https://economix.fr/

Public vs. Private Investments In Network Industries

Document de Travail
Working Paper
2021-4

Marc Bourreau
Jean-Marc Zogheib

EconomiX - UMR7235
Université Paris Nanterre
Bâtiment G - Maurice Allais, 200, Avenue de la République
92001 Nanterre cedex

Email : secretariat@economix.fr

Public vs. Private Investments in Network Industries*

Marc Bourreau[†], Jean-Marc Zogheib[‡]

Abstract

We study the competition between a private firm and public firms on prices and investment in new infrastructures. While the private firm maximizes its profits, public firms maximize the sum of their profits and consumer surplus, subject to a budget constraint. We consider two scenarios of public intervention, with a national public firm and with local public firms. In a monopoly benchmark, we find that the national public firm has the highest coverage and charges a uniform price allowing cross-subsidies between high-cost and low-cost areas. Moreover, the private firm covers as much as local public firms. In a mixed duopoly, a stronger competitive pressure drives firms' prices up while it drives down (up) the national public (private) firm's coverage.

Keywords: public firms, investment, network industries, mixed duopoly.

JEL codes: D43; H44; L20; L33.

*I thank Enrick Arnaud-Jouffray, Maya Bacache, Edmond Baranès, Stéphane Ciriani, Axel Gautier, Lukasz Gryzbowski, François Jeanjean, Laure Jaunaux, Bruno Jullien, Vincente Lagos, Ulrich Laitenberger, Marc Lebourges, Yassine Lefouili, Volci Lesca, Julienne Liang, Thierry Pénard, Adrien Raizonville, Wilfried Sand-Zantman, Jo Seldeslachts, Reinhilde Veugelers, and Christoph Wolf for their useful comments. I also thank the audience at the AFREN Doctoral Workshop conference (Paris, 16 October 2017), at the MACCI conference (Mannheim, 8-9 March 2018), at the IDEI Seminar (Paris, 28 March, 2018) at the 2nd Doctoral Workshop on the Economics of Digitization (Paris, May 4-5 2018), at the CISS Summer School (Ulcinj, 28 May-1 June 2018), at the CRESSE conference (Heraklion, June 29-July 1 2018), at the Digital Economics Summer School (Montpellier, June 28-29 2018), at the Young Economist Seminar (Paris, 14 February 2019), and at the ADRES conference (Lyon, February 3-4 2020).

[†]Telecom Paris, Institut Polytechnique de Paris. Email: marc.bourreau@telecom-paris.fr

[‡]Economix, UPL, University Paris Nanterre, CNRS, University Paris Nanterre and EconomiX-CNRS, Nanterre F92000, France. Email: jmzogheib@parisnanterre.fr

1 Introduction

Due to the high costs of rolling out infrastructures, public authorities often play a key role for the deployment of new networks. In the telecommunications sector, for example, public authorities of many countries are involved in the deployment of the so-called "next-generation networks", capable of delivering high-speed access to the Internet.

In Australia, the National Broadband Network is a nation-wide publicly funded infrastructure project, using the fiber-to-the-home (FTTH) technology, which aims at covering 93% of Australian households and businesses.[1] In the US, the Trump administration is exploring the possibility of building a national 5G mobile network. Private operators and the Federal Communication Commission have raised concerns about this idea on the ground that the market is the best placed to foster innovation and investment.[2] FCC Chairman Ajit Pai stated:

> "I oppose any proposal for the federal government to build and operate a nationwide 5G network. The main lesson to draw from the wireless sector's development over the past three decades is that the market, not government, is best positioned to drive innovation and investment (...). Any federal effort to construct a nationalized 5G network would be a costly and counterproductive distraction from the policies we need to help the United States win the 5G future."

Local authorities are also engaged in the roll-out of high-speed broadband infrastructures. For example, the city of Chattanooga was the first to offer 1Gbit/s high-speed Internet access in the US through a local public company (Electric Power Board).[3] In Europe, the province of Siena (Terrecablate) in Italy, the city of Nuenen (OnsNet) in the Netherlands, and many localities in France, among others, have invested in next-generation access infrastructures (see the evolution for France on Figure 1).[4]

[1]Similarly, in New-Zealand, the Ultra-Fast Broadband Initiative is a public-private partnership of the government with four private companies, aiming at building a fibre-to-the-home network infrastructure covering 87% of the population by the end of 2022.

[2]FTC release (29/01/2018).

[3]Other American cities have deployed public-owned fiber-optic networks offering high-speed Internet

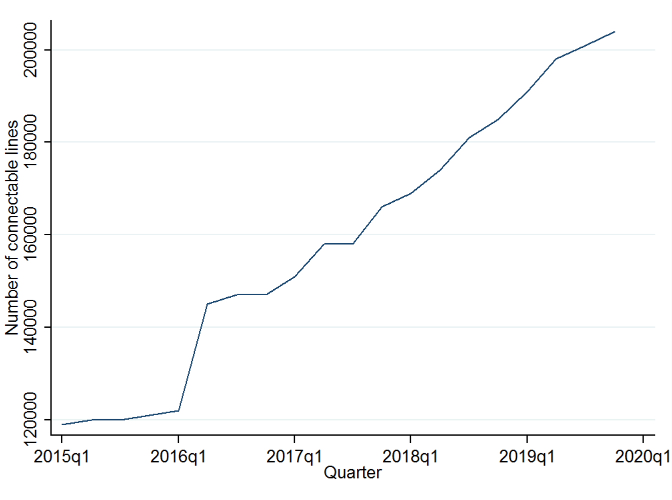


Figure 1: Roll-out of public broadband infrastructure in France (Data from ARCEP, vertically integrated public operators).

Public intervention in the deployment of network infrastructures is justified by a perceived market failure: because they do not internalize all the external effects from high-speed network infrastructures, the argument goes, private operators underinvest, or do not invest fast enough, compared to what would be socially desirable.

However, public investment in next-generation access networks is realized at the same time as private investment in other areas, but also sometimes the same areas. This has raised a hot debate on whether public investment crowds in or rather crowds out private investment. With the concern that public investment could undermine private investment, in the US twenty states have passed legislation banning or restricting public provision of Internet access as of 2016.[5] Tennessee state law has prevented Electric Power Board of Chattanooga from expanding to adjacent communities that lack fast, cheap Internet access.[6] Private operators also question the role of public investment. For example, AT&T wrote to the Federal Communication Commission (FCC) that municipal broadband can bring private ISPs to "operate at a competitive disadvantage", and that there should be

acces, e.g., Lafayette, La., Bristol, Va., etc. See New-York Times (04/01/2014).

[4]These broadband plans are developed in accordance with the Services of General Economic Interest (SGEI) principle. SGEI are economic activities that public authorities identify as being of particular importance to citizens and that would not be supplied (or would be supplied under different conditions) if there were no public intervention.

[5]http://apps.fcc.gov/ecfs/document/view?id=7521826169.

[6]Chattanooga petitioned the Federal Communications Commission to preempt that state law, and the FCC granted the request, using its authority to promote competition in local markets by removing barriers to infrastructure investment. However, the State of Tennessee sued the FCC to overturn its decision. The case is in process.

restrictions on public broadband projects to ensure a "level playing field".[7]

A key issue is therefore to ensure a fair competitive environment for private firms. The Broadband Guidelines in the European Union (European Commission, 2009) follow this objective. Public authority face a complex trade-off here. On the one hand, there is this risk that public investment crowds out private investment, as discussed above. On the other hand, in some areas private investment will not materialize, at least in the short or medium term, due to high costs of infrastructure deployment. In this paper, we provide a theoretical framework to study this trade-off, and characterize the potential benefits and costs of public intervention in the deployment of network infrastructures, when private firms also invest in infrastructures.

We build a model where a private firm competes with public firms in prices and coverage of a new network infrastructure in a given country. Firms decide simultaneously on prices for their services, and on coverage of the country with the new network. The private firm is a profit maximizer, whereas public firms maximize the sum of their profits and consumer surplus, subject to a budget constraint. We consider two scenarios of public intervention: with a national public firm and with local public firms. The national public firm charges the same uniform price all over the country, subject to a global budget constraint. Each local public firm is based in a given area of the country, and charges a price for the service that applies only locally, subject to a local budget constraint. We assume that the private firm charges a uniform price all over the country, as the national public firm. All the areas of the country have the same demand, but the investment cost increases (at an increasing rate) as the operators turn to outlying areas.

First, we examine firms' decisions in a monopoly benchmark. We find that the private firm charges the monopoly price and covers up to a monopoly area where the marginal cost of investment is equal to the (local) monopoly profit. The national public firm charges a price lower than the monopoly price, and cross-subsidizes between low-cost and high-cost areas. Cross-subsidies allow the national public firm to cover a larger share of the country than the private firm, up to the area where the marginal social benefit of investment

[7] http://apps.fcc.gov/ecfs/document/view?id=7521825939.

becomes lower than the marginal cost of investment. Local public firms charge prices that are contingent on the investment cost in their area. They cover the same territory than the private monopoly. Total welfare is always higher with a national public firm or local public firms, compared to a private monopoly. In a specific Shubik-Levitan linear demand model, we also show that welfare is higher with a national public firm than with local public firms.

We then examine firms' decisions in a mixed duopoly framework, when the private firm competes with the national public firm or local public firms in prices and coverage. We focus on the case where public firms lead in investment, that is, invest more than the private firm in equilibrium. In this situation, the private firm competes with the public firms in low-cost areas, while public firms operate as a monopoly in the costlier areas.

When the private firm competes with the national public firm, we find that the private firm reacts to price increase of its rival by increasing its own price (strategic complementarity), whereas the national public firm reacts by decreasing its own price (strategic substitutability). A larger overlap between the private firm's and the public firm's networks drives firms' prices up. The reason is that the competition from the private firm makes it harder for the public firm to break even by setting low prices. Since a larger overlap means more competition, the public has to increase its price to satisfy its budget constraint. The private firm then reacts by increasing its price. In the equilibrium of the coverage-price game, we find that total coverage by the national public firm is lower than in the benchmark. Whereas competition leads the private firm to set lower prices than in the benchmark, the public firm may charge higher prices than in the benchmark. This is because the competition from the private firm makes it harder to sustain low prices.

When the private firm competes with local public firms, we find that a larger overlap between the private firm's network and the local public firms' networks leads to a higher price by the private firm and lower prices by the local public firms. This is because, when the private firm covers a larger territory, with more overlap with the public firms, it faces local public firms that are less aggressive in prices, as they have to charge a higher price to cover their higher investment costs. The private firm reacts to the softer competition (on average) by increasing its uniform price. Since the private firm increases its price, the budget constraint of local public firms is relaxed and they then react by decreasing their own prices. In the equilibrium of the coverage-price game, total coverage is the same than in the benchmark. The private firm sets a lower price than in the benchmark, while the local public firms set higher prices.

The rest of the paper is organized as follows. In Section 2, we discuss the related literature. Section 3 presents the model framework. In Section 4, we solve the model in a monopoly benchmark. We study the mixed duopoly between a private firm and a national public firm in Section 5, and the mixed duopoly between the private firm and local public firms in Section 6. Section 7 concludes.

Related Literature

Our paper contributes to three strands of literature on (i) the impact of regulation on the roll-out of network infrastructures, (ii) mixed oligopolies, and (iii) public intervention for investment in new networks.

In the first strand of literature, various studies have analyzed the impact of universal service obligations (USOs) on the deployment of network infrastructures. Valletti, Hoernig and Barros (2002) examine the impact of universal service obligations as a form of regulation that puts constraints on firms' pricing and coverage decisions. Uniform pricing constraints oblige firms to offer their services at a geographically uniform price, whereas coverage constraints oblige firms to cover at least a given area. They show that there are trade-offs between larger coverage and higher welfare of served consumers, and between consumer welfare in markets with competition or monopoly. In particular, under uniform pricing constraint, the benefits of competition, in terms of lower prices, are distributed to all consumers, even those not served by the entrant. On the other hand, in duopoly areas consumers are charged a higher price and firms' coverage levels are lower. Our analysis focus competition between a private firm and public firms while Valleti et al. examine private competition. Contrary to Valletti et al., we find that more intense competition (through a larger overlap between firms' networks) may drive firms' prices up, particularly in the mixed duopoly with a national public firm.

Other papers on the impact of uniform pricing constraints find comparable results to Valletti et al. (2002), as in Anton et al. (2002), Choné et al. (2000, 2002), Foros and Kind (2003), Hoernig (2006), and Gautier and Wauthy (2010). Gautier and Wauthy show that under uniform pricing obligations, the incumbent has conflicting incentives: on one hand, it may be tempted to "withdraw" to regions with limited competition and charge high prices, leaving the more competitive regions to the entrant; alternatively, it may be willing to undercut the entrant to win market share. The authors show that, due to this conflict, an equilibrium in pure-strategies may fail to exist. In our mixed duopoly framework, the national public firm relies on cross-subsidization between low-cost and high cost areas to cover more widely. Provided that it is able to break even, it is not tempted to withdraw to monopoly areas.

This paper is also related to the literature on access regimes and investment incentives. Some papers examine the impact of access on the investment incentives of incumbent operators (Foros (2004) and Nitsche and Wiethaus (2011)), while other papers analyze entrants' investment incentives (Bourreau and Doğan, 2006). Bourreau, Cambini and Hoernig (2015) investigate the impact of different access regimes on investment in different geographical areas of a country. They find that a duplication-based regime creates more certainty both for firms and regulators and leads to greater welfare, provided that the regulator is fully informed and can fully commit to setting different prices. However, since the regulator may suffer from information asymmetry and commitment problems, a competition-based regime may then become the only feasible alternative to uniform prices. In conclusion, they show that compared to uniform access pricing, the adoption of geographically differentiated access prices can improve welfare and investment. While the above papers model the impact of access regulation on investment incentives, we do not consider the issue of access and rather model an alternative scenario of financial public intervention where (public and private) firms' investment incentives are examined.

Our paper is also related to the literature on wholesale competition between vertically integrated firms. Ordover and Shafer (2007) consider a framework where one of the vertically integrated firms has a larger customer base than the other. The new entrant can enter the market only "service-based", and in two different ways. They find that there is entry in equilibrium and that the wholesale market is perfectly competitive if the entrant cannibalizes the sales of the two integrated firms (in equal proportions), whereas the entrant remains out of the market if it cannibalizes only the sales of its upstream supplier. Brito and Pereira (2010) examine a different setting with circular differentiation between downstream firms , and obtain similar results. Bourreau, Hombert, Pouyet, and Schutz (2011) consider a model of two-tier competition between two vertically integrated firms and one unintegrated downstream firm. They show there are equilibria where the wholesale market is not competitive. Our analysis differs from that of Bourreau et al. (2011) in that we deal with competition between two vertically integrated firms. Moreover, our focus is on the effects of mixed duopoly on firms' investment incentives.

Our paper is also related to the literature on mixed oligopolies. Merill and Schneider (1966) show that the existence of a public firm in an oligopolistic industry can result in improved market performance, with lower prices and increased output. Beato and Mas-Colell (1984) examine a mixed duopoly where the public firm uses marginal cost pricing while the private firm acts as a leader maximizing its profits along the public firm's reaction function. The main conclusion of their analysis is that such behaviour sometimes leads to higher social welfare than what could be obtained in the traditional second-best approach. De Fraja and Delbono (1989) investigate an industry formed by a single public firm and several private firms. They find that social welfare may be higher if the public firm is instructed to maximize profits rather than total surplus. Cremer, Marchand et Thisse (1989) examine to what extent a public firm is a relevant instrument to regulate an oligopolistic market. They find that the nationalization of a single firm in an industry with only private firms can be socially optimal. However, if there are several existing public firms, higher total surplus is likely to be achieved if all but one of the public firms are privatized. We contribute to this literature by examining the issue of public vs. private investments in those areas that one may consider as "underserved". We provide an analysis where public firms compete not only in prices, but also in coverage of territory. Differently from the above studies where public firms maximize total surplus, we consider public firms which maximize only their own profit and consumer surplus.[8]

Another set of papers in the literature on mixed oligopolies, such as in Armstrong and Weeds (2007) and in Jullien, Pouyet, and Sand-Zantman (2010), consider public firms which maximize the sum of their profits and consumer surplus. Armstrong and Weeds (2007) study programme quality in digital television. They show that the public firm obtains a larger audience and broadcasts less advertising than the private competitor. However, the public firm does not necessarily broadcast programs of higher quality.

Jullien et al. (2010) analyze the relationship between a national regulator, an incumbent and a local firm investing in a new network. They model a country divided into "districts", which differ by the level of demand for new services and the cost of building a new network. They find that public investment can be efficient in white areas, but that a ban of local government intervention can be welfare-enhancing in grey areas in presence of externalities, asymmetric information or conflicting goals between regulator and local governments. Therefore, the national regulator has to consider these issues when designing rules for investment of local governments.[9] In the same vein, Jullien, Pouyet and Sand-Zantman (2018) study the link between private firms' incentives to invest and public intervention. Jullien et al. (2010,2018) are close to our analysis. Our main contribution is here to develop a general framework with two levels of public intervention (national and local).

In a recent empirical contribution, Wilson (2019) investigates to what extent public competition from local governments may crowd-in or crowd-out private investment in broadband infrastructures. Wilson finds that public competition decreases the probability of private investment due to a crowding-out effect. However, it increases the probability of private investment through a preemption effect. The overall effect is that public

[8]Other papers consider public firms which maximize a weighted sum of profits and consumer surplus. Matsumura (1998) considers partial privatization with a duopoly involving a private firm and a privatized firm which is jointly owned by the public and private sectors. The author finds that full nationalization is not optimal unless the public firm is a monopolist but that full privatization is not optimal if the public firm is as efficient as the private firm. Lee, Matsumura and Sato (2017) consider a framework where private firms first choose whether to enter the market and then the government chooses the degree of privatization of the public firm. Their main finding is that the equilibrium degree of privatization is too high (low) for both domestic and world welfare if private firms are domestic (foreign).

[9]In a more general treatment, Jullien, Pouyet, and Sand-Zantman (2017) examine the possibility that the local government reaches different types of contractual agreements with the incumbent.

investment crowds-out more private investments than it induces through preemption. Wilson (2019) concludes that banning public provision of internet access increases the profits of incumbent private firms at the expense of both consumers and local governments. We contribute to the empirical analysis of Wilson by providing a theoretical framework to the issue of public and private investments competition in network infrastructures.

Finally, our paper is close to the literature discussing public intervention and investment in new networks. Cave and Martin (2010) analyze the main motives of public investment and show that industrial policy, equity objectives and economic recovery are the main ones. They also question the means of public intervention by studying three broadband plans: Australia, New Zealand and Singapore. Hauge, Jamison and Gentry (2008) compare the types of markets that municipally owned telecommunications providers in the United States serve to the types of markets that competitive local exchange carriers (CLECs) serve. Their results suggest that municipalities may not pose a significant competitive threat to CLECs and do not preclude CLEC participation. Briglauer, Holzleitner and Vogelsang (2016) question the contract practice of determining ex-ante targets of network expansion by governments. Due to information asymmetry and uncertainty, they show that delegating the choice of network expansion to a better informed network operator is efficient.

