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OPTIMAL ALLOCATION OF TRADABLE EMISSION PERMITS UNDER UPSTREAM-DOWNSTREAM STRATEGIC INTERACTION

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In this paper we account for the fact that Cournot equilibrium strategies in the sector under environmental regulation depend on firms' interaction in the permits market (and vice versa). In this context, we show that the cost-effective allocation of permits between firms must compensate the cost-rising strategies exercised by the stronger firm (in the output market). Then, taking into account the previous result, we use a simulation to obtain the optimal allocation of permits between firms as a function of output market characteristics, in particular as a function of goods substitutability that serves as an indicator for the degree of price competition. The simulation allows us to determine how output market characteristics affect differently optimal permit allocation depending on the regulator's objective.

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1. INTRODUCTION

In this paper we investigate how strategic interaction in an olygopolistic output market may affect the effectiveness of environmental regulation based on tradable emissions permits. With this purpose, we assume that two Cournot producers of a (differenciated) polluting good must comply with environmental regulation by holding a tradable emission permit per unit of (non-abated) pollution generated by goods' production. This makes tradable emissions permits an essencial input. We model the interaction between the tradable emissions permits market (upstream) and the output market (downstream) by considering the following three-stages game: in the first stage, one of the firms sets the price of permits alone; in the second stage permits are traded; and, finally, in the third stage, firms compete à la Cournot in the (differenciated) output market. Our model follows Montero (2002) and the traditional theory on upstream-downstream interaction (from Salinger, 1988 to Ordover, Salop and Saloner, 1990), assuming that the upstream (permits) market is cleared first. However, differently from the latter, our model captures the nature of tradable emissions permits markets by allowing firms' position in the upstream market to be endogenous.

Our assumption regarding the fact that a firm has a first-mover advantage over the other in the permits market captures the fact that most permits markets created so far have been organized in subsequent "phases": in a first phase, only the most polluting firms receive tradable permits that can be traded in the permits market, whereas, their less polluting competitors are only included in a second phase (usually two to four years latter). This was the case in the EU Emission Trading Scheme (EU-ETS), in the US Acid Rain Market and in the NOx budget program, among many others. In this respect, Boemare and Quirion (2002) note that EU-ETS coverage is confined to a limited number of sectors, and that there is no provision for voluntary "opt in" by firms below the threshold size that assign firms to the first phase. They argue that such "opt in" provisions could help dilute any emerging market power.

The risk of market power in a tradable permits market covering more than one sector, like the EU-ETS, seems smaller than in the case where only one olygopolistic sector is included[1] or in the case that one olygopolistic sector represents a big portion of the total permits market. According to Kolstad and Wolak (2008), the olygopolistic firms participating in the Californian electricity market (CAISO) behaved strategically in the Los Angeles market for NOx emissions called RECLAIM (Regional Clean Air Incentives Market) to enhace their ability to exercise market power in the CAISO. Such firms, shown to exhert unilateral market power in the CAISO by Wolak (2003), were allocated 56% of total initial stock of permits. In the same line, Chen et al. (2006) compute equilibrium behavior considering the interaction between the NOx budget program and the Pennsylvania-New Jersey-Maryland (PJM) electricity market. Due to the high concentration of the PJM market[2], six large electricity generators alone account for 90% of emissions in the referred permits market. Chen et al. (2006) find that a Stackelberg leader with a long position in the permits market could gain substantial profits by withholding permits and driving up permits costs for rival producers.

The supporters of environmental regulation based on tradable emissions permits argue that the

[1] In this respect Kolstad and Wolak (2008) argue that "although big fossil-fuel generation units produce a fraction of greenhouse gas emissions (GHG), they are likely to be early and substantial participants in any GHG emission permits trading scheme". In the same line, Linares et al. (2006) argue that electricity generators represent more than 50% of emissions covered by the EU-ETS market.

[2] Its Herfindahl-Hirschman Index was 0.154.

creation of an emission permits market allows to reach the pollution reduction target (generally called "the cap") in a cost-effective manner (Montgomery, 1972) and with a minimum information cost for the regulator (namely, concerning information regarding pollution abatement technologies). This argument has been challenged, first, by Hahn's (1985) dominant-fringe model which argues that the existence of market power reduces the cost-effectiveness of tradable emissions permits because a dominant firm manipulates the price of permits to reduce its own emission abatement costs. Misiolek and Elder (1989), inspired in Salop and Scheffman (1987), and Eshel (2005), also relying on a dominant-fringe setting, show that technological linkages between permits and output markets would give raise to rival's cost-rising strategies by the dominant firm, which would introduce an additional type of market distortion. Fehr (1993) and Sartzetakis (1997) have also challenged the effectiveness of environmental regulation based on tradable emissions permits, showing that, in a context of strategic interaction, emission permits markets could lead to monopolization or excessive entry barriers. Although these two last papers have considered strategic interaction in the output market, their objective is just to focus on the previously mentioned corner solutions instead of assessing the effects of strategic interaction on permits prices (or optimal permits allocation). In fact, Fehr (1993) assumes that downstream firms buy permits for a given supply, whereas Sartzetakis (1997) assumes a competitive permits market.

The empirical papers' findings mentioned earlier in this introduction, together with the recent discussion on "windfall" profits in oligopolies[3] subject to environmental regulation based on tradable emissions permits, suggest the need to account for the way this regulation affects profits and firm's behaviour when the downstream market is olygopolistic. This paper wishes to contribute to the literature on non-competitive emissions trading by accounting for an olygopolistic downstream market and discussing how its characteristics determine the optimal permits allocation rule. In this way, we are able to identify three different channels of market distortion. The first market distortion is due to market power in the permits market. Everything else the same, as in Hahn (1985), the first-mover in the permits market manipulates the price of permits to reduce its own environmental costs. The other market distortions are related to firms' market power in the output market, which gives both firms the incentives to underinvest in pollution abatement. By using permits for production, firms become more competitive in the output market as they reduce their own marginal abatement costs, while increasing their rival's marginal abatement costs. Differently from the dominant-fringe literature's findings, we show how strategic interaction in the output market endows the follower in the permits market to adopt rival's cost-rising strategies as well. In this context, we show how the possibility of simultaneous adoption of rival's cost-rising strategies may either aggravate of ameliorate the cost-effectiveness of environmental regulation based on tradable emissions permits markets[4]. The net impact of the simultaneous adoption of rival's cost-rising strategies depends on

[3]The Arcelor-Mittal case in April 2009 is one example from the steel industry under the EU-ETS.

[4]It is worth noting that the conclusions driven in this paper are only concerned with tradable emissions permits

firms' position in the permits market (buyer or seller) as well as on the interplay of firms' actions in the output market.

In this line, we propose an optimal criterion to allocate permits between firms. We argue that the specific features of this criterion are highly sensitive to *(i)* the regulator's objective (cost-effectiveness of pollution abatement, maximization of firms profits, maximization of social welfare,...); and *(ii)* the specific characteristics of the market, in particular the degree of substitution between goods. The latter point underlines how, even if tradable emissions permits may lead to cost-effectiveness of pollution abatement (or any of the mentioned objetives), when the downstream market is olygopolistic this cannot be done at the minnimum information cost.

