# Intra-firm Bargaining and Matching Frictions in a Multiple Equilibria Model

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**Abstract**

In this paper, we combine a monopolistic model à la Dixit and Stiglitz and a matching model derived from Pissarides (2000) in the case of large firms. As in Cahuc and Wasmer (2001), we consider an intra-firm bargaining model for the wage determination. Moreover, we allow for increasing returns to scale in the production function leading to multiple equilibria. Finally, based on numerical simulations, we investigate the effects of stronger competition in the product market, higher unemployment benefits and stronger bargaining power of workers on the labor market performances in the case of unique equilibrium and multiple equilibria.

*Keywords:* matching frictions, monopolistic competition, intra-firm bargaining, multiple equilibria

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# Introduction

The understanding of labor market and its reactions facing economic policies is a widespread subject in economic literature. Since the 1960's, the sharp variations of unemployment rates in European countries have induced economists to consider the possibility of multiple unemployment equilibria. Today, this view is widely accepted. However, the possibility of multiple equilibria involves reconsideration in policy design. Thus, papers on economic policies design of labor market in the case of multiple equilibria have emerged.

Actually, as Cooper (1999) explains it, the existence of multiple equilibria involves the possibility of differentiated policy implications according to the equilibrium achieved by the economy as well as the possibility by an appropriate economic policy to coordinate the economy on the Pareto superior equilibrium in the case of coordination failure[1].

The search and matching models of Pissarides (2000) have permitted to take into account the fact that the employment cannot be increased without time and cost. The firms must post vacancies, what are costly, and commit on wage with potential workers, what are time-consuming, before that any match is doing and production takes place. In his book, Pissarides develops essentially the case of one worker-one firm which is more convenient and whose results can be generalized to a multi-workers firm. However, assuming a standard bargaining process between the firm and the worker, its approach has the main drawback to exclude the strategic interactions within the firm.

Stole and Zwiebel's (1996) paper studies such interactions. Indeed, they provide an intrafirm bargaining model in which contracts cannot commit the firms and its workers to wages and employment. The central assumptions of this wage setting are to consider that the wage is renegotiated with all workers after a hiring or laid off and so to treat each worker as a marginal worker. This bargaining process takes place within a multi-workers firm and, in this way it fills the lack of analysis about the effects of an additional worker wage negotiation on this of incumbents in the Pissarides' model. As Cahuc and Wasmer (2004) proved, these two approaches are complementary due to theirs analyzes of different mechanisms at work in the labor market. Whereas the Pissarides model is explicitly dynamic and analyzes the labor market equilibrium in the presence of search and matching frictions but without strategic interactions within the firm, the Stole and Zwiebel's approach analyzes these latter but in a static framework. Consequently, a model including both approaches will be more adequate to understand the labor markets working.

Furthermore, the size of competition degree on the product market has been also advanced in the understanding of labor markets performances. Blanchard and Giovazzi (2003) show how the product market deregulation, involving lower entry cost and higher competition degree, leads to higher real wages and lower unemployment. Nonetheless, they assume a standard Nash bargaining process.

[1] In the case where multiple pareto ranked equilibria exist, there exists coordination failure if the economy reaches a pareto inferior equilibrium (Cooper, 1999)

Ebell and Haefke (2003) analyse similar issue but consider an intrafirm bargaining and show quantitatively that in such a framework the impact of product market competition on equilibrium unemployment is surprisingly weak.

In this paper, we combine the monopolistic model of Dixit and Stiglitz (1977) and a matching model derived from this of Pissarides (2000) with intrafirm bargaining and large firms. In this way, we use a similar framework of this developed by Ebell and Haefke (2003). Our main contribution is to investigate the case of increasing returns to scale in the production technology in such a framework. Indeed, whereas all papers listed above consider either decreasing returns to scale or constant returns to scale in production technology and so the case of unique equilibrium; our paper follows the Mortensen (1999)'s paper and considers the case of increasing returns involving the existence of multiple equilibria[2]. Then, we show that in the case of multiple equilibria most of comparative statics results are weaker than those of unique equilibrium case.

The paper is organized as follows. In section 1, we develop the mathematical model. After explaining the different assumptions on firms, labor and matching frictions, we analyze the firms' behavior and the wage determination under intrafirm bargaining mechanism. Then, we deduce the general equilibrium at the steady state. In section 2, the possible equilibrium cases according to the size of returns to scale are studied. When returns to scale of production technology are either decreasing or constant, there is a unique equilibrium; whereas there are either multiple equilibria or none in presence of increasing returns to scale. Numerical simulations are run in section 3. These simulations allow us to investigate the effects of a higher degree of competition, higher unemployment benefits and stronger bargaining power of workers. In section 4, we manage a numeric stability analysis and discuss the previous results.

# 1 The model

## 1.1 Hypothesis

### 1.1.1 Economy

We consider a continuous time model of an economy made up agents which have the same discounted rate $r$. The output is produced by multi-workers firms (or large firms). The labor is supplied by workers and each worker supplies one unit of labor. The labor is the only production factor used in this economy. The existence of matching frictions implies that firms need time and ressources to hire workers.

In order to study the dynamic features of our economy, we solve the model both in and out of the steady state. In this way, we obtain the global dynamics of the model (Mortensen, 1999).

[2]Multiple equilibria can be also induced by increasing returns in the matching function, transactions costs, menu costs and others. See Cooper (1999) for a detailed report on this subject.

### 1.1.2 Firms

We assume a monopolistic competition on the product market. We have a continuum of identical firms uniformally distributed on the interval $[0,1]$. Each firm produces an imperfectly substituable good with others. The firm $i$ has the following production function $y_i = An_i^{\alpha}$, with $\alpha > 0$ and where $y_i$ represent its output, $n_i$ its labor employment and $A$ a technology parameter. The demand for the firm $i$'s output is given by $y_i^d = Y(p_i/P)^{-\sigma}$, with $p_i$ the price of the firm $i$, $P$ the general price level, $Y$ the aggregate output and $\sigma > 1$ the demand elasticity for the good supplied by the firm $i$[3].

### 1.1.3 Labor, matching frictions and negotiated wage

Labor is supplied by a continuum of infinitely lived and identical workers of size normalized to one. A worker can be employed or unemployed. All unemployed workers are assumed to have the same search effort which is normalized to one.

The employment cannot be increased instantaneously. The firms must post vacancies in order to recruit. This process incurs a real cost $c$ per unit of time and per unit of vacancy. Furthermore, we assume that the firm can post as many vacancies as necessary and without delay, so vacancies are "jump" variables[4].

Vacancies are matched to the pool of unemployed workers according to the matching technology: $m(u,v) = u^{\eta}v^{1-\eta}$, where $\eta \in [0,1]$ represents the matching elasticity, $v$ the mass of vacancies and $u$ the unemployment rate. This matching function is assumed to be increasing and concave in each argument, and homogenous of degree one.

Let the labor market tightness $\theta = \frac{v}{u}$, the probability to fill a vacant job per unit of time $q(\theta) = \frac{m(u,v)}{v} = \theta^{-\eta}$, with $q'(\theta) < 0$ and $q(0) = +\infty$, and the probability for an unemployed worker to find a job per unit of time $\theta q(\theta) = \frac{m(u,v)}{u} = \theta^{1-\eta}$ with $\frac{d\left[\theta q(\theta)\right]}{d\theta} > 0$. We note that $\theta$ is exogenous to the firms' decision.

At each unit of time, a rate $s$ of existing jobs is destructed. This rate of job destruction is exogenous in our model. Thus, at the firm level, the employment evolves following the law of motion: $\dot{n_i} = q(\theta)v_i - sn_i$. Indeed, at each time, the employment of the firm $i$ increases with the vacancies which are filled and decreases with the existing jobs which are destructed.

The real wage $w_i(n_i)$ is continuously and instantaneously negotiated soon after new information arrives and is a function of firm's employment. Consequently, real wages are also assumed to be jump variables. Although the wage is individually negotiated, it's assumed to be the same for all workers in the firm due to the homogenous labor assumption (approach of Stole and Zwiebel, 1996).

[3] See appendix A for detailed calculations on the determination of demand function.

[4] This assumption makes praticable the investigation of dynamics out-of-steady state hereafter.

### 1.1.4 Sequence of events

The time schedule of the model can be illustrated by the following diagram:

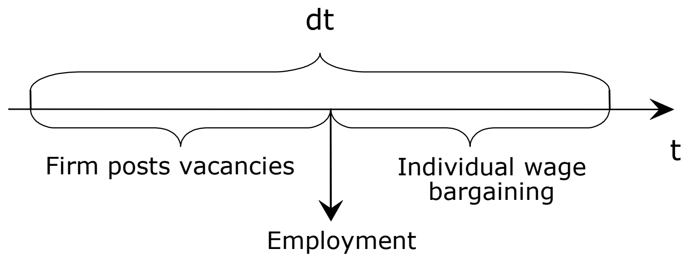


Diagram 1: Sequence of events

For each unit of time, the sequence of events can be described by this story. At the beginning of the period, the firm posts as many vacancies as necessary to hire in expectation the desired number of workers. It takes its decision while considering the wage given by the incoming wage bargaining and expecting the fact that its employment level will have an effect on this. Then, once the employment level determined, the individual bargaining takes place. The real wages are negotiated between the firm and each worker (individual bargaining) even its incumbents. Indeed, when a worker is hired or laid off, the wage is renegotiated with all workers, so each worker is treated as a marginal worker[5].

## 1.2 Firm's behavior

The firm $i$ maximizes the discounted value of future real profits $\pi_i$; its state variable is its current employment of workers $n_i$ and its control variable is its number of posted vacancies $v_i$.

The firm $i$ open as many vacancies as necessary and without delay to have the desired employment level leading to the maximization of the discounted value of future real profits. Its discount rate is $r$.

[5] See Stole and Zwiebel (1996) for more details about the timing of bargaining session.