2 Model

We consider a country composed of different areas $z \in [0, \infty)$, with identical demand but different sunk costs of being covered with a network infrastructure. The cost of covering an area $z \in [0, \infty)$ is $c(z)$, with $c'(z) > 0$, $c(0) = 0$, and $\lim_{z \to \infty} c(z) = +\infty$. The total cost of covering the areas from 0 to z is then $C(z) = \int_0^z c(x)dx$, with $C'(z) = c(z) \geq 0$ and $C''(z) = c'(z) > 0$.

A private firm competes with public firms in coverage and prices all over the country. The private firm, firm P, makes its decisions in order to maximize its profit, whereas public firms maximize the sum of their profit and consumer surplus under a budget constraint.[10]

[10] The public firm cares about its own profit and consumer surplus, but not about the private firm's

We consider two types of public firms: (i) a national public firm, firm N, which decides on prices and coverage for the whole country; and (ii) local public firms based in each area z, making independent price and coverage decisions, which we all call L for simplicity. We assume that all firms, be they private or public, have the same constant marginal cost of production, which we normalize to zero.

Depending on the coverage decisions of the firms, each local market can have no provider, a monopoly provider, or two competing providers. The monopoly demand in a local market for a given price p, $D^m(p)$, is the same for all firms $i \in \{P, N, L\}$, and we denote by $D_i^d(p_i, p_j)$ the duopoly demand for firm i, where p_i denotes the price of firm i and p_j the price of the rival firm j. Firms offer differentiated products. The monopoly demand is downward-sloping, and as usual we assume that a firm's demand decreases in its own price ($\partial D_i^d(p_i, p_j)/\partial p_i \leq 0$) and increases in its rival's price ($\partial D_i^d(p_i, p_j)/\partial p_j \geq 0$). Finally, the duopoly demands are symmetric: $D_i^d(p_i, p_j) = D_j^d(p_i, p_j)$.

We assume that the private firm P and the national public firm N set a uniform price that applies to all their covered areas in the country, which is a standard business practice in network industries. By contrast, each local public firm L sets a price that applies only locally, and depends on the market conditions in the area.

We assume away any subsidies for public deployment of infrastructures; the difference in coverage and pricing incentives between the private firm and the public firms thus only stems from the difference of objective functions.

Social welfare in a given area, gross of investment costs, is defined as the sum of firms' profits and consumer surplus, and denoted by $w^m(p_i)$ and $w^d(p_i, p_j)$ for a monopoly area and a duopoly area, respectively. We assume that it is decreasing in prices, i.e., $\partial w^m(p_i)/\partial p_i \leq 0$ and $\partial w^d(p_i, p_j)/\partial p_k \leq 0$, for $k = i, j$.

Finally, we make the following assumption on profits:

Assumption 1. The monopoly profit $pD^m(p)$ and the duopoly profit $p_i D_i^d(p_i, p_j)$ are concave in prices.

---

profit. This assumption about the public firm's objective function is in line with, for example, Matsumura (1998), Armstrong and Weeds (2007) and Jullien et al. (2010).

We denote the optimal monopoly price by $p^m = \arg\max_p pD^m(p)$ and the monopoly profit by $\pi^m \equiv p^m D^m(p^m)$. Moreover, let $BR^d(p_j) \equiv \arg\max_p pD_i^d(p, p_j)$ denote the duopoly best-response, for $j \neq i$. We assume that prices are strategic complements. The equilibrium in duopoly is assumed to exist and be unique; the duopoly price is given by $p^d = BR^d(BR^d(p_i))$ and the duopoly profit is $\pi^d \equiv p^d D(p^d, p^d)$.

We study the coverage-price game where firms decide simultaneously on coverage and prices. We look for the Nash equilibrium of this game.

3 Monopoly benchmark

In this section, we solve for the equilibrium of the coverage-price game in a monopoly benchmark, where only one firm, private or public, is active. We study the equilibrium outcome under monopoly with (i) a private firm, (ii) a national public firm, and (iii) local public firms.

Private firm. The private firm, P, decides on a uniform price, p, and a coverage, z, to maximize its profit, which is given by:[11]

$$\Pi_P = zpD^m(p) - C(z). \tag{1}$$

Firm P covers the areas from 0 to z for a total investment cost $C(z)$, and obtains the monopoly profit $pD^m(p)$ in each covered area. Its optimal coverage and price decisions are then as follows.

Lemma 1. *The monopoly private firm sets the monopoly price $P_P^m = p^m$ and covers up to the area $Z_P^m = z^m$, with $z^m = c^{-1}(\pi^m)$.*

The private firm sets the monopoly price in all covered areas, since it maximizes its local profits. It then covers up to a marginal area when the marginal private benefit from investment (the monopoly profit) is equal to the marginal cost of investment.

[11]Note that in this monopoly benchmark, the private firm has no incentive to price discriminate between areas since the demand is the same in all areas.

National Public firm. The national public firm, N, chooses a uniform price, p, and a coverage, z, to maximize the sum of its profit and consumer surplus, less the investment cost:

$$W_N = zw^m(p) - C(z), \tag{2}$$

subject to the budget constraint $zpD^m(p) - C(z) \geq 0$. Note that in this monopoly framework, the sum of N's profit and consumer surplus corresponds to social welfare.

Lemma 2. *Under monopoly, the national public firm sets the uniform price $P_N^m \leq p^m$ and covers up to the area $Z_N^m > z^m$, with $w^m(P_N^m) = c(Z_N^m)$ and $Z_N^m P_N^m D^m(P_N^m) - C(Z_N^m) = 0$.*

The public firm covers the areas where the marginal social benefit of investment is greater than the marginal cost of investment, taking into account the uniform price that allows it to break even. This price depends on the marginal area covered, and increases with the level of coverage. As a consequence, the marginal social benefit of investment decreases as the public firm covers more outlying areas. Equilibrium coverage is then defined by the intersection of the (decreasing) marginal social return to investment with the (increasing) marginal investment cost.

Local public firms. Finally, we investigate the case where a continuum of local public firms, based in all the areas of the country, decide independently on the coverage of their area and the local price of the service. We define as $\mathfrak{I}(z) \in \{0, 1\}$ the investment strategy of firm L, based in area z, where $\mathfrak{I}(z) = 1$ if L invests, and $\mathfrak{I}(z) = 0$ if it does not.

Let p_L denote firm L's price in local area z, conditional on coverage. Firm L's objective is to maximize the local area's welfare, which is given by

$$\mathfrak{I}(z) \left[w^m(p_L) - c(z) \right], \tag{3}$$

subject to the local budget constraint

$$p_L D^m(p_L) - c(z) \geq 0. \tag{4}$$

Lemma 3. *Under monopoly, local public firms set the price $P_L^m(z)$, which is increasing in z, so that their local budget constraint binds, and cover up to the area $Z_L^m = z^m$.*

A local public firm L invests only if it can break even. Since the maximum profit it can earn is the monopoly profit, the marginal area with local public firms is the monopoly area z^m. In covered areas, each local public firm sets a price that allows it to break even. The price of the service becomes higher as we move towards more remote areas, because local public firms have to charge a higher price to cover the higher investment cost.

Comparison. We can now compare the market outcome in the three scenarios, with a private firm, a national public firm and local public firms, in terms of total coverage and prices. Using Lemmas 1-3 above, we obtain the following result.

Proposition 1. *In the monopoly benchmark, the market outcome is as follows:*

(i) Total coverage is larger with a national public firm, but it is the same with a private firm or local public firms (i.e., $Z_N^m > Z_P^m = Z_L^m = z^m$).

(ii) The national public firm sets a lower uniform price than the private firm, while the local public firms set local prices that depend on the area and can be either lower or higher than the price set by the national public firm (i.e., $P_N^m < P_P^m = p^m$, while $P_L^m(z) \leq P_N^m$ if z is sufficiently low, and $P_L^m(z) > P_N^m$ otherwise).

The private firm deploys its infrastructure in all the areas $z \leq z^m$ where the monopoly profit covers the investment cost. Due to their local budget constraints, the investment incentives of local public firms are similar. Since a local public firm invests only if it can break even, the marginal area for local public firms is the area where the largest profit, i.e., the monopoly profit, is obtained, that is, area z^m. By contrast, the national public firm covers a larger share of the territory. This is because it has a global budget constraint, and can therefore cross-subsidize investment in high-cost areas with profits obtained in low-cost areas.

We can also compare the market outcome in the three different scenarios in terms of total welfare. Since total coverage is larger with a national public firm than with a private firm ($Z_N^m > Z_P^m$) and prices are lower ($P_N^m < P_P^m$), total welfare is larger with a national public firm than with a private firm ($W_N^m > W_P^m$). Similarly, total coverage is the same with a private firm or with local public firms, but prices are (at least weakly) lower in the latter case, so total welfare is larger with local public firms than with a private firm ($W_L^m > W_P^m$). The comparison of total welfare with a national public firm and local public firms is ambiguous. On the one hand, total coverage is larger with a national public firm. On the other, prices are lower with local public firms in low-cost areas, but higher in high-cost areas.

To illustrate Proposition 1, we adopt a specific Shubik-Levitan demand model and set $c(z) = z$ to compute the equilibrium coverage and prices (See Appendix A for details). Figure 2 shows, for the three scenarios, prices on the left and total coverage on the right. Since $c(0) = 0$, the price set by the local public firm at $z = 0$ is equal to marginal cost, that is, zero with our normalization. The price of local public firms then increases as we move towards costlier areas, until it reaches the monopoly price p^m.

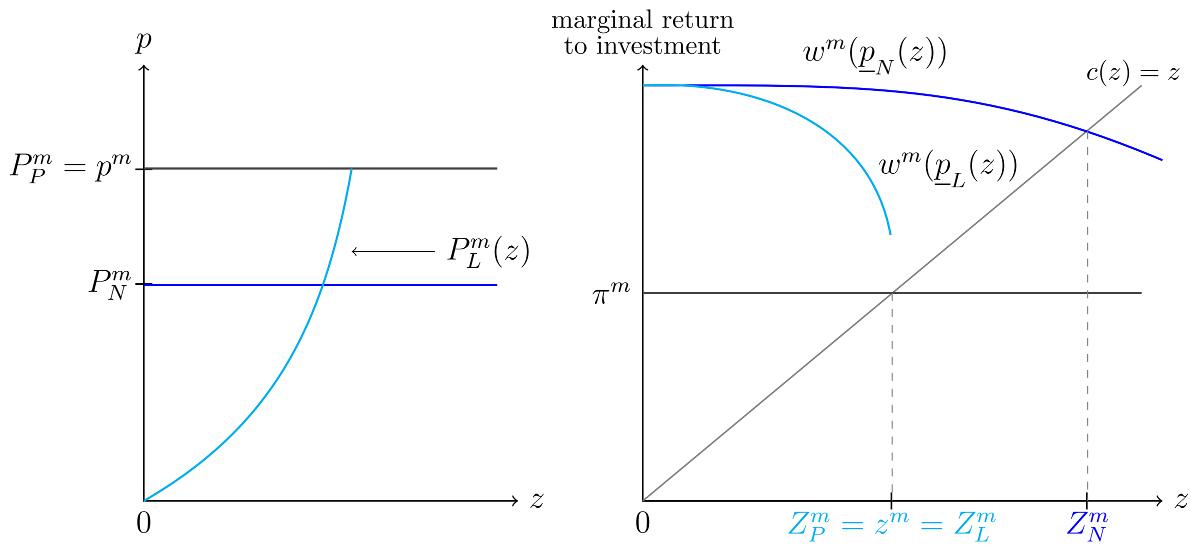


Figure 2: Level of prices (left) and total coverage (right) in the monopoly benchmark.

On the right-hand side, the equilibrium coverage of the private firm, $Z_P^m = z^m$, is given by the intersection of the marginal private return to investment, π^m, with the marginal investment cost, $c(z)$. Similarly, the equilibrium coverage of the national public firm, Z_N^m, is given by the intersection of the marginal social return to investment, $w^m(P_N^m(z))$, with the marginal investment cost, $c(z)$. The marginal social return to investment is decreasing in z, because the public firm has to increase its uniform price when it covers a larger territory. Finally, notice that the marginal social return to investment for local public firms does not intersect with the marginal investment cost; this is because total coverage with local public firms is determined by the budget constraint of the marginal area.

In this specific Shubik-Levitan linear model, we find that $W_N^m > W_L^m > W_P^m$, that is, total welfare is the highest with a national public firm. In other words, it is better to set up a national entity to roll out a new infrastructure than to delegate this task to independent local authorities.

Note that this benchmark model can be readily extended to incorporate subsidies. A subsidy S granted to a public firm relaxes its budget constraint. Since public firms set the lowest price compatible with their budget constraint, subsidies thus lead to lower prices. This, in turn, increases the marginal social return to investment, and therefore public firms also expand their coverage when they receive subsidies.

4 Private Firm vs. National Public Firm

In this section, we analyze the competition in coverage and prices between a private firm and a national public firm.

In this coverage-price game, two equilibria can emerge *a priori*: one where the national public firm leads in investment (that is, invests more than its rival), and another one where the private firm leads in investment. In the context of this paper, it makes sense to consider that the private firm concentrates on low-cost areas, whereas the public firm covers a larger territory, expanding coverage to outlying and costlier areas. We thus focus the analysis on the case where the public firm leads in investment, that is, where $z_N > z_P$ in equilibrium. We solve for the equilibrium of the coverage-price game under this assumption.[12]

[12]The case where the private firm leads in investment is discussed in Appendix B.1.

4.1 Pricing strategies

We start by determining the equilibrium prices of firms P and N for given coverage levels z_N and z_P, with $z_N > z_P$.

Best responses. Consider first the pricing decision of the private firm. Firm P chooses a price p_P to maximize its profit, which is given by

$$\Pi_P = z_P p_P D_P^d(p_P, p_N) - C(z_P). \tag{5}$$

Firm P incurs the total investment cost $C(z_P)$ to cover the areas from 0 to z_P. In each of these areas, it competes with firm N, and obtains the duopoly profit $p_P D_P^d(p_P, p_N)$. From (5), firm P's best-response to a price p_N is the duopoly best-response, $p_P = BR^d(p_N)$.

Consider now the decision of the public firm. Firm N chooses its price p_N to maximize its objective function, which is given by

$$\Pi_N = (z_N - z_P)w^m(p_N) + z_P \left(p_N D_N^d(p_N, p_P) + CS(p_N, p_P)\right) - C(z_N), \tag{6}$$

subject to the budget constraint

$$(z_N - z_P)p_N D^m(p_N) + z_P p_N D_N^d(p_N, p_P) \geq C(z_N). \tag{7}$$

Since it leads in investment, firm N is a (public) monopoly in the areas z_P to z_N, where it cares about total local welfare. In the areas 0 to z_P, firm N competes with firm P; in these areas, it cares about the sum of its duopoly profit and consumer surplus.

Lemma 4. *The best-response of the national public firm N is to set the lowest price such that its budget constraint (7) is binding, that is, $p_N^{BR}(p_P) = \underline{p}_N^d(p_P, z_P, z_N)$, where $\underline{p}_N^d$ satisfies $(z_N - z_P)\underline{p}_N^d D^m(\underline{p}_N^d) + z_P \underline{p}_N^d D_N^d(\underline{p}_N^d, p_P) = C(z_N)$.*

The public firm best responds to a price p_P set by the private firm by setting the lowest price compatible with its budget constraint, and this price is lower than or equal to the uniform price that maximizes its total profit (i.e., $\hat{p}_N$). Note that if at the profit-maximizing price $\hat{p}_N$, firm N does not break even, there is no best-response to the price set by the private firm.

The following lemma shows how the firms react to a price increase of their rival:

Lemma 5. *Whereas firm P reacts to a price increase of its rival by increasing its own price (strategic complementarity), firm N reacts to a price increase of its rival by decreasing its own price (strategic substituability).*

The intuition of this result is that when the private firm increases its price, its demand decreases while the demand, and hence the profit, of the public firm increases. Since the public firm sets the minimum price satisfying its budget constraint, it reacts by decreasing its price. This result shows that compared to the monopoly benchmark with a national public firm, entry and competition from a private firm leads the public firm to *increase* its price.

Equilibrium prices. For given coverage levels, the equilibrium prices P_P and P_N are given by the intersection of the best responses of the private firm and the public firm. If the intersection exists, it is unique, since best responses are continuous, the best response of the private firm is increasing and the best response of the public firm is decreasing. For the rest of the discussion, we assume that this intersection exists.[13] Below, we will discuss this assumption for the linear Shubik-Levitan demand model.

To study how coverage affects pricing decisions, we define $\sigma \equiv z_P/z_N \in (0,1)$ as the ratio of coverage levels. The ratio σ can be interpreted as the overlap between the private firm's and the public firm's covered territories. If σ is low, it means that P covers a small territory compared to N, and we are close to the scenario of a national public monopoly. Conversely, if σ is close to 1, this means that P covers (almost) as much territory as N, and we are close to the scenario of a mixed duopoly all over the country. The equilibrium prices can then be written as functions of z_N and σ: $P_P = P_P(z_N, \sigma)$ and $P_N = P_N(z_N, \sigma)$.

[13] We can provide a necessary and sufficient condition for the best responses to intersect. Let $\widehat{BC} = BC(\hat{p}_N(p_P))$, where BC is the budget constraint (i.e., profit) of the public firm. We have $\partial\widehat{BC}/\partial p_P \geq 0$. If this derivative is positive at $p_P = 0$, it is positive for all $p_P \geq 0$, and the intersection of best responses always exists. If $\partial\widehat{BC}/\partial p_P|_{p_P=0} < 0$, there exists $\underline{p}_P$ such that $\widehat{BC}(\underline{p}_P) = 0$. The intersection of best responses exists in this case if and only if $BR^d(\hat{p}_N(\underline{p}_P)) \geq \underline{p}_P$.

The following proposition characterizes how the coverage by firm N and the degree of overlap between firm P's and firm N's networks affect equilibrium prices.

Proposition 2. *Assume that the price equilibrium exists, for given coverage levels.*

- *For a given coverage z_N by the national public firm, firms' equilibrium prices P_P and P_N increase with the ratio of coverage levels $\sigma = z_P/z_N$ (i.e., with z_P).*
- *For a given coverage z_P by the private firm, equilibrium prices increase when the public firm covers a larger territory (z_N increases) if and only if $P_N D^m(P_N) \leq c(z_N)$.*

Proposition 2 states the counterintuitive result that more intense competition through a larger overlap between the private and public firms' networks drives firms' prices *up*. When the overlap increases, the best response of the private firm, which corresponds to the duopoly best response, is unchanged. By contrast, the best response of the public firm shifts outwards. The idea is that when the duopoly areas expand to the detriment of the monopoly areas, the ability of the public firm to cross-subsidize between low-cost and high-cost areas is lessened. Firm N has to increase its uniform price, and due to strategic complementarity, firm P does the same, which leads to higher equilibrium prices.