2. BASIC SETUP

Consider a duopolistic market, in which firms (firm i and firm j) compete in quantities producing imperfect substitute goods i and j. Quantity y_k represents the production of good $k = i, j$, p_k represents its price and $c_k [y_k]$ its production cost, with $c'_k [y_k] > 0$ and $c''_k [y_k] > 0$.

The production of goods i and j generates polluting emissions as a by-product. The parameter $\beta > 0$ represents output polluting intensity. Firms are subject to environmental regulation based on a cap and trade system. Under such environmental regulation, each firm must hold a number of permits E_k equal to the amount of pollution βy_k emitted during production of y_k. Then, this regulation creates a scarce input, tradable permits, that are available up to the cap S. Then, the total stock of permits S is allocated for free among firms. A percentage α is received by firm i and a percentage $(1 - \alpha)$ by firm j. The percentage $\alpha \in [0, 1]$ and the cap S are chosen exogenously by the regulator, according to the pollution control target. In a framework where only polluting firms trade in rights, the regulator's decisions regarding the allocation of permits between firms and the decision regarding the total cap on emissions can be analyzed independently (Eshel, 2005). In this paper we restrict attention only to the allocation decision[5].

When the permits received for free, αS and $(1 - \alpha) S$ respectively, do not coincide with the emissions provoked by production of the optimal quantity of output βy_k^*, firms choose either to

regulation and cannot be extrapolated to other environmental regulation instruments like taxes or standards.

[5]The cap on pollution is generally fixed by the regulatory authority with the help of experts -like the IPCC 1990 Scientific Assessment in the case of the Kyoto protocol and its european side agreement for the creation of the EU-ETS- that state the impacts of pollution and the pollution reduction required to diminish those impacts to an acceptable level. By creating property rights for an amount equal to the acceptable environmental damage, the authority ensures that the marginal cost of pollution reduction (i.e. the property right's price) equals the marginal benefit of such reduction (i.e. the unit improvement in environmental quality).

abate some of these emissions or to engage in permits trading. That is,

$$E_i = \alpha S - x_i \geq 0, \quad (1)$$
$$E_j = (1-\alpha)S - x_j \geq 0, \quad (2)$$
$$a_k = \beta y_k - E_k \geq 0, \ k = i, j, \quad (3)$$

where x_k denotes the amount of permits sold (when $x_k > 0$) or bought (when $x_k < 0$) by firm k, and a_k stands for the level of emissions abated. Abatement of polluting emissions is costly. To abate a_k polluting emissions, firm k has a cost of $h_k[a_k]$, with $h'_k[a_k] > 0$ and $h''_k[a_k] > 0$.

In this regulatory framework, firms interact in two technologically-linked markets: the permits market (upstream market) and the output market (downstream market). In the output market, both firms exert some degree of market power since firms take their quantity decisions simultaneously. In the permits market, the degree of market power is asymmetric, with one of the firms (say firm j) having a first-mover advantage.

We model interaction in these technologically linked markets using a sequential game theoretical approach. The players are the two firms, the payoffs are firms' profits and strategies correspond to the vector $\left(q, E_i^*(q), \left(y_i^*(q, E_i), y_j^*(q, E_i)\right)\right)$, with $q \in \mathbb{R}^+$, $E_i \in \mathbb{R}^+$, and $y_k \in \mathbb{R}^+$. The timing of the game is the following: in the first stage, firm j sets alone the price of permits (q); in the second stage, firm i observes the posted price of permits (q) and chooses the amount of permits to use for production (E_i), which determines the amount of permits to buy or sell (x_i). Firm j clears the permits market $(x_j = -x_i)$. Finally, in the third stage, given optimal use of permits (E_i) and the corresponding optimal permits trade (x_i), firms simultaneously interact in the output market, strategically competing on quantities (y_i and y_j).

3. CHARACTERIZATION OF THE EQUILIBRIUM

We rely on the notion of subgame perfect Nash equilibrium (SPNE) to investigate firms' optimal behavior. The optimization problem of firms in each stage is detailed in Apendix A.

Definition 1. The SPNE corresponds to the vector of strategies

$$\left(q^*, E_i^*(q), y_i^*(q, E_i), y_j^*(q, E_i)\right)$$

such that
(i) Given q and E_i, y_i^* is the best response of firm i to y_j and vice-versa;
(ii) Given $y_k^*(q, E_i)$, $E_i^*(q)$ is the best response of firm i to q; and
(iii) Given $y_k^*(q, E_i)$ and $E_i^*(q)$, q maximizes total profits of firm j.

We consider that goods are imperfect substitutes and inverse demand functions are such that:

$$\frac{\partial p_i[y_i, y_j]}{\partial y_i} < \frac{\partial p_i[y_i, y_j]}{\partial p_j} < 0. \tag{4}$$

According to the previous definition, in the last stage firms simultaneously choose the output levels that solve problems (17) and (18), respectively. When considering linear[6]demands, the interior[7] equilibrium vector $\left(y_i^*\left(E_i, q\right), y_j^*\left(E_i, q\right)\right)$ is directly obtained from the system of first order conditions (FOCs[8]), i.e.:

$$\begin{array}{c} p_i[y_i, y_j] + \frac{\partial p_i[y_i, y_j]}{\partial y_i} y_i = c_i'[y_i] + \beta h_i'[a_i] \\ p_j[y_i, y_j] + \frac{\partial p_j[y_i, y_j]}{\partial y_j} y_j = c_j'[y_j] + \beta h_j'[a_j] \end{array}. \tag{5}$$

Given outcomes in the permits market and rival's output choice, in equilibrium each firm chooses the equilibrium production level $y_k^*\left(E_i, q\right)$ for which there is a perfect balance between the marginal revenue and the marginal cost (including abatement marginal costs).

According to equilibrium conditions (5), outcomes in the permits markets affect output decisions via marginal abatement costs. The following lemma summarizes this transmission mechanism.

Lemma 1. *The larger the amount of permits used for production by firm k, the larger its equilibrium output level and the lower the equilibrium output level for its rival $-k$. It follows:*

$$\begin{array}{rcl} \frac{\partial y_k^*[E_k]}{\partial E_k} & > & 0; \\ \frac{\partial y_{-k}^*[E_k]}{\partial E_k} & < & 0. \end{array} \tag{6}$$

Proof. See the Appendix. ∎

The result in (6) is due to the fact that, the larger the amount of permits used for production by firm k, the lower its abatement needs a_k, the lower its marginal abatement costs $h_k'(a_k)$, and therefore, the higher its output production. This direct effect is reinforced by strategic substitutability

[6]By considering linear demand functions we ensure that second order effects are zero, i.e. for each firm k it is the case that $\frac{\partial^2 p_k[y_k, y_{-k}]}{\partial y_k^2} = \frac{\partial^2 p_k[y_k, y_{-k}]}{\partial y_k \partial y_{-k}} = \frac{\partial^2 p_k[y_k, y_{-k}]}{\partial y_{-k}^2} = 0$.

[7]Essentiatlly, we restrict our attention to the parameters for which the marginal revenue is higher than the marginal cost when the strategic variable is equal to zero ($q = 0$, $E_k = 0$, $y_k = 0$). When this is not the case, equilibrium outcomes may correspond to corner solutions, in which firms' optimal behavior may differ from the first order conditions derived in this paper. A sufficient condition to guarantee that we are in an interior solution is to assume that, when $y_k = 0$, it is the case that $p_k[y_i, y_j] > c_k'[y_k] + \beta h_k'[y_k]\ \forall y_{-k}$, independently of the rival's output decision.