Its problem is to solve[6]:

$$\begin{aligned}
& V(n_{i0}) &=& \underset{v_i}{Max}\int_0^{+\infty} e^{-rt}\ \pi_i\ dt & \\
s.t\qquad & \pi_i &=& \frac{p_i}{P}(y_i)y_i(n_i) - w_i(n_i)n_i - cv_i & (1)\\
& \dot{n}_i &=& q(\theta)v_i - sn_i, \quad n_i > 0, \quad n_i(0) = n_{i0} & (2)\\
& y_i(n_i) &=& An_i^{\alpha} & (3)\\
& \frac{p_i}{P}(y_i^d) &=& (y_i^d/Y)^{-1/\sigma}, \quad y_i^d = y_i(n_i) & (4)\\
& & & w_i(n_i) \qquad\qquad given & (5)
\end{aligned}$$

We assume that the firm produces exactly the demanded output, so that the good market is clear (condition 4).The condition 5 expresses the fact that the firm expects an effect of its employment level on the bargaining outcome, that's why $w_i$ depends on $n_i$.

In order to solve this problem, we consider the following equivalent variational problem:

$$\underset{n_i}{Max} \int_0^{\infty} e^{-rt} \left( f(n_i) - c\frac{\dot{n}_i + sn_i}{q(\theta)} \right) dt, \tag{6}$$

with $n_i(0) = n_{i0}$ and $0 \leq \dfrac{\dot{n}_i + sn_i}{q(\theta)} \leq v_{im}$, and where $f(n_i) = \dfrac{p_i}{P}(y_i)y_i(n_i) - w_i(n_i)n_i$

For convenience, we rewrite the variational problem as follows:

$$\max_n \int_0^{\infty} G(t, n_i) + H(t, n_i)\dot{n}_i dt \tag{7}$$

where $G(t, n_i) = e^{-rt}\left[f(n_i) - \dfrac{cs}{q(\theta)}n_i\right]$ and $H(t, n_i) = -e^{-rt}\dfrac{c}{q(\theta)}$

The Euler first order condition entails that the optimal solution of the problem of the firm $i$, $n_i^*(t)$, is such that:

$$\frac{\partial G(t, n_i)}{\partial n_i} = \frac{\partial H(t, n_i)}{\partial t} \tag{8}$$

$$\Leftrightarrow \frac{\partial f(n_i)}{\partial n_i} = \frac{(s+r)c}{q(\theta)} + \frac{\eta c\dot{\theta}}{\theta q(\theta)} \tag{9}$$

The expression (9) can be rewritten:

$$\frac{\partial \frac{p_i}{P}(y_i)}{\partial y_i}\frac{\partial y_i(n_i)}{\partial n_i}y_i(n_i) + \frac{p_i}{P}(y_i)\frac{\partial y_i(n_i)}{\partial n_i} - w_i(n_i) - \frac{\partial w_i(n_i)}{\partial n_i}n_i = \frac{(s+r)c}{q(\theta)} + \frac{\eta c\dot{\theta}}{\theta q(\theta)} \tag{10}$$

[6] The $t$ index is removed for more convenient notations.

As $\dfrac{\partial \frac{p_i}{P}(y_i)}{\partial y_i} \dfrac{y_i(n_i)}{\frac{p_i}{P}(y_i)} = -\dfrac{1}{\sigma}$, the previous equality becomes:

$$\frac{p_i}{P}(y_i) = \frac{\sigma}{\sigma - 1} \left\{ \left[ w_i(n_i) + \frac{\partial w_i(n_i)}{\partial n_i} n_i + (r+s)\frac{c}{q(\theta)} + \frac{\eta c \dot{\theta}}{\theta q(\theta)} \right] \left( \frac{\partial y_i(n_i)}{\partial n_i} \right)^{-1} \right\}$$

The firm fix its price by taking a mark-up equal to $\dfrac{\sigma}{\sigma - 1}$. The expression between brackets represents the marginal cost of labor: $w_i(n_i)$ the unit cost of labor, $(r+s)\dfrac{c}{q(\theta)}$ the search cost of an additional worker and $\dfrac{\partial w_i(n_i)}{\partial n_i} n_i$ the effect on the wage bargaining outcome of an additional worker. This equation can be also rewritten as an expression relating the firm's optimal choice of employment and real wage:

$$w_i^D(n_i) = \frac{\sigma - 1}{\sigma} \frac{p_i}{P}(y_i) \frac{\partial y_i(n_i)}{\partial n_i} - \frac{\partial w_i^S(n_i)}{\partial n_i} n_i - (r+s)\frac{c}{q(\theta)} - \frac{\eta c \dot{\theta}}{\theta q(\theta)} \qquad (11)$$

This expression corresponds to the optimal "pseudo" labor demand of the firm $i$. In (11) and hereafter, we note $w_i^D(n_i)$ the real wage of the "pseudo" labor demand and $w_i^S(n_i)$ the real wage of the "pseudo" labor supply, even though both of them result from the bargaining process. The derivative $\dfrac{\partial w_i^S(n_i)}{\partial n_i}$ shows us that the firm expects that the bargaining result is influenced by its employment level. We also note through $\dfrac{\eta c \dot{\theta}}{\theta q(\theta)}$ that the labor demand is driven by the evolution of the labor market thightness $\dot{\theta}$.

Now, we need to model the wage bargaining in order to obtain the "pseudo" supply of labor $w_i^S(n_i)$ and finalize the determination of $w_i^D(n_i)$.

## 1.3 Intra-firm bargaining

The wage negotiation takes place between the worker and the firm (individual bargaining). Given the assumption of identical workers, the negotiated wage is the same for all workers of the firm.

The firm opens vacancies which are matched to the pool of unemployed workers and lead to employment. Given that it takes one period to fill a vacancy, the actual employment is fixed at the same time that the negotiation. That's why we have the derivative $\dfrac{\partial w_i^S(n_i)}{\partial n_i}$ in (11). Thus, the firm determines its employment level heeding its effect on the wage negotiation.

Let $J_i$ and $V_i$ the expected discounted value of profit from an additionnal filled job[7] and a vacant one, and let $E_i$ and $U_i$ the expected discounted value of

[7] We must keep in mind that we have large firms here and not only one job for each firm as in Pissarides (2000).

income stream of an employed and unemployed worker. Consequently, during the bargaining, the firm and its worker share themselves the total surplus of the matching $S_i = J_i + E_i - U_i - V_i$.

The negotiated wage solves the following surplus Nash sharing rule:

$$E_i - U_i = \frac{\gamma}{1-\gamma}\left(J_i - V_i\right) \tag{12}$$

Where $\gamma \in [0,1]$ represents the bargaining power of workers.

Since the firms can open as many vacancies as necessary to obtain its optimal employment level, the free entry condition drives the value of $V_i$ to zero and thus $J_i$, the expected marginal profit for the optimal employment level is equal to the expected recruitment cost of workers[8]:

$$J_i = \frac{\partial V(n_{i0})}{\partial n_{i0}} = \frac{c}{q(\theta)} \tag{13}$$

what entails:

$$\dot{J}_i = \frac{\eta c\dot{\theta}}{\theta q(\theta)} \tag{14}$$

According to the first order conditions of the firm's program, we have also:

$$J_i = \left[\frac{\sigma-1}{\sigma}\frac{p_i}{P}(y_i)\frac{\partial y_i}{\partial n_i} - w_i^S(n_i) - \frac{\partial w_i^S(n_i)}{\partial n_i}n_i + \frac{\eta c\dot{\theta}}{\theta q(\theta)}\right]/(r+s) \tag{15}$$

The expected discounted utility of a job for a worker $E_i$ satisfies the Bellman equation:

$$rE_i = w_i^S(n_i) - s\left[E_i - U_i\right] + \dot{E}_i \tag{16}$$

The expected discounted utility of an unemployed worker $U_i$ satisfies also the following Bellman equation:

$$rU_i = b + \theta q(\theta)\left[E_i - U_i\right] + \dot{U}_i \tag{17}$$

where $b$ represents the unemployment benefits. After substitution of (12) and (13) in (17), we can rewrite the expression $rU_i$ as follows:

$$rU_i = b + \frac{\gamma}{1-\gamma}\theta c + \dot{U}_i \tag{19}$$

After differentiation of (12)[9] we can also write:

$$\dot{E}_i - \dot{U}_i = \frac{\gamma}{1-\gamma}\dot{J}_i = \frac{\gamma}{1-\gamma}\frac{\eta c\dot{\theta}}{\theta q(\theta)} \tag{20}$$

[8] See Appendix B on the firm problem for futher details.

[9] Given that the negotiated wage is a jump variable, this sharing rule holds also in rates of change (see Pissarides , 2000).

The expressions (12), (16), (19) and (20) lead to the following first order differential equation:

$$w_i^S(n_i) = (1-\gamma)\,b + \gamma\left[\frac{\sigma-1}{\sigma}\frac{p_i}{P}(n_i)\frac{\partial y_i}{\partial n_i} - \frac{\partial w_i^S(n_i)}{\partial n_i}n_i + \theta c\right] \tag{21}$$

Its solution[10] gives us the expression of the wage resulting from the wage negotiation:

$$w_i^S(n_i) = (1-\gamma)\,b + \gamma\left[\frac{\sigma-1}{\phi}\frac{p_i}{P}(y_i)\frac{\partial y_i}{\partial n_i} + \theta c\right], \quad with \quad \phi = \sigma\alpha\gamma - \sigma\gamma - \alpha\gamma + \sigma \tag{22}$$

The negotiated wage is independent from any dynamic of theta. Thus, we find the Pissarides' result which asserts that the negotiated wage holds both in and out of steady state[11]. This expression represents the "pseudo" labor supply at the firm level.

Now, we compute the derivative of the equation (22) to insert it in the equation (11):

$$\frac{\partial w_i^S(n_i)}{\partial n_i} = \gamma\frac{\sigma-1}{\phi}\left[(\alpha-1) - \frac{\alpha}{\sigma}\right]\frac{p_i}{P}(n_i)\frac{\partial y_i}{\partial n_i}n_i^{-1} \tag{23}$$

This equation represents the hiring externality due to the intrafirm bargaining wage setting as well as the slope of the individual wage curve (22)[12]. The substitution of (23) in (11) gives us the "pseudo" labor demand at the firm level according to the wage negotiation result:

$$w_i^D(n_i) = \frac{\sigma-1}{\phi}\frac{p_i}{P}(y_i)\frac{\partial y_i}{\partial n_i} - (r+s)\frac{c}{q(\theta)} + \frac{\eta c\dot{\theta}}{\theta q(\theta)} \tag{24}$$

### 1.4 General equilibrium and steady state

At the general symmetric equilibrium all the firms and workers are identical. Thus, given the assumption on the labor force size and firms distribution, we have $\frac{p_i}{P} = 1, y_i = Y$, $n_i = n$, $v_i = v$, $J_i = J$, $E_i = E$, $U_i = U$ and $u = 1-n$.