The second point of the proposition shows that when the public firm extends its coverage, how prices adjust depends on the profitability of the marginal (monopoly) area. If the marginal area is profitable, the public firm can use the incremental profit to reduce its uniform price. Conversely, if it is unprofitable, the public firm has to increase its uniform price to break even. In both cases, the private firm follows the price reaction of the public firm, due to strategic complementarity.

4.2 Coverage strategies

We now consider the coverage decisions of firms N and P, for given prices p_N and p_P. From (5) and (6), the equilibrium coverage levels are solutions to the system of first-order conditions,[14]

$$
\begin{aligned}
p_P D_P^d(p_P, p_N) &= c(z_P), \\
w^m(p_N) &= c(z_N).
\end{aligned}
$$

Replacing for $p_N = P_N(z_P, z_N)$ and $p_P = P_P(z_P, z_N)$, the equilibrium coverage of firms N and P are the solution of

$$
P_P(z_P, z_N) D_P^d(P_P(z_P, z_N), P_N(z_P, z_N)) = c(z_P), \tag{8}
$$

$$
w^m(P_N(z_P, z_N)) = c(z_N). \tag{9}
$$

We assume that the equilibrium exists and is unique, and denote by Z_P^n and Z_N^n the equilibrium coverage of the private firm and the public firm, respectively. The following proposition compares the equilibrium outcome with the monopoly benchmark.

Proposition 3. *Assume that the equilibrium to the coverage-price game exists and is unique. In equilibrium,*

(i) The private firm P covers a smaller territory than in the benchmark and charges a lower price, i.e., $Z_P^n < Z_P^m$ and $P_P^n < P_P^m$;

(ii) The public firm N covers a smaller territory than in the benchmark and charges either a lower or a higher price, i.e., $Z_N^n < Z_N^m$ and $P_N^n \gtreqless P_N^m$.

The proposition shows that the competition between the private firm and the public decreases coverage, compared to the monopoly benchmark. In particular, the public firm, which leads in investment, covers a smaller territory than in the benchmark. This is because, the competition from the private firm forces the public firm to set a higher price to break even. This decreases the marginal social benefit from investment, and thus the public firm covers less of the country than in the benchmark.

In terms of prices, the private firm sets a lower price than in the monopoly benchmark, due to the competition from the public firm. The impact of the competition from the

[14] The second-order conditions are satisfied as $c' > 0$.

private firm on the public firm's price is ambiguous. On the one hand, there is a business stealing effect, which forces the public firm to increase its price. On the other, the competition leads the public firm to cover a smaller territory. Its investment cost is thus lower, allowing the public firm to set a lower price to break even.

4.3 Linear demand example

In the linear Shubik-Levitan demand model, for given coverage levels, firm P's best-response is the duopoly best-response,

$$p_P = BR^d(p_N) = \frac{2 + \gamma p_N}{2(2+\gamma)}, \tag{10}$$

whereas firm N's best-response is given by $p_N = \underline{p}_N^d(p_P, z_P, z_N)$, where $\underline{p}_N^d$ is the lowest price such that the budget constraint holds:

$$\underline{p}_N^d(p_P, z_P, z_N) = \hat{p}_N(p_P)\left(1 - \sqrt{1 - \frac{8C(z_N)}{(4z_N + z_P(\gamma p_P - 2))\hat{p}_N(p_P)}}\right), \tag{11}$$

and $\hat{p}_N$ is the price that maximizes N's profit,

$$\hat{p}_N(p_P) = \frac{4 + \sigma(\gamma p_P - 2)}{8 + 2\sigma(\gamma - 2)}.$$

Firm N's best-response to a given price p_P, $\underline{p}_N^d(\cdot)$, is defined if at the profit-maximizing price $\hat{p}_N$, firm N breaks even.

The equilibrium prices P_P and P_N are then given by the intersection of the best-responses, given in (10) and (11).[15]

Figure 2 below shows the equilibrium prices for given coverage levels, for $\gamma = 1$, $C(z) = c_0 z^2/2$, $z_N = 10$, $\sigma \in [0, 1]$, and $c_0 = 0.01$ (left) and $c_0 = 0.02$ (right). The figure on the left shows a case where equilibrium prices are always defined. On the figure on the right, by contrast, the equilibrium prices exist only if the degree of overlap is not too high. If the coverage of the private firm is close to that of the public firm, the public firm cannot

[15] We omit the expressions of P_P and P_N, which are algebraically complex.

break even when it sets its profit-maximizing price. In this case, the competition between the private firm and the public firm is not sustainable. In both cases, prices increase with the degree of overlap σ, and the price charged by the private firm is also always larger than the price set by the public firm.

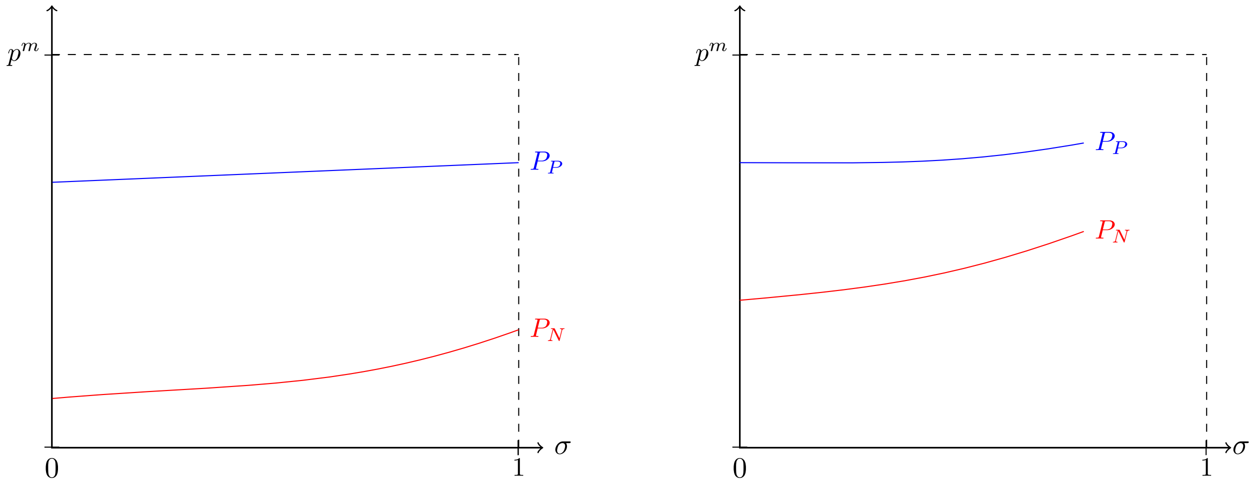


Figure 3: Prices as a function of the ratio of coverage levels σ.

Using (9), we compute the best response coverage for firm N to a coverage z_P by the private firm. Plugging in this best response into the first-order condition (8), we solve for the equilibrium coverage levels Z_P^n and Z_N^n, which then gives the equilibrium prices.

Figure 4 shows the equilibrium prices and coverage levels as a function of the degree of substitutability γ for the investment cost function $c(z) = c_0 z$, with $c_0 = 0.01$. We see that compared to the monopoly benchmark, competition between a private firm and a national public firm leads to lower coverage. However, competition tends to drive prices down. Note if the price charged by the public firm is lower with competition than in the monopoly benchmark, it is because in the former case the firm covers a smaller territory, and has thus lower costs to recover.

Figure 5 shows the equilibrium welfare as a function of the degree of substituability γ for the investment cost function $c(z) = c_0 z$, with $c_0 = 0.01$. We see that compared to the private monopoly, competition between a private and a national public firm leads to a lower welfare when competition is soft (i.e., low γ). On the contrary, as competition intensifies, welfare is higher with a mixed duopoly. Note that while welfare under mixed duopoly is always lower that under national monopoly, we see that there is a level of γ for

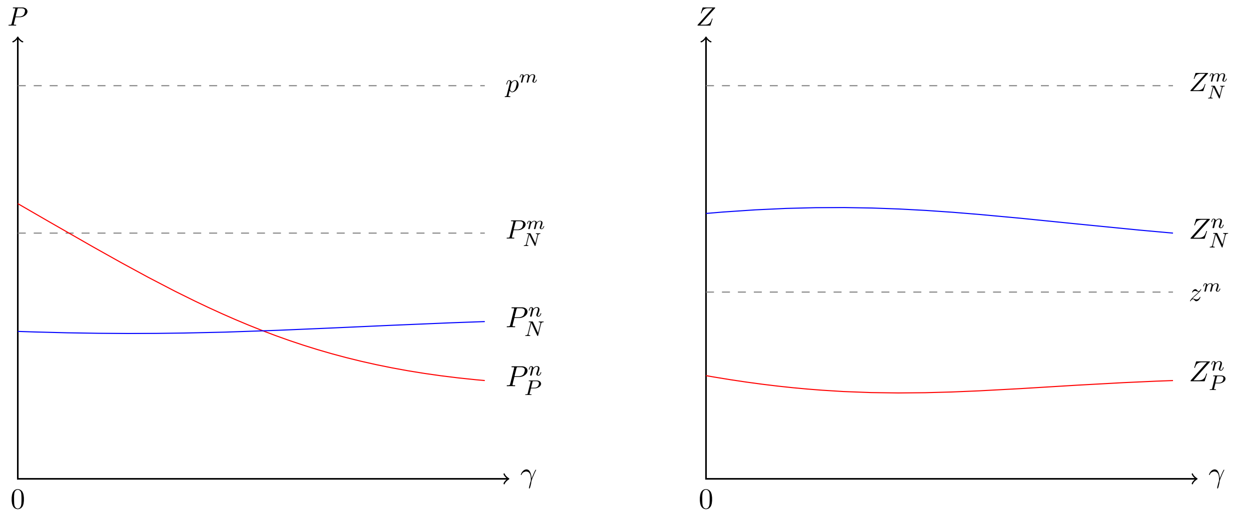


Figure 4: Equilibrium price and coverage levels if the national public firm leads in investment.

which welfare levels W_N^m and W^n are very close.

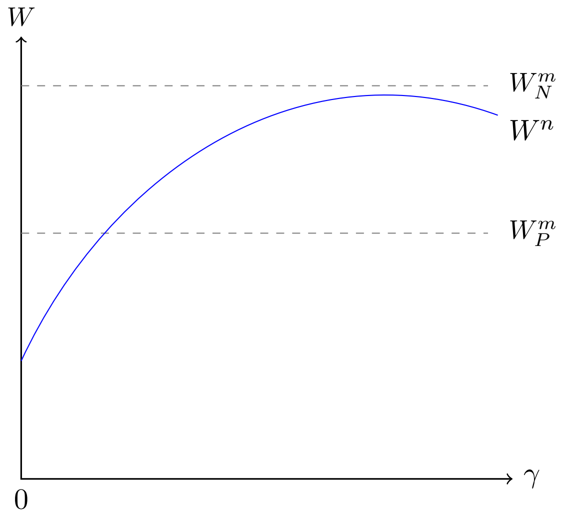


Figure 5: Equilibrium welfare levels if the national public firm leads in investment.

5 Private Firm vs. Local Public Firms

In this section, we study the competition in prices and coverage between a private firm and local public firms. As in Section 5, we focus on the case where the local public firms lead in investment, that is, where $z_L \geq z_P$. We solve for the equilibrium of the coverage-price game under this assumption.[16]

[16]The other case, where the private firm leads in investment, is analyzed in Appendix B.2.

5.1 Pricing strategies

To begin with, we analyze firms' pricing strategies for given coverage levels z_P and z_L, with $z_P \leq z_L$.

Best responses. Consider first the pricing decision of a local public firm L based in a given area $z \leq z_L$. We have to distinguish two cases: (i) if $z \in (z_P, z_L]$, L is a monopoly in its local market; (ii) otherwise, if $z \leq z_P$, L competes with firm P for local consumers.

In case (i), firm L, which is a monopoly in its area z, chooses a price p_L to maximize

$$w^m(p_L) - c(z), \quad \text{subject to} \quad p_L D^m(p_L) \geq c(z). \tag{12}$$

Firm L thus sets the same minimum price $p_L(z) = \underline{p}_L(z)$ than in the monopoly benchmark, such that its budget constraint binds, i.e., $\underline{p}_L(z) D^m(\underline{p}_L(z)) = c(z)$. In these monopoly areas, L's best response increases with z, but does not depend on z_P and p_P.

In case (ii), firm L competes with firm P in its local market, and chooses its price p_L to maximize

$$p_L D^d_L(p_L, p_P) + CS(p_L, p_P) - c(z), \quad \text{subject to} \quad p_L D^d_L(p_L, p_P) \geq c(z). \tag{13}$$

Since the objective function is decreasing in p_L,[17] L sets the lowest price such that its budget constraint holds. Therefore, its best-response is given by $p_L^{BR}(p_P, z) = \underline{p}^d_L(p_P, z)$, where $\underline{p}^d_L$ is the solution to

$$\underline{p}^d_L D^d_L(\underline{p}^d_L, p_P) = c(z). \tag{14}$$

Since $D^d_L(p_L, p_P)$ increases with p_P, firm L's best-response $\underline{p}^d_L(p_P, z)$ is decreasing in p_P: when firm P increases its price, firm L reacts by decreasing its own price. Furthermore, L's best response increases with z. The best response exists if L breaks even when it sets its profit-maximizing price, $p_L = BR^d(p_P)$.

We now determine firm P's best-response to prices $p_L(z)$ set by the local public firms.

[17] See the proof of Lemma 4.

Firm P's profit is given by

$$\Pi_P = \int_0^{z_P} p_P D_P^d \left(p_P, p_L(z)\right) dz - C(z_P). \tag{15}$$

Its best-response is then given by the first-order condition

$$\frac{\partial \Pi_P}{\partial p_P} = \int_0^{z_P} \left[p_P \frac{\partial D_P^d}{\partial p_P}(p_P, p_L(z)) + D_P^d(p_P, p_L(z)) \right] dz = 0. \tag{16}$$

We assume that there is a unique solution $p_P^{BR}(z_P, p_L(.))$ to this first-order condition, and that the second-order condition,

$$\frac{\partial^2 \Pi_P}{\partial p_P^2} = \int_0^{z_P} \left[2 \frac{\partial D_P^d}{\partial p_P}(p_P, p_L(z)) + p_P \frac{\partial^2 D_P^d}{\partial p_P^2}(p_P, p_L(z)) \right] dz \leq 0,$$

holds. If demand is linear, P's best-response is the duopoly best-response to the average price set by L in the areas 0 to z_P, as we will see in the linear demand example below.

Due to the strategic complementarity assumption, P's best-response $p_P^{BR}(z_P, p_L(.))$ increases if L's prices increase. Furthermore, assuming that $p_L(z)$ is increasing in z (we know that this is true for L's best response), P's best-response increases with z_P, as

$$\left. \frac{\partial^2 \Pi_P}{\partial p_P \partial z_P} \right|_{p_P = p_P^{BR}} = p_P^{BR} \frac{\partial D_P^d}{\partial p_P}(p_P^{BR}, p_L(z_P)) + D_P^d(p_P^{BR}, p_L(z_P)) > 0.$$

The inequality comes from the fact that due to strategic complementarity, if the FOC (16) holds, it means that the integrand of (16) is negative for low values of z and positive for high values of z, and thus positive at $z = z_P$. The intuition for the positive relation between p_P^{BR} and z_P is that if $p_L(z)$ is increasing, when P covers costlier areas (with a higher z_P), it faces local firms setting higher prices. P then reacts by increasing its uniform price.

Equilibrium prices. For given coverage levels, the equilibrium prices P_P and $P_L(z)$ are the solution to (14) and (16). We assume that this solution exists and is uniquely defined.

The following proposition characterizes how the coverage of the private firm, and hence the overlap between the private firm's and the public firms' networks, affect equilibrium prices.

Proposition 4. *Assume that the price equilibrium exists, for a given coverage $z_P \leq z_L$. Then, the private firm's equilibrium price P_P increases with z_P, whereas the local public firms prices $P_L(z)$ decrease with z_P. Furthermore, $P_L(z)$ is increasing in z.*

This proposition shows that a larger coverage by the private firm has opposite effects on firms' prices. As the private firm covers a larger territory, it faces local public firms that are less aggressive in prices, because they have to charge a higher price to cover their higher investment cost. The private firm thus reacts by increasing its uniform price. By contrast, since the private firm becomes less agressive in its pricing strategy, local public firms react by decreasing their own prices.

Note that, while each local firm decreases its price when P expands coverage, the average price of local firms can either decrease or increase with z_P. Let

$$\overline{P}_L \equiv \frac{1}{z_P}\int_0^{z_P} P_L(z)dz = \frac{1}{z_P}\int_0^{z_P} \underline{p}_L^d(P_P, z)dz.$$

The variations of the average local price $\overline{P}_L$ with respect to z_P are given by:

$$\frac{\partial \overline{P}_L}{\partial z_P} = \frac{1}{z_P}\left(\underbrace{P_L(z_P) - \overline{P}_L}_{(+)} + \int_0^{z_P} \underbrace{\frac{\partial P_P}{\partial z_P}}_{(+)} \underbrace{\frac{\partial \underline{p}_L^d(P_P, z)}{\partial p_P}}_{(-)} dz\right).$$

The first positive term on the right-hand side represents the idea that, when z_P increases, local firms with higher investment costs, and hence, higher prices, enter the duopoly areas, which drives the average local price up. However, this is mitigated by a second, opposite, effect, which is represented by the second term: when z_P increases, firm P increases its price and local firms react by lowering theirs, which tends to reduce the average local price. In the Shubik-Levitan example, we find that the first effect dominates the second one, and hence, the average local price increases with z_P.

5.2 Coverage strategies

We now solve for firms P and L's coverage decisions.

Since we have assumed that they lead in investment, the local public firms L cover the monopoly areas. Their investment problem in these areas is the same than in the monopoly benchmark: they thus cover up to the area z^m (see Lemma 3).

Firm P covers necessarily less than the local public firms. This is because, in the areas covered by local public firms, P faces competition and thus makes less profit than the monopoly profit. P's equilibrium coverage, z^P, if it exists, is therefore lower than $z^L = z^m$.

Replacing for P_P and $P_L(z)$, P's profit can be written as

$$\Pi_P = \int_0^{z_P} \pi_P \left(P_P, P_L(z)\right) dz - C(z_P). \tag{17}$$

Using the envelope theorem, P's equilibrium coverage is given by the following first-order condition:

$$\frac{d\Pi_P}{dz_P} = \pi_P \left(P_P, P_L(z_P)\right) - c(z_P) = 0$$

The two terms represent the direct effect of coverage expansion: P earns an incremental profit in the marginal area, less the investment cost for this area.

Let $z^d \equiv \arg\max_z zp^d D(p^d, p^d) - C(z)$. We can state the following result:

Proposition 5. *Assume that the equilibrium to the coverage-price game exists and is unique. In equilibrium,*

(i) The private firm P covers up to $Z_P^l \leq z^d$ and charges the price $P_P^l \leq p^d$;

(ii) The local public firms L cover up to the area $Z_L^l = z^m$. In monopoly areas, they set the same price $P_L^m(z) \leq p^m$ than in the monopoly benchmark. In duopoly areas, they set the price $P_L^l(z)$ such that $P_L^m(z) \leq P_L^l(z) \leq p^d$.