[8]Second order conditions are fulfilled as long as: $\frac{\partial^2 p_k[y_i, y_j]}{\partial y_k^2} y_k \leq -2\frac{\partial p_k[y_i, y_j]}{\partial y_k} + c_k'[y_k] + \beta_k'[a_k]$. In the case of linear demands and strictly convex cost functions (production costs and abatement costs), this condition is always met.

of firms' output decisions. Accordingly, the result in (6) is independent of which of the two rival firms is the most efficient in terms of abatement.

In the second stage, firm i chooses the amount of permits to use for production (E_i) after observing the price of permits (q). When deciding E_i, firm i anticipates the strategic interaction that will take place in the output market. Therefore, when deciding E_i, firm i takes into account the marginal profitability of permits transactions ($q - h_i'[a_i]$) as well as the impact of E_i in the output market outcome. In the interior equilibrium, $E_i^*[q]$ is obtained from the first order condition[9] to problem (15), that is,

$$\frac{dp_i^*[E_i]}{dE_i} y_i^*[E_i] + \mu_i^*[E_i] \frac{\partial y_i^*}{\partial E_i} = q - h_i'[y_i^*[E_i]], \tag{7}$$

with $\mu_i^*[E_i] = p_i^*[E_i] - c_i'[y_i^*[E_i]] - \beta h_i'[y_i^*[E_i]]$ and $\frac{dp_i}{dE_i} = \frac{\partial p_i}{\partial y_i}\frac{\partial y_i^*[E_i]}{\partial E_i} + \frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*[E_i]}{\partial E_i}$.

In equilibrium, $E_i^*[q]$ is such that firm i does not have any opportunity to increase its total profits by trading-off output profits by profits due to permits' transactions. More precisely, in equilibrium, any variation in the profits from permits transactions caused by a marginal variation of E_i (equal to $q - h_i'$, in the RHS of (7)) is perfectly offset by an equivalent variation in output profits (which is given by the LHS of (7)).

Lemma 2. *For a given price of permits q, firm i always abates less than efficiently. The difference between permits price and marginal abatement costs is given by:*

$$q - h_i'[E_i] = \frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*[E_i]}{\partial E_i} y_i^*[E_i] > 0. \tag{8}$$

Proof. See the Appendix. ∎

The lemma shows that, in a scenario of strategic interaction in the output market, abatement decisions of firm i are such that $q > h_i'[E_i]$. Accordingly, in equilibrium, firm i always abates less than what a competitive firm would abate. Firm i strategically chooses to forego profits from permits transactions to benefit from a better position in relation to its rival in the output market. Due to strategic interaction in the output market, despite being a price-taker in the permits market, firm i is able to use its decisions concerning E_i to reduce its overall marginal costs, while increasing its rival marginal costs (since the rival is responsible for clearing the permits market). Firm i is then able to increase its market share in the output market, obtaining higher output profits than the profit level corresponding to the choice of E_i that leads to $q = h_i'$.

[9] Our analysis is valid when the second order condition $\frac{\partial^2 \pi_i^*[E_i,q]}{\partial E_i^2} < 0$ holds, i.e. when:

$$2\frac{\partial y_i^*}{\partial E_i}\frac{dp_i^*}{dE_i} < c_i'' \left(\frac{\partial y_i^*}{\partial E_i}\right)^2 + \left(1 - \beta\frac{\partial y_i^*}{\partial E_i}\right)^2 h_i''.$$

Finally, in the first stage, firm j quotes the price q^* that solves the following first order condition:

$$q - \frac{x_i^*[q]}{\frac{\partial E_i^*[q]}{\partial q}} - h_j' = -\left(\frac{dp_j^*}{dE_i} y_j^* + \mu_j^*[E_i^*[q]] \frac{\partial y_j^*}{\partial E_i}\right). \tag{9}$$

where we still focus exclusively on interior solutions.

The equilibrium price of permits q^* that satisfies (9) guarantees that firm j is exploiting all existing profit opportunities (considering the permits market as well as the output market). In equilibrium, variations in output profits induced by marginal variations in q (given by the RHS in condition (9)) are exactly compensated by variations in the profits associated with permits' transactions (given by the LHS in condition (9)).

Lemma 3. *Everything else the same, strategic competition in output quantities leads to a permits price that may be either higher or lower than the efficient one $\widehat{q} = h_j'$. In equilibrium, the difference between the price of permits and the marginal abatement cost of firm j is equal to:*

$$q^* - h_j' = \frac{x_i^*[q]}{\frac{\partial E_i^*[q]}{\partial q}} - \frac{\partial p_j[y_i, y_j]}{\partial y_i} \frac{\partial y_i^*}{\partial E_i} y_j^*[E_i[q]]. \tag{10}$$

Proof. See the Appendix. ∎

According to (10), we could observe a price of permits q^* equal to h_j' only by chance. The difference between q^* and h_j' can be decomposed in two effects. The first contribution to the difference between permits price and marginal abatement costs is due to the market power of firm j in the permits market (this corresponds to $\frac{x_i^*[q]}{\frac{\partial E_i^*[q]}{\partial q}}$ in condition (10)). Focusing exclusively on this effect, condition (10) implies that, everything else the same, when firm j is a net-seller of permits ($x_i^* < 0$), it enjoys a positive mark-up over h_j' as a result of its first mover advantage in the permits market. In contrast, when firm j is a net-buyer of permits ($x_i^* > 0$), firm j will mark down permits price (price discount).

Besides the market power exerted by firm j in the permits market, there is an additional effect that contributes to the difference between price of permits q^* and its marginal abatement costs h_j'. This additional source of inefficiency corresponds to the positive term $-\frac{\partial p_j[y_i, y_j]}{\partial y_i} \frac{\partial y_i^*}{\partial E_i} y_j > 0$ in condition (10). Independently of firms' position in the permits market, the latter effect derives from the technological link that exists between the permits market and the output market and is always positive. All the rest being equal, firm j has incentives to make permits more expensive for its rival i, reducing firm i's output production, while increasing its own market share in the output market.

4. COST-EFFECTIVENESS OF POLLUTION ABATEMENT

From Lemma 2 and Lemma 3, it follows that market mechanisms based on tradable emissions permits may not entail an efficient allocation of abatement efforts between firms.

This result is in line with previous literature. In a dominant-fringe setting, Hahn (1985) shows how market power in the permits market could damages cost-effectiveness in pollution abatement. Also using a dominant-fringe setting, Misiolek and Elder (1989) and Eshel (2005), among others, have suggested that the cost-effectiveness in pollution abatement could also be affected by dominant firms' ability to use emission permits to raise rivals' costs. These two effects correspond to the effects described in Lemma 3.

