cost of care used in section 6, assuming that each firm operates at a cost given by $c(x)$ that satisfies: $c(0) > 0$, $c'(x) > 0$, $c''(x) \geq 0$, and $c'''(x) \geq 0$ $\forall\ x$. The different conditions we verify here *a fortiori* hold in the more restrictive case $c(x) = x$ of section 3 (i.e. when $c''(x) = 0 = c'''(x)\ \forall\ x$).

Social welfare maximization. The first-order conditions for an interior solution (q^{sw}, x^{sw}) are written:

$$\begin{aligned} a - bnq &= 2nqh(nx) && \text{(2')} \\ -h'(nx).(nq)^2 &= c'(x) && \text{(3')} \end{aligned}$$

Second order conditions for social welfare maximization (where usual notations are used for second order derivatives) require that at: $\Delta^{SW} \equiv SW_{qq}.SW_{xx} - (SW_{qx})^2 > 0$ where:

$$\begin{aligned} SW_{qq} &= -n(b + 2h(nx)) < 0 \\ SW_{qx} &= SW_{xq} = -2n^2qh'(nx) > 0 \\ SW_{xx} &= -(h''(nx)n\,(nq)^2 + c''(x)) < 0 \end{aligned}$$

Substituting, we obtain:

$$\begin{aligned}\Delta^{SW} &= n(b+2h(nx))(h''(nx)n\,(nq)^2+c''(x)) - 4n^4q^2\,(h'(nx))^2 \\ &> 2n^4q^2\left[h(nx)h''(nx) - 2\,(h'(nx))^2\right]\end{aligned}$$

Thus under (C2), $\Delta^{SW} > 0$ holds.

Let us now denote as $q^{sw}(x)$ the output level that solves (2') for any given x, and $x^{sw}(q)$ the care level that solves (3') for any given q, which is the level of care which is social welfare maximizing, for any given value of output. Straightforward calculations show that these functions are positively sloped in the space (q,x) since:

$$\frac{d\tilde{x}^{sw}}{dq}(q) = -\frac{SW_{qq}}{SW_{qx}} > 0\ ,\ \frac{dx^{sw}}{dq}(q) = -\frac{SW_{xq}}{SW_{xx}} > 0$$

where we denote $\tilde{x}^{sw}(q)$ for the inverse of $q^{sw}(x)$; thus, care and activity levels are strategic complements at the social optimum. The stability of the Nash equilibrium described by (2')-(3') requires that $\frac{dq^{sw}}{dx}(x) \times \frac{dx^{sw}}{dq}(q) < 1 \Leftrightarrow \frac{d\tilde{x}^{sw}}{dq}(q) > \frac{dx^{sw}}{dq}(q)$. Observe that according to the second order conditions $\Delta^{SW} > 0 \Leftrightarrow -\frac{SW_{qq}}{SW_{qx}} > -\frac{SW_{xq}}{SW_{xx}} \Rightarrow \frac{d\tilde{x}^{sw}}{dq} > \frac{dx^{sw}}{dq}$ – thus any Nash equilibrium (q^{sw}, x^{sw}) satisfying (2')-(3') is stable. This also implies the unicity of (q^{sw}, x^{sw}). To see why, consider that the condition $\frac{d\tilde{x}^{sw}}{dq}(q) > \frac{dx^{sw}}{dq}(q)$ means that the slope of $q^{sw}(x)$ is larger than the slope of $x^{sw}(q)$ in the space (q,x), and they cross at least once for (q^{sw}, x^{sw}) that satisfy (2')-(3'). Should they cross twice, this would require that there exists another local maximum $(\tilde{q}^{sw}, \tilde{x}^{sw})$ satisfying (2')-(3') and for which the condition $\frac{d\tilde{x}^{sw}}{dq}(q) < \frac{dx^{sw}}{dq}(q)$ now holds – implying that $\Delta^{SW} > 0$ does not hold, which is a contradiction.

When the marginal cost of care satisfies $c''(x) > 0\ \forall x$, it is easy to see that the aggregate output and care levels depend on n; for that purpose, we write conditions (2')-(3') in an equivalent form as:

$$a - bQ = 2Qh(X) \tag{2'bis}$$

$$-h'(X).Q^2 = c'\left(\frac{X}{n}\right) \tag{3'bis}$$

We will denote as $Q^{sw}(X)$ the aggregate output level that solves (2'bis), for any given X; while $X^{sw}(Q)$ is the aggregate care level that solves (3'bis), for any given Q. Condition (2'bis) does not depend on n, and thus $Q^{sw}(X)$ is independent of n; in contrast, the RHS in condition (3'bis) decreases with n, thus $X^{sw}(Q)$ is increasing in n. As a result (since output and care are strategic complements) an increase in n yields an increase in both the optimal level of aggregate output and aggregate care expenditures, (Q^{sw}, X^{sw}). Moreover, it is easy to verify that Social Welfare also increases with the number of firms at optimum, since evaluating (1) at (Q^{sw}, X^{sw}) and differentiating with respect to n yields (according to the envelop theorem):

$$\frac{dSW}{dn}(Q^{sw}, X^{sw}) = -c\left(\frac{X^{sw}}{n}\right) + c'\left(\frac{X^{sw}}{n}\right).\frac{X^{sw}}{n} > 0$$

by the convexity of $c(x)$. As far as the expenditures in care are the unique source of fixed costs for a firm (which is consistent with our interpretation that care is durable), this implies that it is socially optimal to have an infinite number of firms engaged in the activity (choosing both a level of output and care that are infinitively small). Finally, remark that when $n \to \infty$ then the socially optimal aggregate output and care reach some upper limit values $(Q^{sw}_{\infty}, X^{sw}_{\infty})$ satisfying $Q^{sw}_{\infty} = \frac{a}{b+2h(X^{sw}_{\infty})}$ and $-h'(X^{sw}_{\infty}) = \frac{c'(0)}{\left(\frac{a}{b+2h(X^{sw}_{\infty})}\right)^2}$.

Strict liability and the per capita rule. Denoting by $q_{-i} = \sum\limits_{s\neq i} q_s$ and $x_{-i} = \sum\limits_{s\neq i} x_s$, the first-order conditions for firm i are:

$$\begin{aligned} a - 2bq_i - bq_{-i} &= \frac{2\alpha}{n}(q_i + q_{-i})h(x_i + x_{-i}) \\ -h'(x_i + x_{-i}).\frac{\alpha}{n}(q_i + q_{-i})^2 &= c'(x_i) \end{aligned}$$

Second order conditions at firm level require that (we use usual notations for second order derivatives) $\Delta^{pc} \equiv \Pi^{pc}_{qq}.\Pi^{pc}_{xx} - \left(\Pi^{pc}_{qx}\right)^2 > 0$ at (q^{pc}, x^{pc}):

$$\begin{aligned} \Pi^{pc}_{qq} &= -2(b + \frac{\alpha}{n}h(nx)) < 0 \\ \Pi^{pc}_{qx} &= \Pi^{pc}_{xq} = -2\alpha nqh'(nx) > 0 \\ \Pi^{pc}_{xx} &= -(h''(nx)\alpha nq^2 + c''(x)) < 0 \end{aligned}$$

Substituting, we obtain:

$$\begin{aligned} \Delta^{pc} &= 2(b + \frac{\alpha}{n}h(nx))(\alpha h''(nx)nq^2 + c''(x)) - 4q^2\left(\alpha h'(nx)\right)^2 \\ &> 2\alpha^2 q^2\left[h(nx)h''(nx) - 2\left(h'(nx)\right)^2\right] \end{aligned}$$

Thus under (C2), $\Delta^{pc} > 0$ holds.

The symmetric Cournot-Nash equilibrium where $q_1 = ... = q_n = q^{pc}$ and $x_1 = ... = x_n = x^{pc}$ solves the system:

$$a - b(1+n)q = 2\alpha q h(nx) \tag{5'}$$

$$-\alpha h'(nx).nq^2 = c'(x) \tag{6'}$$

Let us now denote as $q^{pc}(x)$ the output level that solves (5') for any given x; similarly $x^{pc}(q)$ is the care level that solves (6') for any given q. Straightforward calculations show that these functions are positively sloped in the space (q, x); denoting $\tilde{x}^{pc}(q)$ the inverse of $q^{pc}(x)$. Let us denote the first order conditions for profit function at equilibrium as: $\hat{\Pi}_q^{pc} = 0 = a - b(1+n)q - 2\alpha h(nx)q$ and $\hat{\Pi}_x^{pc} = 0 = -h'(nx).\alpha nq^2 - c'(x)$; we obtain:

$$\frac{d\tilde{x}^{pc}}{dq} = -\frac{\hat{\Pi}_{qq}^{pc}}{\hat{\Pi}_{qx}^{pc}} > 0 \ , \ \frac{dx^{pc}}{dq} = -\frac{\hat{\Pi}_{qx}^{pc}}{\hat{\Pi}_{xx}^{pc}} > 0$$

since:

$$\begin{aligned} \hat{\Pi}_{qq}^{pc} &= -((1+n)b + 2\alpha h(nx)) < 0 \\ \hat{\Pi}_{qx}^{pc} &= \hat{\Pi}_{xq}^{pc} = -2\alpha nqh'(nx) > 0 \\ \hat{\Pi}_{xx}^{pc} &= -(\alpha h''(nx)\,(nq)^2 + c''(x)) < 0 \end{aligned}$$

Hence output and care are strategic complements under the per capita rule. Moreover note that $-\frac{\hat{\Pi}_{qq}^{pc}}{\hat{\Pi}_{qx}^{pc}} > -\frac{\hat{\Pi}_{qx}^{pc}}{\hat{\Pi}_{xx}^{pc}}$ is equivalent to the condition:

$$((1+n)b + 2\alpha h(nx)).(\alpha h''(nx)\,(nq)^2 + c''(x)) - 4\,(nq)^2\,(\alpha h'(nx))^2 > 0$$

which obviously holds under (C2) since:

$$\begin{aligned} & ((1+n)b + 2\alpha h(nx)).(\alpha h''(nx)\,(nq)^2 + c''(x)) - 4\,(nq)^2\,(\alpha h'(nx))^2 \\ > \ & 2\,(\alpha nq)^2\left[h(nx).h''(nx) - 2\,(h'(nx))^2\right] > 0 \end{aligned}$$

hence implying $\frac{d\tilde{x}^{pc}}{dq} > \frac{dx^{pc}}{dq}$ – any Nash equilibrium (q^{pc}, x^{pc}) satisfying (5')-(6') is thus stable. This also implies the unicity of (q^{pc}, x^{pc}). To see why, consider that the condition $\frac{d\tilde{x}^{pc}}{dq}(q) > \frac{dx^{pc}}{dq}(q)$ means that the slope of $q^{pc}(x)$ is larger than the slope of $x^{pc}(q)$ in the space (q, x), and they cross at least once for (q^{pc}, x^{pc}) that satisfy (5')-(6'). Should they cross twice, this would require that there exists another local maximum $(\tilde{q}^{pc}, \tilde{x}^{pc})$ satisfying (5')-(6') and for which the condition $\frac{d\tilde{x}^{pc}}{dq}(q) < \frac{dx^{pc}}{dq}(q)$ now holds – implying that $\Delta^{pc} > 0$ does not hold, which is a contradiction.