The competition between a private firm and local public firms leads to the same total coverage than in the benchmark. This is because local public firms are independent from each other, and in the monopoly areas they thus have the same investment incentives than in the monopoly benchmark.

In duopoly areas, prices are lower compared to the benchmark with a monopoly private firm, but higher compared to the benchmark with monopoly local public firms. Indeed, public firms face the competition from the private firm, and have to increase their prices in order to break even.

5.3 Linear demand example

As an example, we solve for the equilibrium in the linear Shubik-Levitan demand model. To start with, we determine the equilibrium prices for given coverage levels. If it is located in a monopoly area $z \in (z_P, z_L]$, firm L's best-response is given by:

$$p_L(z) = \underline{p}_L(z) = p^m \left(1 - \sqrt{1 - \frac{c(z)}{\pi^m}}\right),$$

where $\underline{p}_L(z)$ is the lowest price such that the budget constraint holds with a local monopoly.

If firm L is located in a duopoly area $z \in [0, z_P]$, its best-response is given by:

$$p_L^{BR}(p_P, z) = \underline{p}_L^d(p_P, z) = BR^d(p_P) \left(1 - \sqrt{1 - \frac{4c(z)}{(2+\gamma)(BR^d(p_P))^2}}\right).$$

Firm P's best-response is given by the first-order condition (16). Since demand is linear, P's best-response to a price scheme $p_L(z)$ set by the local public firms is simply the duopoly best response to the average price set by L in the areas 0 to z_P, that is,

$$p_P = \frac{1}{z_P} \int_0^{z_P} BR^d(p_L(z))dz = BR^d(\overline{p}_L), \tag{18}$$

where $\overline{p}_L = \int_0^{z_P} p_L(z)dz/z_P$ is the average price set by L in duopoly areas.

To solve for the equilibrium prices for given coverage levels, we replace for $\underline{p}_L^d(p_P, z)$ into (18). The equilibrium price P_P then solves

$$P_P = \frac{1}{z_P} \int_0^{z_P} BR^d(\underline{p}_L^d(P_P, z))dz = BR^d(\overline{p}_L^{BR}(P_P)), \tag{19}$$

with $\overline{p}_L^{BR}(P_P) = \int_0^{z_P} \underline{p}_L^d(p_P, z)dz/z_P$. The equilibrium price for a firm L based in area z is then $P_L(z) = \underline{p}_L^d(P_P, z)$.

From (19), we can see that everything is as if firm P were competing with an "average" local public firm, with best response $\overline{p}_L^{BR}(P_P)$. In particular, the equilibrium prices are simply given by the intersection of the duopoly best response $BR^d(\overline{p}_L)$ with the average best response of local firms, $\overline{p}_L^{BR}(p_P)$.

Figure 6 shows the equilibrium prices, for given coverage levels, as a function of the private firm's coverage z_P. The investment cost is $c(z) = c_0 z$, with $c_0 = 0.01$, and we set the degree of substitutability to $\gamma = 1$. The equilibrium price of the private firm is P_P, whereas $\bar{P}_L$ represents the average price of local firms in the duopoly areas 0 to z_P. For comparison, we show the average price of local firms if they cover the same areas and act as local monopolists, $\bar{P}_L^m$. We see that both P_P and $\bar{P}_L$ increase when the private firm extends coverage. Firm P's price is lower than the duopoly price, while the local firms set higher prices than in a situation where they don't face competition (i.e., $\bar{P}_L > \bar{P}_L^m$).

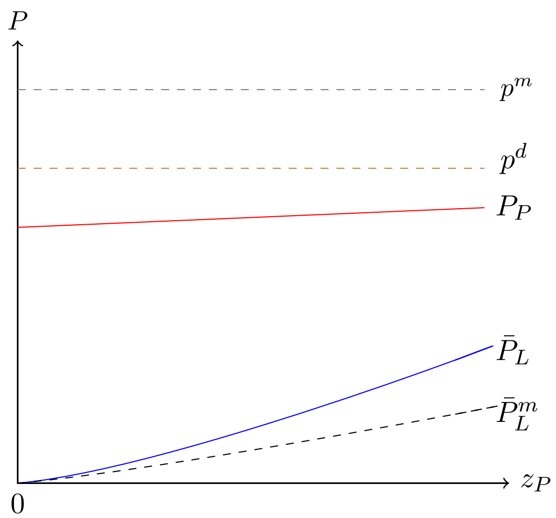


Figure 6: Variation of prices with respect to z_P.

We plug in the equilibrium prices for given coverage levels into firm P's profit function, which is given by (17). We then solve for the coverage z_P that maximizes P's profit, assuming that $c(z) = c_0 z$. Figure 7 shows the equilibrium prices and coverage levels as a function of the degree of substitutability γ, for $c_0 = 0.01$. For the local public firms, we show the average local price in duopoly areas, $\bar{P}_L$. In equilibrium, the private firm and the local public firms set lower prices than the duopoly price p^d, and cover less than the duopoly coverage z^d. As competition intensifies (i.e., γ increases), prices decline and the private firm invests less in coverage.

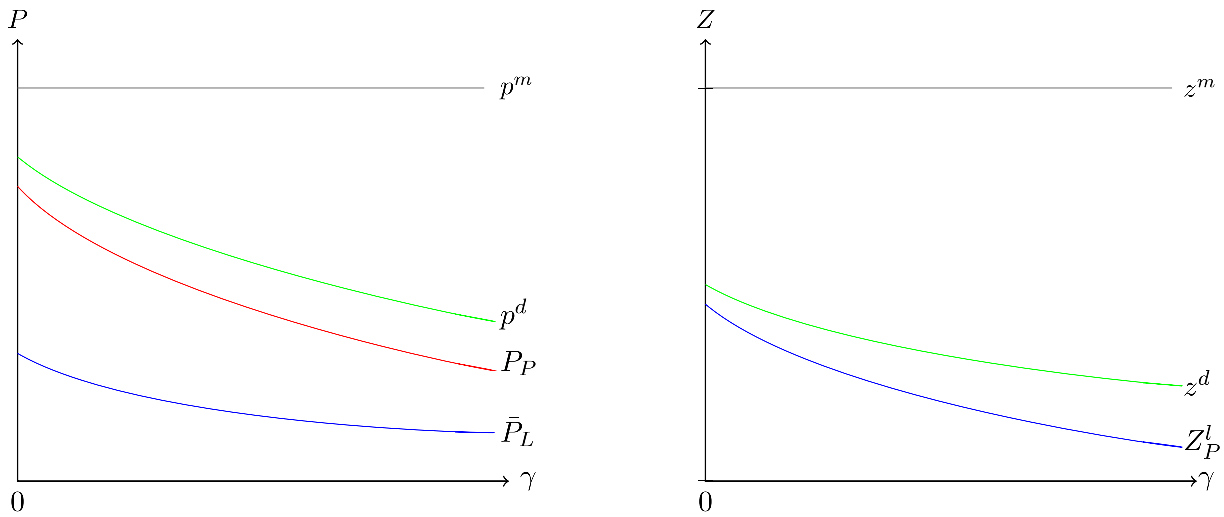


Figure 7: Equilibrium price and coverage levels if local public firms lead in investment.

Figure 8 shows the equilibrium welfare level as a function of the degree of substitutability γ, for $c_0 = 0.01$. We see that if competition is soft (i.e., low γ), welfare is lower compared to the private monopoly where it is the contrary as competition intensifies.

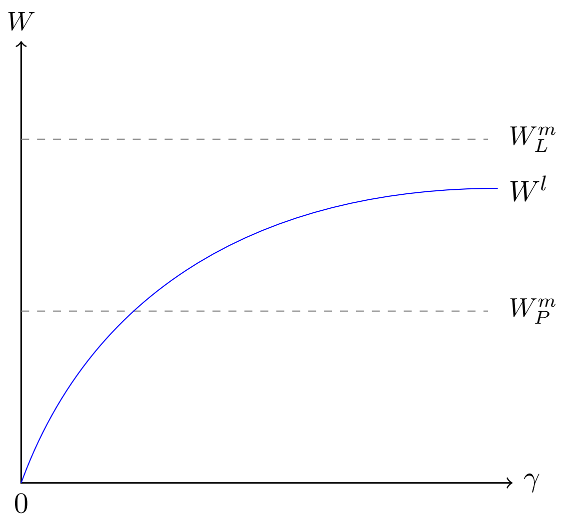


Figure 8: Equilibrium welfare levels if local public firms lead in investment.

6 Conclusion

In this paper, we examine a model where a private firm competes with public firms in prices and coverage of a new network infrastructure in a given country. In a monopoly benchmark, we find that the private firm charges the monopoly price and covers up to a monopoly area where the marginal cost of investment is equal to the (local) monopoly profit. A national public firm charges a price lower than the monopoly price, and cross-subsidizes between low-cost and high-cost areas, which allows it to cover more than the private firm, i.e., up to the area where the marginal social benefit of investment becomes lower than the marginal cost of investment. Local public firms, which charge prices that are contingent on market conditions in their area, cover the same territory than the private monopoly, but charge lower prices.

We then examine a mixed duopoly framework, where the private firm competes with the national public firm or local public firms in prices and coverage and focus on the case where public firms lead in investment, that is, invest more than the private firm in equilibrium. When the private firm competes with the national public firm, we find that the private firm reacts to a price increase of its rival by increasing its own price (strategic complementarity), whereas the national public firm reacts by decreasing its own price (strategic substitutability). Moreover, a larger overlap between the private firm's and the public firm's networks drives firms' prices up. We find that, at the equilibrium, total coverage by the national public firm is lower than in the benchmark. Competition leads the private firm to set lower prices than in the benchmark, while the public firm may charge higher prices than in the benchmark. This is due to the competition from the private firm, which makes it harder to sustain low prices.

When the private firm competes with local public firms, a larger overlap between the private firm's network and the local public firms' networks leads to a higher price by the private firm and lower prices by the local public firms. When the private firm covers a larger territory, it faces local competitors with higher costs, and therefore it becomes more accommodating. Since the private firm increases its price, the budget constraint of local public firms is relaxed and they react by decreasing their own prices. We find that, at the equilibrium, total coverage is the same than in the benchmark. The private firm sets a lower price than in the benchmark though, but local public firms set higher prices.

We have studied how the competition between a private firm and public firms in prices and coverage of a territory may affect market equilibria. In order to gain more intuition, it would be interesting to compare the mixed duopoly setting with the private duopoly. Moreover, we focussed on firms which are vertically integrated, that is, which operate at both the wholesale and retail levels. The case where public firms operate only upstream, at the wholesale level, also exists. One direction for further research may be to investigate in our framework a scenario where the public firm operates only at the wholesale level, while providing access to competitors at the retail level.

References

Anton, James J, James H Vander Weide, and Nikolaos Vettas (2002) "Entry auctions and strategic behavior under cross-market price constraints," *International Journal of Industrial Organization*, Vol. 20, pp. 611–629.

Armstrong, Mark and Helen Weeds (2007) "Programme quality in subscription and advertising-funded television," *Unpublished working paper.*

Beato, P and A Mas-Colell (1984) "The marginal cost pricing as a regulation mechanism in mixed markets": in Marchand, M, Pestieau, P., and H. Tulkens, eds., The Performance of Public Enterprises."

Bourreau, Marc, Carlo Cambini, and Steffen Hoernig (2015) "Geographic access markets and investments," *Information Economics and Policy*, Vol. 31, pp. 13–21.

Bourreau, Marc and Pinar Doğan (2006) "Build or buy strategies in the local loop," *American Economic Review*, Vol. 96, pp. 72–76.

Bourreau, Marc, Johan Hombert, Jerome Pouyet, and Nicolas Schutz (2011) "Upstream competition between vertically integrated firms," *The Journal of Industrial Economics*, Vol. 59, pp. 677–713.

Briglauer, Wolfgang, Christian Holzleitner, and Ingo Vogelsang (2016) "The need for more efficient public funding of new communications infrastructure in EU member states," *Information Economics and Policy*, Vol. 36, pp. 26–35.

Brito, Duarte and Pedro Pereira (2010) "Access to bottleneck inputs under oligopoly: a prisoners' dilemma?" *Southern Economic Journal*, Vol. 76, pp. 660–677.

Cave, Martin and Ian Martin (2010) "Motives and means for public investment in nationwide next generation networks," *Telecommunications Policy*, Vol. 34, pp. 505–512.

Chone, Philippe, Laurent Flochel, and Anne Perrot (2000) "Universal service obligations and competition," *Information Economics and Policy*, Vol. 12, pp. 249–259.

——— (2002) "Allocating and funding universal service obligations in a competitive market," *International Journal of Industrial Organization*, Vol. 20, pp. 1247–1276.

Cremer, Helmuth, Maurice Marchand, and Jacques-Francois Thisse (1989) "The public firm as an instrument for regulating an oligopolistic market," *Oxford Economic Papers*, pp. 283–301.

De Fraja, Giovanni and Flavio Delbono (1989) "Alternative strategies of a public enterprise in oligopoly," *Oxford Economic Papers*, Vol. 41, pp. 302–311.

European Commission (2009) "Community guidelines for the application of state aid rules in relation to rapid deployment of broadband networks."

Foros, 0ystein (2004) "Strategic investments with spillovers, vertical integration and foreclosure in the broadband access market," *International journal of industrial organization*, Vol. 22, pp. 1–24.

Foros, Oystein and Hans Jarle Kind (2003) "The broadband access market: Competition, uniform pricing and geographical coverage," *Journal of Regulatory Economics*, Vol. 23, pp. 215–235.

Gautier, Axel and Xavier Wauthy (2010) "Price competition under universal service obligations," *International Journal of Economic Theory*, Vol. 6, pp. 311–326.

Hauge, Janice A, Mark A Jamison, and Richard J Gentry (2008) "Bureaucrats as entrepreneurs: Do municipal telecommunications providers hinder private entrepreneurs?" *Information Economics and Policy*, Vol. 20, pp. 89–102.

Hoernig, Steffen H (2006) "Should uniform pricing constraints be imposed on entrants?" *Journal of Regulatory Economics*, Vol. 30, pp. 199–216.

Jullien, Bruno, Jerome Pouyet, and Wilfried Sand-Zantman (2010) "Public and private investments in regulated network industries: coordination and competition issues," *Review of Network Economics*, Vol. 9.

——— (2017) "An offer you can't refuse: early contracting with endogenous threat," *The RAND Journal of Economics*, Vol. 48, pp. 733–748.

——— (2018) "Quel role pour les acteurs publics dans l'incitation privee aux investissements?" *Revue Economique*, Vol. 69, pp. 985–1007.

Lee, Sang-Ho, Toshihiro Matsumura, and Susumu Sato (2018) "An analysis of entry-then-privatization model: welfare and policy implications," *Journal of Economics*, Vol. 123, pp. 71–88.

Matsumura, Toshihiro (1998) "Partial privatization in mixed duopoly," *Journal of Public Economics*, Vol. 70, pp. 473–483.

Merrill, William C and Norman Schneider (1966) "Government firms in oligopoly industries: a short-run analysis," *The Quarterly Journal of Economics*, Vol. 80, pp. 400–412.

Nitsche, Rainer and Lars Wiethaus (2011) "Access regulation and investment in next generation networks: A ranking of regulatory regimes," *International Journal of Industrial Organization*, Vol. 29, pp. 263–272.

Ordover, Janusz and Greg Shaffer (2007) "Wholesale access in multi-firm markets: When is it profitable to supply a competitor?" *International Journal of Industrial Organization*, Vol. 25, pp. 1026–1045.

Valletti, Tommaso M, Steffen Hoernig, and Pedro P Barros (2002) "Universal service and entry: The role of uniform pricing and coverage constraints," *Journal of Regulatory Economics*, Vol. 21, pp. 169–190.

Wilson, Kyle (2019) "Does public competition crowd out private investment? Evidence from municipal provision of internet access," *Working Paper*.

Appendix A: Shubik-Levitan demand model

Following Shubik and Levitan (1980), we introduce a representative consumer with the following quasi-linear preferences:

$$U(q_i, q_j, y) = q_i + q_j - \frac{1}{1+\gamma}\left(q_i^2 + q_j^2 + \frac{\gamma}{2}(q_i + q_j)^2\right) + y,$$

where y si the numeraire good and $\gamma \in [0, \infty)$ represents the degree of substitutability between products i and j, a lower γ corresponding to a higher degree of differentiation. If products i and j are both available to the consumer (i.e., we have a duopoly), the demand of firm i is given by:

$$D_i^d(p_i, p_j) = \frac{1}{4}\left(2 - (2+\gamma)p_i + \gamma p_j\right).$$

If only one product is available to the consumer, the monopoly demand of firm i is:

$$D^m(p_i) = 1 - p_i.$$

Illustrative model for monopoly benchmark

We solve for the equilibrium of the coverage-price game in the linear Shubik-Levitan demand model. We assume that the investment cost in a given area z is $c(z) = z$.

Private firm. The equilibrium price and coverage of firm P are

$$P_P^m = p^m = 1/2 \text{ and } Z_P^m = z^m = c^{-1}(p^m D(p^m)) = c^{-1}(1/4) = 1/4.$$

Total welfare is $W_P^m = z^m w(p^m) - C(z^m) = 1/16$.

National public firm. For a given coverage z, firm N sets the lowest price such that its budget constraint holds, which is given by

$$\underline{p}_N(z) = p^m \left(1 - \sqrt{1 - \frac{C(z)/z}{\pi^m}}\right).$$

Firm N's optimal coverage is then the solution of $w(\underline{p}_N(z)) = c(z) = z$. We find that firm N's equilibrium coverage is $Z_N^m = 4/9$ and its uniform price is $P_N^m = 1/3$. Total welfare is $W_N^m = Z_N^m w(P_N^m) - C(Z_N^m) = 8/81$.

Local public firms. In a given area z, firm L sets the lowest price such the local budget constraint holds, which is given by

$$\underline{p}_L(z) = p^m \left(1 - \sqrt{1 - \frac{c(z)}{\pi^m}}\right) = P_L^m(z).$$

As shown in Lemma 3, firm L invests if and only if $z \leq z^m = Z_L^m$. Total welfare is $W_L^m = \int_0^{z^m} \left(w(\underline{p}_L(z)) - c(z)\right) dz = 17/192$.

Appendix B: the private firm leads in investment

In this section, we briefly analyze the case where the private firm has invested more than the national public firm (Appendix B.1), and the case where it has invested more than the local public firms (Appendix B.2).

Appendix B.1: National public firm vs. private firm

Pricing strategies

We start by determining the equilibrium prices of firms P and N for given coverage levels z_N and z_P , with $z_P > z_N$.

Best responses. Consider first the pricing decision of the private firm. P chooses a price p_P to maximize its profit, which is given by

$$\Pi_P = (z_P - z_N)p_P D^m(p_P) + z_N p_P D_P^d(p_P, p_N) - C(z_P), \tag{A.1}$$

Firm P's best-response is given by

$$\frac{z_N}{z_P}\frac{\partial \left[pD_P^d(p, p_N)\right]}{\partial p} + \left(1 - \frac{z_N}{z_P}\right)\frac{\partial \left[pD^m(p)\right]}{\partial p} = 0. \tag{A.2}$$

From the above equation , we observe that if $z_N = 0$, firm P's best-reponse is the monopoly price, p^m, whereas if $z_N \to z_P$, its best-response is close to the duopoly best-response, $BR^d(p_N)$. Therefore, P's best-response to a price p_N, is $p_P^{BR}(p_N, z_N, z_P) \in (BR^d(p_N), p^m)$.