However, in the context of our model, even when firm i is a price taker in the permits market, strategic interaction in the output markets creates a new effect, which a priori might either aggravate or ameliorate the cost-effectiveness of abatement efficiency. This effect stems from the fact that we observe cost-rising strategies also in the case of firm i (see Lemma 2). More precisely, in equilibrium, we obtain that

$$h_i'[a_i^*] - h_j'[a_i^*] = \underbrace{\frac{x_i^*[q]}{\frac{\partial E_i^*[q]}{\partial q}}}_{\lesseqgtr 0} + \Big(\underbrace{-\frac{\partial p_j}{\partial y_i}\frac{\partial y_i^*}{\partial E_i}y_j^*}_{>0}\Big) + \Big(\underbrace{-\frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*}{\partial E_i}y_i^*}_{<0}\Big)\,. \tag{11}$$

According to (11), the equilibrium differential between firms' abatement costs can be decomposed in three effects. The first effect is given by $\frac{x_i^*[q]}{\frac{\partial E_i^*[q]}{\partial q}}$ and it stems from firm j's market power in the permits market (Hahn, 1985). The second effect is given by $-\frac{\partial p_j}{\partial y_i}\frac{\partial y_i^*}{\partial E_i}y_j^*$ and it stems from firm j's ability to use its decision regarding the price of permits to raise firm i's marginal costs. This effect is equivalent to the output market effect emphasized in the dominant-fringe literature, since firm j is also strategic in the output market. Finally, the third effect is given by $-\frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*}{\partial E_i}y_i^*$ and stems from firm i's ability to use its decision regarding permits' use to raise firm j's marginal costs, i.e. to compensate the previous effects. This third effect due to strategic interaction in the output market was not considered in previous literature, like Misiolek and Elder (1989) or Eshel (2005). Whether this third effect aggravates or ameliorates cost-effectiveness of abatement depends on firms' position in the permits market, as well as on the interplay of firms in the permits market and in the output market.

When firm j is a net-seller of permits, permits are always over-priced[10] ($q > h_j'$). Ceteris paribus, the higher price of permits induces a reduction in permits demanded by firm i due to a

[10] When firm j is a net-seller of permits, permits are over-priced for two reasons: according to (10), on the one hand, there is the output market effect corresponding to $-\frac{\partial p_j[y_i,y_j]}{\partial y_i}\frac{\partial y_i^*}{\partial E_i}y_j > 0$ and, on the other hand, there is an addicional mark-up of $\frac{(\alpha S - E_i^*[q])}{\frac{\partial E_i^*[q]}{\partial q}}$ due to firm i's market power in the permits market (first mover advantage).

move alo*ng* firm i's demand of permits. But firm i is also strategic. As described in Lemma 2, firm i's strategic behavior *shifts* permits' demand upwards, offsetting (at least partially) the market distortion associated with the market power of firm j. However, when firm i's strategic influence in this market is very strong (i.e. $\frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*}{\partial E_i}y_i^*$ is very large), this effect may yield instead market distortions with an opposite nature, i.e. with firm i under-investing in pollution abatement (such that $h_i'[a_i^*] < h_j'[a_i^*]$)[11] even more than firm j.

The previous analysis remains valid when firm j is a net buyer of permits but firm j's market power in the permits market is relatively limited and its incentives to adopt a raising rival's costs strategy more than compensate the incentives to mark-down permits price, i.e. $\frac{x_i^*[q]}{\frac{\partial E_i^*[q]}{\partial q}} - \frac{\partial p_j}{\partial y_i}\frac{\partial y_i^*}{\partial E_i}y_j^* > 0$.

When firm j is a net buyer of permits and it exerts a substantial degree of market power in the permits markets, permits prices are under-priced, i.e. the first term in the RHS of (10) is higher in absolute value that the second term.

Ceteris paribus, this yields a downward move along firm i's supply of permits. In this case, firm i's strategic effect aggravates the market distortion provoked by firm j by shifting supply of permits upward, which makes the difference in (11) larger.

To the light of the previous results, we conclude that market mechanisms alone may not (and in general do not) lead to cost-effectiveness of pollution abatement. The extent to which firms' deviate from the cost-effective pollution abatement solution depends on firms' initial permits' endowments, together with the incentives for rising rival's costs. Therefore, for a given cap on permits (S), the regulator may use the allocation of permits among firms (α) as a policy instrument to reestablish the cost-efficient result or to promote welfare-enhancing policies. Such choice depends on the regulator's objective. The policy implications of our results are analyzed with further detail in the next section.

5. REGULATION AND POLICY IMPLICATIONS

This section investigates how the environmental regulator may rely on his decision regarding firms' initial endowment of permits to restablish cost-effectiveness in the allocation of abatement efforts between firms. Afterwards, we consider a broader regulatory objective, investigating the impact of permits allocation rules on total social welfare.

From Lemma 2, we conclude that strategic interaction in the output market results in an upward shift of firm i's demand (or supply) of permits, which allows firm i to under-invest in pollution abatement, for any given price of permits. Depending on the interplay between the market power in the permits market and the incentive to adopt a rival's cost-raising strategy, firm j might be interested in under-invest or over-invest in pollution abatement.

[11] From a theoretical point of view, such situation may occur. Nevertheless, there should be a considerable degree of asymmetry between firms so that firm i's strategic effect in the output market more than compensates firm j's strategic effect in the output market as well as its market power in the permits market.

PROPOSITION 1. *A regulator is able to use the permits' allocation rule to restore cost-effectiveness of pollution abatement. The cost effective allocation rule* $\left(\alpha^{CE}\right)$ *is implicitly given by the value of* α *which solves:*

$$\frac{\partial p_j}{\partial y_i}\frac{\partial y_i^*}{\partial E_i}y_j^* + \frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*}{\partial E_i}y_i^* = \frac{S\alpha^{CE} - E_i^*[q]}{\frac{\partial E_i^*[q]}{\partial q}}. \tag{12}$$

The sign of the LHS of (12) is a priori undetermined. If the effect of firm i's cost-raising strategy dominates the effect of firm j's cost-raising strategy, i.e. $\frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*}{\partial E_i}y_i^* > -\frac{\partial p_j}{\partial y_i}\frac{\partial y_i^*}{\partial E_i}y_j^*$, when the cost-effective allocation rule is applied by the regulator, firm j must end up being a net seller of permits to eliminate firm i's incentive to overuse emission permits in production. In contrast, when $\frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*}{\partial E_i}y_i^* < -\frac{\partial p_j}{\partial y_i}\frac{\partial y_i^*}{\partial E_i}y_j^*$, the LHS of (12) is negative, and therefore, the effect of firm j's cost-raising strategy by itself is more than enough to compensate the effect inherent to firm i's cost-raising strategy. To avoid under-investment in pollution abatement by firm j, the regulator must allocate permits in such a way that firm j becomes a net-buyer of permits so that its market power in the permits market partially offsets its cost-raising strategy in the output market.

However, the cost-effectiveness of pollution abatement may not be the only aspect that the regulator considers when deciding permits' allocation rules. In fact, given the technological linkages between permits and output markets, we observe that outcomes in the output market are also affected by the regulator's choices with respect to the permits allocation.