Strict liability with the market share rule. The first-order conditions for firm i are:

$$\begin{aligned} a - 2bq_i - bq_{-i} &= \alpha(2q_i + q_{-i})h(x_i + x_{-i}) \\ -h'(x_i + x_{-i}).\alpha q_i Q &= c'(x_i) \end{aligned}$$

Second order conditions at firm level require that: $\Delta^{ms} \equiv \Pi^{ms}_{qq}.\Pi^{ms}_{xx} - \left(\Pi^{ms}_{qx}\right)^2 > 0$ at (q^{ms}, x^{ms}), where:

$$\begin{aligned} \Pi^{ms}_{qq} &= -2(b + \alpha h(nx)) < 0 \\ \Pi^{ms}_{qx} &= -(1+n)q\alpha h'(nx) > 0 \\ \Pi^{ms}_{xx} &= -(\alpha h''(X)nq^2 + c''(x)) < 0 \end{aligned}$$

Substituting, we obtain:

$$\begin{aligned} \Delta^{ms} &= 2(b + \alpha h(nx))(\alpha h''(nx)nq^2 + c''(x)) - (1+n)^2 q^2 \left(\alpha h'(nx)\right)^2 \\ &> \alpha^2 q^2 \left[2nh(nx)h''(nx) - (1+n)^2 \left(h'(nx)\right)^2\right] \end{aligned}$$

Thus the convexity assumption of the individual liability cost function $L^{ms}_i(q_i, x_i) = q_i Q.h(X)$ implies that this last term is positive; thus $\Delta^{ms} > 0$ holds.

The symmetric Cournot-Nash equilibrium $q_1 = ... = q_n = q^{ms}$ and $x_1 = ... = x_n = x^{ms}$ solves the system:

$$a - b(1+n)q = \alpha(1+n)qh(nx) \tag{8'}$$

$$-\alpha h'(nx).nq^2 = c'(x) \tag{9'}$$

Let us now denote as $q^{ms}(x)$ the output level that, for any given value of care, solves (8'); similarly $x^{ms}(q)$ will denote the care level that, for any given value of the output, solves (9'). Straightforward calculations show now that these functions are positively sloped in the space (q, x). Let us denote the first order conditions for profit maximization at equilibrium $\hat{\Pi}^{ms}_q = 0 = a - b(1+n)q - (1+n)\alpha h(nx)q$ and $\hat{\Pi}^{ms}_x = 0 = -h'(nx).\alpha nq^2 - c'(x)$, we have:

$$\frac{d\tilde{x}^{ms}}{dq} = -\frac{\hat{\Pi}^{ms}_{qq}}{\hat{\Pi}^{ms}_{qx}} > 0 \ , \ \frac{dx^{ms}}{dq} = -\frac{\hat{\Pi}^{ms}_{xq}}{\hat{\Pi}^{ms}_{xx}} > 0$$

since:

$$\begin{aligned}
\hat{\Pi}^{ms}_{qq} &= -(1+n)(b+\alpha h(nx)) < 0 \\
\hat{\Pi}^{ms}_{qx} &= -nq(1+n)\alpha h'(nx) > 0 \\
\hat{\Pi}^{ms}_{xq} &= -2\alpha h'(nx)nq > 0 \\
\hat{\Pi}^{ms}_{xx} &= -(\alpha h''(nx)\,(nq)^2 + c''(x)) < 0
\end{aligned}$$

Hence output and care are strategic complements under the market share rule. Moreover $-\frac{\hat{\Pi}^{ms}_{qq}}{\hat{\Pi}^{ms}_{qx}} > -\frac{\hat{\Pi}^{ms}_{xq}}{\hat{\Pi}^{ms}_{xx}}$ is equivalent to the next condition:

$$(1+n)(b+\alpha h(nx))(\alpha h''(nx)\,(nq)^2 + c''(x)) - (1+n)\,(nq)^2\,(\alpha h'(nx))^2 > 0$$

which holds under (C2) since:

$$\begin{aligned}
& (1+n)(b+\alpha h(nx))(\alpha h''(nx)\,(nq)^2 + c''(x)) - (1+n)\,(nq)^2\,(h'(nx))^2 \\
> \quad & (1+n)\,(nq)^2\,\alpha^2\left[(h''(nx).h(nx)) - (h'(nx))^2\right] > 0
\end{aligned}$$

hence $\frac{d\tilde{x}^{ms}}{dq} > \frac{dx^{ms}}{dq}$ – any Nash equilibrium (q^{ms}, x^{ms}) satisfying (8')-(9') is thus stable. This also implies the unicity of (q^{ms}, x^{ms}). To see why, consider that the condition $\frac{d\tilde{x}^{ms}}{dq}(q) > \frac{dx^{ms}}{dq}(q)$ means that the slope of $q^{ms}(x)$ is larger than the slope of $x^{ms}(q)$ in the space (q, x), and they cross at least once for (q^{ms}, x^{ms}) that satisfy (8')-(9'). Should they cross twice, this would require that there exists another local maximum $(\tilde{q}^{ms}, \tilde{x}^{ms})$ satisfying (8')-(9') and for which the condition $\frac{d\tilde{x}^{ms}}{dq}(q) < \frac{dx^{ms}}{dq}(q)$ now holds – implying that $\Delta^{ms} > 0$ does not hold, which is a contradiction.

APPENDIX 2

Sensibility of the equilibrium to the number of firms under Joint Liability with strict liability (propositions 2, 4, 5, 8)

In this appendix, we consider the general specification for the cost of care used in section 6, assuming that each firm operates at a cost given by $c(x)$, with $c(0) > 0$, $c'(x) > 0$, $c''(x) \geq 0$, and $c'''(x) \geq 0\ \forall\ x$. When necessary, we contrast the case $c(x) = x\ \forall\ x$ of section 3 (where $c''(x) = 0 = c'''(x)\ \forall\ x$).

Firms entry with an exogenous damages multiplier.

The per capita rule. Let us denote (5')-(6') the first order derivatives of the profit function at equilibrium as $\hat{\Pi}_q^{pc} = a - b(1+n)q - 2\alpha h(nx)q = 0$ and $\hat{\Pi}_x^{pc} = -h'(nx).\alpha nq^2 - c'(x) = 0$. Differentiating in n, given that $\hat{\Pi}_{qq}^{pc}.\hat{\Pi}_{xx}^{pc} - \hat{\Pi}_{qx}^{pc}.\hat{\Pi}_{xq}^{pc} > 0$, we obtain:

$$
\begin{aligned}
sign\frac{dq^{pc}}{dn} &= sign\left(-\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc}\right) \\
sign\frac{dx^{pc}}{dn} &= sign\left(-\hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qq}^{pc} + \hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xq}^{pc}\right)
\end{aligned}
$$

with:

$$
\begin{aligned}
\hat{\Pi}_{qq}^{pc} &= -((1+n)b + 2\alpha h(nx)) < 0 \\
\hat{\Pi}_{qx}^{pc} &= \hat{\Pi}_{xq}^{pc} = -2\alpha nqh'(nx) > 0 \\
\hat{\Pi}_{xx}^{pc} &= -(\alpha h''(nx)\,(nq)^2 + c''(x)) < 0 \\
\hat{\Pi}_{qn}^{pc} &= -q(b + 2\alpha xh'(nx)) \gtreqless 0 \\
\hat{\Pi}_{xn}^{pc} &= -\alpha q^2(h'(nx) + xnh''(nx)) \gtreqless 0
\end{aligned}
$$

Proposition 2. Part i) Assume $c''(x) = 0\ \forall x$. Developing we obtain:

$$
\begin{aligned}
-\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc} &= -\alpha\,(nq)^2\,q\left(b - \frac{2\alpha}{n}\frac{(h'(nx))^2}{h''(nx)}\right).h''(nx) \\
-\hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qq}^{pc} + \hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xq}^{pc} &= \alpha q^2\left([(n-1)b - 2\alpha h(nx)]\,h'(nx)\right) \\
&\quad -\alpha nxq^2\left((1+n)bh''(nx) + 2\left[h(nx).h''(nx) - 2\,(h'(nx))^2\right]\right)
\end{aligned}
$$

According to assumption H1a: $b > 2h(0) > 2h(nx)$ for any $x > 0$, and under (C2): $h(nx) > 2\frac{(h'(nx))^2}{h''(nx)} > \lambda\frac{(h'(nx))^2}{h''(nx)}$ for any $x > 0$ and $\lambda \leq 1$. Hence it comes that for $1 \leq \alpha < n-1 : -\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc} < 0$ and $-\hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qq}^{pc} + \hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xq}^{pc} < 0$. This implies $\frac{dq^{pc}}{dn} < 0$ and $\frac{dx^{pc}}{dn} < 0$ for α not too large, i.e. for $1 \leq \alpha < n-1$.

Remark: Assume $c''(x) > 0\ \forall x$. This does not change $-\hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qq}^{pc} + \hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xq}^{pc}$ – thus: $\frac{dx^{pc}}{dn} < 0$ still holds. Moreover, $\frac{dq^{pc}}{dn} < 0$ still holds for α low enough. To see this, remark that we have now: $\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc} = -\alpha\,(nq)^2\,q\left(b - \frac{2\alpha}{n}\frac{(h'(nx))^2}{h''(nx)}\right).h''(nx) - q\,(b + 2\alpha x.h'(nx))\,.c''(x)$; under (C1) we have $b > -h'(nx).nx$, and as long as $1 \leq \alpha \leq \frac{n}{2}$ it comes that $-h'(nx).nx \geq -h'(nx).2\alpha x$: as a result $b > -h'(nx).2\alpha x$ holds, which implies finally that $\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc} < 0$; hence $\frac{dq^{pc}}{dn} < 0$ for α not too large ($1 \leq \alpha \leq \frac{n}{2}$).