Consider now the pricing decision of the public firm. Firm N chooses its price p_N to maximize its objective function, which is given by

$$\Pi_N = z_N \left(p_N D_N^d(p_P, p_N) + CS(p_P, p_N)\right) - C(z_N), \tag{A.3}$$

subject to the budget constraint

$$z_N p_N D_N^d(p_P, p_N) \geq C(z_N). \tag{A.4}$$

Proceeding in a similar way than as in Lemma 4, N's best-response to a price p_P is to set the lowest price such that its budget constraint (A.4) binds, that is, $p_N^{BR}(p_P) = \underline{p}_N^d(p_P, z_N)$, where $\underline{p}_N^d$ satisfies $z_N \underline{p}_N^d D_N^d(\underline{p}_N^d, p_P) = C(z_N)$. This price is lower than or equal to the uniform price that maximizes its total profit (i.e., $\hat{p}_N = BR^d(p_N)$). If at the profit-maximizing price $\hat{p}_N$, firm N does not break even, there is no best-response to the price set by the private firm.

Proceeding in a similar way as in Lemma 5, prices are strategic complements for firm P and strategic substitutes for firm N.

Equilibrium prices. The equilibrium prices P_P and P_N are given by the intersection of the best responses of the private firm and the public firm. We assume that this intersection exists. To study how coverage affects pricing decisions, we define the ratio of coverage levels $\delta \equiv z_N/z_P \in (0,1)$. The equilibrium prices can then be written as functions of z_N and δ: $P_P = P_P(z_N, \delta)$ and $P_N = P_N(z_N, \delta)$.

The following proposition characterizes how the coverage by firm P and the degree of overlap ($\delta = z_N/z_P$) between firm P's and firm N's networks affect equilibrium prices.

Proposition 2′. *Assume that the price equilibrium exists, for given coverage levels.*

- *For a given coverage z_N by the national public firm, firm N's equilibrium price P_N decreases with z_P, whereas firm P's equilibrium price increases with z_P.*
- *For given coverage z_P by the private firm, $\partial P_N/\partial\delta \gtreqless 0$ and $\partial P_P/\partial\delta < 0$ if and only if $P_N D^d(P_N, P_P) \geq c(z_N)$ while $\partial P_N/\partial\delta > 0$ and $\partial P_P/\partial\delta \gtreqless 0$ if and only if $P_N D^d(P_N, P_P) \leq c(z_N)$.*

Coverage strategies

We now consider the coverage decisions of firms N and P, for given prices p_N and p_P. From (A.1) and (A.3), the equilibrium coverage levels are solutions to the system of first-order conditions,

$$p_P D_P^m(p_P) = c(z_P),$$
$$p_N D_N^d(p_N, p_P) + CS(p_N, p_P) = c(z_N),$$

Replacing for $p_N = P_N(z_P, z_N)$ and $p_P = P_P(z_P, z_N)$, the equilibrium coverage of firms N and P are the solution of

$$P_P(z_P, z_N)D_P^m(P_P(z_P, z_N)) = c(z_P), \quad \text{(A.5)}$$

$$P_N(z_P, z_N)D_N^d(P_N(z_P, z_N), P_P(z_P, z_N)) + CS(P_N(z_P, z_N), P_P(z_P, z_N)) = c(z_N). \quad \text{(A.6)}$$

Proposition 3′. *Assume that the equilibrium to the coverage-price game exists and is unique. In equilibrium,*

(i) The private firm P covers a smaller territory than in the benchmark and charges a lower price, i.e., $Z_P^n < Z_P^m$ and $P_P^n < P_P^m$.

(ii) The public firm N covers a smaller territory than in the benchmark and charges a lower price, i.e., $Z_N^n < Z_N^m$ and $P_N^n < P_P^m$.

Appendix B.2: Private vs. local public firms

Pricing strategies

We analyze firms' pricing strategies for given coverage levels z_P and z_L, with $z_P \geq z_L$.

Best responses. Consider first the pricing decision of a local public firm L based in a given area $z \leq z_L$. As in Section 6.1 (case (ii)), L competes with P in its local market, and chooses its price p_L to maximize

$$p_L D_L^d(p_L, p_P) + CS(p_L, p_P) - c(z), \quad \text{subject to} \quad p_L D_L^d(p_L, p_P) \geq c(z). \quad \text{(A.7)}$$

Firm L's best-response is then $p_L^{BR}(p_P, z) = \underline{p}_L^d(p_P, z)$, where $\underline{p}_L^d$ is the solution to (15).

We find that when P increases its price, L reacts by decreasing its own price and that L's best response increases with z. The best response exists if L breaks even when it sets its profit-maximizing price, $p_L = BR^d(p_P)$.

We now determine firm P's best-response to prices $p_L(z)$ set by the local public firms.

Firm P's profit is given by

$$\Pi_P = \int_0^{z_L} p_P D_P^d\left(p_P, p_L(z)\right) dz + \int_{z_L}^{z_P} p_P D_P^m(p_P) dz - C(z_P). \tag{A.8}$$

Its best-response is given by the first-order condition

$$\frac{\partial \Pi_P}{\partial p_P} = \int_0^{z_L} \left[p_P \frac{\partial D_P^d}{\partial p_P}(p_P, p_L(z)) + D_P^d(p_P, p_L(z)) \right] dz + (z_P - z_L) \left[p_P \frac{\partial D_P^m}{\partial p_P}(p_P) + D_P^m(p_P) \right] = 0. \tag{A.9}$$

We assume that there is a unique solution $p_P^{BR}(z_L, z_P, p_L(.))$ to this first-order condition, and that the second-order condition,

$$\frac{\partial^2 \Pi_P}{\partial p_P^2} = \int_0^{z_L} \left[2\frac{\partial D_P^d}{\partial p_P}(p_P, p_L(z)) + p_P \frac{\partial^2 D_P^d}{\partial p_P^2}(p_P, p_L(z)) \right] dz \\ + (z_P - z_L) \left[2\frac{\partial D_P^m}{\partial p_P}(p_P) + p_P \frac{\partial^2 D_P^m}{\partial p_P^2}(p_P) \right] \leq 0,$$

holds.

Due to the strategic complementarity assumption, P's best-response $p_P^{BR}(z_L, z_P, p_L(.))$ increases if L's prices increase. Furthermore, assuming that $p_L(z)$ is increasing in z, P's best-response increases with z_P, as

$$\left.\frac{\partial^2 \Pi_P}{\partial p_P \partial z_P}\right|_{p_P = p_P^{BR}} = p_P^{BR} \frac{\partial D_P^m}{\partial p_P}(p_P^{BR}) + D_P^m(p_P^{BR}) > 0.$$

Firm P's best-response p_P^{BR} increases with z_P as covering more increases the investment cost of P, thereby increasing P's uniform price.

Firm P's best-response price decreases with z_L as

$$\left.\frac{\partial^2 \Pi_P}{\partial p_P \partial z_L}\right|_{p_P = p_P^{BR}} = p_P^{BR} \frac{\partial D_P^d}{\partial p_P}(p_P, p_L(z_L)) + D_P^d(p_P, p_L(z_L)) - \left(p_P^{BR} \frac{\partial D_P^m}{\partial p_P}(p_P^{BR}) + D_P^m(p_P^{BR}) \right) < 0.$$

As z_L increases, the investment cost of local public firms covering (the new areas) increases. If $p_L(z)$ is increasing, firm P faces local public firms setting higher prices and thus reacts by increasing its uniform price (first two terms on the right). On the other hand, a higher z_L means that P benefits less from monopoly areas and faces competition from local public firms, affecting negatively its price (last two terms on the right).

Equilibrium prices. The equilibrium prices $P_P(z_L, z_P)$ and $P_L(z, z_L, z_P)$ are the solution to (15) and (A.9). We assume that this solution exists and is uniquely defined.

The following proposition characterizes how the coverage of the private firm, and hence the overlap between the private firm's and the public firms' networks, affect equilibrium prices.

Proposition 4′. *Assume that the price equilibrium exist, for a given coverage $z_L \leq z_P$. Then, the private firm's equilibrium price P_P increases (decreases) with z_P (z_L), whereas the local public firms prices $P_L(z)$ decrease (increase) with z_P (z_L). Furthermore, $P_L(z)$ is increasing in z.*

Coverage strategies

We now solve for firms P and L's coverage decisions. Replacing for $P_P(z_L, z_P)$ and $P_L(z, z_L, z_P)$, P's profit is given by

$$\Pi_P = \int_0^{z_L} \pi_P^d \left(P_P, P_L(z)\right) dz + \int_{z_L}^{z_P} \pi_P^m \left(P_P\right) dz - C(z_P).$$

Using the envelope theorem, P's equilibrium coverage is given by the following first-order condition:

$$\frac{d\Pi_P}{dz_P} = \pi_P^m \left(P_P\right) - c(z_P)$$

The two terms on the right hand side represent the direct effect of coverage expansion: P earns an incremental profit in the marginal (monopoly) area, less the investment cost for this area.

We now state the following result:

Proposition 5′. *Assume that the equilibrium to the coverage-price game exists and is unique. In equilibrium,*

(i) The private firm P covers up to $Z_P^l < Z_P^m = z^m$ and charges the price $P_P^l < p^m$;

(ii) The local public firms cover up to $Z_L^l < Z_L^m = z^m$. They set the price $P_L^l(z)$ such that $P_L^m(z) \leq P_L^l(z) < p^m$.

Appendix B.3: Private competition

To compare mixed duopoly outcomes with private competition, we let firm N behave as a private firm. Let firm $i = N, P$ lead in investment, i.e., $z_i > z_j$, where $i \neq j$. For the sake of exposition, let $z_N > z_P$.

Pricing strategies

We start by determining the equilibrium prices of firms P and N for given coverage levels z_N and z_P, with $z_P > z_N$.

Best responses. Consider first the pricing decision of firm P. P chooses a price p_P to maximize its profit (5). Firm P's best-response to a price p_N is the duopoly best-response $p_P^{BR} = BR^d(p_N)$.

Consider now the pricing decision of firm N. Firm N chooses its price p_N to maximize its objective function, which can be writed in a similar way as equation (A.1). Using the first-order condition (A.9), firm N's best-response to a price p_P is then $p_N^{BR}(p_P, z_N, z_P) \in \big(BR^d(p_P), p^m\big)$.

Equilibrium prices. Using equation (A.9), we observe that if $z_N \to z_P$, we are close to a private duopoly (i.e., close the private duopoly price p^d), whereas if $z_P \to 0$, we are close to a private monopoly (i.e., close to the monopoly price p^m for firm N).

To study how coverage affects pricing decisions, we define the ratio of coverage levels $k \equiv z_P/z_N \in (0, 1)$.The equilibrium prices can then be written as a function of k, that is, $P_i = P_i(k)$.

The following proposition characterizes how the coverage by firm N and the degree of overlap ($k = z_P/z_N$) between firms' networks affect equilibrium prices.

Proposition 2″. *Assume that the price equilibrium exists, for given coverage levels.*

- *For given coverage z_N, firms' equilibrium prices decreases with k (i.e., with z_P)*
- *For given coverage z_P, equilibrium prices increases with z_N.*

Coverage strategies

We now consider the coverage decisions of firms N and P, for given prices p_N and p_P. From firms' objective functions, the equilibrium coverage levels are solutions to the system of first-order conditions,

$$p_P D_P^d(p_P, p_N) = c(z_P),$$
$$p_N D_N^m(p_N) = c(z_N),$$

By replacing for $p_N = P_N(z_P, z_N)$ and $p_P = P_P(z_P, z_N)$, the equilibrium coverage of firms N and P are the solution of

$$P_P(z_P, z_N) D_P^d(P_P(z_P, z_N), P_N(z_P, z_N)) = c(z_P), \tag{A.10}$$
$$P_N(z_P, z_N) D_N^m(P_N(z_P, z_N)) = c(z_N). \tag{A.11}$$

Proposition 3″. *Assume that the equilibrium to the coverage-price game exists and is unique. In equilibrium,*

(i) $p^d < P_i^p < p^m$ *where* $i = P, N$*;*

(ii) $z^d < Z_P^p < Z_N^p < z^m$.

Linear demande example

Appendix C: Proofs

Proof of Lemma 1

For a given coverage z, firm P's profit, which is given by (1), is maximized at the monopoly price $p = p^m$. Replacing for $p = p^m$ into (1), the optimal coverage for the private firm is then given by $c(z) = p^m D^m(p^m) = \pi^m$, i.e., $z = c^{-1}(\pi^m) \equiv z^m$.

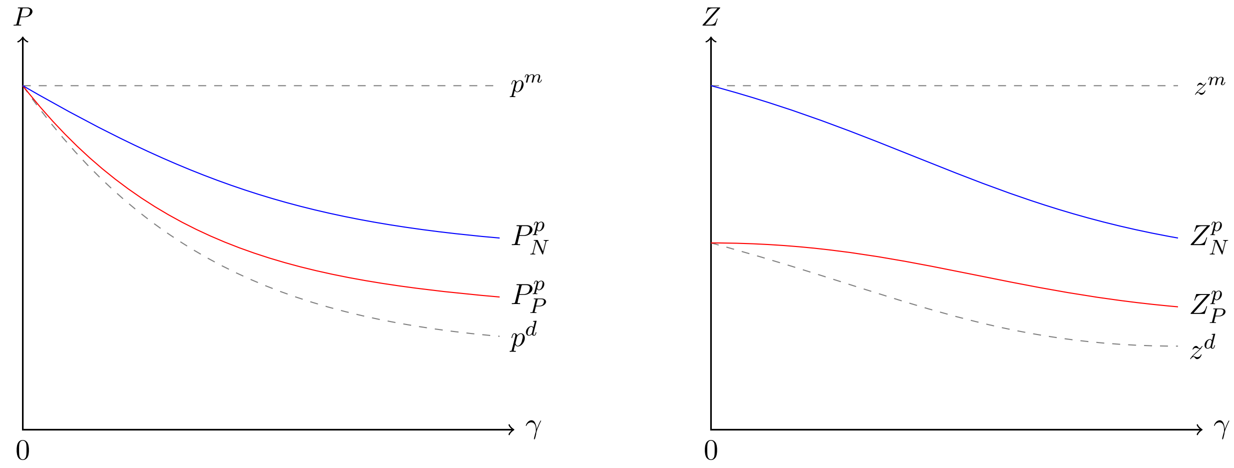


Figure 9: Equilibrium price and coverage levels if private competition ($c(z) = c_0 z$ and $c_0 = 0.01$).

Proof of Lemma 2

Consider first firm N's pricing decision for a given coverage z. From our assumptions, for a given coverage z, total welfare, which is given by (2), is decreasing in the price p, whereas the public firm's profit, $zpD^m(p) - C(z)$, is increasing in p, up to the monopoly price p^m. Firm N thus sets the minimum price such that its budget constraint still holds. Let $\underline{p}_N(z)$ denote the lowest price p such that $zpD^m(p) - C(z) \geq 0$. The limit price $\underline{p}_N(z)$ exists if and only if $z\pi^m \geq C(z)$. If it does, N's optimal price, for a given coverage z, is $p_N = \underline{p}_N(z)$. If $\underline{p}_N(z)$ does not exist, there is no price allowing the firm to break even while covering the areas from 0 to z.

Using the implicit function theorem on the budget constraint and the fact that $c' > 0$, we find that $\partial \underline{p}_N(z)/\partial z \geq 0$. Furthermore, we have $\underline{p}_N(z) \leq p^m$. Intuitively, if the public firm covers a larger part of the country, it must set a higher uniform price to break even. This price cannot be larger than p^m, since the monopoly profit is decreasing for $p \geq p^m$.

Consider now firm N's choice of coverage for a given uniform price p. Using (2), N's optimal coverage is the solution of $w^m(p) = c(z)$. The optimal coverage for the public firm is then the solution of $w^m(\underline{p}_N(z)) = c(z)$. Since $\underline{p}_N(z)$ is increasing in z and $w(p)$ is decreasing in p, $w^m(\underline{p}_N(z))$ is decreasing in z, whereas $c(z)$ is increasing in z, with $c(0) = 0$ and $\lim_{z\to\infty} c(z) = +\infty$. Therefore, there exists a unique Z_N^m such that $w^m(\underline{p}_N(Z_N^m)) = c(Z_N^m)$. The public firm's optimal uniform price is then $P_N^m = \underline{p}_N(Z_N^m)$.

Finally, we have $P_N^m \leq p^m$ as $P_N^m = \underline{p}_N(Z_N^m)$ and $\underline{p}_N(z) \leq p^m$ for all z. Furthermore, since $w^m(p)$ is decreasing and $\underline{p}_N(Z_N^m) \leq p^m$, Z_N^m is larger than the coverage z' defined by $w^m(p^m) = c(z')$. Since $w^m(p^m) > p^m D^m(p^m) = c(z^m)$, $z' > z^m$ and hence $Z_N^m > z^m$.

Proof of Lemma 3

Consider first firm L's pricing decision in a given area z. Since $\partial w^m(p)/\partial p < 0$, firm L sets the lowest price compatible with its budget constraint (4). Let $\underline{p}_L(z)$ denote the lowest price such that $p_L D^m(p_L) - c(z) \geq 0$, which exists if and only if $\pi^m \geq c(z)$. If it exists, firm L's optimal price for a given coverage z is $p_L = \underline{p}_L(z)$. Since the investment cost $c(z)$ is increasing, $\underline{p}_L(z)$ is increasing too.

Note that $\underline{p}_L(z) > \underline{p}_N(z)$. Indeed, the budget constraint for N can be written as $p_N D^m(p_N) - C(z)/z \geq 0$, and

$$c(z) - \frac{C(z)}{z} = \frac{1}{z}\int_0^z (c(z) - c(t))\, dt > 0,$$

as $c' > 0$. We now solve for firm L's coverage decision. Replacing for $\underline{p}_L(z)$ into (3) and (4), firm L decides to invest if and only if

$$w^m(\underline{p}_L(z)) - c(z) \geq 0, \quad \text{with} \quad \underline{p}_L(z) D^m(\underline{p}_L(z)) - c(z) = 0, \tag{A.12}$$

that is, the local public firm invests if the social benefit from investment is larger than the investment cost, for a price set so that it just breaks even.

We find that L invests if and only if $z \leq z^m$. Indeed, the maximum gross profit that L can make is the monopoly profit π^m, and therefore, the marginal area such that the budget constraint is satisfied is the monopoly area z^m. In this area, we have $w^m(p^m) - c(z^m) > 0$, since $w^m(p^m) = \pi^m + CS(p^m) > \pi^m = c(z^m)$. Hence, L invests if and only if $z \leq z^m = Z_L^m$. If it does, it sets the local price $P_L^m(z) = \underline{p}_L(z)$ defined above.