In particular, along the equilibrium path, the output level of the firm receiving more permits increases while the output of the firm receiving less permits decreases:

$$\frac{\partial y_i^*}{\partial \alpha} > 0 \text{ and } \frac{\partial y_j^*}{\partial \alpha} < 0 \tag{13}$$

The mechanism behind this result is the following: first, an increases of α leads to a decreases in permits' price. This reduction of permits price entails a downward move along firm i's demand of permits (or supply of permits, if firm i is a net-seller of permits), which entails an increase in the use of permits for production by firm i. As a consequence, firm i's marginal abatement costs are lower than before, yielding an increase in firm i's output production. The opposite occurs to firm j, that looses market share after an increase of α. The impact on total output and consumers' welfare depends on whether the increase of y_k more than compensates the decrease of y_{-k} after the variation of α.

The cost-effective permits allocation rule (α^{CE}) fails to internalize this type of considerations. In fact, optimal allocation rules may change considerably depending on the regulator's objective. Some examples of possible objectives to be pursued by the regulator are: abatement cost-effectiveness, maximization of joint profits, maximization of consumers' surplus (even with some preference over some type of consumers), maximization of total welfare, or other. Depending on the scope of the regulator's activity and the specific objective pursued by the regulator, there may be contradictory recommendations regarding the optimal allocation rule α^*.

In the following section we introduce an example that illustrates the previous point. In particular, the following section shows how output demand characteristics influences optimal allocation of permits between firms.

6. THE IMPORTANCE OF OUTPUT DEMAND CHARACTERISTICS

To illustrate how sensitive optimal allocation rules may be both in relation to the specific characteristics of the industry under regulation and to the objective pursued by the regulator, consider an industry with the following characteristics: *(i)* the inverse demand for good k is given by $p_k[y_k, y_{-k}] = 25 + \varepsilon_j - 2y_k - \delta y_{-k}$, with $0 < \delta < 2$ and $\varepsilon_j = 0.1$ if $k = j$; *(ii)* firms' production technology is similar, with $c_k[y_k] = \frac{(y_k)^2}{4}$; *(iii)* the intensity of pollution is equal to $\beta = 0.8$; *(iv)* the total stock of emission permits is exogenously fixed by the regulator to meet the pollution control target and we consider it to be $S = 3$, which constrains firms' production plans. Finally, *(v)* firms' abatement technologies are given by $h_i[a_i] = \frac{1.1(a_i)^2}{2}$ and $h_j[a_j] = \frac{(a_j)^2}{2}$. From *(v)* follows that the first-mover in the permits market (firm j) owns a more efficient abatement technology, even if the degree of asymmetry between firms' abatement technologies is rather limited. This is in line with Hahn (1985) and Eshel (2005), among others, that define the dominant firm in their dominant-fringe setting as the firm with the lowest marginal abatement cost function.

In this context, we compute the equilibrium outcomes as in Section 3 but now considering the specific functions detailed in points *(i)* to *(v)*. We then illustrate the degree of output substitutability, as a measure of competition between firms in the output market, as well as the objective pursued by the regulator, affect optimal allocation rules. Regarding, the impact of output substitutability, we compute optimal allocation rules for progressively higher degrees of substitutability between goods, comparing outcomes for $\delta = 0.01$; $\delta = 0.4$; $\delta = 1.5$; and $\delta = 1.99$. Concerning the impact of the objective pursued by the regulator, we compare optimal allocation rules under four different regulatory objectives: *(i)* cost-effectiveness of abatement effort (CE); *(ii)* maximization of firms' joint profits (JP); and, finally, *(iii)* maximization of total social welfare (W). Regarding the way we compute consumer's surplus to be able to consider it when calculating social welfare, we use the partial equilibrium analysis from Belleflamme and Peitz (2010). Table 1 summarizes our results:

Reading each line separately, we conclude that the optimal allocation of permits varies according to the regulator's objective. When the regulator wants to promote cost-effectiveness of pollution abatement (**CE**), he voluntary chooses to ignore the impact of permits decisions on the output market. Table 1 shows that, under a cost-effective allocation rule, the regulator must allocate more permits to the firm that owns the less efficient abatement technology. Since in this example we consider $h'_j < h'_i$, the asymmetric allocation in favor of firm i is needed to reduce its abatement needs (in comparison to firm j's abatement needs) up to the point in which $h'_i[a^*_i] = h'_j[a^*_j]$. In relation to

Substitution	CE	JP	W
$\delta = 0.01$	$\alpha^*_{CE} = 54.1\%$	$\alpha^*_{JP} = 54\%$	$\alpha^*_{W} = 69\%$
$\delta = 0.4$	$\alpha^*_{CE} = 54.4\%$	$\alpha^*_{JP} = 53\%$	$\alpha^*_{W} = 61\%$
$\delta = 1.5$	$\alpha^*_{CE} = 54.9\%$	$\alpha^*_{JP} = 51\%$	$\alpha^*_{W} = 60\%$
$\delta = 1.99$	$\alpha^*_{CE} = 55.2\%$	$\alpha^*_{JP} = 51\%$	$\alpha^*_{W} = 60\%$

TABLE 1

Optimal permits allocation as a function of substituibility according to regulatory objective

the way output market characteristics influence the cost-effective allocation rule, as sustituability decreases, rising-rival's cost strategies are weaker (due to less competition) and therefore the lower must be the compensation (through a higher permits allocation) to the less efficient firm. However, the cost-effective allocation rule is not very sensitive to changes in the degree of output substituability. This is due to the fact that the latter only affects the former indirectly, i.e. through the effect that a change in output substitutability has on output production, and consequently in abatement needs.

Turning now to the maximization of joint profits (**JP**), the optimal allocation of permits once more favours the least efficient firm i. This is the case because, when giving more permits to the least efficient firm, the increase in its output production (and its revenues) more than compensates the decrease in its rival's production (and profits). The changes in the profit-maximizing allocation as substituability changes are stronger than in the cost-effective allocation. In particular, we observe that, as substituability decreases, firm i's increase in production (and profits) after an increase in α (keep in mind equation (13)) is accompaigned by a smaller decrease in j's production (and profits). Then, the lower the substitution between goods, the higher the amount of permits that should be allocated to the less efficient firm.

Finally, the welfare maximizing (**W**) allocation rule gives even more permits to the least efficient firm than the profit maximizing allocation rule. This is the case because the latter only accounts for the fact that the increase in the least efficient firm's output production after an increase in α more than compensates the decrease in the most efficient firm's output production. Instead, the welfare maximizing allocation rule additionally accounts for the fact that the mentioned changes in quantities also affect prices and consequently consumer's surplus. In particular, the negative variation in the least efficient firm's price p^*_i (thin line in Figure 1) more than compensates the positive variation in the most efficient firm's price p^*_j. These price changes make consumers better-off as α increases.