Thus, we obtain the following general equilibrium equalities:

$$w^D(n) = \frac{\sigma-1}{\phi}\frac{\partial y}{\partial n} - (r+s)\frac{c}{q(\theta)} + \frac{\eta c\dot{\theta}}{\theta q(\theta)} \tag{25}$$

$$w^S(n) = (1-\gamma)\,b + \gamma\left[\frac{\sigma-1}{\phi}\frac{\partial y}{\partial n} + \theta c\right] \tag{26}$$

With $\phi = \sigma\alpha\gamma - \sigma\gamma - \alpha\gamma + \sigma$.

[10] See appendix C for the detailed resolution.

[11] Indeed, the wage curve is independent of the law of motion of $n_i$ and $\theta$.

[12] See appendix D for an analysis of the sign of (23).

Given that $w^D(n) = w^S(n)$, we can deduce the law of motion of the labor market tightness $\theta$ at the general equilibrium and, thus, obtain the following dynamic system:

$$\begin{cases} \dot{\theta} = \dfrac{\theta q(\theta)}{\eta c}\left[(1-\gamma)\,b + \gamma\theta c + (r+s)\dfrac{c}{q(\theta)} - (1-\gamma)\,\dfrac{\sigma-1}{\phi}\dfrac{\partial y}{\partial n}\right] \\ \dot{n} = q(\theta)v - sn \end{cases} \tag{27}$$

At the steady state, the employment and labor market tightness are constant, so $\dot{n} = \dot{\theta} = 0$. Hence, the general equilibrium equalities at the steady state becomes:

$$w^D(n) = \frac{\sigma-1}{\phi}\frac{\partial y}{\partial n} - (r+s)\frac{c}{q(\theta)} \tag{28}$$

$$w^S(n) = (1-\gamma)\,b + \gamma\left[\frac{\sigma-1}{\phi}\frac{\partial y}{\partial n} + \theta c\right] \tag{29}$$

Thus, we obtain the same equations than those found in Cahuc and Wasmer (2004).

Since $u = 1-n$, the tightness of labor market can be written: $\theta = \dfrac{v}{u} = \dfrac{v}{1-n}$. And given the constancy of employment rate at the steady state[13], we have:

$$\dot{n} = q(\theta)v - sn = 0 \Leftrightarrow v = \frac{sn}{q(\theta)} \tag{30}$$

As $q(\theta) = \theta^{-\eta}$, the expression (30) can be written : $v = sn\theta^{\eta}$.

Using the previous expressions of $\theta$ and $v$, we obtain the following expression of $\theta$ and $q(\theta)$ as functions of $n$ only:

$$\theta(n) = \left[\frac{sn}{1-n}\right]^{1/1-\eta} \tag{31}$$

$$q(\theta(n)) = \left[\frac{sn}{1-n}\right]^{-\eta/1-\eta} \tag{32}$$

With $\dfrac{\partial q(\theta(n))}{\partial n} < 0$ and $\dfrac{\partial \theta(n)}{\partial n} > 0$. Thanks to the equations (31) and (32), we can also rewrite the equilibrium equalities as functions of employment rate only:

$$w^D(n) = \frac{\sigma-1}{\phi}A\alpha n^{\alpha-1} - (r+s)c\left[\frac{sn}{1-n}\right]^{\eta/1-\eta} \tag{33}$$

$$w^S(n) = (1-\gamma)\,b + \gamma\left[\frac{\sigma-1}{\phi}A\alpha n^{\alpha-1} + c\left(\frac{sn}{1-n}\right)^{1/1-\eta}\right] \tag{34}$$

with $\phi = \sigma\alpha\gamma - \sigma\gamma - \alpha\gamma + \sigma$.

[13] This corresponds to the flow equilibrium condition which implies a pseudo Beveridge Curve that is to say a positive relation between the vacancies and employment.

# 2 Equilibrium

In this section, we study the possible equilibrium cases which can occur according to the nature of returns to scale. However, before doing this, we must define clearly what equilibrium means here. In our model, an equilibrium corresponds to any couple of variable $(n, w)$ at the steady state for which both firms and workers behave optimally. As a consequence, it corresponds to the intersections between the curves represented by the equation (33), the product market equilibirum locus, and the equation (34),the labor market equilibrium locus.

## 2.1 Constant or decreasing returns to scale and unique equilibrium

**Proposition 1** *When the returns to scale are decreasing or constant (*$0 < \alpha \leqslant 1$*), there is an unique equilibrium on the interval* $n \in ]0, 1[$.

In the space $(n, w)$, the expression $w^D(n)$ is decreasing when $0 < \alpha \leqslant 1$. Indeed, we have:

$$\begin{aligned}\frac{\partial w^D(n)}{\partial n} &= \frac{\sigma-1}{\phi}A\alpha(\alpha-1)n^{\alpha-2} - (r+s)c\left[-\frac{\partial q(\theta(n))}{\partial n}q(\theta(n))^{-2}\right] \qquad (35)\\ &= \frac{\sigma-1}{\phi}A\alpha(\alpha-1)n^{\alpha-2} - (r+s)c\left[\left(\frac{\eta}{1-\eta}\right)\frac{s}{(1-n)^2}\left(\frac{sn}{1-n}\right)^{\frac{2\eta-1}{1-\eta}}\right]\end{aligned}$$

Consequently, $\frac{\partial w^D(n)}{\partial n}$ is always negative when $0 < \alpha \leqslant 1$ because $\frac{\sigma-1}{\phi}A\alpha(\alpha-1)n^{\alpha-2}$ is always negative and $-\frac{\partial q(\theta(n))}{\partial n}q(\theta(n))^{-2}$ is always positive. On the interval $n \in ]0, 1[$, the expression $w^S(n)$ is U shaped when $0 < \alpha < 1$ and increasing when $\alpha = 1$. We can conclude it according its limits' computations:

$$\begin{aligned}\lim_{n\to 0} w^S(n) &= +\infty \quad \text{and} \quad \lim_{n\to 1^-} w^S(n) = +\infty, \quad \text{when} \quad 0 < \alpha < 1\\ \lim_{n\to 0} w^S(n) &= (1-\gamma)\,b + \gamma\frac{\sigma-1}{\phi}A \text{ and } \lim_{n\to 1^-} w^S(n) = +\infty, \text{ when } \alpha = 1\end{aligned}$$

Now, to show the existence of an unique equilibrium and so of one intersection between the curves in the space $(n, w)$, we must demonstrate that the curve $w^D(n)$ is above the curve $w^S(n)$ when $n$ tends toward zero.

In the case where $0 < \alpha < 1$, both expressions tend to $+\infty$ when $n$ tends to zero. This results from the following part of the expressions: $\frac{\sigma-1}{\phi}A\alpha n^{\alpha-1}$ for $w^D(n)$ and $\gamma\frac{\sigma-1}{\phi}A\alpha n^{\alpha-1}$ for $w^S(n)$, but given that $\gamma \in [0, 1]$ we have always $w^D(n) > w^S(n)$ when $n$ tends to zero.

In the case where $\alpha = 1$, $w^D(0) = \frac{\sigma-1}{\phi}A > w^S(0) = (1-\gamma)\,b + \gamma\frac{\sigma-1}{\phi}A$ if and only if $b < \frac{\sigma-1}{\phi}A$.

## 2.2 Increasing returns to scale and multiple equilibria

**Proposition 2** *When the returns to scale are increasing ($\alpha > 1$), there are either multiple distinct equilibria or none.*

To show the existence of multiple equilibria, we need to show the multiplicity of intersections between the curves $w^D(n)$ and $w^S(n)$ in the space $(n, w)$.

The curve $w^S(n)$ is strictly increasing in the space $(n, w)$ when the returns to scale are increasing. Indeed, its derivatives gives:

$$\begin{aligned}\frac{\partial w^S(n)}{\partial n} &= \gamma\left[\frac{\sigma-1}{\phi}A\alpha(\alpha-1)n^{\alpha-2} + c\left(\frac{\partial\theta(n)}{\partial n}\right)\right] \qquad (36)\\ &= \gamma\frac{\sigma-1}{\phi}A\alpha(\alpha-1)n^{\alpha-2} + \gamma c\left(\frac{\eta}{1-\eta}\right)\frac{s}{(1-n)^2}\left(\frac{sn}{1-n}\right)^{\dfrac{\eta}{1-\eta}}\end{aligned}$$

Given that $\dfrac{\partial\theta(n)}{\partial n} > 0$, (33) is always positive when $\alpha > 1$.

The curve $w^D(n)$ is increasing and, then, decreasing when $n$ becomes high (near to one). We can note this through its partial derivatives (35) which becomes negative at a certain treshold of $n$ named $\overline{n}$[14]. Truly, when $n > \overline{n}$, $\dfrac{\sigma-1}{\phi}A\alpha(\alpha-1)n^{\alpha-2}$ becomes inferior to $(r+s)c\left[\left(\dfrac{\eta}{1-\eta}\right)\dfrac{s}{(1-n)^2}\left(\dfrac{sn}{1-n}\right)^{\frac{2\eta-1}{1-\eta}}\right]$ and (35) becomes negative.

Thus, if the slope of $w^D(n)$ is steeper than $w^S(n)$ for $n < \overline{n}$ , i.e. $\dfrac{\partial w^D(n)}{\partial n} > \dfrac{\partial w^S(n)}{\partial n}$ when $n < \overline{n}$, and if $w^D(0) < w^S(0)$, the curves must intersect at least twice in the space $(n, w)$. For increasing returns to scale ($\alpha > 1$), we have always: $w^D(0) = 0 < w^S(0) = (1-\gamma)b$. However, when the returns to scale become to high, the inequality $\dfrac{\partial w^D(n)}{\partial n} > \dfrac{\partial w^S(n)}{\partial n}$ is not verified[15]. As a consequence, when the returns to scale are too increasing, the economy exhibits no equilibrium, otherwise it exhibits at least two.