Proof of Lemma 4

We first show that the objective function of the public firm, which is given by (6), is decreasing in p_N. This is because, (i) $w^m(p_N)$ is decreasing in p_N from our assumptions, and (ii) $p_N D_N^d(p_N, p_P) + CS(p_N, p_P)$ is decreasing in p_N too. To prove point (ii), we rewrite $p_N D_N^d(p_N, p_P) + CS(p_N, p_P) = w^d(p_N, p_P) - p_P D_P^d(p_P, p_N)$. Therefore,

$$\frac{\partial[p_N D_N^d(p_N, p_P) + CS(p_N, p_P)]}{\partial p_N} = \underbrace{\frac{\partial w^d(p_N, p_P)}{\partial p_N}}_{\leq 0} - p_P \underbrace{\frac{\partial D_P^d(p_P, p_N)}{\partial p_N}}_{\geq 0} \leq 0.$$

The public firm thus sets the lowest price compatible with its budget constraint. From Assumption 1, $pD^m(p)$ and $p_N D_N^d(p_N, p_P)$ are concave, which implies that the budget constraint (7) is concave in p_N too. Let $\hat{p}_i(p_j) \equiv \arg\max_p z_j p D_i^d(p, p_j) + (z_i - z_j) p D^m(p)$. We have $\hat{p}_N(p_P) \in [BR^d(p_P), p^m]$. The left-hand side of the budget constraint (7) is then increasing in p_N for $p_N \in [0, \hat{p}_N]$. For given coverage z_N and z_P, we define $\underline{p}_N^d(p_P, z_P, z_N)$ such that $z_P \underline{p}_N^d D_N^d(\underline{p}_N^d, p_P) + (z_N - z_P)\underline{p}_N^d D^m(\underline{p}_N^d) = C(z_N)$. This price $\underline{p}_N^d$ exists if and only if firm N breaks even when it sets its profit-maximizing price, that is, if $z_P \hat{p}_N D_N^d(\hat{p}_N, p_P) + (z_N - z_P)\hat{p}_N D^m(\hat{p}_N) \geq C(z_N)$. Firm N's best-response to a price p_P set by the private firm is then $p_N^{BR}(p_P) = \underline{p}_N^d(p_P, z_P, z_N)$, with $\underline{p}_N^d \leq \hat{p}_N$.

Proof of Lemma 5

The first point of the proposition simply re-states our assumption that prices are strategic complements in a duopoly with two private firms. To prove the second point, let $BC(p_N, p_P) \equiv z_P p_N D_N^d(p_N, p_P) + (z_N - z_P) p_N D^m(p_N) - C(z_N)$ represent the budget constraint of the public firm. Firm N's best-response satisfies $BC(p_N^{BR}, p_P) = 0$. From the implicit function theorem, we have

$$\frac{\partial p_N^{BR}}{\partial p_P} = - \left. \frac{\partial BC/\partial p_P}{\partial BC/\partial p_N} \right|_{p_N = p_N^{BR}}.$$

Since $p_N^{BR} \leq \hat{p}_N$, we have $\partial BC/\partial p_N|_{p_N = p_N^{BR}} \geq 0$. Furthermore, we have

$$\left. \frac{\partial BC}{\partial p_P} \right|_{p_N = p_N^{BR}} = z_P p_N^{BR} \frac{\partial D_N}{\partial p_P} \geq 0.$$

This proves that $\partial p_N^{BR}/\partial p_P \leq 0$.

Proof of Proposition 2

For given coverage levels, and hence, for a given ratio $\sigma = z_P/z_N$, firms' equilibrium prices are the solution of the following system of equations:

$$A \equiv (z_N - z_P)p_N D^m(p_N) + z_P p_N D_N^d(p_N, p_P) - C(z_N) = 0,$$
$$B \equiv D_P^d(p_P, p_N) + p_P \frac{\partial D_P^d(p_P, p_N)}{\partial p_P} = 0.$$

The first equation, which defines firm N's best response, corresponds to the budget constraint of the public firm, whereas the second equation is the first-order condition for firm P. Using Cramer's rule, we find that

$$\frac{\partial P_N}{\partial z_P} = \frac{-p^N(D^m(P_N) - D_N^d(P_N, P_P))}{D} \frac{\partial^2 \pi_P^d}{\partial p_P^2},$$
$$\frac{\partial P_P}{\partial z_P} = \frac{p^N(D^m(P_N) - D_N^d(P_N, P_P))}{D} \frac{\partial^2 \pi_P^d}{\partial p_P p_N},$$

where

$$D = \underbrace{\frac{\partial A}{\partial p_P}}_{(+)} \underbrace{\frac{\partial B}{\partial p_N}}_{(+)} - \underbrace{\frac{\partial A}{\partial p_N}}_{(+)} \underbrace{\frac{\partial B}{\partial p_P}}_{(-)} > 0.$$

Since at the equilibrium prices, $\partial^2 \pi_P^d/\partial p_P^2 \leq 0$ (concavity), $\partial^2 \pi_P^d/\partial p_P p_N \geq 0$ (strategic complementarity), and $D^m(P_N) > D_N^d(P_N, P_P)$, we have $\partial P_N/\partial z_P \geq 0$ and $\partial P_P/\partial z_P \geq 0$, and hence for a given z_N, $\partial P_N/\partial \sigma \geq 0$ and $\partial P_P/\partial \sigma \geq 0$.

Using a similar approach, we find that $\partial P_N/\partial z_N \geq 0$ and $\partial P_P/\partial z_N \geq 0$ if $P_N D^m(P_N) \leq c(z_N)$; otherwise, if $P_N D^m(P_N) \geq c(z_N)$, then $\partial P_N/\partial z_N \leq 0$ and $\partial P_P/\partial z_N \leq 0$.

Proof of Proposition 3

First, we compare the equilibrium coverage levels to the benchmark. The private firm's coverage is given by the first-order condition, $P_P^n D_P^d(P_P^n, P_N^n) = c(Z_P^n)$. Since $P_P^n D_P^d(P_P^n, P_N^n) < \pi^m$, then $Z_P^n < c^{-1}(\pi^m) = Z_P^m$. Using (9), we see that since P_N increases with z_P (from Proposition **??**) and $w^m(\cdot)$ is decreasing, a higher z_P leads to a lower coverage z_N by the public firm. Since for $z_P = 0$, the public firm sets the benchmark coverage z^N, this proves that $Z_N^n < Z_N^m$.

Second, we compare the equilibrium prices to the prices in the benchmark. Firm N's equilibrium price satisfies $P_N^n = \underline{p}_N^d(P_P^n) \leq \hat{p}_N(P_P^n) < p^m$. Since $P_N^n < p^m$, we have $P_P^n = BR^d(P_N^n) < p^m = P_P^m$. As for the public firm, in the benchmark, firm N's price is given by

$$Z_N^m P_N^m D^m(P_N^m) = C(Z_N^m).$$

In the duopoly setting, firm N's equilibrium price satisfies $P_N^n = \underline{p}_N^d(P_P^n)$, that is,

$$Z_N^n P_N^n D^m(P_N^n) - Z_P^n \underbrace{\left(P_N^n D^m(P_N^n) - P_N^n D_N^d(P_N^n, P_P^n)\right)}_{\text{business stealing effect}} = C(Z_N^n).$$

Assume that $Z_N^n = Z_N^m$. Then, firm N has to set a higher price to break even in duopoly, which means that $P_N^n > P_N^m$. However, this effect is counter-balanced by the fact that N covers less in duopoly than in the monopoly benchmark (i.e., $Z_N^n < Z_N^m$). Its investment cost is thus lower, allowing to set a lower price. Firm N's price can thus be either lower or higher in duopoly compared to the benchmark.

Proof of Proposition 4

Let $\pi_P \equiv p_P D_P^d(p_P, p_L)$ denote P's profit in a local area with prices p_P and p_L. Plugging in L's best response, the first-order condition for firm P can be rewritten as

$$\frac{\partial \Pi_P}{\partial p_P} = \int_0^{z_P} \frac{\partial \pi_P}{\partial p_P}(p_P, \underline{p}_L^d(p_P, z))dz \equiv F(p_P, z_P) = 0.$$

From the implicit function theorem, we thus have

$$\frac{\partial P_P}{\partial z_P} = -\frac{\partial F/\partial z_P}{\partial F/\partial p_P}.$$

Note that we have

$$\frac{\partial F}{\partial p_P} = \int_0^{z_P} \Bigg[\underbrace{\frac{\partial^2 \pi_P}{\partial p_P^2}}_{(-)} + \underbrace{\frac{\partial \pi_P}{\partial p_L}}_{(+)} \underbrace{\frac{\partial \underline{p}_L^d}{\partial p_P}}_{(-)} \Bigg] dz < 0.$$

Therefore, $\partial P_P/\partial z_P$ has the sign of $\partial F/\partial z_P$, which we find to be positive:

$$\frac{\partial F}{\partial z_P} = \frac{\partial \pi_P}{\partial p_P}(P_P, \underline{p}_L^d(P_P, z_P)) > 0,$$

from strategic complementarity and the fact that $\underline{p}_L^d(p_P, z)$ is increasing in z. Therefore, P_P increases with z_P. It follows that $P_L(z)$ decreases with z_P, because $P_L(z)$ gets lower with a higher P_P. Finally, from the definition of $\underline{p}_L^d(p_P, z)$, it is immediate that $P_L(z)$ is increasing in z.

Proof Proposition 5

Let us first characterize the equilibrium prices. In a given area z, firm L sets the lowest price such that its budget constraint, $p_L D_L^d(p_L, p_P) \geq c(z)$, holds. This price is always lower than the duopoly best-response, $BR^d(p_P)$. Therefore, due to strategic complementarity, in equilibrium firm P will always set a price lower than the duopoly price p^d, and so similarly for firm L (in any area). Finally, since $D_L^d(p_L, p_P) \leq D^m(p_L)$, in each area z the price set by firm L is higher in the mixed duopoly compared to the monopoly benchmark.

Since firms P and L set prices that are lower than the duopoly price, P makes a profit, gross of investment cost, which is lower than the duopoly profit $\pi^d \equiv p^d D^d(p^d, p^d)$. Its coverage is thus lower than the duopoly coverage $z^d = c^{-1}(\pi^d)$.

Proof of Proposition 2'

For given coverage levels, and hence, for a given ratio $\delta = z_N/z_P$, firms' equilibrium prices are the solution of the following system of equations:

$$A \equiv z_N p_N D_N^d(p_P, p_N) - C(z_N) = 0,$$
$$B \equiv (z_P - z_N)\frac{\partial[p_P D^m(p_P)]}{\partial p_P} + z_N \frac{\partial[p_P D_P^d(p_P, p_N)]}{\partial p_P} = 0.$$

Using Cramer's rule, we find that:

$$\frac{\partial P_P}{\partial z_P} = \frac{1}{D}\frac{\partial A}{\partial p_N}\frac{\partial \pi_P^m}{\partial p_P},$$
$$\frac{\partial P_N}{\partial z_P} = -\frac{1}{D}\frac{\partial A}{\partial p_P}\frac{\partial \pi_P^m}{\partial p_P},$$

where

$$D = \underbrace{\frac{\partial A}{\partial p_P}}_{(+)}\underbrace{\frac{\partial B}{\partial p_N}}_{(+)} - \underbrace{\frac{\partial A}{\partial p_N}}_{(+)}\underbrace{\frac{\partial B}{\partial p_P}}_{(-)} > 0.$$

Then, we have $\partial P_P/\partial z_P \geq 0$ and $\partial P_N/\partial z_P \leq 0$.

Let $C = c(z_N) - p_N D_N^d(p_N, p_P)$ and $E = \partial \pi_P^m/\partial p_P - \partial \pi_P^d/\partial p_P \geq 0$. Using a similar approach, we find that:

$$\frac{\partial P_P}{\partial z_N} = \frac{\frac{\partial B}{\partial p_N}C - \frac{\partial A}{\partial p_N}E}{D},$$
$$\frac{\partial P_N}{\partial z_N} = \frac{\frac{\partial A}{\partial p_P}E - \frac{\partial B}{\partial p_P}C}{D}.$$

Then, if $C \geq 0$, that is, $p_N D_N^d(p_P, p_N) \leq c(z_N)$, we have that $\partial P_P/\partial z_N \gtreqless 0$ and $\partial P_N/\partial z_N \geq 0$, and hence for given z_P, $\partial P_P/\partial \delta \gtreqless 0$ and $\partial P_N/\partial \delta \geq 0$. On the other hand, if if $C \leq 0$, that is, $p_N D_N^d(p_P, p_N) \geq c(z_N)$, we have that $\partial P_P/\partial z_N \leq 0$ and $\partial P_N/\partial z_N \gtreqless 0$, and hence for given z_P, $\partial P_P/\partial \delta \leq 0$ and $\partial P_N/\partial \delta \gtreqless 0$.

Proof of Proposition 3'

Let compare equilibrium prices and coverages to the benchmark. The private firm P charges the monopoly price in the benchmark, whereas in the mixed duopoly we have $p_P^{BR}(p_N, z_N, z_P) \in (BR^d(p_N), p^m)$. Then, $P_P^n < P_P^m = p^m$. Therefore, $P_P^n D_P^m(P_P^n) < \pi^m$, then $Z_P^n < c^{-1}(\pi^m) = Z_P^m$.

Comparing the public firm N's best-reply prices in the benchmark $\underline{p}_N(z_N)$ and in the duopoly setting $\underline{p}_N^d(z_N)$, we can show that as $D_N^d(p_N, p_P) < D^m(p_N)$, $\underline{p}_N^d(z_N) < \underline{p}_N(z_N)$. Moreover, a higher z_P leads the national public firm to decrease its price. It then follows that firm N charges a lower price in the duopoly setting, i.e., $P_N^n < P_P^m$.

Consequently, we have $P_N^n D_N^d(P_N^n, P_P^n) < P_N^m D^m(P_N^m)$. Therefore, compared to the benchmark, firm N, covers necessarily less in the duopoly setting, i.e., $Z_N^n < Z_N^m$.

Proof of Proposition 4'

Let $\pi_P^d \equiv p_P D^d(p_P, p_L)$ and $\pi_P^m \equiv p_P D^m(p_P)$ denote P's duopoly and monopoly profit in a local area, respectively.

Plugging in L's best-response, the first-order condition of firm P can be rewritten as

$$\frac{\partial \Pi_P}{\partial p_P} = \int_0^{z_L} \frac{\partial \pi_P^d}{\partial p_P}(p_P, \underline{p}_L^d(p_P, z))dz + \int_{z_L}^{z_P} \frac{\partial \pi_P^m}{\partial p_P}(p_P)dz \equiv F(p_P, z_P) = 0.$$

From the implicit function theorem, we thus have

$$\frac{\partial P_P}{\partial z_P} = -\frac{\partial F/\partial z_P}{\partial F/\partial p_P}.$$

Note that we have

$$\frac{\partial F}{\partial p_P} = \int_0^{z_L} \left(\underbrace{\frac{\partial^2 \pi_P^d}{\partial p_P^2}}_{(-)} + \underbrace{\frac{\partial \pi_P^d}{\partial p_L}}_{(+)} \underbrace{\frac{\partial \underline{p}_L^d}{\partial p_P}}_{(-)} \right) dz + (z_P - z_L) \underbrace{\frac{\partial \pi_P^m}{\partial p_P^2}}_{(-)} < 0.$$

Therefore, $\partial P_P/\partial z_P$ has the sign of $\partial F/\partial z_P$, which we find to be positive:

$$\frac{\partial F}{\partial z_P} = \frac{\partial \pi_P^m}{\partial p_P}(p_P) > 0.$$

Therefore, P_P increases with z_P. It follows that $P_L(z)$ decreases with z_P (by strategic substituability).

Let now $\partial \Pi_P/\partial p_P \equiv F(p_P, z_L) = 0$. In a similar way, we use the implicit function theorem and thus have

$$\frac{\partial P_P}{\partial z_L} = -\frac{\partial F/\partial z_L}{\partial F/\partial p_P},$$

where

$$\frac{\partial F}{\partial z_L} = \frac{\partial \pi_P^d}{\partial p_P}(p_P, \underline{p}_L^d(p_P, z_L)) - \frac{\partial \pi_P^m}{\partial p_P}(p_P) < 0,$$

from strategic complementarity and the fact that $\underline{p}_L^d(p_P, z_L)$ is increasing in z. Therefore, P_P decreases with z_L and it follows that $P_L(z)$ increases in z_L.

Finally, from the definition of $\underline{p}_L^d(p_P, z)$, it is immediate that $P_L(z)$ is increasing in z.

Proof of Proposition 5'

Let us first characterize the equilibrium prices. If $z_L = 0$, firm P would charge the monopoly price $P_P^m = p^m$. Since firm P's price is decreasing in z_L, we have $P_P^l < P_P^m$. Moreover, the best-reply price of firm L being downward sloping, we have $P_L(z) < p^m$. Finally, since $D_L^d(p_L, p_P) \leq D^m(p_L)$, in each area z the price set by firm L is higher in the mixed duopoly compared to the monopoly benchmark.

Since $P_P^l < p^m$, then $P_P^l D^m(P_P^l) < \pi^m$ meaning that $Z_P^l < z^m$. As firm L's price $P_L^l(z)$ is lower than the monopoly price, firm L covers up to the area $Z_L^l < z^m$.

Proof of Proposition 3"

Let us first characherize the equilibrium prices. Given the reaction function of firm N and since the reaction function of firm P is upward sloping, then firms's equilibrium prices are given by $p^d < P_i^p < p^m$.

Given that $P_P^p > p^d$, then $\pi_P^d(p_P, p_N) > \pi^d$, which implies that $Z_P^p > z^d$. Moreover, since $P_N^p < p^m$, then $\pi^m(p_N) < \pi^m$, which implies that $Z_N^p < z^m$.

The Policy Drivers of Self-Employment: New Evidence from Europe

Annabelle Mourougane[#]

Balázs Égert[§]

Mark Baker[$]

Gábor Fülöp[*,1]

Abstract

Using cross-country time series panel regressions for the last two decades, this paper seeks to identify the main policy and institutional factors that explain the share of self-employment across European countries. It looks at the aggregate share of self-employed as well as its breakdown by age, skill and gender. The generosity of unemployment benefits, and to a lesser extent, spending on active labour market policies appear to be robust determinants of the long-term share of self-employed in European countries. No significant relation could be identified between the stringency of employment protection and aggregate self-employment. However, there are significant, and oppositely signed, impacts on high- and low-skilled self-employed separately. Both the tax wedge and the minimum wage appear to be related positively to the share of self-employed in the long term, but the relation holds for some categories of workers only.

JEL Classification: J01, J21, J41, J48.

Keywords: self-employment, labour market, labour market regulations, labour market institutions, Europe.

[#]OECD Economics Department.

[§]OECD Economics Department; EconomiX at the University of Paris X-Nanterre; CESifo. Corresponding author: balazs.egert@oecd.org.

[$]OECD Economics Department.

[*]OECD Education Directorate.

[1] The authors would like to thank Pierre Beynet, Mathilde Pak and Rory O'Farrell for their comments and suggestions. The usual disclaimer applies.