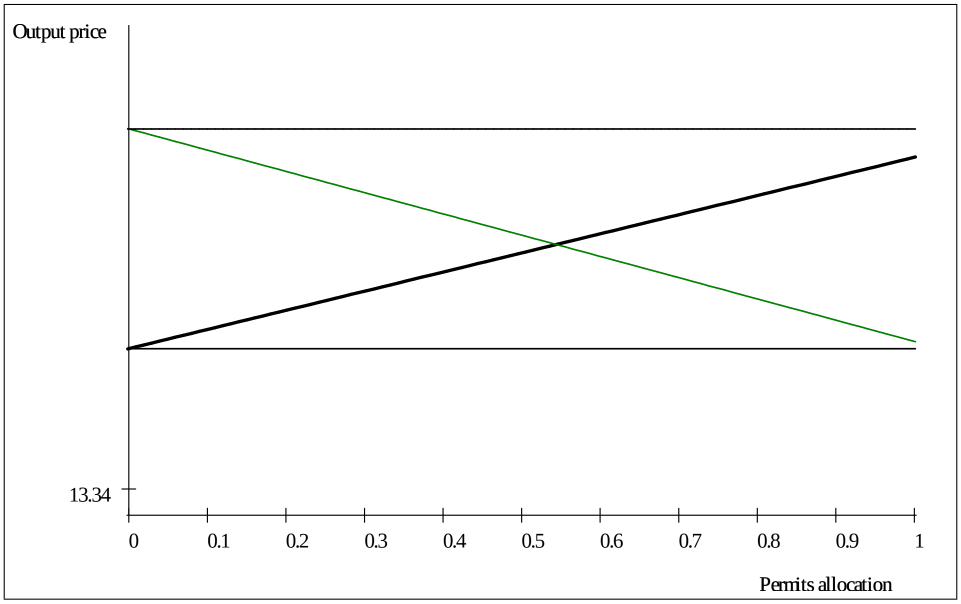

Figure 1: Output prices as a function of α.

Concerning the impact of output substitutability on the welfare-maximizing allocation rule, we find that as the substituability between goods decreases, the higher the gap between the negative variation of p_i^* with an increase in α and the positive variation of p_j^* with an increase in α. As a result, as substituability decreases, the higher it must be the fraction of permits allocated to firm i to compensate the fact that p_i^* is higher than p_j^* (due to higher marginal abatement costs).

It is worth noting that the amount of permits allocated to the least efficient firm increases as we move rightwise in Table 1. This is because, when giving more permits to the least efficient firm, the cost-effective allocation rule takes into account the benefit in terms of cost minimization while the profit-maximizing allocation takes into account, additionally, the positive effect on overall quantity produced and, finally, the welfare-maximizing allocation additionally considering the benefits for consumers in terms of lower prices due to the latter increase in production.

The analysis regarding the regulatory possibilities as a function of goods substituability becomes particularly relevant when thinking of environmental policy that may affect the redistribution of production between goods and, through that redistribution, may harm least favoured consumers.

7. CONCLUDING REMARKS

We have investigated how strategic interaction in the output market together with market power in the permits market may affect the effectiveness of environmental regulation based on tradable emissions permits. We propose a model of upstream-downstream competition, whose major features are the following: *(i)* explicit consideration of the technological linkages between the permits market (upstream) and the output market (downstream); *(ii)* strategic interaction in the output market, with firms competing *à la Cournot;* and *(iii)* market power in the permits market.

In line with previous literature dealing with the impact of market power on the effectiveness of environmental regulation based on tradable emissions permits, we also conclude that emission permits markets may lead to outcomes that differ from the cost-effective pollution abatement solution.

Besides market distortions associated with market power in the permits market (as in Hahn, 1985) and market distortions associated with the possibility of having a dominant firm adopting rival's cost-rising strategies (as in Misiolek and Elder, 1989 and Eshel, 2005), we show that strategic interaction in the output market may give raise to an additional source of market distortion. This market distortion is associated with the possibility that a follower in the permits market (exerting some degree of market power in the output market) may adopt a rival's cost-rising strategy as well.

The net effect of the three market distortions is a priori unknown. Strategic effects associated with followers' cost-rising strategies might ameliorate or aggravate the cost-effectiveness of emission permits markets but it may as well aggravate it, depending on the interplay of firms' actions in the permits and in the output market, as well as on firms' position in the permits market (buyer/ seller of permits). Since firms' position in the permits market is directly related to firms' initial endowments of permits, to the light of the regulator's specific objective, it is possible to design optimal allocation rules that restore cost-effectiveness (or achieves another policy objective).

In relation to the design of optimal allocation rules, this paper highlights that the optimal allocation rule may be extremely sensitive to the specific characteristics of the output market, in particular to goods substituability. Then, in line with Sartzetakis (1997), this paper emphasizes that the regulator is often faced with extremely demanding information needs.

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8. APPENDIX

Appendix A

In the first stage, firm j quotes the price of permits (q) that solves the following optimization problem

$$\max_{q} \{p_j y_j - c_j[y_j] + q\,(E_i - \alpha S) - h_j[\beta y_j - S + E_i]\} \tag{14}$$
$$\text{s.t.}$$
$$p_j = p_j^*[E_i^*[q]],$$
$$y_j = y_j^*[E_i^*[q]],$$
$$E_i = E_i^*[q],$$
$$q \geq 0.$$

where (i) $x_j = -x_i = E_i - \alpha S$ corresponds to the amount of permits sold (or bought, when $x_j < 0$) by firm j; and (ii) $a_j = \beta y_j - S + E_i$ corresponds to the amount of polluting emissions abated[12] by firm j.

In the second stage, firm i observes q and chooses the part of polluting emissions it wishes to cover with permits E_i by solving the following optimization problem.

$$\max_{E_i} \{p_i y_i - c_i[y_i] + q\,(\alpha S - E_i) - h_i[\beta y_i - E_i]\} \tag{15}$$
$$\text{s.t.}$$
$$p_i = p_i^*[E_i],$$
$$y_i = y_i^*[E_i],$$
$$E_i \geq 0.$$

where (i) $x_i = \alpha S - E_i$ corresponds to the amount of permits sold (or bought, when $x_i < 0$) by firm i; and (ii) $a_i = \beta y_i - E_i$ corresponds to the amount of polluting emissions abated by firm i. The solution to problem (15) defines the equilibrium level of permits used for production conditional on permits price, $E_i^*[q]$. The choice of E_i will then determine the amount of permits to buy. The amount of permits traded in equilibrium, conditional on permits price is equal to:

$$x_i^*[q] = -x_j^*[q] = \alpha S - E_i^*[q]. \tag{16}$$

Finally, in the third stage, given firms' choices regarding optimal use of permits for production and the consequent amount of permits bought or sold, firms strategically compete *à la Cournot* in

[12] From the market clearing condition, $x_i = -x_j$. Since $x_i = \alpha S - E_i$ and $x_j = (1-\alpha)S - E_j$, we obtain that $E_i + E_j = S$ and, accordingly, $a_j = \beta y_j - E_j$ can be written as $a_j = \beta y_j - S + E_i$.

the output market. Given q and $E_i[q]$, the equilibrium vector $\left(y_i^*[E_i,q], y_j^*[E_i,q]\right)$ simultaneously solves the following optimization problems:

$$\max_{y_i} \left\{p_i y_i - c_i[y_i] + q\left(\alpha S - E_i\right) - h_i[\beta y_i - E_i]\right\}, \quad (17)$$
$$s.t.$$
$$y_i \geq 0$$

$$\max_{y_j} \left\{p_j y_j - c_j[y_j] + q\left(E_i - \alpha S\right) - h_j[\beta y_j - S + E_i]\right\}, \quad (18)$$
$$s.t.$$
$$y_j \geq 0$$

Relying on backward induction techniques, the previous optimization problems can be solved sequentially starting from the last stage of interaction. In our analysis, we focus exclusively on interior solutions, in which the first order conditions lead to $y_k^* > 0$, $E_k^* > 0$, and $a_k^* > 0$. The variable x_k can be positive or negative. When firm k is a net seller of permits, it follows $x_k > 0$. Conversely, when firm k is a net buyer of permits, $x_k < 0$.