# 3 Numerical simulations

In this section, we run numerical simulations in order to give support to the previous propositions and to assess the effects of:

- the size of returns to scale on the equilibrium possibilities
- the competition degree on the product market

[14] This feature will be illustrated in the numeric simulations. The treshold for which (35) becomes negative depend on the value of various parameters.

[15] This statement will be demonstrated in the section of numerical simulations.

- an unemployment benefits increase
- an increase of workers' bargaining power

on labor market perfomances.

The model period is one year. We don't calibrate the model according to empirical studies on a particular economy, but our parametrization choices are based on previous works (Ebell and Haefke, 2003) and corresponds to realistic one. However, some difficulties appear regarding some parameters. We will discuss this issue below.

In all following graphs, the black curves represent the equation $w^D(n)$ and the grey curves the equation $w^S(n)$.

### 3.1 Equilibrium

In this part, we fix a set of parameters and we move the size of returns to scale in order to verified the propositions 1 and 2. The parametrization is given in the following table:

*Table 1:* Parameters values

| | |
|---|---|
| $A = 1$ | Technological level (normalized) |
| $\gamma = 0.5$ | Workers bargaining power (standard) |
| $\eta = 0.5$ | Elasticity of matching function |
| $s = 0.1$ | Job destruction rate |
| $c = 0.2$ | Vacancy cost |
| $r = 0.04$ | Discounted rate |
| $b = 0.3$ | Unemployment benefit |

For more simplicity, we normalized to one $A$, the parameter of technological level. Furthermore, we set $\gamma = \eta = 0.5$ and, thus, we use standard values and impose Hosios condition (1990) for constant returns to scale case[16]. The job desctruction rate value corresponds to a destruction of 10% of existing jobs each year and the discounted rate one to a 4% annual interest rate. The parameter $b$ is commonly interpreted as the monetary compensation for the unemployed. Usually, it's more easy to consider the replacement rate, i.e. the ratio of benefits to wage, during calibration. According OECD reports, the average replacement rate can be very different from a country to another and from a family type to another but in average it can be included between 20% and 80%. In our model, the wage is determined endogenously and so when we fix a value for $b$ we can give the replacement rate value after its determination. Here we fix the parameter

[16] Hosios (1999) identified a general condition underwhich all the externalities of search process are internalised and all decisions are efficient: the matching elasticity with respect to unemployment must be equal to the worker's share of the match surplus in the case of constant returns to scale in the production technology.

$b$ to 0.3. The same problem appears for the parameter of vacancy cost $c$, we fix its value at 0.2 here. The parameter $\sigma > 1$, which represents the demand elasticity of the good supplied by the firms, translates the degree of competition on the good market in our economy[17]. Indeed, when the competition is strongly monopolistic it tends to 1 and, inversely, when the degree of competition comes near to perfect competition, it tends to $+\infty$. Through the expression "degree of compention" we means the degree of product market competition, that is to say the power that the firms have on the good prices. In this first subsection, this parameter is fixed to 2 (monopolistic competition). Later, we will increase its value in order to check the effects of the good market competition.

### 3.1.1 Decreasing and constant returns to scale

In this part, we will demonstrate graphically thanks to numerical simulations that we have always an unique equilibrium in the case where the returns to scale are decreasing or constant.

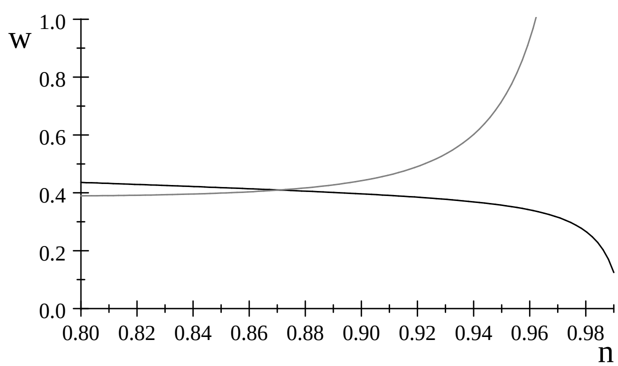


*Figure 1:* Decreasing returns to scale $\alpha = 0.5$

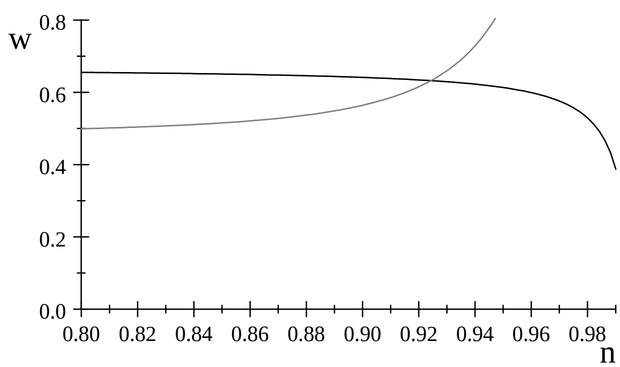


*Figure 2:* Constant returns to scale $\alpha = 1$

[17]This feature comes from the assumption on firms distribution on the interval $[0, 1]$.

The figures 1 & 2 confirm the algebra proof of unique equilibrium of the previous section and give support to the proposition 1. We find the usual results that the strategic complementarity introduced by the monopolistic competition isn't strong enough to involve the presence of multiple equilibria. The graphs show also that the equilibrium in the case of decreasing returns to scale is worse than the one in the case of constant returns to scale. Indeed, the employment rate and the wage are higher in the second one than in the first one.

According to our parameterization and equilibrium outcomes, we compute the reservation wage of workers $U$[18] to compare our findings with those of Stole and Zwiebel (1996), and Pissarides (2000). In the case of decreasing returns, we find a mix of them. Indeed, as in Stole and Zwiebel (1996) where returns to scale are decreasing, we note the absence of rents for workers[19]. Due to the presence of hiring externality which depress wages when an additional worker is hired[20], the firms exploit the decreasing returns and overemploy in order to moderate the workers' wages. However, we find that equilibrium wages is lower than marginal labor productivity in both two cases due to the presence of hiring cost. Thus, our findings are similar to those of Pissarides (2000) on this point. In the case of constant returns, our results are also consistent with those of Pissarides (2000) concerning workers' rents. In spite of intra-firm bargaining, workers get rents due to the constancy of returns ruling out strategic employment level which depress wages until reservation wage[21].

### 3.1.2 Increasing returns to scale

Here, we run several numerical simulations with different values of $\alpha > 1$. By this way, we give support to the presence of multiple equilibria and to absence of them (proposition 2). These simulations also allow to investigate the effects of the size of returns to scale on the equilibrium outcomes.

[18] At the each equilibrium, the reservation wage can be deduced from the equilibrium values of $w$ and $n$ .

[19] The equilibrium wage is equal to the reservation wage in the case of decreasing returns what translates an absence of rents given that the workers earn what they would be expected earn if they were unemployed.

[20] We show easily that $\frac{\partial w_i^S}{\partial n_i} < 0$ when $\alpha \leq 1$, so hiring an additional worker will decrease the wage for all workers.

[21] We have $\left|\frac{\partial w_i^S}{\partial n_i}\right|_{\alpha<1} > \left|\frac{\partial w_i^S}{\partial n_i}\right|_{\alpha=1}$. As a consequence, even if the firms overemploy, wages cannot diminish until the reservation wage.

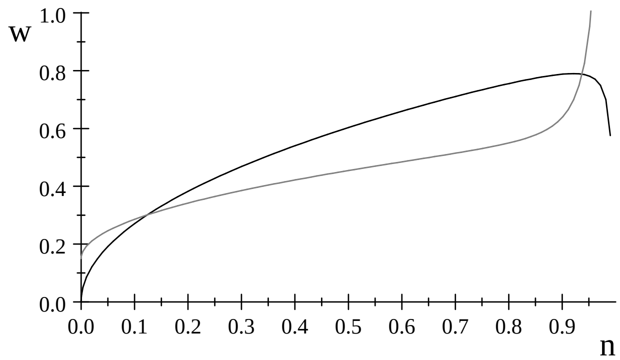


*Figure 3:* Increasing returns to scale $\alpha = 1.5$

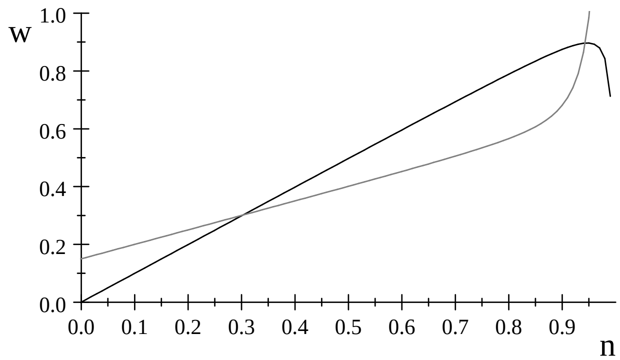


*Figure 4*: Increasing returns to scale $\alpha = 2$

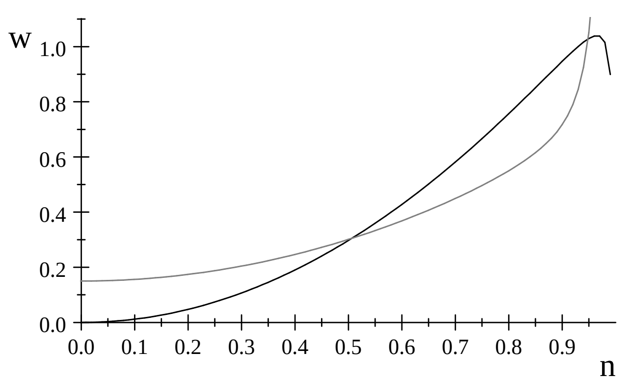


*Figure 5*: Increasing returns to scale $\alpha = 3$

The previous graphs show the existence of two positive equilibria. These can be characterized as follow: one as the "low equilibirum", where both employment rate and wage are low, and another as the "high equilibrium", where both employment rate and wage are high.