1. Introduction

Self-employed individuals represent around 15% of total employment in OECD countries. The self-employed are a highly heterogeneous category. Many of them provide business services on contract and have high-skilled and high-income jobs while others have much poorer working conditions, lower wages and little job security. To the extent that self-employment reflects the entrepreneurial activity of individuals, by facilitating the adoption and creation of new technologies and innovations, a high share of self-employment would be positive for economic growth. Furthermore, self-employment can also be an avenue for individuals to enjoy a more flexible working environment, can act as a transition to more formal employment position for new entrants, migrants and younger workers or can allow for work on a more marginally attached basis. At the same time, the high share of self-employed in some countries has raised concerns of increased labour-market duality between employees and self-employed and the increase in precarious jobs. Against this backdrop, economists have long sought to understand the individual characteristics such as age, sex, family background, marital status or education influencing the choice of becoming self-employed (Taylor, 1996; Katz and Krueger, 2016; Henley, 2015; Dvoulety and Lukes, 2016; Dvolety, 2018).

The resurgence of self-employment in many industrialised countries in the 1990s sparked further interest about the underlying drivers, including the decline in the manufacturing sector, dominated by large firms (Evans and Leighton, 1989), and the rise of the ICT sector, digitalisation and the emergences of the gig economy (Shevchuk and Strebkov, 2015; OECD, 2016; Krueger, 2018). Cyclical conditions may also encourage workers to switch to self-employment. High unemployment and poor hiring prospects during downturn can generate necessity-driven self-employment (Bögenhold and Staber, 1991; Alba-Ramirez, 1994), whereas good economic conditions can create opportunity-driven self-employment (Henley, 2015). Taylor (1996) shows that higher expected earnings relative to paid employment and the freedom from managerial constraints that it offers push individuals into self-employment. Part of the trend of rising self-employment can also be the result of companies misclassifying workers (Weil, 2014).

Bogus self-employment avoids labour-market regulations and institutions, and paying social security and pension contribution. High levels of self-employment and a significant gap in social security payments between different worker types have implications for government revenues and could imply a lack of social security coverage for a larger share of the workforce, which could result in a large contingent liability to the public sector.

Labour-market institutions could also play an important role in individuals' decisions to opt for self-employment. Work based on household surveys has identified policies such as the unemployment benefit replacement ratio (Zouhar and Lukeš, 2015), active labour market policies (Rodríguez-Planas, 2010) or the stringency of employment protection legislation (Román et al., 2013) as important drivers of unemployed individual becoming self-employed. This paper contributes to this literature by looking at the main policy and institutional factors that could drive the share of self-employment at the aggregate level for a panel of European countries. The paper looks at the aggregate share of self-employed as well as its breakdown by age, gender and skill. A wide range of policy indicators is considered, such as employment protection legislation (for permanent contracts); the differential between tax and social security treatment of self-employed vis-à-vis employees; the tax wedge; the relative minimum wage rate; the unemployment benefit replacement rate; and the level of spending on activation policies on unemployed (ALMP).

The paper is organised as follows. Section 2 describes recent developments in self-employment. Section 3 reviews the policy drivers of self-employment. Section 4 deals with model selection and modelling issues. Section 5 describes the data. Section 6 presents the estimation results. Finally, Section 7 provides concluding remarks.

2. Recent developments in self-employment

The self-employed represent a sizeable share of total employment in a number of OECD countries, amounting to slightly less than 15% on average (Figure 1). Self-employment is particularly prevalent in Greece, Turkey and Italy where it exceeds 20%. By contrast, the share was lower or close to 10% in some Nordic countries.

Although these shares tend to be quite stable recently in most countries, longer-term trends have varied widely across countries. Since 2005, most countries experienced a decline in the share of self-employed, with Portugal and Turkey witnessing the largest decline in the share over the past decade. Not all countries experienced a decline however, and countries like the Netherlands and United Kingdom witnessed a considerable increase in the share.

Own-account workers (without employees) have made up an increasing share of the self-employed in many countries, with the rise relatively larger in those countries that have experienced an increase in the share of total self-employment over the past decade (Figure 2). To the extent that this trend continues, and if own-account workers do not scale up their businesses by hiring employees, then the potential positive impact to aggregate productivity associated with increased entrepreneurial activity would diminish. Indeed, in the Netherlands where own-account workers have seen a very large rise in the share of self-employed, only around 2-3% of individuals annually scale up their operations by taking on new employees (ter Weel et al., 2017).

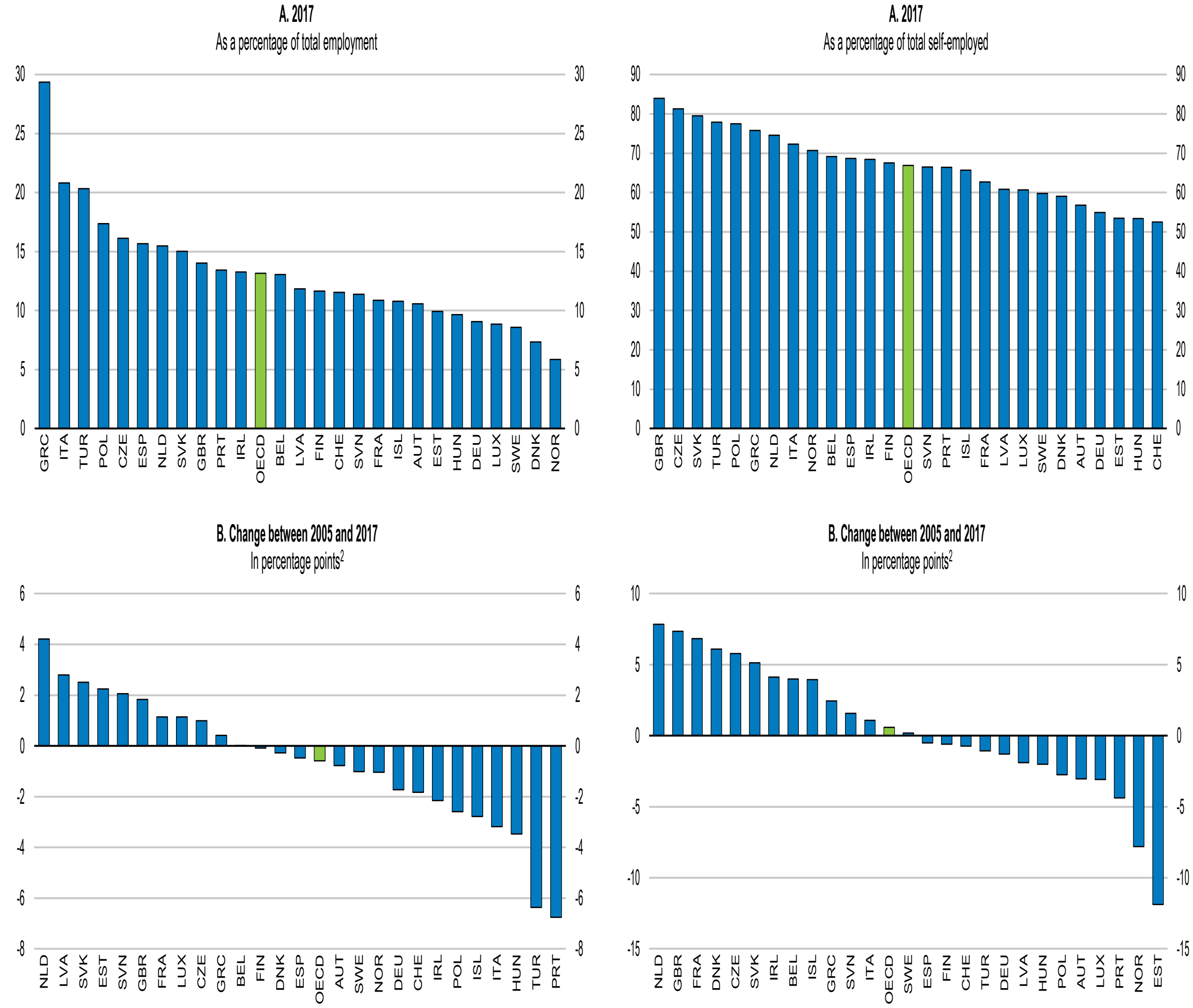
Figure 1. Self-employment (aged 15-64)[1]

Panel A

Share of self-employed: overall

Panel B

Share of self-employed persons without employees (own-account workers): overall



1. The OECD aggregate is calculated as an unweighted average of the data shown.
2. Change between 2006 and 2017 for Turkey.

Source: Eurostat (2018), "Employment and unemployment (Labour force survey)", *Eurostat Database*, May.

3. The policy drivers of self-employment

A general insight of the literature is that more developed countries tend to have lower self-employment rates (Acs et al., 1994). Nevertheless, varying policies and institutions can explain the diversity of self-employment rates across countries at comparable levels of development. This section provides an overview of the literature on the two main types of policy drivers: i.) labour market regulations and institutions, and ii.) tax policies.

3.1. Labour market policies and institutions

The *generosity of unemployment benefits* has *a priori* an ambiguous effect on the share of self-employment. The extent to which employers fund benefits through social security contributions can act as a deterrent to hiring workers, potentially leading to higher levels of self-employment. Alternatively, generous unemployment benefits could act as suitable income support for workers who have separated from earlier employment and encourage them to stay unemployed rather than to start up their own business venture. Empirically, it seems that the second effect prevails given that generous unemployment benefits appear to be inversely related to the share of self-employment (Koellinger & Minniti, 2009; Zouhar and Lukeš, 2015).

Similarly, *spending on active labour market policy (ALMP) measures*, which reflect primarily spending on Public Employment Services (PES) and on training, could help workers build up their human capital and find a more suitable job at the end, reducing the necessity to opt for self-employment. Empirical evidence suggests that high-skill workers are more willing to become self-employed and start a business with employees if there is a greater supply of skilled workers graduating from ALMP programmes (Zouhar and Lukeš, 2015). There is also evidence for direct effects. Rodríguez-Planas (2010) shows that unemployed persons benefitting from ALMP programmes are more likely to exit unemployment and become self-employed, compared to those not participating.

Government programmes designed to encourage the growth of self-employed can also have significant impacts in some countries (Baumgartner and Caliendo, 2008; Wolff et al. 2016). Since the financial crisis, a growing number of countries have introduced schemes to help unemployed create their own firm combining financial aids with counselling. Those schemes have usually limited objectives such as encouraging entrepreneurship. They are rarely fully evaluated making it difficult to assess the extent to which they have contributed to self-employment growth. Those programmes represent only a very small part of spending on active labour market for unemployed.

The role that *employment protection legislation* (EPL) could play in incentivising the choice to work self-employed has also been explored in detail, although the findings have yielded mixed results. A number of studies have shown that EPL restrictiveness has little impact on aggregate self-employment (Robson, 2003; Torrini, 2005; Kannaiainen and Vesala, 2005). However, highlighting the heterogeneity of self-employed as a group, studies that focus on specific categories of self-employment – including a negative impact from the interaction between protections and educational attainment (Baumann and Brädle, 2012) - find a significant impact of EPL. Román et al. (2011, 2013) show the positive role that strict employment protections can have on levels of ‘dependent’ self-employment – a term used to characterise individuals who are classified as self-employed contractors yet remain, for work purposes, employees. High job protections can discourage hiring by employers and encourages subcontracting of work instead if there is a discrepancy between the degree of protections on temporary and permanent contracts.

Self-employed are very often not subject to minimum wage legislation. A higher wage floor increases the cost of hiring employees, and makes self-employed workers relatively more attractive as a source of labour. All else equal, employers are thus likely to respond by substituting employees for self-employed workers. Empirically, however, there is only weak evidence of such a link at least in the United Kingdom (D'Arcy, 2017; Cominetti, 2019). One reason is that a higher minimum wage could also spillover over the wages of self-employed.

Policies that target different demographic groups could have an influence on the growth in self-employment. Self-employment as an alternative to unemployment plays an important role for immigrant populations, although the incidence of self-employment differs across different host and origin countries, ethnicities and skill levels (Volery, 2007; Baycan-Levent and Nijkamp, 2009; Kanas et al., 2009).

3.2. Tax policies

Self-employment offers greater opportunities for a reduction in the burden of taxation. The impact that tax policies can have on self-employment has been thoroughly analysed, although the focus has particularly been on the extent to which self-employed individuals mis-report their income to minimise their tax burden (Guyton et al., 2018; Astebro and Chen, 2014; Kleven et al. 2011; and Bárány, 2017). The role that complexities in the labour taxation system can have on self-employment has been explored in great detail in Aghion et al. (2017). [develop]

OECD countries where the incidence of self-employment is particularly high, are often those where the *tax wedge* between self-employed and employees are larger. In most countries it is possible to deduct some form of business expenses or investment from self-employed income subject to personal income tax. It is also often possible to allow losses in one year to be offset against income from another or to benefit from the timing of tax payment. In the Netherlands for instance, a large gap between the fiscal treatment of employees and self-employed have had a strong influence on the rising incidence of self-employment (IBO, 2015; ter Weel et al., 2017).

4. Modelling Issues

4.1. Model selection

The paper seeks to estimate the impact of a variety the policy drivers of self-employment. The policy drivers selected for the empirical analysis are based on the discussion in Section 3. Our long-run empirical model can be written as follows:

$$\frac{selfemployed}{total\_employment} = f(EPL, UBRR, ALMP, MINW, DIFF\_SSC, TAXW, TOPRATE) \quad (1)$$

Where EPL, UBRR and ALMP stand for employment protection legislation for permanent contracts, the unemployment benefit replacement ratio and active labour market policies, respectively. MINW and DIFF_SSC denote the relative minimum wage and the difference between social security contributions for regular employment and the self-employed. TAXW and TOPRATE represent the tax wedge and the top personal income tax rate. Table 1 summarises the expected relationship between the self-employed and policies. Based on the discussion in Section 3, more stringent EPL, a larger difference in social security contribution and higher tax wedge and higher top marginal income tax rate are expected to be associated with a higher share of self-employed in total employment. Reducing unemployment benefits should go in tandem with a rise in the share of self-employed. The sign of the relationship between active labour market policies or the minimum wage and the share of self-employed is ambiguous.

Table 1. Regulation and institutional design affecting the share of self-employed in total employment

Variable	Expected relationship with self-employed
Employment protection legislation (EPL) regular contracts	+
Unemployment benefit replacement rate	-
ALMP	?
Minimum wage to median	?
Difference in social security contribution rate (total-self-employed)	+
Tax wedge, single earner, couple with two children.	+
Top marginal tax rate	+

4.2. Estimation issues

The share of self-employed in total employment is modelled as a function of labour market regulations and policies. The relation is estimated at the aggregate levels and looking at the gender, age and skill breakdown. The long-term coefficients are estimated on the basis of the Dynamic OLS (DOLS) estimator. It has the advantage that it corrects for the possible endogeneity of the regressors and autocorrelation in the residuals by incorporating leads and lags of the regressors in first differences (Stock and Watson, 1993).

$$Y_{j,t} = \beta_0 + \sum_{i=1}^{n} \beta_n X_{j,i,t} + \sum_{i=1}^{n} \sum_{l=-k_1}^{k_2} \gamma_{i,l} \Delta X_{j,i,t-l} + \varepsilon_t \qquad (2a)$$

where Y_t represents a number of self-employed groups including: the aggregate share of self-employed in total employment , young or elderly self-employed, male or female, or the share of low, medium or high-skilled self-employed. $\overline{X}$ is the set of labour market regulation and policies described in section 4.1, and variables controlling for the business cycle, for long-term trends with regard to the share of ICT value-added in the total and the share of manufacturing or services.

j stands for individual countries, i for the regressors, and k_1 and k_2 represent respectively leads and lags. In the empirical analysis, one lead and one lag of the covariates will be used. Equation (1a) will be estimated using country and time fixed effects to avoid omitted variable bias.

Whether or not the variables of interest are cointegrated can be tested in a second step error correction model. The residuals obtained from the long-term relationship (ε_t) can be used to estimate the error correction model in the second stage. There is weak evidence for the presence of cointegration when the error correction term in this second stage is statistically significant and has a negative sign. In the short term, the model is expressed as a standard error-correction model:

$$\Delta Y_{j,t} = \delta * \epsilon_{it-1} + \sum_k \alpha^k \, \Delta X_{i,t}^k + \vartheta_{it} \qquad (2b)$$

5. Data issues

The dataset used in this paper covers 21 European countries over the period 1995-2013[2]. The panel is unbalanced: regional coverage and the time sample vary depending on data availability. Data for self-employed are taken from the Eurostat databsae. Both aggregate self-employed and the breakdown by age, gender, and skills are used. The self-employed data from Eurostat allows us to look at own-account self-employed as well as aggregate self-employed. Data from the OECD, whose definitions differ slightly from those of Eurostat – reflecting the treatment of unpaid family members – and do not have a separateown-account workers category, are used to investigate the robustness of the analysis.

Data for labour market and tax policies are drawn from the OECD's SPIDER database (Égert, Gal and Wanner, 2017). The analysis is limited to institutional variables that have been found important determinants of the share of self-employed in the economic literature (See section 3; Table 2).

Simple correlations provide preliminary insights on the link between labour-market institutions or tax and developments in self-employed. Statistical evidence points to a weak positive relation between the difference in employee and employer social contributions and self-employed social security contribution rates across countries. There is also little evidence of a relationship between minimum wages and developments in self-employment. Generous unemployment benefits appear to be inversely related to the share of self-employment, suggesting that generous unemployment benefits could act as suitable income support for workers who have separated from earlier employment and encourage them to stay unemployed rather than to start-up their own business venture. Statistical evidence also suggests a positive but weak relationship between the share of self-employed in total employment and the stringency of employment protection legislation, as measured by the OECD indicator of employment protection legislation for permanent workers. By contrast, cross-country evidence does not point to a strong correlation between the top marginal income tax rate and the share of self-employed. Countries such as Denmark where the top income rate is high experience a low share of self-employed. There is also no strong evidence that cuts in the top marginal tax rate have been associated with the fall in the number of self-employed. Lastly, union density and excess coverage appear to be well correlated with the share of self-employed, but as the direction of causality between these two variables is ambiguous it was judged preferable not to include them in the analysis.

[2] Austria, Belgium, Switzerland, Czech Republic, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Hungary, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Sweden, Slovakia.

Table 2. Descriptive statistics

Sample of 21 European countries, 1985-2013

	Average	Minimum	Maximum
Dependent variables			
Self-employed (Eurostat, share in total employment)	14.6	6.1	35.1
Self-employed (OECD share in total employment)	18.2	6.5	68.2
Elderly	22.0	5.4	52.4
Young	4.3	0.6	13.5
Male	17.4	6.0	41.4
Female	9.4	2.8	25.4
Low-skilled	14.6	2.4	46.5
Medium-skilled	13.2	6.3	28.9
High-skilled	13.9	4.0	29.2
Independent variables			
Unemployment benefit replacement ratio (share of last income)	26.3	0.0	65.2
Employment protection legislation (permanent contracts)	2.4	1.0	5.0
Minimum wage (ratio to median wage)	19.0	0.0	85.1
Active labour market policy (spending per unemployed as a share of per capita income)	26.3	1.3	182.9
Social security contribution rate (difference between regulator workers and self-employed)	10.7	-17.4	40.8
Top marginal tax rate	48.9	13.5	81.6
Tax wage (single earner, couple with two children)	49.2	22.5	81.6

Source: Authors' calculations

Finally a set of controls, including the share of ICT, manufacturing or services value added and indicator of the business cycles (output gap, unemployment gap, unemployment rate), have been used to correct for structural changes in the economy and the cyclical position which may also affect the share of self-employed. These data are taken from the latest Economic Outlook, Eurostat and the STAN databases.