Proof of Lemma 1

The first order conditions in (5) correspond to firms best response functions $y_i[E_i, y_j]$ and $y_j[E_i, y_i]$. These functions define firms' optimal output conditional on the strategy of the rival. To compute the derivatives $\frac{\partial y_i^*[E_i]}{\partial E_i}$ and $\frac{\partial y_j^*[E_i]}{\partial E_i}$, one must take into account the *direct effect* of E_i (directly incorporated in firms' best response functions) but also the indirect effect of E_i, which affects firms' decisions through the output decision of the rival (the so-called *strategic effect*[13]). More precisely, the derivatives can be obtained from firms' best response functions as follows:

$$\frac{\partial y_i^*[E_i]}{\partial E_i} = \frac{\frac{\partial y_i[E_i,y_j]}{\partial E_i} + \frac{\partial y_i[E_i,y_j]}{\partial y_j}\frac{\partial y_j[E_i,y_i]}{\partial E_i}}{1 - \frac{\partial y_i[E_i,y_j]}{\partial y_j}\frac{\partial y_j[E_i,y_i]}{\partial y_i}}, \quad (19)$$

$$\frac{\partial y_j^*[E_i]}{\partial E_i} = \frac{\frac{\partial y_j[E_i,y_i]}{\partial E_i} + \frac{\partial y_j[E_i,y_i]}{\partial yi}\frac{\partial y_i[E_i,y_j]}{\partial E_i}}{1 - \frac{\partial y_j[E_i,y_i]}{\partial y_i}\frac{\partial y_i[E_i,y_j]}{\partial y_j}}, \quad (20)$$

where the application of the theorem of the implicit function to the first order conditions allows us to obtain the derivatives $\frac{\partial y_i[E_i,y_j]}{\partial E_i}$; $\frac{\partial y_j[E_i,y_i]}{\partial E_i}$; $\frac{\partial y_i[E_i,y_j]}{\partial y_j}$; and $\frac{\partial y_j[E_i,y_i]}{\partial y_i}$.

[13]For a rigorous differentiation between direct effect and strategic effect, see Tirole (1988).

Conditions in system (5) define two equalities: $\phi_i(E_i, y_i, y_j) = 0$ and $\phi_j(E_i, y_i, y_j) = 0$, where:

$$\left\{ \begin{array}{l} \phi_i(E_i, y_i, y_j) = p_i[y_i, y_j] + \frac{\partial p_i[y_i,y_j]}{\partial y_i} y_i - c_i'[y_i] - \beta h_i'[a_i] \\ \phi_j(E_i, y_i, y_j) = p_j[y_i, y_j] + \frac{\partial p_j[y_i,y_j]}{\partial y_j} y_j - c_j'[y_j] - \beta h_j'[a_j] \end{array} \right. \tag{21}$$

By the theorem of the implicit function:

$$\frac{\partial y_i[E_i, y_j]}{\partial E_i} = -\frac{\frac{\partial \phi_i(E_i,y_i,y_j)}{\partial E_i}}{\frac{\partial \phi_i(E_i,y_i,y_j)}{\partial y_i}} = -\frac{\beta h_i''}{\frac{\partial^2 \pi_i(y_i,y_j,E_i)}{\partial y_i^2}} > 0, \tag{22}$$

with $\frac{\partial^2 \pi_i(y_i,y_j,E_i)}{\partial y_i^2} = 2\frac{\partial p_i[y_i,y_j]}{\partial y_i} - c_i'' - \beta^2 h_i'' < 0$.

Similarly, the derivative:

$$\frac{\partial y_i[E_i, y_j]}{\partial y_j} = -\frac{\frac{\partial \phi_i(E_i,y_i,y_j)}{\partial y_j}}{\frac{\partial \phi_i(E_i,y_i,y_j)}{\partial y_i}} = -\frac{\frac{\partial p_i[y_i,y_j]}{\partial y_j}}{\frac{\partial^2 \pi_i(y_i,y_j,E_i)}{\partial y_i^2}}, \tag{23}$$

which is negative due to the negative denominator (second order conditions) and the negative numerator (given the properties of the demand functions). In the light of the negative sign of the derivative $\frac{\partial y_i[E_i,y_j]}{\partial y_j}$, firms' decisions are strategic substitutes, which is always the case in the context of quantity competition. In addition, we observe

$$-1 < \frac{\partial y_i[E_i, y_j]}{\partial y_j} < 0, \tag{24}$$

given that $\left| \frac{\partial^2 \pi_i(y_i,y_j,E_i)}{\partial y_i^2} \right| > \left| \frac{\partial p_i[y_i,y_j]}{\partial y_j} \right|$.

The derivative

$$\frac{\partial y_j[E_i, y_i]}{\partial E_i} = -\frac{\frac{\partial \phi_j(E_i,y_i,y_j)}{\partial E_i}}{\frac{\partial \phi_j(E_i,y_i,y_j)}{\partial y_j}} = \frac{\beta h_j''}{\frac{\partial^2 \pi_j(y_i,y_j,E_i)}{\partial y_j^2}}, \tag{25}$$

which is negative.

Concerning strategic substitutability of quantity decisions, we obtain:

$$\frac{\partial y_j[E_i, y_i]}{\partial y_i} = -\frac{\frac{\partial \phi_j(E_i,y_i,y_j)}{\partial y_i}}{\frac{\partial \phi_j(E_i,y_i,y_j)}{\partial y_j}} = -\frac{\frac{\partial p_j[y_i,y_j]}{\partial y_i}}{\frac{\partial^2 \pi_j(y_i,y_j,E_i)}{\partial y_j^2}}, \tag{26}$$

with $-1 < \frac{\partial y_j[E_i,y_i]}{\partial y_i} < 0$.

In the light of the signs of the previous derivatives we obtain that

$$\frac{\partial y_i^*[E_i]}{\partial E_i} = \frac{\underbrace{\frac{\partial y_i[E_i,y_j]}{\partial E_i}}_{+} + \underbrace{\frac{\partial y_i[E_i,y_j]}{\partial y_j}\frac{\partial y_j[E_i,y_i]}{\partial E_i}}_{+}}{\underbrace{1-\frac{\partial y_i[E_i,y_j]}{\partial y_j}\frac{\partial y_j[E_i,y_i]}{\partial y_i}}_{+}} > 0. \tag{27}$$

Similarly,

$$\frac{\partial y_j^*[E_i]}{\partial E_i} = \frac{\underbrace{\frac{\partial y_j[E_i,y_i]}{\partial E_i}}_{-} + \underbrace{\frac{\partial y_j[E_i,y_i]}{\partial y_i}\frac{\partial y_i[E_i,y_j]}{\partial E_i}}_{-}}{\underbrace{1-\frac{\partial y_j[E_i,y_i]}{\partial y_i}\frac{\partial y_i[E_i,y_j]}{\partial y_j}}_{+}} < 0, \tag{28}$$

as stated in Lemma 1.■

Proof of Lemma 2

From the first order condition in (5) follows that:

$$\mu_i^*[E_i] = p_i[y_i,y_j] - c_i'[y_i] - \beta h_i'[a_i] = -\frac{\partial p_i[y_i,y_j]}{\partial y_i} y_i. \tag{29}$$