Proposition 2. Part ii) Let us substitute with $Q = nq$ and $X = nx$ in (5')-(6'), and denote:

$\Pi_q^{pc} = 0 = a - b\left(1+\frac{1}{n}\right)Q - \frac{2\alpha}{n}h(X)Q$ and $\Pi_x^{pc} = 0 = -h'(X).\frac{\alpha}{n}Q^2 - c'\left(\frac{X}{n}\right)$. Differentiating in n yields, and using that $\Pi_{qQ}^{pc}.\Pi_{xX}^{pc} - \Pi_{qX}^{pc}.\Pi_{xQ}^{pc} > 0$, we obtain:

$$\begin{aligned} sign\frac{dQ^{pc}}{dn} &= sign\left(-\Pi_{qn}^{pc}.\Pi_{xX}^{pc} + \Pi_{xn}^{pc}.\Pi_{qX}^{pc}\right) \\ sign\frac{dX^{pc}}{dn} &= sign\left(-\Pi_{xn}^{pc}.\Pi_{qQ}^{pc} + \Pi_{qn}^{pc}.\Pi_{xQ}^{pc}\right) \end{aligned}$$

where:

$$\begin{aligned} \Pi_{qQ}^{pc} &= -\left(\left(1+\frac{1}{n}\right)b + \frac{2\alpha}{n}h(X)\right) < 0 \\ \Pi_{qX}^{pc} &= \Pi_{xQ}^{pc} = -h'(X)\frac{2\alpha}{n}Q > 0 \\ \Pi_{xX}^{pc} &= -h''(X)\frac{\alpha}{n}Q^2 - \frac{1}{n}c''\left(\frac{X}{n}\right) < 0 \\ \Pi_{qn}^{pc} &= \frac{Q}{n^2}(b + 2\alpha h(X)) > 0 \\ \Pi_{xn}^{pc} &= \alpha h'(X)\left(\frac{Q}{n}\right)^2 + \frac{1}{n^2}c''\left(\frac{X}{n}\right) \lesseqgtr 0 \end{aligned}$$

Assume $c''(x) = 0\ \forall x$. Let us develop to obtain:

$$\begin{aligned} -\Pi_{qn}^{pc}.\Pi_{xX}^{pc} + \Pi_{xn}^{pc}.\Pi_{qX}^{pc} &= \frac{\alpha Q^3}{n^2}\left[bh''(X) + 2\alpha\left(h(X).h''(nx) - (h'(X))^2\right)\right] \\ -\Pi_{xn}^{pc}.\Pi_{qQ}^{pc} + \Pi_{qn}^{pc}.\Pi_{xQ}^{pc} &= \frac{\alpha}{n}\left(\frac{Q}{n}\right)^2\left[b(n-1) - 2\alpha h(X)\right].h'(X) \end{aligned}$$

The bracketed term on the first line is positive under (C2); the bracketed term on the second line is also positive if $1 \le \alpha < n-1$, since by H1a: $b > 2h(0) > 2h(X)\ \forall X > 0$. As a result, we obtain $\frac{dQ^{pc}}{dn} > 0$ and $\frac{dX^{pc}}{dn} < 0$.

Remark: Assume $c''(x) > 0\,\forall x$. We obtain now: $-\Pi_{qn}^{pc}.\Pi_{xX}^{pc}+\Pi_{xn}^{pc}.\Pi_{qX}^{pc} = \left(-\Pi_{qn}^{pc}.\Pi_{xX}^{pc} + \Pi_{xn}^{pc}.\Pi_{qX}^{pc}\right)_{c''(x)=0} + \frac{1}{n}.c''\left(\frac{X}{n}\right)\left(n.\Pi_{qn}^{pc} + \Pi_{qX}^{pc}\right) > 0$ given that $n.\Pi_{qn}^{pc} + \Pi_{qX}^{pc} > 0$; hence $\frac{dQ^{pc}}{dn} > 0$ still holds. On the other hand, we have now $-\Pi_{xn}^{pc}.\Pi_{qQ}^{pc} + \Pi_{qn}^{pc}.\Pi_{xQ}^{pc} = \left(-\Pi_{xn}^{pc}.\Pi_{qQ}^{pc} + \Pi_{qn}^{pc}.\Pi_{xQ}^{pc}\right)_{c''(x)=0} + \frac{1}{n^2}.c''\left(\frac{X}{n}\right)\left(-\Pi_{qQ}^{pc}\right)$ where $-\Pi_{qQ}^{pc} > 0$; hence, the effect is ambiguous. It turns out that if $c''(x)$ is low then $\frac{dX^{pc}}{dn} < 0$ still holds; but as $c''(x)$ becomes large enough then it is more likely that $\frac{dX^{pc}}{dn} > 0$ may hold.

The market share rule. Let us use (8')-(9') the first order derivatives at equilibrium, and write $\hat{\Pi}_q^{ms} = a - b(1+n)q - (1+n)\alpha q h(nx) = 0$ and $\hat{\Pi}_x^{ms} = -h'(nx).\alpha n q^2 - c'(x) = 0$. Differentiating in n yields, given that $\hat{\Pi}_{qq}^{ms}.\hat{\Pi}_{xx}^{ms} - \hat{\Pi}_{qx}^{ms}.\hat{\Pi}_{xq}^{ms} > 0$:

$$sign\frac{dq^{ms}}{dn} = sign\left(-\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xx}+\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qx}\right)$$
$$sign\frac{dx^{ms}}{dn} = sign\left(-\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qq}+\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xq}\right)$$

with:

$$\begin{aligned}
\hat{\Pi}^{ms}_{qq} &= -(1+n)(b+\alpha h(nx))<0\\
\hat{\Pi}^{ms}_{qx} &= -nq(1+n)\alpha h'(nx)>0\\
\hat{\Pi}^{ms}_{xq} &= -2\alpha h'(nx)nq>0\\
\hat{\Pi}^{ms}_{xx} &= -(\alpha h''(nx)\,(nq)^2+c''(x))<0\\
\hat{\Pi}^{ms}_{qn} &= -q(b+\alpha h(nx))-(1+n)\alpha qxh'(nx)\lesseqgtr 0\\
\hat{\Pi}^{ms}_{xn} &= -\alpha q^2(h'(nx)+nxh''(nx))\lesseqgtr 0
\end{aligned}$$

Proposition 2. Part i) Assume $c''(x)=0\ \forall x$. Developing we obtain:

$$\begin{aligned}
-\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xx}+\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qx} &= -\alpha\,(nq)^2\,q\left(b+\alpha.\left[h(nx).h''(nx)-\frac{1+n}{n}\,(h'(nx))^2\right]\right)\\
-\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qq}+\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xq} &= \alpha q^2(n-1)\,(b+\alpha h(nx))\,h'(nx)-(1+n)\alpha nxq^2\left[bh''(nx)+\alpha\left(h(nx).h''(nx)-(h'(nx))^2\right)\right]
\end{aligned}$$

Given (C2), it comes that $-\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xx}+\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qx}<0$ (the bracketed term is positive given that $\frac{1+n}{n}<2$) and $-\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qq}+\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xq}<0$ (the bracketed term is positive for any $\alpha\geq 1$). Hence $\frac{dq^{ms}}{dn}<0$ and $\frac{dx^{ms}}{dn}<0$.

Remark: Assume $c''(x)>0\ \forall x$. This does not change $-\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qq}+\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xq}$ (thus, same effect: $\frac{dx^{ms}}{dn}<0$). However, the effect on $\frac{dq^{ms}}{dn}$ is more demanding. We have now: $-\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xx}+\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qx}=-\alpha\,(nq)^2\,q\left(b+\alpha.\left[h(nx).h''(nx)-\frac{1+n}{n}\,(h'(nx))^2\right]\right)-q\,(b+\alpha h(nx)+\alpha(1+n)x.h'(nx))\,.c''(x)$; hence under (C2), a new condition: $h(nx)+(1+n)x.h'(nx)>0$, is sufficient for $-\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xx}+\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qx}<0$ to hold: as a result, it comes that $\frac{dq^{ms}}{dn}<0$ – remark that assuming that the marginal cost of liability, $(1+n)q.h(nx)$, increases with n, implies that $h(nx)+(1+n)x.h'(nx)>0$.

Proposition 2. Part iii) Let us substitute with $Q=nq$ and $X=nx$ in (8')-(9'), and denote: $\Pi^{ms}_q=0=a-b\left(1+\frac{1}{n}\right)Q-\left(1+\frac{1}{n}\right)\alpha h(X)Q$ and $\Pi^{ms}_x=0=-h'(X).\frac{\alpha}{n}Q^2-c'\left(\frac{X}{n}\right)$. Differentiating in n yields, given that $\Pi^{ms}_{qQ}.\Pi^{ms}_{xX}-\Pi^{ms}_{qX}.\Pi^{ms}_{xQ}>0$:

$$\begin{aligned}
sign\frac{dQ^{ms}}{dn} &= sign\left(-\Pi^{ms}_{qn}.\Pi^{ms}_{xX}+\Pi^{ms}_{xn}.\Pi^{ms}_{qX}\right)\\
sign\frac{dX^{ms}}{dn} &= sign\left(-\Pi^{ms}_{xn}.\Pi^{ms}_{qQ}+\Pi^{ms}_{qn}.\Pi^{ms}_{xQ}\right)
\end{aligned}$$

where:

$$\begin{aligned}
\Pi^{ms}_{qQ} &= -\left(1+\frac{1}{n}\right)(b+\alpha h(X))<0\\
\Pi^{ms}_{qX} &= -\left(1+\frac{1}{n}\right)\alpha h'(X)Q>0\\
\Pi^{ms}_{xX} &= -h''(X)\frac{\alpha}{n}Q^2-\frac{1}{n}c''\left(\frac{X}{n}\right)<0\\
\Pi^{ms}_{xQ} &= -\frac{2\alpha}{n}h'(X)Q>0\\
\Pi^{ms}_{qn} &= \frac{Q}{n^2}(b+\alpha h(X))>0\\
\Pi^{ms}_{xn} &= \alpha h'(X)\left(\frac{Q}{n}\right)^2+\frac{1}{n^2}c''\left(\frac{X}{n}\right)\lesseqgtr 0
\end{aligned}$$

Assume $c''(x)=0\ \forall x$. Let us write:

$$\begin{aligned}
-\Pi^{ms}_{qn}.\Pi^{ms}_{xX}+\Pi^{ms}_{xn}.\Pi^{ms}_{qX} &= \alpha\left(\frac{Q}{n}\right)^3\left[bh''(X)+\alpha\left(h(X).h''(X)-(1+n)(h'(X))^2\right)\right]\\
-\Pi^{ms}_{xn}.\Pi^{ms}_{qQ}+\Pi^{ms}_{qn}.\Pi^{ms}_{xQ} &= \alpha\,(n-1)\left(\frac{Q}{n}\right)^2\frac{1}{n}(b+\alpha h(X)).h'(X)
\end{aligned}$$

Remark that the convexity of $L^{ms}_i(q_i,x_i)$ implies that $\frac{2n}{1+n}h(X).h''(X)-(1+n)\left(h'(X)\right)^2>0$ (where $\frac{2n}{1+n}>1$); thus, the bracketed term on the first line may be positive or negative (the convexity of the individual liability cost under the market share is not sufficient to sign this term); in contrast, the second line is negative without ambiguity. As a result, we obtain $\frac{dQ^{ms}}{dn}>(<)0$ if b is large (small) enough; while $\frac{dX^{ms}}{dn}<0$ always holds.