6. Empirical results

6.1. Unemployment benefits and active labour market policies are important drivers of the developments in self-employed

A summary of estimates from equations (2a) and (2b) is reported in Table 3[3]. The unemployment benefit replacement rate and spending on active labour market policies are estimated to have a significant negative impact on the share of self-employed in the long term, and to a lesser extent in the short term. More generous unemployment benefits significantly reduce the share of own-account workers over the long-term. The effect of active labour market spending is also negative but not significant. By contrast, the

[3] A complete set of estimation results is reported in Annex 2 of the working paper version of this paper (Baker et al., 2018)

stringency of employment protection legislation on permanent contract does not seem to play a major role in explaining the decision to move to self-employment in the short or the long term. The result is consistent with Torrini (2005) and Robson (2003). The lack of significance of results is likely to reflect to a large extent the limitation of the measure of employment protections, which is a de jure indicator and captures only imperfectly the stringency of labour-market regulations faced by firms.

These results appear to be robust to a change in the definition of self-employed, using the OECD measure, rather than the Eurostat measure of self-employed. They also hold when the sample period is expanded or when alternative business cycle indicators (unemployment rate, unemployment gap) are used to control for the position in the economic cycle.

Other labour market institutions are estimated to influence the share of self-employed, but their impact is less robust. The tax wedge appears to have a positive and significant impact on the share of self-employed, suggesting that workers are encouraged to become self-employed when there is relative tax advantages compared to regular employment. The ratio of the minimum wage to the median is found to be positively related to the share of the self-employed. However, both indicators loose significance when the OECD definition of self-employed is used. The minimum wage does not also appear to be associated with the share of self-employed over a longer time sample

Other labour-market institutions did not appear to play a significant role in determining the share of self-employed. This includes the top marginal tax rate and the difference in social contributions for employees and the self-employed, the number of maternity leave weeks, or the amount of in-kind transfers.

Table 3. Share of self-employed, different measures

	Share of self-employed, Eurostat	Share of self-employed, Eurostat	Share of self-employed, Eurostat	Share of self-employed, Eurostat	Share of self-employed, Eurostat longer sample	Share of self-employed, own account	Share of self-employed, OECD data
Long term							
Constant	10.555**	9.388**	9.616**	14.88**	11.019**	72.948**	15.476**
Employment protection	0.145	0.032	-0.394	-0.143	-	-6.391	0.984
Tax wedge	0.098**	0.104**	0.131**	0.109**	0.087*	0.245	-0.023
Unemployment benefit	-0.078**	-0.064**	-0.052**	-0.093**	-0.08**	-0.198**	-0.076**
Minimum wage	0.025*	0.029**	0.038**	0.025*	0.019	0.146*	0.009
ALMP	-0.03*	-0.031**	-0.039**	-0.037**	-0.028*	-0.067	-0.06**
Output gap	-0.038			-0.056	-0.029	-0.365*	-0.333**
Unemployment gap		0.006					
Unemployment rate			-0.078				
Share of ICT	0.543*	0.685**	0.713**		0.594**	0.94	0.717
Share of manufacturing				-0.046			
Error correction term	-0.132**	-0.138**	-0.151**	-0.13**	-0.126**	-0.35**	-0.071**
Adjusted R-squared	0.979	0.979	0.98	0.979	0.98	0.898	0.983
Country fixed effects	yes	yes	yes	yes	yes	yes	yes
Time fixed effects	yes	yes	yes	yes	yes	yes	yes
No. of observations	244	244	244	246	251	244	212
No. of countries	21	21	21	21	21	21	19

Note: Employment protection is for regular workers. Tax wedge is for the single earner, couple with two children. Minimum wage is the ratio to median. ALMP stands for active labour market policies. * means significant at 10%, ** at 5% and *** at 1%.

Source: Authors' calculations.

6.2. Less generous unemployment benefits increase self-employment for all categories

Looking separately at different demographic groups, the results do not differ markedly from those observed at the aggregate level. The unemployment benefit replacement rate and active labour market spending are found to be negatively related to the share of self-employed of all the categories of workers (except for youth in the case of active labour market policies). Employment protection legislation on permanent contract is in no case found to play a role (Table 4).

By contrast, the impact of tax wedge appears to be stronger for male than female self-employed and nil for youth. In the same vein, the minimum wage is not found to play a role for any worker categories. Nevertheless, the results on demographic groups should be interpreted with care as the number of workers in some categories is quite small.

Table 4. Share of self-employed by age and gender

	Share of self-employed, Eurostat	Young	Elderly	Female	Male
Long term					
Constant	10.555**	1.05	14.442**	5.741*	12.733**
Employment protection	0.145	0.54	-0.549	0.923	0.844
Tax wedge	0.098**	0.044	0.136*	0.077*	0.153**
Unemployment benefit	-0.078**	-0.035**	-0.102**	-0.07**	-0.11**
Minimum wage	0.025*	0.01	0.039	0.019	0.025
ALMP	-0.03*	0.008	-0.051**	-0.028**	-0.048**
Output gap	-0.038	0.019	0.007	0.055	-0.09
Share of ICT	0.543*	0.121	1.524**	0.22	0.312
Error correction term	-0.132**	-0.245**	-0.106**	-0.207**	-0.168**
Adjusted R-squared	0.979	0.908	0.983	0.963	0.962
Country fixed effects	yes	yes	yes	yes	yes
Time fixed effects	yes	yes	yes	yes	yes
No. of observations	244	228	244	244	244
No. of countries	21	20	21	21	21

Note: Employment protection is for regular workers. Tax wedge is for the single earner, couple with two children. Minimum wage is the ratio to median, ALMP stands for active labour market policies. * means significant at 10%, ** at 5% and *** at 1%.

Source: Authors' calculations.

6.3. High-skilled self-employed are different from the mid- and low-skilled self-employed

The generosity of unemployment benefit and active labour market spending, as well as the tax wedge and the relative minimum wage, continue to explain the share of self-employed for most skills. There are two exceptions: active labour market spending does not explain self-employment of high-skilled workers and the minimum wage does not appear to play a role in the share of self-employed of medium-skilled workers (Table 5). Contrary to what is observed at the aggregate level, strict employment protection is associated with lower levels of high-skilled self-employment and higher levels of low-skilled self-employment. It is probable that high-skilled workers are more likely to be on permanent contracts than low and mid-skilled workers. Therefore, when protection is high high-skilled workers opt for regular employment to benefit from such a protection. By contrast, the stringency of employment protection may encourage low-skilled workers or employees to circumvent the resulting high labour costs by moving to self-employment.

Table 5. Self-employed by skills

	Share of self-employed, Eurostat	High skill	Medium skill	Low skill
Long term				
Constant	10.555**	20.621**	14.255**	1.769
Employment protection	0.145	-3.764**	-0.682	2.075**
Tax wedge	0.098**	0.118**	0.079*	0.171**
Unemployment benefit	-0.078**	-0.07**	-0.049**	-0.074**
Minimum wage	0.025*	0.038**	0.021	0.044**
ALMP	-0.03*	-0.03	-0.043**	-0.055**
Output gap	-0.038	-0.145**	0.213**	-0.136*
Share of ICT	0.543*	0.021	-0.051	1.023**
Error correction term	-0.132**	-0.352**	-0.177**	-0.257**
Adjusted R-squared	0.979	0.963	0.966	0.99
Country fixed effects	yes	yes	yes	yes
Time fixed effects	yes	yes	yes	yes
No. of observations	244	238	238	238
No. of countries	21	21	21	21

Note: Employment protection is for regular workers. Tax wedge is for the single earner, couple with two children. Minimum wage is the ratio to median, ALMP stands for active labour market policies. * means significant at 10%, ** at 5% and *** at 1%.

Source: Authors' calculations.

7. Concluding remarks

This paper examines the main policy drivers of self-employment. The main insights from the empirical analysis are as follows. First, the generosity of the unemployment benefits – measured by the replacement ratio – appears to be a robust determinant of the long-term share of self-employed in European countries (Table 1). It also affects short-term developments of the share of self-employed, but not in all the specifications tested. One interpretation of this result would be that unemployed workers might be more willing to take on the risks of starting their own business if income support supplied to unemployed is low. The negative impact of the unemployment benefits replacement ratio on the share of self-employed is found to be robust to the use of different measures of self-employment, and holds for own-account workers – those individuals who work for themselves without taking on staff – as well as for different categories of workers broken down by age, gender and skills.

Second, spending on active labour market policies is also found to negatively impact the long-term share of self-employed for most categories of worker, own-account workers and youth being an exception. Enhanced job matching through training and job-seeking measures, which represent the bulk of active labour market measures, increases the chances of finding a new job and reduces the necessity to opt for self-employment.

Third, the stringency of employment protection legislation is found to have a negative impact on self-employment amongst high-skilled workers and is positively associated with self-employment amongst low-skilled workers. The contrasting impact on self-employment across skill types results in no impact of employment-protection stringency on aggregate self-employment. High-skilled workers are likely to benefit more from strict employment protections and therefore opt for regular employment. Self-employment can act as an avenue for low-skilled workers, and for businesses hiring these workers, to circumvent the higher costs associated with strict regulation, perhaps explaining the positive impact.

Fourth, both the tax wedge and the minimum wage appear to be positively related to the share of self-employed in the long term, but the relation holds for some categories of workers only.

Table 5. Effect of institutions on the share of self-employment

	Long term	Short term
Employment protection legislation	0	0
Unemployment benefits	-	-/0
ALMP	-	-/0
Tax wedge	+/0	0
Minimum wage	+/0	0

Note: Employment protection legislation is for regular workers. Unemployment benefit stands for the unemployment benefit replacement ratio. ALMP stands for active labour market policies. Tax wedge is for the single earner, couple with two children. Minimum wage is the ratio to median.

Overall, these results need to be interpreted with care, in particular when the age, gender or skilled categories are examined as the number of workers in those categories is sometimes limited. Moreover, only linear relations have been tested in the paper, while some institutions could have an effect on the share of self-employed only after they reach a certain threshold. In the same vein, interactions between institutions have also not been investigated.

One important area for further research would be a more nuanced investigation of the role that labour taxation across different types of working types plays in influencing self-employment. Our work uses the difference in social security contributions but does not account for potential differences in pension contributions, or potentially tax breaks put in place to stimulate self-employment that are used across countries. For instance, in the Netherlands there is no obligation for the self-employed to make second pillar pension contributions, which account for a large share of gross income of salaried employees, and there exist a number of tax deductions available to stimulate entrepreneurship, which contribute to a very large difference in the net incomes of employees and self-employed individuals. It would be useful to test whether those features of the tax system influence developments in the share of self-employment.

References

Acs, Z. et al. (1994), “Why Does the Self-Employment Rate Vary Across Countries and Over Time?”, https://econpapers.repec.org/paper/cprceprdp/871.htm (accessed on 11 May 2018).

Aghion, P. et al. (2017), “Tax simplicity and heterogeneous learning”, *NBER Working Paper Series*, National Bureau of Economic Research, Cambridge, http://www.nber.org/papers/w24049 (accessed on 11 May 2018).

Alba-Ramirez, A. (1994), “Self-employment in the midst of unemployment: the case of Spain and the United States”, *Applied Economics*, Vol. 26/3, pp. 189-204, http://dx.doi.org/10.1080/00036849400000001.

Åstebro, T. and J. Chen (2014), “The entrepreneurial earnings puzzle: Mismeasurement or real?”, *Journal of Business Venturing*, Vol. 29/1, pp. 88-105, http://dx.doi.org/10.1016/J.JBUSVENT.2013.04.003.

Baker, M., B. Égert, G. Fülöp and A. Mourougane (2018), To what extent do policies contribute to self-employment, OECD Economics Department Working Paper No. 1512.

Bárány, Z. (2017), “Taxation and self-employment”, *LIS Working papers*, https://ideas.repec.org/p/lis/liswps/723.html (accessed on 05 September 2018).

Baumann, F. and T. Brändle (2012), “Self-employment, educational attainment and employment protection legislation”, *Labour Economics*, Vol. 19/6, pp. 846-859, http://dx.doi.org/10.1016/J.LABECO.2012.08.008.

Baumgartner, H. and M. Caliendo (2008), “Turning Unemployment into Self-Employment: Effectiveness of Two Start-Up Programmes”, *Oxford Bulletin of Economics and Statistics*, Vol. 70/3, pp. 347-373, http://dx.doi.org/10.1111/j.1468-0084.2008.00505.x.

Baycan-Levent, T. and P. Nijkamp (2009), “Characteristics of migrant entrepreneurship in Europe”, *http://dx.doi.org/10.1080/08985620903020060*, http://dx.doi.org/10.1080/08985620903020060.

Borjas, G. (1987), *Self-Selection and the Earnings of Immigrants*, National Bureau of Economic Research, Cambridge, MA, http://dx.doi.org/10.3386/w2248.

Cominetti N. (2019), “Is the Minimum Wage Pushing People into Self-Employment?”, Resolution foundation blog, 29 July.

D’Arcy C. (2017), “Minimum Required? Minimum Wage and Self-Employed”, Resolution Foundation Briefing, https://www.resolutionfoundation.org/app/uploads/2017/07/The-minimum-required.pdf.

Dögenhold, D. and U. Staber (1991), The decline and rise of self-employment, Work Employment Society, 5:223.

Égert, B., P. Gal and I. Wanner (2017), “Structural policy indicators database for economic research (SPIDER)”, *OECD Economics Department Working Papers*, No. 1429, OECD Publishing, Paris, http://dx.doi.org/10.1787/39d69dff-en.

Evans, D. et al. (1989), “Some Empirical Aspects of Entrepreneurship”, *American Economic Review*, Vol. 79/3, pp. 519-35, https://econpapers.repec.org/article/aeaaecrev/v_3a79_3ay_3a1989_3ai_3a3_3ap_3a519-35.htm (accessed on 11 May 2018).

Evans, D. S. and L. S. Leighton (1989), The determinants of changes in US self-employment, 1968-1987, Small Business Economics,1(2), 111-119.

Guyton, J. et al. (2018), *Tax Enforcement and Tax Policy: Evidence on Taxpayer Responses to EITC Correspondence Audits*, National Bureau of Economic Research, Cambridge, MA, http://dx.doi.org/10.3386/w24465.

Henley, A. (2015), The post crisis growth in the self-employed, IZA Discussion Paper No. 9323.

IBO (2015), *Eindrapport Zelfstandigen zonder personeel (Interdepartment report on the self-employed without personnel)*, IBO, The Hague, https://www.rijksoverheid.nl/documenten/rapporten/2015/10/02/eindrapport-ibo-zelfstandigen-zonder-personeel (accessed on 07 May 2018).

Kanas, A., F. Tubergen and T. Lippe (2009), "Immigrant Self-Employment: Testing Hypotheses About the Role of Origin- and Host-Country Human Capital and Bonding and Bridging Social Capital", *Work and Occupations*, Vol. 36/3, http://dx.doi.org/10.1177/0730888409340128.

Kanniainen, V. and T. Vesala (2005), "Entrepreneurship and labor market institutions", *Economic Modelling*, Vol. 22/5, pp. 828-847, http://dx.doi.org/10.1016/J.ECONMOD.2005.05.002.

Katz, L. and A. Krueger (2016), "The rise and nature of alternative work arrangements in the United States, 1995–2015", National Bureau of Economic Research Working Paper No. 22667, Cambridge, MA.

Kleven, H. et al. (2011), "Unwilling or Unable to Cheat? Evidence From a Tax Audit Experiment in Denmark", *Econometrica*, Vol. 79/3, pp. 651-692, http://dx.doi.org/10.3982/ECTA9113.

Koellinger, P. and M. Minniti, (2009), Unemployment benefits crowd out entrepreneurial activity. *Economics Letters*, 103: 96–98.

Krueger, A. B. (2018), "Independent Workers: What Role for Public Policy?", The Annals of the American Academy of Political and Social Science, 675(1), 8–25. https://doi.org/10.1177/0002716217741109.

Le, A. (1999), "Empirical Studies of Self-Employment", *Journal of Economic Surveys*, Vol. 13/4, pp. 381-416, http://dx.doi.org/10.1111/1467-6419.00088.

OECD (2016), New Forms of Work in the Digital Economy, OECD Publishing, Paris.

Robson, M. (2003), "Does Stricter Employment Protection Legislation Promote Self-employment?", *Small Business Economics*, Vol. 21/3, pp. 309-319, http://dx.doi.org/10.1023/A:1025742423308.

Román, C., E. Congregado and J. Millán (2011), "Dependent self-employment as a way to evade employment protection legislation", *Small Business Economics*, Vol. 37/3, pp. 363-392, http://dx.doi.org/10.1007/s11187-009-9241-3.

Román, C., E. Congregado and J. Millán (2013), "Start-up incentives: Entrepreneurshop policy or active labour market programme?" Journal of Business Venturing, 28: 151-175.

Shevchuk, A. and D. Strebkov (2015), "The rise of freelance contracting on the Russian-language internet." *Small Enterprise Research*, 22(2–3), 146–158.

Stock, J. and M. Watson (1993), "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems", *Econometrica*, Vol. 61 (4), p. 783-820, http://dx.doi.org/10.2307/2951763.

ter Weel, B. et al. (2017), "Evaluatie fiscale ondernemersregelingen (Evalution of entrepreneurial tax schemes: Final report)", *SEO-rapport*, p. 167, http://dare.uva.nl/search?identifier=230b508a-c0a6-475b-bd95-32a5b23ea6d1 (accessed on 07 May 2018).

Taylor M. (1996), “Earning independence or unemployment: Why become self-employed?”, Oxford Bulletin, https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-0084.1996.mp58002003.x

Torrini, R. (2005), “Cross-country differences in self-employment rates: the role of institutions”, *Labour Economics*, Vol. 12/5, pp. 661-683, http://dx.doi.org/10.1016/J.LABECO.2004.02.010.

Volery, T. (2007), “Ethnic entrepreneurship: A theoretical framework”, in Dana, L. (ed.), *Handbook of Research on Ethnic Minority Entrepreneurship: a Co-evolutionary View on Resource Management*, Edward Elgar, https://www.alexandria.unisg.ch/39658/ (accessed on 19 September 2018).

Wolff, J., A. Nivorozhkin and S. Bernhard (2016), “You can go your own way! The long-term effectiveness of a self-employment programme for welfare recipients in Germany”, *International Journal of Social Welfare*, Vol. 25/2, pp. 136-148, http://dx.doi.org/10.1111/ijsw.12176./////