Replacing $\mu_i^*[E_i]$ by $-\frac{\partial p_i[y_i,y_j]}{\partial y_i} y_i$ in (7), we obtain:

$$q - h_i'[y_i^*[E_i]] = \frac{dp_i^*[E_i]}{dE_i} y_i^*[E_i] + \left(-\frac{\partial p_i[y_i,y_j]}{\partial y_i} y_i\right)\frac{\partial y_i^*}{\partial E_i}. \tag{30}$$

Introducing in the previous equation $\frac{dp_i^*[E_i]}{dE_i} = \frac{\partial p_i}{\partial y_i}\frac{\partial y_i^*}{\partial E_i} + \frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*}{\partial E_i}$ where, for the sake of simplicity, we denote $p_i[y_i,y_j]$ as p_i, it follows that:

$$q - h_i'[y_i^*[E_i]] = \left(\frac{\partial p_i}{\partial y_i}\frac{\partial y_i^*}{\partial E_i} + \frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*}{\partial E_i}\right) y_i^*[E_i] + \left(-\frac{\partial p_i}{\partial y_i} y_i^*\right)\frac{\partial y_i^*}{\partial E_i}, \tag{31}$$

which simplifies to

$$q - h_i'[y_i^*[E_i]] = \frac{\partial p_i}{\partial y_j}\frac{\partial y_j^*}{\partial E_i} y_i^*[E_i] \tag{32}$$

as stated in Lemma 2.

Since *(i)* $\frac{\partial p_i}{\partial y_j} < 0$; *(ii)* $\frac{\partial y_j^*}{\partial E_i} < 0$; and *(iii)* $y_i^*[E_i] > 0$, it follows that for $E_i^*[q] : q > h_i'[y_i^*[E_i]]$.■

Proof of Lemma 3

From the first order condition in (5) follows that:

$$\mu_j^*[E_i] = p_j[y_i, y_j] - c_j'[y_j] - \beta h_j'[a_j] = -\frac{\partial p_j[y_i, y_j]}{\partial y_j} y_j. \quad (33)$$

Replacing $\mu_j^*[E_i]$ by $-\frac{\partial p_j[y_i,y_j]}{\partial y_j} y_j$ in (9), and introducing $\frac{dp_j^*[E_i]}{dE_i} = \frac{\partial p_j}{\partial y_i}\frac{\partial y_i^*}{\partial E_i} + \frac{\partial p_j}{\partial y_j}\frac{\partial y_j^*}{\partial E_i}$, we obtain:

$$q^* - \frac{(\alpha S - E_i^*[q])}{\frac{\partial E_i^*[q]}{\partial q}} - h_j' = -\frac{\partial p_j}{\partial y_i}\frac{\partial y_i^*}{\partial E_i} y_j^*, \quad (34)$$

with $-\frac{\partial p_j}{\partial y_i}\frac{\partial y_i^*}{\partial E_i} y_j^* > 0$.■

Proof of (13)

Applying the chain rule, the derivatives $\frac{\partial y_k^*}{\partial \alpha}$ can be decomposed as follows:

$$\frac{\partial y_k^*}{\partial \alpha} = \frac{\partial y_k^*[q, E_i]}{\partial E_i}\frac{\partial E_i^*[q]}{\partial q}\frac{\partial q^*}{\partial \alpha}, \quad (35)$$

where the derivatives $\frac{\partial y_k^*[q,E_i]}{\partial E_i}$ have already been obtained in Lemma , with $\frac{\partial y_k^*[E_k]}{\partial E_k} > 0$;and $\frac{\partial y_{-k}^*[E_k]}{\partial E_k} < 0$. Accordingly, it remains to obtain $\frac{\partial E_i^*[q]}{\partial q}$ and $\frac{\partial q^*}{\partial \alpha}$. The derivative $\frac{\partial E_i^*[q]}{\partial q}$ can be directly obtained from the application of the theorem of the implicit function to the first order condition associated with problem (15). Let us re-write this first order condition as $\zeta_i(E_i, q) = 0$, where the function $\zeta_i(E_i, q)$ is given by:

$$\zeta_i(E_i, q) = \frac{dp_i^*[E_i]}{dE_i} y_i^*[E_i] + \mu_i^*[E_i]\frac{\partial y_i^*}{\partial E_i} - q + h_i'[y_i^*[E_i]]. \quad (36)$$

From the theorem of the implicit function it follows that:

$$\frac{\partial E_i^*}{\partial q} = -\frac{\frac{\partial \zeta_i(E_i,q)}{\partial q}}{\frac{\partial \zeta_i(E_i,q)}{\partial E_i}} = -\frac{-1}{\frac{\partial \pi_i^2(E_i,q)}{\partial^2 E_i}} < 0, \quad (37)$$

where $\frac{\partial \pi_i^2(E_i,q)}{\partial^2 E_i}$ is necessarily negative due to the second order conditions associated with problem (15). Accordingly $\frac{\partial E_i^*}{\partial q} < 0$, which means that the higher permits' price, the less permits firm i uses in production (and therefore the more polluting emissions must be abated in order to maintain output production).

Analogously, the derivative $\frac{\partial q^*}{\partial \alpha}$ can be directly obtained from the application of the theorem of the implicit function to the first order condition associated with problem (14). First, re-write the first order condition as $\zeta_j(q, \alpha) = 0$, where:

$$\zeta_j(q, \alpha) = \frac{\partial E_i^*[q]}{\partial q}\left(q - h_j' + \frac{dp_j^*}{dE_i} y_j^* + \mu_j^*[E_i^*[q]]\frac{\partial y_j^*}{\partial E_i}\right) - (\alpha S - E_i^*[q]). \quad (38)$$

Now, apply the theorem of the implicit function, obtaining:

$$\frac{\partial q^*}{\partial \alpha} = -\frac{\frac{\partial \zeta_j(q,\alpha)}{\partial \alpha}}{\frac{\partial \zeta_j(q,\alpha)}{\partial q}} = -\frac{-S}{\frac{\partial^2 \pi_j(q,\alpha)}{\partial q^2}}. \tag{39}$$

This derivative is negative since the second order conditions associated with problem (14) require the denominator to be negative.

Accordingly, along the equilibrium path

$$\frac{\partial y_i^*}{\partial \alpha} = \underset{+}{\frac{\partial y_i^*[q, E_i]}{\partial E_i}} \underset{-}{\frac{\partial E_i^*[q]}{\partial q}} \underset{-}{\frac{\partial q^*}{\partial \alpha}} > 0, \tag{40}$$

and

$$\frac{\partial y_j^*}{\partial \alpha} = \underset{-}{\frac{\partial y_i^*[q, E_i]}{\partial E_i}} \underset{-}{\frac{\partial E_i^*[q]}{\partial q}} \underset{-}{\frac{\partial q^*}{\partial \alpha}} < 0. \tag{41}$$

As stated in (13) after a marginal variation of α, the equilibrium output of firm i changes in the same direction, while the equilibrium output of firm j changes in the opposite direction. ■