Remark: Assume $c''(x)>0\ \forall x$. We have: $-\Pi^{ms}_{qn}.\Pi^{ms}_{xX}+\Pi^{ms}_{xn}.\Pi^{ms}_{qX}=\left(-\Pi^{ms}_{qn}.\Pi^{ms}_{xX}+\Pi^{ms}_{xn}.\Pi^{ms}_{qX}\right)_{c''(x)=0}+\frac{1}{n}.c''\left(\frac{X}{n}\right)\left(n.\Pi^{ms}_{qn}+\Pi^{ms}_{qX}\right)$ with $n.\Pi^{ms}_{qn}+\Pi^{ms}_{qX}>0$; hence, the effect is again ambiguous; $\frac{dQ^{ms}}{dn}>0$ may hold for values of b much lower than before, and generally (or equivalently) as long as $c''(x)$ is large enough. On the other hand, $-\Pi^{ms}_{xn}.\Pi^{ms}_{qQ}+\Pi^{ms}_{qn}.\Pi^{ms}_{xQ}=\left(-\Pi^{ms}_{xn}.\Pi^{ms}_{qQ}+\Pi^{ms}_{qn}.\Pi^{ms}_{xQ}\right)_{c''(x)=0}+\frac{1}{n^2}.c''\left(\frac{X}{n}\right)\left(-\Pi^{ms}_{qQ}\right)$ where $-\Pi^{ms}_{qQ}>0$; hence, the effect is now ambiguous; nevertheless, it is more likely $\frac{dX^{ms}}{dn}>0$ now holds as long as $c''(x)$ is large enough.

Proposition 4 (The per capita rule and firms entry with an optimal damages multiplier).

Proposition 4. Part i) When $\alpha^* = n$, the first order derivatives of the profit function at equilibrium (5')-(6') are equivalently $\hat{\Pi}_q^{pc} = 0 = a - b(1+n)q - 2nh(nx)q$ and $\hat{\Pi}_x^{pc} = 0 = -h'(nx).(nq)^2 - c'(x)$. Differentiating in n, given that $\hat{\Pi}_{qq}^{pc}.\hat{\Pi}_{xx}^{pc} - \hat{\Pi}_{qx}^{pc}.\hat{\Pi}_{xq}^{pc} > 0$, we obtain:

$$
\begin{aligned}
sign\frac{dq^{pc}}{dn} &= sign\left(-\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc}\right) \\
sign\frac{dx^{pc}}{dn} &= sign\left(-\hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qq}^{pc} + \hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xq}^{pc}\right)
\end{aligned}
$$

with:

$$
\begin{aligned}
\hat{\Pi}_{qq}^{pc} &= -((1+n)b + 2nh(nx)) < 0 \\
\hat{\Pi}_{qx}^{pc} &= \hat{\Pi}_{xq}^{pc} = -2n^2qh'(nx) > 0 \\
\hat{\Pi}_{xx}^{pc} &= -(nh''(nx)(nq)^2 + c''(x)) < 0 \\
\hat{\Pi}_{qn}^{pc} &= -q(b + 2h(nx) + 2nxh'(nx)) \gtrless 0 \\
\hat{\Pi}_{xn}^{pc} &= -nq^2(2h'(nx) + nxh''(nx)) \gtrless 0
\end{aligned}
$$

Assume $c''(x) = 0 \; \forall x$. Developing we obtain:

$$
\begin{aligned}
-\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc} &= -(nq)^3\left((b - nxh'(nx)).h''(nx) + 2\left[h(nx).h''(nx) - 2(h'(nx))^2\right]\right) \\
-\hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qq}^{pc} + \hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xq}^{pc} &= -(nq)^2\left(\frac{b}{n}\left[2h'(nx) + (1+n)h''(nx)nx\right] + 2nx\left[h(nx).h''(nx) - 2(h'(nx))^2\right]\right)
\end{aligned}
$$

Under (C2), $-\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc} < 0$: hence $\frac{dq^{pc}}{dn} < 0$. The second bracketed term in $-\hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qq}^{pc} + \hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xq}^{pc}$ is also positive under (C2); however, the first bracketed term has an ambiguous sign; thus the sign of $\frac{dx^{pc}}{dn}$ is generally ambiguous. Remark however that:

$$
[2h'(nx) + h''(nx)nx] > 0 \Rightarrow [2h'(nx) + (1+n)h''(nx)nx] > 0
$$

Thus, under (C3) we obtain $\frac{dx^{pc}}{dn} < 0$.

Remark: Assume $c''(x) > 0 \; \forall x$. Hence, the expression for $-\hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qq}^{pc} + \hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xq}^{pc}$ does not change; thus, under (C3) we still obtain $\frac{dx^{pc}}{dn} < 0$. In contrast, we obtain now $-\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc} = \left(-\hat{\Pi}_{qn}^{pc}.\hat{\Pi}_{xx}^{pc} + \hat{\Pi}_{xn}^{pc}.\hat{\Pi}_{qx}^{pc}\right)_{c''(0)} - q.c''(x).(b + 2h(nx) + 2nxh'(nx))$. Remark that if (C1) holds, then we can write equivalently $b + 2h(nx) + 2nxh'(nx) > 2h(0) + 2nx.h'(nx) > 2h(nx) > 0$ implying that under (C1) we obtain $\frac{dq^{pc}}{dn} < 0$.

Proposition 4. Part ii) Let us substitute with $Q = nq$ and $X = nx$ in (5')-(6') when $\alpha^* = n$, to obtain:

$$
\begin{aligned}
a-b\left(1+\frac{1}{n}\right)Q &= 2Qh(X) && \text{(5'bis)}\\
-h'(X)Q^2 &= c'\left(\frac{X}{n}\right) && \text{(6'bis)}
\end{aligned}
$$

We will denote as $Q^{pc}(X)$ the aggregate output level that solves (5'bis), for any given value of aggregate care; while $X^{pc}(Q)$ is the aggregate care level that solves (6'bis), for any given value of the aggregate output. (5'bis) shows that an increase in n yields an increase in the aggregate output $Q^{pc}(X)$ at any level of X, since it shifts upward the marginal market proceeds. According to (6'bis), one verifies that i) either $c'(x)=1$ at any x, and thus $X^{pc}(Q)$ does not depend on n; or ii) $c''(x)>0$ at any x, and then an increase in n yields an increase in aggregate care expenditures $X^{pc}(Q)$, since it shifts downward the marginal cost of aggregate care expenditures. As a result when $c''(x)\geq 0$, an increase in n yields an increase in Q^{pc} and X^{pc}.

Proposition 5 (The market share rule and firms entry with an optimal damages multiplier when $c''(x)=0$).

Proposition 5. Part i) When $\alpha^*=n$, the first order derivatives at equilibrium (8')-(9') are $\hat{\Pi}_q^{ms}=a-b(1+n)q-(1+n)nqh(nx)=0$ and $\hat{\Pi}_x^{ms}=-h'(nx).\left(nq\right)^2-c'(x)=0$. Differentiating in n yields, given that $\hat{\Pi}_{qq}^{ms}.\hat{\Pi}_{xx}^{ms}-\hat{\Pi}_{qx}^{ms}.\hat{\Pi}_{xq}^{ms}>0$:

$$
\begin{aligned}
sign\frac{dq^{ms}}{dn} &= sign\left(-\hat{\Pi}_{qn}^{ms}.\hat{\Pi}_{xx}^{ms}+\hat{\Pi}_{xn}^{ms}.\hat{\Pi}_{qx}^{ms}\right)\\
sign\frac{dx^{ms}}{dn} &= sign\left(-\hat{\Pi}_{xn}^{ms}.\hat{\Pi}_{qq}^{ms}+\hat{\Pi}_{qn}^{ms}.\hat{\Pi}_{xq}^{ms}\right)
\end{aligned}
$$

with:

$$
\begin{aligned}
\hat{\Pi}_{qq}^{ms} &= -(1+n)(b+nh(nx))<0\\
\hat{\Pi}_{qx}^{ms} &= -n^2(1+n)h'(nx)q>0\\
\hat{\Pi}_{xq}^{ms} &= -2h'(nx)n^2q>0\\
\hat{\Pi}_{xx}^{ms} &= -(nh''(nx)\left(nq\right)^2+c''(x))<0\\
\hat{\Pi}_{qn}^{ms} &= -(b+(1+2n)h(nx))q-(1+n)nxh'(nx)q\lesseqgtr 0\\
\hat{\Pi}_{xn}^{ms} &= -nq^2(2h'(nx)+nxh''(nx))\lesseqgtr 0
\end{aligned}
$$

Assume $c''(x)=0\ \forall x$. Developing we obtain:

$$
\begin{aligned}
-\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xx}+\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qx} &= -(nq)^3\left(\left(b+h(nx)\right)h''(nx)+(1+n)\left(h(nx).h''(nx)-2\left(h'(nx)\right)^2\right)\right)\\
-\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qq}+\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xq} &= -(nq)^2\left(\frac{b}{n}\left(2h'(nx)+(1+n)h''(nx)nx\right)\right)\\
&\quad -(nq)^2\,n\left[-2h(nx).h'(nx)+(1+n)\,x\left(h(nx).h''(nx)-2\left(h'(nx)\right)^2\right)\right]
\end{aligned}
$$

Under (C2), it comes that $-\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xx}+\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qx}<0$; hence $\frac{dq^{ms}}{dn}<0$. Regarding $-\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qq}+\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xq}$ the bracketed term on the second line is also positive under (C2), while the bracketed term on the first line is also positive under (C3); hence $\frac{dx^{ms}}{dn}<0$.

Remark: Assume instead that $c''(x)>0\ \forall x$. As a result $-\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qq}+\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xq}$ does not change; hence $\frac{dx^{ms}}{dn}<0$ under (C2) and (C3). In contrast, we obtain now:

$-\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xx}+\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qx}=\left(-\hat{\Pi}^{ms}_{qn}.\hat{\Pi}^{ms}_{xx}+\hat{\Pi}^{ms}_{xn}.\hat{\Pi}^{ms}_{qx}\right)_{c''(0)}-q.c''(x).\left(b+(1+2n)(h(nx)+\left(\frac{1+n}{1+2n}\right)nxh'(nx))\right)$.

As a result, the sign of $\frac{dq^{ms}}{dn}$ is generally ambiguous; but it is sufficient that $b+nh(nx)+(1+n)(h(nx)+nxh'(nx))>0$ to obtain $\frac{dq^{ms}}{dn}<0$ – remark once more that assuming that the marginal cost of liability, $(1+n)q.h(nx)$, increases with n, implies that $h(nx)+(1+n)x.h'(nx)>0$; thus $nh(nx)+(1+n)nx.h'(nx)>0$, implying that $b+nh(nx)+(1+n)(h(nx)+nxh'(nx))>0$.