Concerning the workers' rents, we find Stole and Zwiebel's (1996) result at the low equilibrium and the Pissarides' (2000) one at the high equilibrium. At both equilibria, the marginal productivity of labor is always greater than the equilibrium wage. At the high equilibrium, the workers get rents which are increasing with the size of returns. Indeed, we observe a higher wage when the returns' size increases, whereas the equilibrium employment remains unchanged. Nonetheless, at the low equilibrium, the firms behave strategically and exploit increasing returns to scale of labor by underemploying and, as a consequence, workers get no rents. At this equilibrium, an increase of returns to scale have just an effect on employment, it increases, but the wage remains unchanged and equal to reservation wage.

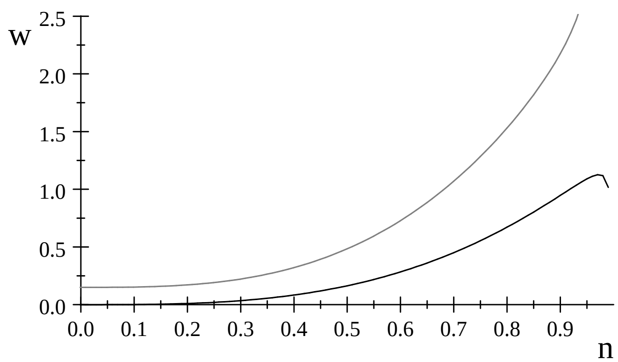


*Figure 6*: Increasing returns to scale $\alpha = 4$

In the case where the returns to scale are too high we have no equilibrium. This case gives additional support to the proposition 2.

In what follows, due to the complexity of equilibrium equalities, we run numerical simulations with the intention of managing a comparative statics analysis.

## 3.2 Higher degree of competition

In this section, we investigate the effects of a higher competition degree on the good market on the labor market outcomes. In order to do that, we use the parameters values given in the table 1 as well as two disctinct values of $\sigma$ translating the case of monopolistic competition and of more competitive good market. The solid lines represent the case where $\sigma = 2$ (the benchmark case), that is to say when the competition is "very" monopolistic and the dashed lines represent the case where the degree of competition is stronger, $\sigma = 100$.

In their paper, Ebell and Haefke (2003) identify two channels by which competition affects employment[22] in a model with intrafirm bargaining. The

[22] They study the effect of competition on unemployment; but, given our assumption on the

first one is the output expansion channel: a stronger competition involves a higher profit-maximizing output and so a higher employment level. The second one is the hiring externality channel. We have seen in the previous subsection how the firms behave strategically in presence of intrafirm bargaining due to the presence of hiring externality. This second channel can either reinforce or diminish the first channel effect on employment[23].

### 3.2.1 Decreasing returns to scale

In the case of decreasing returns to scale as well as in the case of constant returns to scale[24], a higher competition on the product market improves both employment rate and real wage. These results are quite similar to those of Blanchard and Giavazzi (2003) which state that stronger competition on good market leads to higher real wages and lower equilibrium unemployment rates.

The figure 7 illustrates the case of decreasing returns to scale.

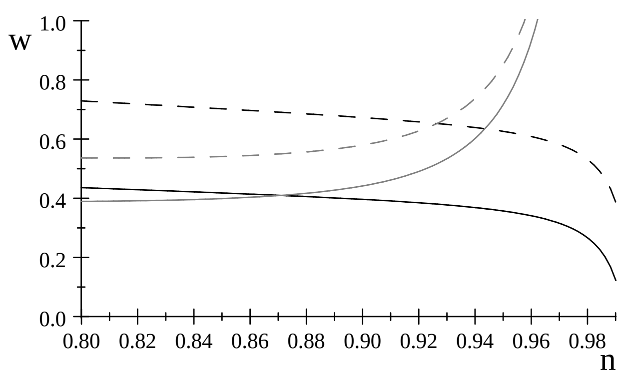


*Figure 7:* Stronger competition when $\alpha = 0.5$

A stronger competition on the product market decreases the market power on price of each firm. As a result, at the general equilibrium, the general price level is lower and the real wages increases. Then, involving a higher profit-maximizing output, stronger competition leads to higher employment level (first channel). Furthermore, we show easily that the hiring externality is less important when the degree of competition increases[25] in this case. Given that the employment rate increases here, we conclude that the first channel is stronger than the second one here.

labor force, we can compare their findings with ours.

[23] This depends on the returns to scale features.

[24] The case of constant returns to scale is not illustrated here, but the results are the same than in the case of decreasing returns, that is to say a higher wage and a higher employment.

[25] We show in the appendix E that $\dfrac{\partial\left[\dfrac{\partial w_i^S(n_i)}{\partial n_i}\right]}{\partial \sigma} < 0$ when $\alpha \leqslant 1$.

### 3.2.2 Increasing returns to scale

In the case where the returns to scale are increasing, we find the same results than Blanchard and Giavazzi (2003) at the high equilibrium only. Indeed, according to the figures below, we note that with a stronger competition on the good market the high equilibrium displays a higher wage rate than in the benchmark case and an unchanged employment rate, whereas the low equilibrium displays a lower employment rate than in the benchmark case and an unchanged wage.

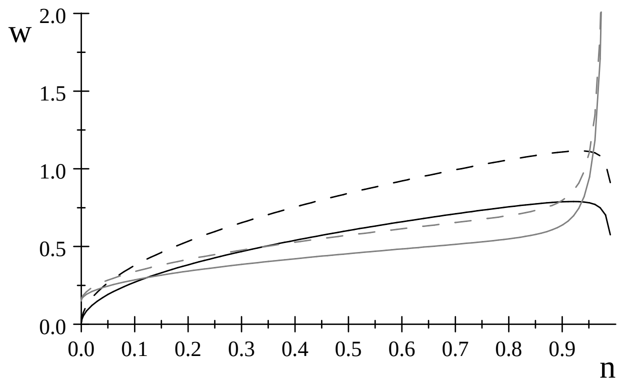


*Figure 8:* Stronger competition when $\alpha = 1.5$

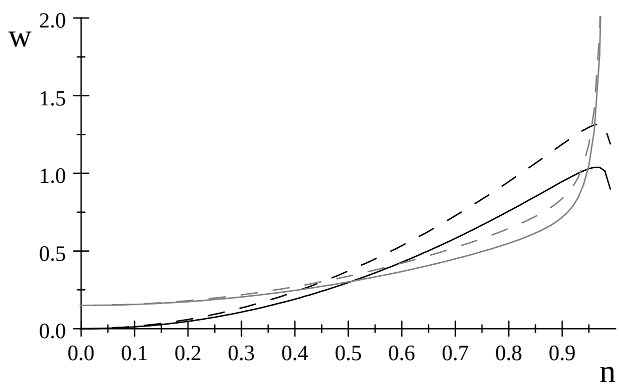


*Figure 9:* Stronger competition when $\alpha = 3$

## 3.3 Unemployment benefits increase

Here, we investigate the effects of an increase in unemployment benefits. We still use the parameters values of table 1 and $\sigma = 2$ in the benchmark case (solid curves) and we compare it with the case where unemployment benefits are higher, $b = 0.4$ (dashed curves). As previously, we first check the case of decreasing returns and, then, the case of increasing returns.

### 3.3.1 Decreasing returns to scale

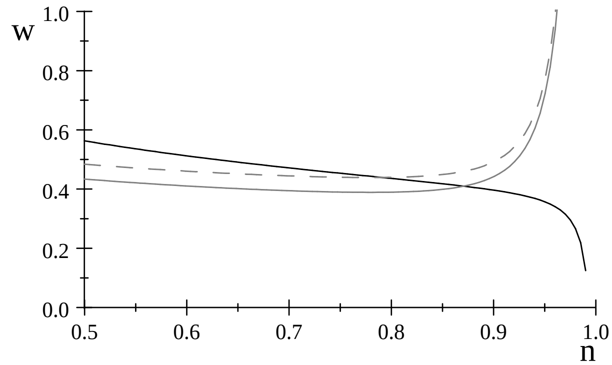


*Figure 10:* Higher allocations $\alpha = 0.5$

The figure 11 shows that raising unemployment benefits involves a reduction in the employment rate and an increase in real wage. Actually, higher unemployment benefits imply higher reservation wage for workers and reduce incentives to work. As a result, the new wage is higher and always equal to the reservation wage here; while the employment rate is lower. We note the same widespread outcome when the returns to scale are constant.

### 3.3.2 Increasing returns to scale

In the case of increasing returns to scale, we don't find the previous result. The figures 12 & 13 show us that an increase in unemployment benefits has:

- a tiny negative effect on employment at the high equilibrium
- a positive effect both on the employment rate and on the wage at the low equilibrium.

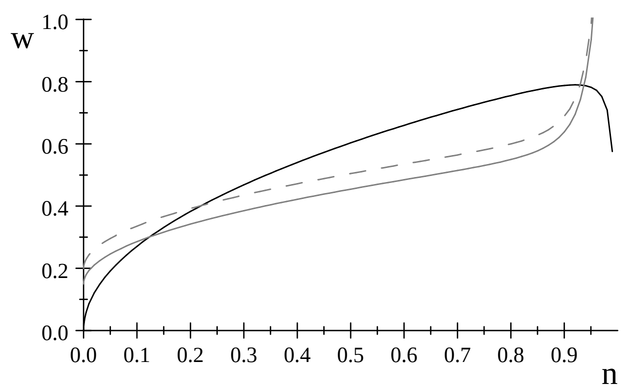


*Figure 11:* Higher allocations $\alpha = 1.5$

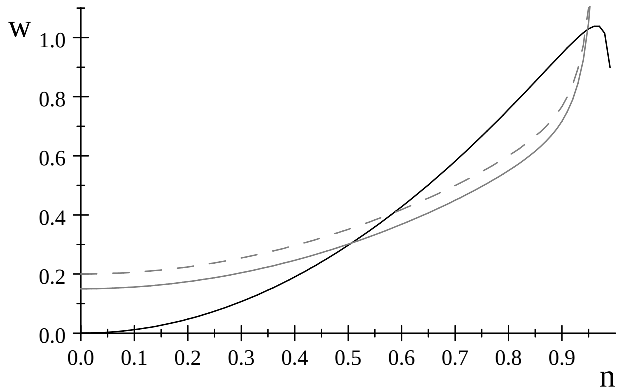


*Figure 12*: Higher allocations $\alpha = 3$

We have found in the subsection 3.1 that workers get rent at the high equilibrium which is increasing with the returns' size. The extent of this rent implies a non significant effect of a moderate increase in unemployment benefits. Indeed, the rent is so high that higher unemployment benefits have a tiny effect on incentives to work and, the greater the returns' size is, the tinier this effect is. Only an unemployment benefits increase superior to the workers' rent can have an effect at the high equilibrium[26].