Proposition 5. Part ii) Let us substitute with $Q=nq$ and $X=nx$ in (8')-(9') when $\alpha^*=n$, to obtain:

$$
\begin{aligned}
a-b\left(1+\frac{1}{n}\right)Q &= (1+n)Qh(X) && \text{(8'bis)}\\
-h'(X).Q^2 &= c'\left(\frac{X}{n}\right) && \text{(9'bis)}
\end{aligned}
$$

We will denote as $Q^{ms}(X)$ the aggregate output level that solves (8'bis), for any given value of aggregate care; while $X^{ms}(Q)$ is the aggregate care level that solves (9'bis), for any given value of the aggregate output. By construction (9'bis) is identical to (6'bis), and thus aggregate care expenditures $X^{ms}(Q)$ also increase with n, at any level of Q. In contrast, according to (8ter) increasing the number of firms for any given level of care has two opposite effects on the output level: on the one hand, the marginal cost of liability increases with n, which drives downward the output; on the other hand, the marginal market proceeds also increases with n, driving upward the output. Thus, $Q^{ms}(X)$ may increase or decrease at any level of X, depending on whether the impact of n on the market proceeds is larger or smaller than on the cost of liability. In all, the impact of firms entry on the equilibrium levels of aggregate output and care depends on how aggregate output evolves with n, and on the feedback influence exerted by the choice of care.

Let us denote (8'bis)-(9'bis) as : $\Pi_q^{ms} = 0 = a - b\left(1+\frac{1}{n}\right)Q - (1+n)h(X)Q$ and $\Pi_x^{ms} = 0 = -h'(X).Q^2 - c'\left(\frac{X}{n}\right)$. Differentiating in n yields, given that $\Pi_{qQ}^{ms}.\Pi_{xX}^{ms} - \Pi_{qX}^{ms}.\Pi_{xQ}^{ms} > 0$, we obtain:

$$\begin{aligned}
sign\frac{dQ^{ms}}{dn} &= sign\left(-\Pi_{qn}^{ms}.\Pi_{xX}^{ms} + \Pi_{xn}^{ms}.\Pi_{qX}^{ms}\right) \\
sign\frac{dX^{ms}}{dn} &= sign\left(-\Pi_{xn}^{ms}.\Pi_{qQ}^{ms} + \Pi_{qn}^{ms}.\Pi_{xQ}^{ms}\right)
\end{aligned}$$

where:

$$\begin{aligned}
\Pi_{qQ}^{ms} &= -(1+n)\left(\frac{b}{n} + h(X)\right) < 0 \\
\Pi_{qX}^{ms} &= -(1+n)h'(X)Q > 0 \\
\Pi_{xX}^{ms} &= -h''(X)Q^2 - \frac{1}{n}c''\left(\frac{X}{n}\right) < 0 \\
\Pi_{xQ}^{ms} &= -2h'(X)Q > 0 \\
\Pi_{qn}^{ms} &= \frac{Q}{n^2}(b - n^2h(X)) \gtrless 0 \\
\Pi_{xn}^{ms} &= c''\left(\frac{X}{n}\right)\frac{X}{n^2} > 0
\end{aligned}$$

Let us write:

$$\begin{aligned}
-\Pi_{qn}^{ms}.\Pi_{xX}^{ms} + \Pi_{xn}^{ms}.\Pi_{qX}^{ms} &= \frac{Q}{n^2}(b - n^2h(X)).\left(h''(X)Q^2 + \frac{1}{n}c''\left(\frac{X}{n}\right)\right) + c''\left(\frac{X}{n}\right)\frac{X}{n^2}.(1+n)\left(-h'(X)\right)Q \qquad \text{(A)} \\
-\Pi_{xn}^{ms}.\Pi_{qQ}^{ms} + \Pi_{qn}^{ms}.\Pi_{xQ}^{ms} &= \frac{Q}{n^2}(b - n^2h(X)).2\left(-h'(X)\right)Q + c''\left(\frac{X}{n}\right)\frac{X}{n^2}.(1+n)\left(\frac{b}{n} + h(X)\right) \qquad \text{(B)}
\end{aligned}$$

Assume $c''(x) = 0\ \forall x$. Then:

$-\Pi_{qn}^{ms}.\Pi_{xX}^{ms} + \Pi_{xn}^{ms}.\Pi_{qX}^{ms} = \frac{Q}{n^2}(b - n^2h(X)).h''(X)Q^2$ and $-\Pi_{xn}^{ms}.\Pi_{qQ}^{ms} + \Pi_{qn}^{ms}.\Pi_{xQ}^{ms} = \frac{Q}{n^2}(b - n^2h(X)).2\left(-h'(X)\right)Q$.
As a result:

$b > n^2h(X)$ implies $\frac{dQ^{ms}}{dn} > 0$ and $\frac{dX^{ms}}{dn} > 0$. To the converse, $b < n^2h(X)$ implies $\frac{dQ^{ms}}{dn} < 0$ and $\frac{dX^{ms}}{dn} < 0$.

Proposition 5. Part v). Assume that $c''(x) = 0\ \forall x$. Evaluating (1) at (Q^{ms}, X^{ms}) and differentiating in n yields:

$$\frac{dSW}{dn}(Q^{ms}, X^{ms}) = \left(\frac{b}{n} + (n-1)h(X^{ms})\right).Q^{ms}.\frac{dQ^{ms}}{dn}$$

One can verify that as $n \to \infty$ then $-\Pi_{qn}^{ms}.\Pi_{xX}^{ms} + \Pi_{xn}^{ms}.\Pi_{qX}^{ms} \to -h(X)).Qh''(X)Q^2 < 0$ and thus $\frac{dQ^{ms}}{dn} < 0$. Moreover, $\left(\frac{b}{n} + (n-1)h(X^{ms})\right) \to \infty$, such that $\lim_{n\to\infty} \frac{dSW}{dn}(Q^{ms}, X^{ms}) \to -\infty$. Now, setting $n = 1$, we have $-\Pi^{ms}_{qn}.\Pi^{ms}_{xX} + \Pi^{ms}_{xn}.\Pi^{ms}_{qX} = (b - h(X)).\left(h''(X)Q^2\right) > 0$ since by assumption $b > h(0) > h(X)$. Thus $\left(\frac{dQ^{ms}}{dn}\right)_{n=1} > 0$. Moreover $\left(\frac{b}{n} + (n-1)h(X^{ms})\right) = b$ when $n = 1$. Hence $\left(\frac{dSW}{dn}(Q^{ms}, X^{ms})\right)_{n=1} > 0$. As a consequence, the number of firms that maximizes social welfare is finite, larger than 1, and satisfies:

$$\frac{dSW}{dn}(Q^{ms}, X^{ms}) = 0 \Rightarrow \left(\frac{b}{n} + (n-1)h(X^{ms})\right).Q^{ms}.\frac{dQ^{ms}}{dn} = 0 \Leftrightarrow \frac{dQ^{ms}}{dn} = 0$$

or equivalently: $b = n^2 h(X) \Rightarrow n^{ms} = \sqrt{\frac{b}{h(X^{ms})}}$.

Proposition 8. (The market share rule and firms entry with an optimal damages multiplier when $c''(x) > 0\ \forall x$)

Proposition 8 Part i). Let us go back to (A)-(B) when $c''(x) > 0\ \forall x$. Then:

i) it is obvious that $b > n^2 h(X)$ is sufficient to obtain $-\Pi^{ms}_{qn}.\Pi^{ms}_{xX} + \Pi^{ms}_{xn}.\Pi^{ms}_{qX} > 0$ and thus $\frac{dQ^{ms}}{dn} > 0$ and $\frac{dX^{ms}}{dn} > 0$ (whatever the size of $h''(x)$ and $c''(x)$).

ii) to the converse, when $b < n^2 h(X)$, the effects of n depend in a complex way on the relative size of b, $c''(x)$ and $h''(x)$. To illustrate, let us focus on some limit cases:

- assume $c''(x)$ is low enough (i.e. as in proposition 5, to the limit if $c''(x) \to 0$ at any x) then $-\Pi^{ms}_{qn}.\Pi^{ms}_{xX} + \Pi^{ms}_{xn}.\Pi^{ms}_{qX} \to \left(b - n^2 h(X)\right) h''(X)\frac{Q^3}{n^2}$ and $-\Pi^{ms}_{xn}.\Pi^{ms}_{qQ} + \Pi^{ms}_{qn}.\Pi^{ms}_{xQ} \to 2\left(b - n^2 h(X)\right)(-h'(X))\left(\frac{Q}{n}\right)^2$ such that $sign\frac{dQ^{ms}}{dn} = sign\left(b - n^2 h(X)\right) = sign\frac{dX^{ms}}{dn}$. Hence, $b < n^2 h(X)$ implies that we have at the same time $\frac{dQ^{ms}}{dn} < 0$ and $\frac{dX^{ms}}{dn} < 0$.

- in contrast $c''(x)$ large enough makes more likely that $\frac{dQ^{ms}}{dn} > 0$ as well as $\frac{dX^{ms}}{dn} > 0$ despite $b < n^2 h(X)$.

Proposition 8. Part iv). Assume that $c''(x) > 0\ \forall x$. Evaluating (1) at (Q^{ms}, X^{ms}) and differentiating in n yields:

$$\frac{dSW}{dn}(Q^{ms}, X^{ms}) = \left(\frac{b}{n} + (n-1)h(X^{ms})\right).Q^{ms}.\frac{dQ^{ms}}{dn} + \left(-c\left(\frac{X^{ms}}{n}\right) + c'\left(\frac{X^{ms}}{n}\right).\frac{X^{ms}}{n}\right)$$

Note that $\left(\frac{b}{n} + (n-1)h(X^{ms})\right) > 0$. One can verify that as $n \to \infty$ then $-\Pi^{ms}_{qn}.\Pi^{ms}_{xX} + \Pi^{ms}_{xn}.\Pi^{ms}_{qX} \to -h(X).Q.\left(h''(X)Q^2\right) < 0$ still holds since $\lim_{n\to\infty}\frac{1+n}{n^2} = \lim_{n\to\infty}\frac{1}{n} = 0$ and thus $\frac{dQ^{ms}}{dn} < 0$. Moreover $\left(-c\left(\frac{X^{ms}}{n}\right) + c'\left(\frac{X^{ms}}{n}\right).\frac{X^{ms}}{n}\right) \to -c(0) < 0$ as $n \to \infty$. This implies that $\lim_{n\to\infty}\frac{dSW}{dn}(Q^{ms}, X^{ms}) < 0$. Now, setting $n = 1$, we obtain:

$$-\Pi^{ms}_{qn}.\Pi^{ms}_{xX} + \Pi^{ms}_{xn}.\Pi^{ms}_{qX} = (b - h(X)).\left(h''(X)Q^2 + c''(X)\right) + c''(X)\,X.2\,(-h'(X))\,Q > 0$$

Given that by assumption $b > h(0) > h(X)$ for any $X > 0$, we obtain $\left(\frac{dQ^{ms}}{dn}\right)_{n=1} > 0$. On the other hand, under (C4) $-c\left(\frac{X^{ms}}{n}\right) + c'\left(\frac{X^{ms}}{n}\right).\frac{X^{ms}}{n} > 0$ holds for any finite n and $X > 0$. Thus, it is still true that $\left(\frac{dSW}{dn}(Q^{ms}, X^{ms})\right)_{n=1} > 0$. As a consequence, the number of firms that maximizes social welfare is finite and larger than 1, solving now (i.e. $\frac{dSW}{dn}(Q^{ms}, X^{ms}) = 0$):

$$-c\left(\frac{X^{ms}}{n^{ms}}\right) + c'\left(\frac{X^{ms}}{n^{ms}}\right).\frac{X^{ms}}{n^{ms}} = -\frac{dQ^{ms}}{dn}.\left(\frac{b}{n^{ms}} + (n^{ms}-1)h(X^{ms})\right).Q^{ms}$$

which is condition (19) in the paper. Obviously, there may exist several values of n that satisfy $\frac{dSW}{dn}(Q^{ms}, X^{ms}) = 0$ given that this condition is highly non linear, all being local maximum.