## 3.4 Stronger bargaining power of workers

Here, we investigate the effects of an increase in the workers' bargaining power $\gamma$. We still use the parameters values of table 1 and $\sigma = 2$ for the benchmark case (the solid curves) and, next, we raise the parameter $\gamma$ at the value 0.8 to translate a stronger bargaining power of workers[27] (the dashed curves).

[26] In this case, the effect would be the same that the common result, that is to say a decrease in employment rate and a wage increase.

[27] Note that now the Hosio (1990) condition do not hold.

### 3.4.1 Decreasing returns to scale

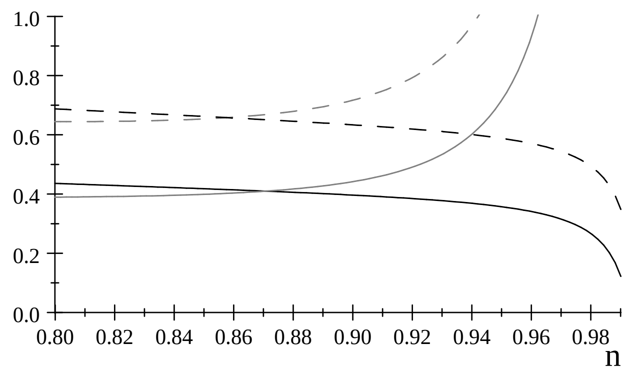


*Figure 13:* Stronger bargaining power $\alpha = 0.5$

In this case, a stronger bargaining power of workers involves a higher wage and a lower employment rate and implies the same results than those found in the literature.

### 3.4.2 Increasing returns to scale

Here, whatever the equilibrium, the effect of a stronger bargaining power on the wage goes in the same direction, whereas it is opposite on the employment rate.

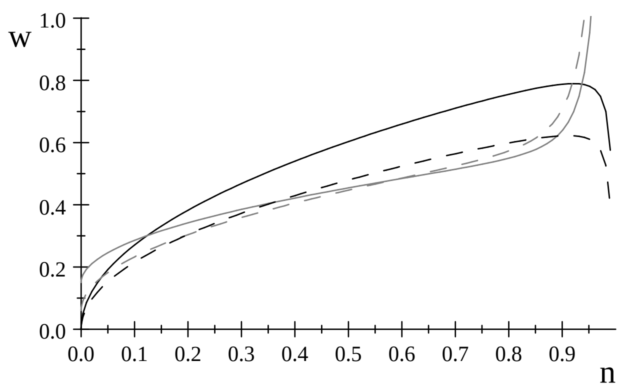


*Figure 14:* Stronger bargaining power $\alpha = 1.5$

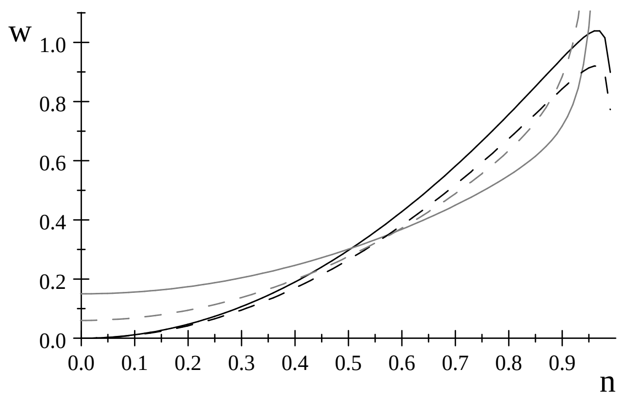


*Figure 15*: Stronger bargaining power $\alpha = 3$

Indeed, according the figures 15 & 16, we note that at the high and low equilibrium a stronger bargaining power implies a lower wage[28]. Inversely, it implies an higher employment rate at the low equilibrium and a lower one at the high equilibrium.

# 4 Stability analysis and discussion

## 4.1 Stability analysis

Running a stability analysis of the model at the general equilibrium implies the specification of the dynamics of $\theta$ and $n$. These are given in the system (27):

$$\begin{cases} \dot{\theta} = \dfrac{\theta q(\theta)}{\eta c}\left[(1-\gamma)\,b + \gamma\theta c + (r+s)\dfrac{c}{q(\theta)} - (1-\gamma)\,\dfrac{\sigma-1}{\phi}\dfrac{\partial y}{\partial n}\right] \\ \dot{n} = q(\theta)v - sn \end{cases}$$

According to the propositions 1 & 2 and the previous numerical simulations, we note that there exists an unique equilibrium when the returns to scale are decreasing or constant and two disctinct equilibria, a low equilibrium and a high equilibrium, when the returns to scale are increasing (but not too). On the basis of the parameters' values used in the previous section, we can find numerically that:

- If $\alpha \leq 1$, the unique interior equilibrium is a saddle point.
- If $\alpha > 1$, the low interior equilibrium is unstable and the high one is a saddle point[29].

[28] At the low equilibrium, its negative effect on wage is tiny.

[29] The saddle point arises in both cases because $n$, the employment rate, is a sticky and stable variable; whereas $v$, vacancies, is a forward-looking and unstable variable (Pissarides, 2000).

- For all $\alpha$, there exists two corner equilibria which are stable and for which $n = 0$ or 1.

The equilibrium paths of the variables $\theta$ and $n$ are plotted (arrows) in the figures 16 & 17 in case of decreasing and increasing returns to scale.

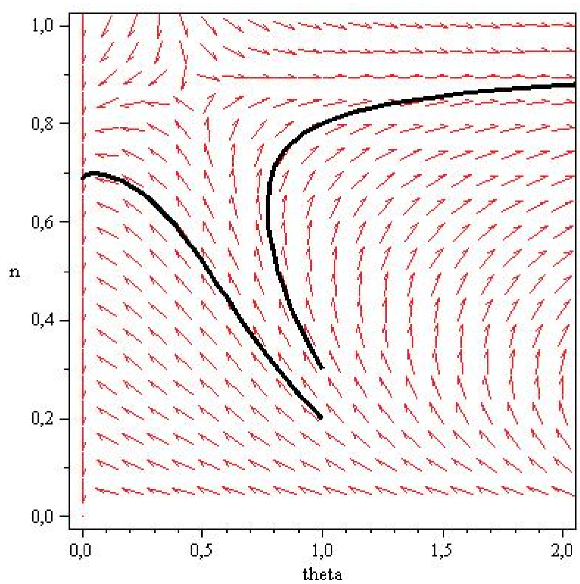


Figure 16: Equilibrium paths when $\alpha = 0.5$

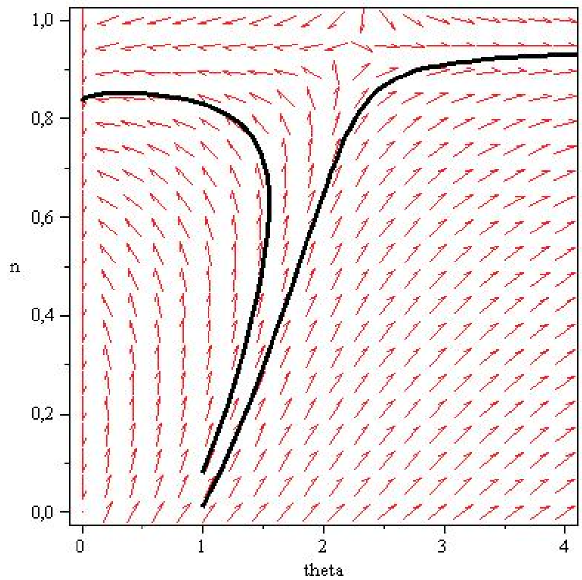

Figure 17: Equilibrium paths when $\alpha = 1.5$

We can see on the two figures that there exists only one equilibrium path (or saddle path) for which $n$ converges to the saddle point in both cases. As in Pissarides (2000), this saddle path corresponds to the curve $\dot{\theta} = 0$. According to the first order condition of the firm's problem, the optimal employment of the firm $i$, as well as the initial state of its employment level $n_{i0}$, is always on the $\theta$-stationary (i.e. the curve $\dot{\theta} = 0$) in the absence of expected change in the exogenous variable $\theta$[30]. Thus, at our general symmetric equilibrium, the economy is always on the saddle path and converges to the saddle point through variations of employment only (and open vacancies). We focus on this case here[31].

## 4.2 Discussion

In this part, we discuss the comparative statics results. Since the high equilibrium is the one where the economy converges in the case of multiple equilibria with perfect foresight, we focus the discussion on this equilibrium and rule out the others. In the case of unique equilibrium, we ruled out also the corner equilibria and consider only the interior one Thus, we compare both the sign and the extent of the effects on employment and wage of the three economic policies

[30] We must keep in mind that the variable $\theta$ is exogenous at the firm level.

[31] The case where the firms expect any change in $\theta$ is worth to be investigated, but it's not the purpose of the paper.

reported previously in the case of unique and multiple (interior) equilibria. The tables 2 & 3 summarize our findings.