APPENDIX 3

First and second order conditions for negligence under a standard of care

In this appendix, we consider the general specification for the cost of care used in section 6, assuming that a firm i $(= 1, ..., n)$ operates at a cost given by $C(q_i, x_i) = c(x_i)$, with $c(0) > 0$, $c'(x_i) > 0$, $c''(x_i) \geq 0$, and $c'''(x_i) \geq 0 \ \forall \ x_i$. When necessary, we contrast the cases where $c(x_i) = x_i \ \forall \ x_i$ of section 3 (where $c''(x_i) = 0 = c'''(x_i) \ \forall \ x_i$).

The maximization of $(a - bQ)q_i - c(x_i(q))$ under the constraint (11) with respect to q_i yields that any firm will chose a level of output and care x_i satisfying the conditions:

$$\begin{aligned} a - 2bq_i - bq_{-i} &= c'(x_i).\frac{dx_i}{dq_i} \\ -h'(x_i + x_{-i}).(q_i + q_{-i})^2 &= c'(x_i) \end{aligned}$$

with $\frac{dx_i}{dq_i} = \frac{-h'(x_i+x_{-i}).2(q_i+q_{-i})}{h''(x_i+x_{-i}).(q_i+q_{-i})^2+c''(x_i)} > 0$. The second order condition requires that:

$$\pi_{qq} = -2b - c'(x_i).\frac{d^2x_i}{dq_i^2} - c''(x_i).\left(\frac{dx_i}{dq_i}\right)^2 < 0$$

In a symmetric Cournot-Nash equilibrium, $q_1 = ... = q_n = q^{neg}$ and $x_1 = ... = x_n = x^{neg}$ solve the system:

$$\begin{aligned} a - b(1+n)q &= 2nq\left(\frac{-h'(nx).c'(x)}{h''(nx)\,(nq)^2 + c''(x)}\right) \\ -h'(nx).\,(nq)^2 &= c'(x) \end{aligned}$$

Then substituting the second line in the first one gives us:

$$\begin{aligned} a-b(1+n)q &= 2nq\left(\frac{\left(h'(nx)\right)^2.\,(nq)^2}{h''(nx)\,(nq)^2+c''(x)}\right) && (13') \\ -h'(nx).\,(nq)^2 &= c'(x) && (3') \end{aligned}$$

Thus, when $c(x)=x$, one obtains conditions (13)-(3) as in the text setting $c'(x)=1$ and $c''(x)=0$.

APPENDIX 4

Comparison between negligence with a standard of care and the social optimum

(proposition 6)

Social optimum vs negligence with a standard of care. Let us defined in a generic way the following system encompassing both systems (2')-(3') and (13')-(3'), defining a scale parameter $k\in[0,1]$:

$$\begin{aligned} \pi_q &= a-(1-k)\left(bnq+2nqh(nx)\right)-k\left(b(1+n)q+2nq\left(\frac{\left(h'(nx)\right)^2.\,(nq)^2}{h''(nx)\,(nq)^2+c''(x)}\right)\right)=0 && \text{(C)} \\ \pi_x &= -h'(nx).\,(nq)^2-c'(x)=0 && \text{(D)} \end{aligned}$$

For $k=0$, then (C)-(D) is equivalent to (2')-(3'), whereas for $k=1$, then (C)-(D) is equivalent to (13')-(3'). Differentiating (C)-(D) in k, it comes that:

$$\begin{aligned} \pi_{qq}\frac{dq}{dk}+\pi_{qx}\frac{dx}{dk} &= -\pi_{qk} \\ \pi_{xq}\frac{dq}{dk}+\pi_{xx}\frac{dx}{dk} &= -\pi_{xk} \end{aligned}$$

where:

$$\begin{aligned} \pi_{qq}= \quad & -(1-k)\left(bn+2nh(nx)\right) \\ & -k\left(b(1+n)+2n\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)+2nq\frac{d}{dq}\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)\right)<0 \\ \pi_{xx}= \quad & -h''(nx)n\,(nq)^2-c''(x)<0 \\ \pi_{qx}= \quad & -2nq\left[(1-k)nh'(nx)+k\frac{d}{dx}\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)\right]>0 \\ \pi_{xq}= \quad & -2h'(nx)n^2q>0 \\ \pi_{qk}= \quad & -bq+2nq\left(h(nx)-\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)\right)\gtreqless 0 \\ \pi_{xk}= \quad & 0 \end{aligned}$$

with $\frac{d}{dq}\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)>0$ and $\frac{d}{dx}\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)<0$. We obtain, given that by second order conditions $\Delta=\pi_{qq}.\pi_{xx}-\pi_{qx}.\pi_{xq}>0$ (which holds for b large enough):

$$sign\frac{dq}{dk} = sign\left\{-\pi_{qk}.\pi_{xx}\right\} \tag{E}$$
$$sign\frac{dx}{dk} = sign\left\{\pi_{qk}.\pi_{xq}\right\} \tag{F}$$

Thus, the sign of $\pi_{qk}=-bq+2nq\left(h(nx)-\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)\right)$ is determinant for the result, with under (C2): $h(nx)-\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)>0$. As a result:

i) either b is large enough, i.e. $b>2n\left(h(nx)-\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)\right)$ such that $\pi_{qk}<0$ and then we obtain: $\frac{dq}{dk}<0$ and $\frac{dx}{dk}<0$, meaning that starting from $k=0$ and increasing continuously k (up to $k=1$) yields a decrease in the levels of output and care; hence $\hat{q}^{neg}<q^{sw}$ and $\hat{x}^{neg}<x^{sw}$;

ii) or b is small, i.e. $b<2n\left(h(nx)-\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx)(nq)^2+c''(x)}\right)\right)$ such that $\pi_{qk}>0$ and then we obtain: $\frac{dq}{dk}>0$ and $\frac{dx}{dk}>0$, meaning that starting from $k=0$ and increasing continuously k (up to $k=1$) yields an increase in the levels of output and care; hence $\hat{q}^{neg}>q^{sw}$ and $\hat{x}^{neg}>x^{sw}$.

APPENDIX 5

Sensibility of negligence with a standard of care to the number of firms

(propositions 7 and 9)

Proposition 7. Part i). Let us denote (13')-(3') as $\hat{\Pi}_q^{neg}=0=a-b(1+n)q-2nq.v$ where $v=\left(\frac{\left(h'(nx)\right)^2.(nq)^2}{h''(nx).(nq)^2+c''(x)}\right)$ ($<h(nx)$ by convexity of the expected harm) and $\hat{\Pi}_x^{neg}=0=-h'(nx).\left(nq\right)^2-c'(x)$. Differentiating in n, given that $\Pi_{qq}^{neg}.\Pi_{xx}^{neg}-\Pi_{qx}^{neg}.\Pi_{xq}^{neg}>0$, yields:

$$sign\frac{dq^{neg}}{dn} = sign\left(-\Pi_{qn}^{neg}.\Pi_{xx}^{neg}+\Pi_{xn}^{neg}.\Pi_{qx}^{neg}\right)$$
$$sign\frac{dx^{neg}}{dn} = sign\left(-\Pi_{xn}^{neg}.\Pi_{qq}^{neg}+\Pi_{qn}^{neg}.\Pi_{xq}^{neg}\right)$$

with:

$$
\begin{aligned}
\Pi_{qq}^{neg} &= -\left((1+n)\,b+2n.\left(v+q\frac{dv}{dq}\right)\right)<0\\
\Pi_{qx}^{neg} &= -2n.\frac{dv}{dx}q>0\\
\Pi_{xq}^{neg} &= -2n^{2}h'(nx)q>0\\
\Pi_{xx}^{neg} &= -h''(nx)n\,(nq)^{2}-c''\,(x)<0\\
\Pi_{qn}^{neg} &= -\left(b+2\left(v+n\frac{dv}{dn}\right)\right)q\gtrless 0\\
\Pi_{xn}^{neg} &= -nq^{2}\left(h''(nx)nx+2h'(nx)\right)\gtrless 0
\end{aligned}
$$

Let us assume $c''(x)=0$. Then $v=\left(\frac{\left(h'(nx)\right)^{2}}{h''(nx)}\right)$, such that $\frac{dv}{dq}=0$ and $\frac{dv}{dx}x=\frac{dv}{dn}n=2nx.h'(nx)-vnx\frac{h'''(nx)}{h''(nx)}<0$. This implies that:

$$
\begin{aligned}
-\Pi_{qn}^{neg}.\Pi_{xx}^{neg}+\Pi_{xn}^{neg}.\Pi_{qx}^{neg} &= -\left(nq\right)^{3}\left((b+2v).h''(nx)-\frac{4}{n}\frac{\partial v}{\partial x}.h'(nx)\right)\\
&= -\left(nq\right)^{3}h''(nx).\left(b-6v\left[1-\frac{2}{3}\frac{h'(nx).h'''(nx)}{\left(h''(nx)\right)^{2}}\right]\right)\\
-\Pi_{xn}^{neg}.\Pi_{qq}^{neg}+\Pi_{qn}^{neg}.\Pi_{xq}^{neg} &= -(nq)^{2}\frac{b}{n}\left[2h'(nx)+nxh''(nx)\right]\\
&\quad -\left(nq\right)^{2}nx.h''(nx).\left(b-6v\left[1-\frac{2}{3}\frac{h'(nx).h'''(nx)}{\left(h''(nx)\right)^{2}}\right]\right)
\end{aligned}
$$

Hence $b>6v\left[1-\frac{2}{3}\frac{h'(nx).h'''(nx)}{\left(h''(nx)\right)^{2}}\right]\,(>0)\Rightarrow-\Pi_{qn}^{neg}.\Pi_{xx}^{neg}+\Pi_{xn}^{neg}.\Pi_{qx}^{neg}<0$ and thus $\frac{dq^{neg}}{dn}<0$; to the converse $b<6v\left[1-\frac{2}{3}\frac{h'(nx).h'''(nx)}{\left(h''(nx)\right)^{2}}\right]\Rightarrow-\Pi_{qn}^{neg}.\Pi_{xx}^{neg}+\Pi_{xn}^{neg}.\Pi_{qx}^{neg}>0$ and thus $\frac{dq^{neg}}{dn}>0$.

In contrast, the sign of $\frac{dx^{neg}}{dn}$ is more demanding to establish. 1) If $b>6v\left[1-\frac{2}{3}\frac{h'(nx).h'''(nx)}{\left(h''(nx)\right)^{2}}\right]$ then the term on second line in $-\Pi_{xn}^{neg}.\Pi_{qq}^{neg}+\Pi_{qn}^{neg}.\Pi_{xq}^{neg}$ is negative; while (C3) (i.e. $2h'(nx)+nxh''(nx)>0$) implies that the term on first line in $-\Pi_{xn}^{neg}.\Pi_{qq}^{neg}+\Pi_{qn}^{neg}.\Pi_{xq}^{neg}$ is negative – hence $b>6v\left[1-\frac{2}{3}\frac{h'(nx).h'''(nx)}{\left(h''(nx)\right)^{2}}\right]$ and (C3) imply that $\frac{dx^{neg}}{dn}<0$. Otherwise, 2) if $b<6v\left[1-\frac{2}{3}\frac{h'(nx).h'''(nx)}{\left(h''(nx)\right)^{2}}\right]$, the term on second line is positive ; and if (C3) does not hold (i.e. assume to the converse that $2h'(nx)+nxh''(nx)<0$), the term on first line is positive – hence, $b<6v\left[1-\frac{2}{3}\frac{h'(nx).h'''(nx)}{\left(h''(nx)\right)^{2}}\right]$ together with $2h'(nx)+nxh''(nx)<0$ imply that $\frac{dx^{neg}}{dn}>0$.

Using (13')-(3') and substituting with $Q=nq$ and $X=nx$, now we have: $\Pi_{q}^{neg}=0=a-b\left(1+\frac{1}{n}\right)Q-2Q.V$ where we denote $V=\left(\frac{\left(h'(X)\right)^{2}.Q^{2}}{h''(X)Q^{2}+c''\left(\frac{X}{n}\right)}\right)$, and $\Pi_{x}^{neg}=0=-h'(X).Q^{2}-c'\left(\frac{X}{n}\right)$. Differentiating in n, given that $\Pi_{qQ}^{neg}.\Pi_{xX}^{neg}-\Pi_{qX}^{neg}.\Pi_{xQ}^{neg}>0$, yields:

$$
\begin{aligned}
sign\frac{dQ^{neg}}{dn} &= sign\left(-\Pi_{qn}^{neg}.\Pi_{xX}^{neg}+\Pi_{xn}^{neg}.\Pi_{qX}^{neg}\right)\\
sign\frac{dX^{neg}}{dn} &= sign\left(-\Pi_{xn}^{neg}.\Pi_{qQ}^{neg}+\Pi_{qn}^{neg}.\Pi_{xQ}^{neg}\right)
\end{aligned}
$$

with:

$$
\begin{aligned}
\Pi_{qQ}^{neg} &= -\left(\frac{1+n}{n}b+2\left(V+Q\frac{dV}{dQ}\right)\right)<0\\
\Pi_{qX}^{neg} &= -2Q\frac{dV}{dX}>0\\
\Pi_{xX}^{neg} &= -h''(X)Q^2-\frac{1}{n}c''\left(\frac{X}{n}\right)<0\\
\Pi_{xQ}^{neg} &= -2h'(X)Q>0\\
\Pi_{qn}^{neg} &= \left(\frac{b}{n^2}-2\frac{dV}{dn}\right)Q\gtrless 0\\
\Pi_{xn}^{neg} &= c''\left(\frac{X}{n}\right)\frac{X}{n^2}>0
\end{aligned}
$$

where: $\frac{dV}{dQ}=2\frac{V}{Q}\frac{c''\left(\frac{X}{n}\right)}{h''(X)Q^2+c''\left(\frac{X}{n}\right)}>0$ and $\frac{dV}{dX}=V.\left(2\frac{h''(X)}{h'(X)}-\frac{h'''(X)Q^2+c'''\left(\frac{X}{n}\right)\frac{1}{n}}{h''(X)Q^2+c''\left(\frac{X}{n}\right)}\right)<0$; moreover one can verify that $\frac{dV}{dn}=\frac{V}{n}.\frac{c'''\left(\frac{X}{n}\right).\frac{X}{n}}{h''(X)Q^2+c''\left(\frac{X}{n}\right)}$, implying that: if $c'''(x)>0$ for any $x>0$ then $\frac{dV}{dn}>0$; but if $c'''(x)=0$ for any $x>0$ then $\frac{dV}{dn}=0$. Let us write after some manipulations:

$$
\begin{aligned}
&-\Pi_{qn}^{neg}.\Pi_{xX}^{neg}+\Pi_{xn}^{neg}.\Pi_{qX}^{neg}\\
=\ &Q\left(\frac{b}{n^2}-2\frac{dV}{dn}\right)\left(h''(X)Q^2+\frac{1}{n}c''\left(\frac{X}{n}\right)\right)+\frac{X}{n^2}c''\left(\frac{X}{n}\right).2Q\left(-\frac{dV}{dX}\right)
\end{aligned}
$$

and:

$$
\begin{aligned}
&-\Pi_{xn}^{neg}.\Pi_{qQ}^{neg}+\Pi_{qn}^{neg}.\Pi_{xQ}^{neg}\\
=\ &Q\left(\frac{b}{n^2}-2\frac{dV}{dn}\right).(-2h'(X)Q)+c''\left(\frac{X}{n}\right)\frac{X}{n^2}\left[b\left(\frac{1+n}{n}\right)+2\left(V+Q\frac{dV}{dQ}\right)\right]
\end{aligned}
$$

Generally, the effects of n on Q^{neg} and X^{neg} are driven in a complex way by the slope of different curves (respectively, the market demand, the marginal cost of care, the marginal probability of damage): b, $c''(x)$ and $h''(x)$. Proposition 7 considers the case with a constant marginal cost for care ($c''(x)=0\ \forall x$); proposition 9 instead focuses on the case with an increasing marginal cost for care ($c''(x)>0\ \forall x$).

Proposition 7. Part ii). Assume $c''(x)=0\ \forall x$. Then $\frac{dV}{dn}=0$, implying that $-\Pi_{qn}^{neg}.\Pi_{xX}^{neg}+$

$\Pi_{xn}^{neg}.\Pi_{qX}^{neg} = Q\frac{b}{n^2}.h''(X)Q^2 > 0$ and $-\Pi_{xn}^{neg}.\Pi_{qQ}^{neg} + \Pi_{qn}^{neg}.\Pi_{xQ}^{neg} = Q\frac{b}{n^2}.(-2h'(X)Q) > 0$; hence we obtain $\frac{dQ^{neg}}{dn} > 0$ and $\frac{dX^{neg}}{dn} > 0$.

Proposition 7. Part iv). Assume $c''(x) = 0\ \forall x$. Evaluating (1) at (Q^{neg}, X^{neg}) and differentiating in n, we obtain:

$$\frac{dSW}{dn}(Q^{neg}, X^{neg}) = \frac{dQ^{neg}}{dn}.\left(\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right)\right).Q^{neg}$$

Hence $\frac{b}{n} > 2\left(h(X^{neg}) - V^{neg}\right) \Rightarrow \frac{dSW}{dn}(Q^{neg}, X^{neg}) > 0$ and $\frac{b}{n} < 2\left(h(X^{neg}) - V^{neg}\right) \Rightarrow \frac{dSW}{dn}(Q^{neg}, X^{neg}) < 0$.

Proposition 7. Part v). Assume $c''(x) = 0\ \forall x$. One can verify that as $n \to \infty$ then $-\Pi_{qn}^{neg}.\Pi_{xX}^{neg} + \Pi_{xn}^{neg}.\Pi_{qQ}^{neg} \to \frac{1}{n^2} \to 0$ implying that $\lim_{n\to\infty}\left(\frac{dQ^{neg}}{dn}\right) = 0_+$; moreover, $\left(\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right)\right) \to -2\left(h(X^{neg}) - V^{neg}\right) < 0$. Thus $\lim_{n\to\infty}\left(\frac{dSW}{dn}(Q^{neg}, X^{neg})\right) = 0_-$. On the other hand, when $n = 1$ we have: $\left(\frac{dQ^{neg}}{dn}\right)_{n=1} > 0$, and $\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right) = b - 2\left(h(X^{neg}) - V^{neg}\right) > 0$ since $b > 2.h(0)$ by assumption. Hence $\left(\frac{dSW}{dn}(Q^{neg}, X^{neg})\right)_{n=1} > 0$. As a consequence, the number of firms that maximizes social welfare is finite, larger than 1, and satisfies:

$$\frac{dSW}{dn}(Q^{neg}, X^{neg}) = 0 \Rightarrow \left(\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right)\right).\frac{dQ^{neg}}{dn}.Q^{neg} = 0 \Leftrightarrow \frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right) = 0$$

since $\frac{dQ^{neg}}{dn} > 0$. Solving yields $n^{neg} = \frac{b}{2(h(X^{neg}) - V^{neg})}$.

Proposition 9. Proposition 9 is substituted to proposition 7 when we assume $c''(x) > 0$, and $c'''(x) \geq 0\ \forall x$.

Proposition 9. Part i) If $c''(x) > 0$ but $c'''(x) = 0\ \forall x > 0$ (then $\frac{dV}{dn} = 0$ still holds), we obtain $-\Pi_{qn}^{neg}.\Pi_{xX}^{neg} + \Pi_{xn}^{neg}.\Pi_{qX}^{neg} = Q\frac{b}{n^2}\left(h''(X)Q^2 + \frac{1}{n}c''\left(\frac{X}{n}\right)\right) + \frac{X}{n^2}c''\left(\frac{X}{n}\right).2Q\left(-\frac{dV}{dX}\right) > 0$ and: $-\Pi_{xn}^{neg}.\Pi_{qQ}^{neg} + \Pi_{qn}^{neg}.\Pi_{xQ}^{neg} = Q\frac{b}{n^2}.(-2h'(X)Q) + c''\left(\frac{X}{n}\right)\frac{X}{n^2}\left[b\left(\frac{1+n}{n}\right) + 2\left(V + Q\frac{dV}{dQ}\right)\right] > 0$; hence $\frac{dQ^{neg}}{dn} > 0$ and $\frac{dX^{neg}}{dn} > 0$.