*Table 2:* Effects' sign

| | $\alpha \leqslant 1$ | | $\alpha > 1$ | |
|---|---|---|---|---|
| Product market deregulation: $\Delta\sigma > 0$ | $\frac{\partial n^*}{\partial \sigma} > 0$ | $\frac{\partial w^*}{\partial \sigma} > 0$ | $\frac{\partial n^*}{\partial \sigma} \geqslant 0$ | $\frac{\partial w^*}{\partial \sigma} > 0$ |
| Unemployment benefits increase: $\Delta b > 0$ | $\frac{\partial n^*}{\partial b} < 0$ | $\frac{\partial w^*}{\partial b} > 0$ | $\frac{\partial n^*}{\partial b} \leqslant 0$ | $\frac{\partial w^*}{\partial b} \simeq 0$ |
| Workers' bargaining power increase: $\Delta\gamma > 0$ | $\frac{\partial n^*}{\partial \gamma} < 0$ | $\frac{\partial w^*}{\partial \gamma} > 0$ | $\frac{\partial n^*}{\partial \gamma} < 0$ | $\frac{\partial w^*}{\partial \gamma} < 0$ |

*Table 3:* Comparison of the effects' extent

| | Employment Rate | Real Wage |
|---|---|---|
| $\Delta\sigma > 0$ | $\left[\frac{\partial n^*}{\partial \sigma}\right]_{\alpha\leqslant 1} > \left[\frac{\partial n^*}{\partial \sigma}\right]_{\alpha>1}$ | $\left[\frac{\partial w^*}{\partial \sigma}\right]_{\alpha\leqslant 1} > \left[\frac{\partial w^*}{\partial \sigma}\right]_{\alpha>1}$ |
| $\Delta b > 0$ | $\left[\frac{\partial n^*}{\partial b}\right]_{\alpha\leqslant 1} > \left[\frac{\partial n^*}{\partial b}\right]_{\alpha>1}$ | – |
| $\Delta\gamma > 0$ | $\left[\frac{\partial n^*}{\partial \gamma}\right]_{\alpha\leqslant 1} < \left[\frac{\partial n^*}{\partial \gamma}\right]_{\alpha>1}$ | – |

When the effects of a economic policy go in the same direction in both case, they are always higher in the case of unique equilibrium than in the case of multiple equilibria except for an increase in the workers' bargaining power (table 3). Indeed, we note that higher returns to scale entail lower impact of some economic policies.

We note also that in the case of multiple equilibria an increase of unemployment benefits has no effect on the real wage and a tiny one on employment whereas it entails a decrease in employment and an increase in real wage in the case of unique equilibrium according to our calibration. This feature comes from the size of workers rents. Indeed, an increase of unemployment benefits will has as much more impact as it will reduce the workers wage rents.

# Conclusion

In this paper, we have used a model similar to this of Ebell and Haefke (2003) with monopolistic competition on the product market, matching frictions and intra-firm bargaining on the labor market. The main contribution of our paper is to allow for increasing returns to scale in production technologies and to investigate in a such case the effects of deregulation on product market, increase of unemployment benefits and stronger bargaining power of workers on labor market performances in a such framework. To do that, we have also specify the out of steady state dynamics of the labor market tightness as in the Mortensen's paper. Our findings have shown that some results of Stole and Zwiebel don't hold in the case of multiple equilibria (absence of rent for workers due to the intra-firm bargaining). We have also demonstrated that the impact of some economic policy are lower in the case of multiple equilibria than in the case of unique equilibrium. Future researches should more investigate how the expectations of agents on the exogenous variable as $\theta$ influence the dynamics of employment and unemployment in this kind of economy.

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# Appendix

## Appendix A: Determination of the demand for the firm $i$'s output

Households are both consumers and workers. They are risk neutral and have Dixit-Stiglitz preferences over a continuum of $i$ differenciated goods uniformally distributed on the interval $[0, 1]$. A representative household derives its demand in good $i$ by solving:

$$\underset{c_i}{Max}\left(\int_0^1 c_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{A1}$$

under the budget constraint $\int_0^1 \frac{p_i}{P} c_i di = I$.

Where $\sigma > 1$ represents the elasticity of substitution between goods, $p_i$ the price of good $i$ , $P$ the price index and $I$ the real income of the representative household.

Given the absence of saving and the fact that the population of indentical households is normalized to one, we obtain the aggregate demand for the good $i$ given as:

$$y_i^d = c_i = \left(\frac{p_i}{P}\right)^{-\sigma} I \tag{A2}$$

Furthermore, assuming a clear product market, we have $PI = PY$ where $I$ represents the aggregate real income here and $Y$ the aggregate output. Thus, we can write (A2) as following:

$$y_i^d = \left(\frac{p_i}{P}\right)^{-\sigma} Y \tag{A3}$$

with the price index $P = \left[\int_0^1 p_i^{\frac{\sigma-1}{\sigma}} di\right]^{\dfrac{\sigma}{\sigma-1}}$.

The expression (A3) represents a standard monopolisitic competition demand function with an elasticity of subsitution among differenciated goods given by $-\sigma$.

## Appendix B: Existence and uniqueness of the firm's problem

The firm solves:

$$\max_{n_i} \int_0^\infty e^{-rt}\left(f(n_i) - c\frac{\dot{n}_i + sn_i}{q(\theta)}\right)dt, \quad n_i(0) = n_{i0}, \quad 0 \leq \frac{\dot{n}_i + sn_i}{q(\theta)} \leq v_{im}. \tag{B1}$$

The turnpike solution is given by $n_i^*(t)$:

$$n_i^*(t) \quad \text{such that} \quad f'(n_i) = \frac{(s+r)c}{q(\theta)} + \frac{\eta c\dot{\theta}}{\theta q(\theta)} \tag{B2}$$

The following theorem called the most rapid approach theorem or turnpike theorem (Clark,1990 and, Hart and Feightinger, 1987) gives the optimal solution for this problem:

**Theorem 3** *If*

$$G_n(t,n) > (<)H_t(t,n), \quad when \quad n > (<)n^*(t),$$

*where $n_i^*(t)$ is the unique solution of (B2) and if for all admissible paths $n_i(t)$, the following condition holds:*

$$\lim_{t\to\infty} e^{-rt}\int_{n(t)}^{n^*(t)} H(t,x)dx = \lim_{t\to\infty} e^{-rt}\frac{c}{q(\theta)}(n_i^*(t) - n_i(t)) \geq 0,$$

*then, the optimal solution of problem (B1) is the most rapid approach to $n_i^*(t)$ i.e.:*

$$v_i^*(t) = \begin{cases} 0 & \text{si} \quad n_i > n_i^*(t) \\ \\ v_{im} & \text{si} \quad n_i < n_i^*(t) \\ \\ \dfrac{\dot{n_i^*} + sn_i^*}{q(\theta)} & \text{si} \quad n_i = n_i^*(t). \\ \end{cases} \tag{B3}$$

We are going to verify if the hypothesis of this theorem holds with the wage function $w_i^S(n)$ given by (22). To begin with, let us compute

$$J_i = \frac{\partial V(n_{i0})}{\partial n_{i0}}$$

which is used in the wage negotiation.
Suppose $n_{i0} > n_i^*(0)$, in this case the optimal control is zero and the optimal path $n_i^*(t)$ is the solution of:

$$\dot{n}_i = -sn_i, \; n_i(0) = n_{i0}$$

Let $n_i^{opt}(t) = n_{i0}e^{-st}$ this solution and

$$tm \quad \text{such that } n_i^*(tm) = n_i^{opt}(tm). \tag{B4}$$

where $tm$ is the time at which the optimal path meets $n_i^*(t)$ and note that $tm = tm(n_{i0})$. Consequently, we have

$$V(n_{i0}) = \int_0^{tm} e^{-rt}f(n_i^{opt}(t))dt + \int_{tm}^{\infty} e^{-rt}\left(f(n_i^*(t)) - c\,\frac{\dot{n_i^*}(t) + sn_i^*(t)}{q(\theta)}\right)dt \tag{B5}$$

and differentiating with respect to $n_{i0}$, yields

$$\frac{\partial V(n_{i0})}{\partial n_{i0}} = \int_0^{tm} e^{-rt}\frac{\partial f(n_i^{opt}(t))}{\partial n_{i0}}dt + e^{-rtm}f(n_i^{opt}(tm))\frac{\partial tm}{\partial n_{i0}} +$$

$$\int_{tm}^{\infty} e^{-rt}\frac{\partial\left(f(n_i^*(t)) - c\frac{\dot{n_i^*}(t)+sn_i^*(t)}{q(\theta)}\right)}{\partial n_0}dt - e^{-rtm}\left(f(n_i^*(tm)) - c\frac{\dot{n_i^*}(tm)+sn_i^*(tm)}{q(tm)}\right)\frac{\partial tm}{\partial n_0}. \tag{B6}$$

As $n_i^*$ does not depend on $n_{i0}$, $n_i^*(tm) = n_i^{opt}(tm) = n_{i0}e^{-s\ tm}$ and by (B4):

$$\dot{n_i^*}(tm)\frac{\partial tm}{\partial n_0} = e^{-rtm} - n_{i0}se^{-rtm}\frac{\partial tm}{\partial n_0},$$

we obtain that

$$\frac{\partial tm}{\partial n_{i0}} = \frac{e^{-rtm}}{\dot{n_i^*}(tm) + n_{i0}se^{-rtm}},$$

so that (B6) becomes:

$$\frac{\partial V(n_{i0})}{\partial n_{i0}} = \int_0^{tm} e^{-(r+s)t} f'^{opt}(t))\ dt + e^{-(r+s)tm}\frac{c}{q(\theta)}. \tag{B7}$$

When $tm$ goes to zero with $n_{i0} > n_i^*(0)$, we have:

$$\frac{\partial V(n_i^*(0)^+)}{\partial n_{i0}} = \frac{c}{q(\theta)}. \tag{B8}$$

The same argument is valid when $n_{i0} < n_i^*(0)$, and

$$\frac{\partial V(n_i^*(0)^-)}{\partial n_{i0}} = \frac{c}{q(\theta)}.$$

**Remark 4** *Note that for this singular control problem, (but only when $n_{i0} = n_i^*(0)$) or, in other words, when starting on $n_i^*(t)$) $\frac{c}{q(\theta)}$ is the value of the co-state variable of the control problem only when $n_{i0} = n_i^*(0)$ or, in other words, when starting on $n_i^*(t)$.*

$$H(n_i, v_i, \lambda,) = (f(n_i) - cv_i) + \lambda(q(\theta)v_i - sn_i),$$

*actually:*

$$\frac{\partial H(n_i, v_i, \lambda,)}{v_i} = 0 \quad\iff\quad \lambda = \frac{c}{q(\theta)}.$$

Now, we verify that the equation (B2) (or equation (10) in the paper's body) has an unique solution when the wage is given by the negotiated wage (equation (22)). The substitution of (22) in (B2) gives:

$$n^{-\frac{\alpha}{\sigma}+\alpha-1} = \phi\frac{(1-\gamma)b + \gamma\theta c + c(r+s)/q(\theta) - \eta c\dot{\theta}/q(\theta)}{(1-\gamma)(\sigma-1)A^{1-1/\sigma}\alpha Y^{1/\sigma}} = K. \tag{B9}$$

As a result, there exists an unique positive solution to this equation when $K > 0$.Sine $\sigma > 1$,this statement always holds when $\eta c\dot{\theta} < (1-\gamma)bq(\theta) + \gamma\theta q(\theta)c + c(r+s)$.

## Appendix C: Solving the differential equation

The differential equation to be solved is:

$$w_i(n_i) = (1-\gamma)\, b + \gamma\theta c + \gamma \left[ \frac{\sigma - 1}{\sigma} \frac{p_i}{P}(n_i) \frac{\delta y_i}{\delta n_i} - \frac{\delta w_i(n_i)}{\delta n_i} n_i \right] \tag{C1}$$

The method of resolution is standard and follows Cahuc and Wasmer (2004). Initially, we can disregard the term which don't depend on $n_i$ (the constant term) and add it back in later. Given that $\frac{p_i}{P}(y_i)\frac{\delta y_i}{\delta n_i} = \alpha A^{1-\frac{1}{\sigma}} n_i^{\alpha-1-\frac{\alpha}{\sigma}} Y^{\frac{1}{\sigma}}$, the equation (C1) becomes:

$$w_i(n_i) = \gamma \frac{\sigma - 1}{\sigma} \alpha A^{1-\frac{1}{\sigma}} n_i^{\alpha-1-\frac{\alpha}{\sigma}} Y^{\frac{1}{\sigma}} - \gamma \frac{\delta w_i(n_i)}{\delta n_i} n_i \tag{C2}$$

And can be rewritten as follows:

$$\frac{w_i(n_i)}{\gamma n_i} + \frac{\delta w_i(n_i)}{\delta n_i} - \frac{\sigma - 1}{\sigma} \alpha A^{1-\frac{1}{\sigma}} n_i^{\alpha-2-\frac{\alpha}{\sigma}} Y^{\frac{1}{\sigma}} = 0 \tag{C3}$$

The homogenous version of (C3) is:

$$\frac{w_i(n_i)}{\gamma n_i} + \frac{\delta w_i(n_i)}{\delta n_i} = 0 \tag{C4}$$

which has the solution:

$$w_i(n_i) = K n_i^{-\frac{1}{\gamma}} \tag{C5}$$

We take the derivative of (C5), using the fact that $K$ may depend upon $n_i$:

$$\frac{\delta w_i(n_i)}{\delta n_i} = -K \frac{1}{\sigma} n_i^{-\frac{1}{\gamma}-1} + n_i^{-\frac{1}{\gamma}} \frac{\delta K}{\delta n_i} \tag{C6}$$

Now, we substitute (C5) and (C6) in (C3) and we obtain:

$$n_i^{-\frac{1}{\gamma}} \frac{\delta K}{\delta n_i} - \frac{\sigma - 1}{\sigma} \alpha A^{1-\frac{1}{\sigma}} n_i^{\alpha-2-\frac{\alpha}{\sigma}} Y^{\frac{1}{\sigma}} = 0$$

$$\Leftrightarrow \frac{\delta K}{\delta n_i} = \frac{\sigma - 1}{\sigma} \alpha A^{1-\frac{1}{\sigma}} n_i^{\alpha-2-\frac{\alpha}{\sigma}+\frac{1}{\gamma}} Y^{\frac{1}{\sigma}} \tag{C7}$$

Given that $\frac{p_i}{P}(y_i)\frac{\delta y_i}{\delta n_i} = \alpha A^{1-\frac{1}{\sigma}} n_i^{\alpha-1-\frac{\alpha}{\sigma}} Y^{\frac{1}{\sigma}}$, the integral over both sides of (B7) gives:

$$K = \gamma \frac{\sigma - 1}{\phi} \frac{p_i}{P}(y_i) \frac{\delta y_i}{\delta n_i} n_i^{\frac{1}{\gamma}} + J \tag{C8}$$

where $J$ is a constant of integration and $\phi = \sigma\alpha\gamma - \sigma\gamma - \alpha\gamma + \sigma$. Now, we substitute (C8) in (C5) and we obtain:

$$w_i(n_i) = \gamma \frac{\sigma - 1}{\phi} \frac{p_i}{P}(y_i) \frac{\delta y_i}{\delta n_i} + J n_i^{-\frac{1}{\gamma}} \tag{C9}$$

Following Cahuc and Wasmer (2004), the terminal condition $\lim_{n_i \to 0} n_i w_i = 0$, which reports the fact that the firm-level bargained wage should not explode as firm-level employment $n_i$ approaches zero, implies that $J = 0$. As a consequence, the constant of integration can be withdrawn and ,after adding back the constant term, we obtain the following solution for the differential equation (C1):

$$w_i(n_i) = (1-\gamma)\, b + \gamma\theta c + \gamma \left[ \frac{\sigma - 1}{\phi} \frac{p_i}{P}(y_i) \frac{\delta y_i}{\delta n_i} \right] \tag{C10}$$

with $\phi = \sigma\alpha\gamma - \sigma\gamma - \alpha\gamma + \sigma$.

## Appendix D: Hiring externality analysis

The sign of the hiring externality given by the expression () depends on the sign of the following expression:

$$a(\alpha, \sigma) = (\alpha - 1) - \frac{\alpha}{\sigma} \tag{D1}$$

According the values of $\alpha$ and $\sigma$, the function $a(\alpha, \sigma)$ is positive, negative or null. Indeed, we have:

- $a(\alpha, \sigma) = 0$ if $\alpha = \dfrac{\sigma}{\sigma - 1}$
- $a(\alpha, \sigma) > 0$ if $\alpha > \dfrac{\sigma}{\sigma - 1}$
- $a(\alpha, \sigma) < 0$ if $\alpha < \dfrac{\sigma}{\sigma - 1}$

And graphically, we have:

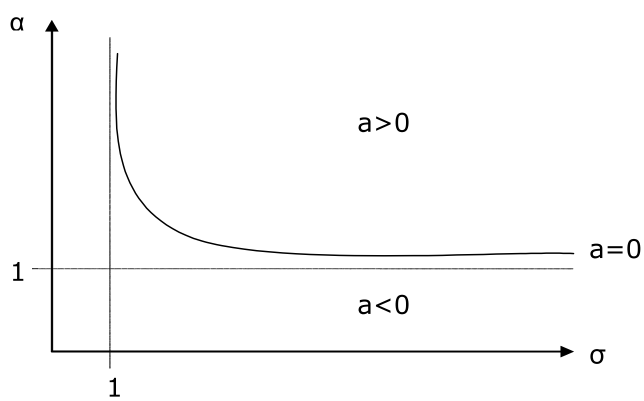


Figure D1: Sign of $a(\alpha, \sigma)$

Thus, when the size of returns to scale is inferior (superior) to the price mark-up of the firm, the hiring externality is negative (positive).

We can also conclude that whatever the degree of competition on the product market, the hiring externality is always negative when the returns to scale are decreasing or constant.

## Appendix E: Hiring externality and competition degree

We investigate here how the hiring externality varies when the competition degree increase. To do that, we need to compute the following derivative:

$$\frac{\partial\left[\dfrac{\partial w_i^S(n_i)}{\partial n_i}\right]}{\partial\sigma}=\frac{\partial\left[\gamma\dfrac{\sigma-1}{\phi}\left[(\alpha-1)-\dfrac{\alpha}{\sigma}\right]\dfrac{p_i}{P}(n_i)\dfrac{\partial y_i}{\partial n_i}n_i^{-1}\right]}{\partial\sigma} \tag{E1}$$

Let:

$$\begin{aligned}
u &= \gamma\frac{\sigma-1}{\phi}\\
v &= (\alpha-1)-\frac{\alpha}{\sigma}\\
w &= \frac{p_i}{P}(n_i)\frac{\partial y_i}{\partial n_i}n_i^{-1}
\end{aligned}$$

The derivative of (1) is given by $u'vw+uv'w+uvw'$.

Given that:

$$\begin{aligned}
u' &= \gamma\frac{\phi+(\sigma-1)(\alpha\gamma-\gamma+1)}{\phi^2}>0,\ \text{since } \gamma\in[0,1]\\
v' &= \frac{\alpha}{\sigma^2}>0\\
w' &= \frac{1}{\sigma^2}\ln\left[\frac{p_i}{P}(n_i)\right]\frac{p_i}{P}(n_i)\frac{\partial y_i}{\partial n_i}n_i^{-1}=0,\ \text{since } \frac{p_i}{P}(n_i)=1 \text{ due to the symmetrical firms assumption}
\end{aligned}$$

The derivatives of (E1) is given by this following expression after arrangements:

$$\frac{\partial\left[\dfrac{\partial w_i^S(n_i)}{\partial n_i}\right]}{\partial\sigma}=\left[\frac{\phi+(\sigma-1)(\alpha\gamma-\gamma+1)}{\phi(\sigma-1)}\right]\left[\frac{\partial w_i^S(n_i)}{\partial n_i}\right]+\gamma\frac{(\sigma-1)}{\phi}\frac{\alpha}{\sigma^2}\frac{p_i}{P}(n_i)\frac{\partial y_i}{\partial n_i}n_i^{-1}$$

Thus, (E1) is negative when the hiring externality is negative, that is to say $\dfrac{\partial w_i^S(n_i)}{\partial n_i}<0$ and positive otherwise.

When the returns to scale are decreasing or constant ($\alpha\leqslant 1$), the hiring externality is always negative[32] and the expression (E1) too.

[32] See appendix D.