Proposition 9. Part ii) Otherwise when $c'''(x) > 0\ \forall x$ (then $\frac{dV}{dn} > 0$), the signs of $-\Pi_{qn}^{neg}.\Pi_{xX}^{neg} + \Pi_{xn}^{neg}.\Pi_{qX}^{neg}$ and $-\Pi_{xn}^{neg}.\Pi_{qQ}^{neg} + \Pi_{qn}^{neg}.\Pi_{xQ}^{neg}$ depend on the sign of $\left(\frac{b}{n^2} - 2\frac{dV}{dn}\right)$ but also on terms depending on the relative size of $h''(x)$ and $c''(x)$. i) $b > 2n^2\frac{dV}{dn}$ is sufficient for that $\frac{dQ^{neg}}{dn} > 0$ and $\frac{dX^{neg}}{dn} > 0$. ii) To the converse, if $b < 2n^2\frac{dV}{dn}$ then the signs of $\frac{dQ^{neg}}{dn}$ and $\frac{dX^{neg}}{dn}$ are indeterminate; let us consider some polar cases:

- obviously as $c''(x) > 0$ is small enough compared to $h''(x)$ (to the limit $c''(x)$ is close to 0 , but with $c'''(x) > 0$ at any x), then $-\Pi_{qn}^{neg}.\Pi_{xX}^{neg} + \Pi_{xn}^{neg}.\Pi_{qQ}^{neg} \to h''(X)\frac{Q^3}{n^2}\left(b - 2n^2\frac{dV}{dn}\right)$ and $-\Pi_{xn}^{neg}.\Pi_{qQ}^{neg} + \Pi_{qn}^{neg}.\Pi_{xQ}^{neg} \to (-h'(X))\left(\frac{Q}{n}\right)^2\left(b - 2n^2\frac{dV}{dn}\right)$ such that $b < n^2\frac{dV}{dn}$ implies that we have both $\frac{dQ^{neg}}{dn} < 0$ and $\frac{dX^{neg}}{dn} < 0$.

- in contrast, as $c''(x) > 0$ is large enough compared to $h''(x)$ (i.e. to the limit $h''(x)$ is close to 0 at any x), then $\frac{dQ^{neg}}{dn} > 0$ and $\frac{dX^{neg}}{dn} > 0$ is more likely to occur.

On the other hand, substituting with $\frac{dV}{dn}$ and $\frac{dV}{dX}$, we can write (we omit the dependant variables, to alleviate the notations):

$$\begin{aligned}
& -\Pi_{qn}^{neg}.\Pi_{xX}^{neg} + \Pi_{xn}^{neg}.\Pi_{qX}^{neg} \\
= \; & Q\frac{b}{n^2}\left(h''Q^2 + \frac{1}{n}c''\right) - 2\frac{dV}{dn}\left(h''Q^2 + \frac{1}{n}c''\right) + \frac{X}{n^2}c''\frac{X}{n}.2Q\left(-\frac{dV}{dX}\right) \\
= \; & Q\frac{b}{n^2}\left(h''Q^2 + \frac{1}{n}c''\right) + 2\frac{V}{n}\frac{X}{n}Qc''.\underbrace{\left[-\frac{c'''}{c''}.\frac{h''Q^2 + c''.\frac{1}{n}}{h''Q^2 + c''} - 2\frac{h''}{h'} + \frac{h'''Q^2 + c'''.\frac{1}{n}}{h''Q^2 + c''}\right]}_{A}
\end{aligned}$$

It can be verify that the bracketed term denoted A takes a non negative sign under the next different conditions:

(a) $h'' = h''' = 0$ at any x implies that $A = 0$;

(b) $\frac{h'''}{h''} > \frac{c'''}{c''}$ implies that $A > 0$.

Hence, whether (a) or (b) holds, it comes $-\Pi_{qn}^{neg}.\Pi_{xX}^{neg} + \Pi_{xn}^{neg}.\Pi_{qX}^{neg} > 0$ and thus $\frac{dQ^{neg}}{dn} > 0$. We did not find equivalent conditions for $\frac{dX^{neg}}{dn}$.

Consider now the impact of firms entry on social welfare. Evaluating (1) at (Q^{neg}, X^{neg}) and differentiating in n (using (13'bis)-(3'bis)), we obtain:

$$\frac{dSW}{dn}(Q^{neg}, X^{neg}) = \frac{dQ^{neg}}{dn}.\left(\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right)\right).Q^{neg} + \left(-c\left(\frac{X^{neg}}{n}\right) + c'\left(\frac{X^{neg}}{n}\right).\frac{X^{neg}}{n}\right)$$

with $-c\left(\frac{X^{neg}}{n}\right) + c'\left(\frac{X^{neg}}{n}\right).\frac{X^{neg}}{n} > 0$ for any finite n and $X > 0$ under (C4).

Proposition 9. Part iii) If $c'''(x) = 0 \; \forall x$, given that $\frac{dQ^{neg}}{dn} > 0$, then if $\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right) > 0$ we obtain that $\frac{dSW}{dn}(Q^{neg}, X^{neg}) > 0$; in contrast if $\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right) < 0$ then $\frac{dSW}{dn}(Q^{neg}, X^{neg}) <$ *or* > 0.

If $c'''(x) > 0 \; \forall x$, then $\left(\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right)\right) \gtrless 0$ for a given set of values for b, while $\frac{dQ^{neg}}{dn} \lesseqgtr 0$ for another set of values for b. Hence, generally $\left(\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right)\right).\frac{dQ^{neg}}{dn} > 0 \Rightarrow \frac{dSW}{dn}(Q^{neg}, X^{neg}) > 0$ while $\left(\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right)\right).\frac{dQ^{neg}}{dn} < 0 \Rightarrow \frac{dSW}{dn}(Q^{neg}, X^{neg}) < 0$.

Proposition 9. Part v) To establish that there exists a finite number of firms that maximizes $SW(Q^{neg}, X^{neg})$, let us consider two cases:

1) If $c'''(x) = 0 \; \forall x$, then $\frac{dQ^{neg}}{dn} > 0$ always holds for any finite n since $\frac{dV}{dn} = 0$; as $n \to \infty$, then $-\Pi_{qn}^{neg}.\Pi_{xX}^{neg} + \Pi_{xn}^{neg}.\Pi_{qX}^{neg} \to 0$ and thus $\frac{dQ^{neg}}{dn} \to 0$. We also verify that as $n \to \infty$ then $\left(\frac{b}{n} - 2\left(h(X^{neg}) - V^{neg}\right)\right) \to -2\left(h(X^{neg}) - V^{neg}\right) < 0$ while $\left(-c\left(\frac{X^{ms}}{n}\right) + c'\left(\frac{X^{ms}}{n}\right).\frac{X^{ms}}{n}\right) \to -c(0) < 0$. As a result, $\lim_{n\to\infty} \frac{dSW}{dn}(Q^{neg}, X^{neg}) <$

0. On the other hand, if $n=1$, then $\left(\frac{b}{n}-2\left(h(X^{neg})-V^{neg}\right)\right)=\left(b-2\left(h(X^{neg})-V^{neg}\right)\right)>0$ given that $b>2.h(0)$; moreover $\left(\frac{dQ^{neg}}{dn}\right)_{n=1}>0$. As a consequence, given that $-c\left(X\right)+c'\left(X\right).X>0$, we obtain $\left(\frac{dSW}{dn}(Q^{neg},X^{neg})\right)_{|n=1}>0$. It follows that n^{neg} the number of firms that maximizes $SW(Q^{neg},X^{neg})$ is finite, larger than 1, and defined as the solution to:

$$-c\left(\frac{X^{neg}}{n^{neg}}\right)+c'\left(\frac{X^{neg}}{n^{neg}}\right).\frac{X^{neg}}{n^{neg}}=-\frac{dQ^{neg}}{dn}.\left(\frac{b}{n^{neg}}-2\left(h(X^{neg})-V^{neg}\right)\right).Q^{neg}$$

which is condition (16) in the text. Thus it must be that $-\frac{dQ^{neg}}{dn}.\left(\frac{b}{n^{neg}}-2\left(h(X^{neg})-V^{neg}\right)\right)>0$. Note that given $\frac{dQ^{neg}}{dn}>0$ always holds, this implies that at n^{neg} we have $b-2.n^{neg}\left(h(X^{neg})-V^{neg}\right)<0$.

2) If $c'''(x)>0\ \forall x$, then $\frac{dQ^{neg}}{dn}\lesseqgtr 0$ since $\frac{dV}{dn}>0$. Remark that as $n\to\infty$ then $-\Pi_{qn}^{neg}.\Pi_{xX}^{neg}+\Pi_{xn}^{neg}.\Pi_{qX}^{neg}\to 0$, hence $\lim_{n\to\infty}\frac{dQ^{neg}}{dn}=0$. We also verify that as $n\to\infty$ then $\left(\frac{b}{n}-2\left(h(X^{neg})-V^{neg}\right)\right)\to -2\left(h(X^{neg})-V^{neg}\right)<0$ while $\left(-c\left(\frac{X^{ms}}{n}\right)+c'\left(\frac{X^{ms}}{n}\right).\frac{X^{ms}}{n}\right)\to -c(0)<0$. As a result, $\lim_{n\to\infty}\frac{dSW}{dn}(Q^{neg},X^{neg})<0$ is still true. On the other hand, if $n=1$, then $\left(\frac{b}{n}-2\left(h(X^{neg})-V^{neg}\right)\right)=\left(b-2\left(h(X^{neg})-V^{neg}\right)\right)>0$ given that $b>2.h(0)$; however it comes that $\left(\frac{dQ^{neg}}{dn}\right)_{n=1}\lesseqgtr 0$. As a consequence, given that $-c\left(X\right)+c'\left(X\right).X>0$:

- Either $\left(\frac{dQ^{neg}}{dn}\right)_{n=1}>0$ (remark that as shown above, $\frac{h'''}{h''}>\frac{c'''}{c''}$ is sufficient to obtain $\frac{dQ^{neg}}{dn}>0$ for any given finite n): thus $\left(\frac{dSW}{dn}(Q^{neg},X^{neg})\right)_{|n=1}>0$ holds for certainty. Hence under $\left(\frac{dQ^{neg}}{dn}\right)_{n=1}>0$, there exists a finite number of firms maximizing $SW(Q^{neg},X^{neg})$, that satisfies : $n^{neg}>1$, and defined as the solution to (16); this implies that at n^{neg} we must have $b-2.n^{neg}\left(h(X^{neg})-V^{neg}\right)<0$.

- Or $\left(\frac{dQ^{neg}}{dn}\right)_{n=1}<0$; then the number of firms that maximizes social welfare is either $n^{neg}=1$ (when $\left(\frac{dSW}{dn}(Q^{neg},X^{neg})\right)_{n=1}<0\Leftrightarrow -c\left(\frac{X^{neg}}{n^{neg}}\right)+c'\left(\frac{X^{neg}}{n^{neg}}\right).\frac{X^{neg}}{n^{neg}}+\left(\frac{dQ^{neg}}{dn}\right)_{n=1}.\left(\frac{b}{n^{neg}}-2\left(h(X^{neg})-V^{neg}\right)\right).Q^{neg}<0$) or a $n^{neg}>1$ satisfying (16) – in which case it must be that at n^{neg} we have $b-2.n^{neg}\left(h(X^{neg})-V^{neg}\right)>0$ now.

Obviously, there may exist several values of n that satisfy $\frac{dSW}{dn}(Q^{neg},X^{neg})=0$ given that this condition is highly non linear, all being local maximum.