Environmental Policy and Inequality: A Matter of Life and Death*

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Abstract

This paper analyzes the economic implications of an environmental policy when we take into account the life expectancy of heterogeneous agents. In a framework where everyone suffers from pollution, but health status depends also on individual human capital, we find that the economy may be stuck in a trap where inequalities persistently grow, when the initial pollution intensity is too high. Moreover, it is emphasized that such inequalities are costly in the long run for the economy, notably in terms of health and growth. Therefore, we study whether a tax on pollution associated with an investment in pollution abatement can be used to overcome this situation. We show that a tighter environmental policy may allow the economy to escape the inequality trap, while it enhances its long-term growth rate, when initial inequalities are not too large.

JEL Classification: I14; O44; Q56; Q58

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1 Introduction

While the average life expectancy has globally increased during the last decades, health inequalities have not only persisted but widened sharply. For example, Singh and Siahpush (2006) highlight that the absolute difference in life expectancy between less-deprived groups and more deprived groups has risen by over 60% between 1980 and 2000 in the United States. Moreover, such disparities in terms of life expectancy represent a worldwide phenomenon. OECD (2013) reports that on average the gap in expected years of life between men with the highest level and the lowest level of education was of 7.8 years in 2010.[1] In addition to the cost of such inequalities in terms of wellbeing, their extent may have crucial economic consequences, through increased

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[1]More precisely, this value corresponds to the average expected years of life remaining at age 30 among 14 OECD countries.

health and social costs, lowered productivity, discouraged investments in education and savings, etc. Therefore, addressing health disparities has become a major political issue that many governments, including the United States, explicitly aim to eliminate (see the report of the U.S. Department of Health and Human Services , 2000).

In this paper, we seek to study if an environmental policy can represent a useful tool for removing existing health inequalities. The reason why we are interested in the role of the environment in this issue is twofold. First, regarding the determinants of life expectancy, there is considerable evidence that pollution has a positive and significant effect on mortality (see e.g. Bell and Davis , 2001 ; Pope et al. , 2002 ; Bell et al. , 2004 or Evans and Smith , 2005). At an aggregate level, studies estimate that 23 to 40% of all premature deaths can be attributed to environmental factors (see Pimentel et al. , 1998 and WHO , 2006), while air pollution alone was found to be responsible for around 7 million of premature deaths in 2012, representing 1 in 8 of total global deaths (WHO , 2014).[2]

Second, a key characteristic of the health effects of pollution seems to be their unequal repartition across population. In this regard, it is often stated that "environmental degradation is everyone's problem but it is especially a problem for the poor, who are less able to respond effectively".[3] This observation is broadly supported by empirical studies, which provide evidence of an increased susceptibility to mortality from pollution of disadvantaged populations in particular in terms of education (see e.g. Cifuentes et al. , 1999 ; Health Effects Institute , 2000 ; Pope et al. , 2002 ; O'Neill et al , 2003 ; Laurent et al. , 2007 or Cakmak et al. , 2011). For example, Zeka *et al.* (2006) reveal that the mortality risk associated with particulate matter PM_{10} for low educated individuals in the United States is more than twice the size of the risk for individuals with high education.[4] Moreover, education alone is also identified as an important determinant of mortality (see among others Elo and Preston , 1996 ; Lleras-Muney , 2005 ; Cutler and Lleras-Muney , 2010 or Miech et al , 2011). It stems from the fact that more educated individuals are more likely to live and work in better socio-economic conditions, but also to enjoy better information leading to healthier behaviors and to have a better access to healthcare. Through these channels, one can also readily see how human capital conditions the exposure and the susceptibility of an individual to pollution and hence the way she is affected by it. Therefore, it seems crucial to consider both pollution and human capital when dealing with health inequalities.

So far, from a theoretical point of view, there has been an increasing interest for life expectancy and its interaction with human capital and/or pollution. The positive effect of human capital on longevity has been considered in few contributions as Blackburn and Cipriani (2002), Castello-Climent and Domenech (2008) or Mariani et al. (2010), while the effect of pollution on mortality has been studied for example in Pautrel (2008) ; Jouvet et al. (2010) ; Mariani et al.

[2]Specifically, air pollution plays an important role in the development of respiratory and heart diseases (asthma, cancer, stroke...) which can be fatal.

[3]See e.g. the Resources 2020 lecture of Joseph E. Stiglitz given in October 2012 (http://www.rff.org).

[4]This study deals with US population in twenty cities between 1989 and 2000. Low education corresponds to less than 8 years of schooling, while high education refers to 13 years or more.

(2010) ; Varvarigos (2010) ; Raffin and Seegmuller (2014) or Palivos and Varvarigos (2015).[5] These contributions emphasize notably the risk of an underdevelopment trap with short life expectancy, the interest to implement an environmental policy and its consequences (positive or negative) on growth. However, no study has been conducted on the economic consequences of the unequal effects of pollution on longevity and only very little consideration has been given to the uneven distribution of health in general. Among the latter, Castello-Climent and Domenech (2008) analyze the relationship between inequality and a longevity index determined by parents' human capital. Focusing on a form of human capital depending only on an investment in time, they find that an economy may converge to a long-term equilibrium where two kinds of agents are highly unequal. Here, we extent this work in order to take into account a more complete relationship between longevity and inequalities. For that, we consider additional determinants of human capital accumulation that drive its convergence or divergence among population, i.e. the intergenerational transmission and the quality of the school system as in Tamura (1991) or de la Croix and Doepke (2003), and we care about the role of pollution on health.

Two recent papers, Aloi and Tournemaine (2013) and Schaefer (2015), take also into account the interplay between health, inequalities and pollution. In their models, households are heterogeneous in terms of human capital and suffer from different health effects of pollution. On one hand, Aloi and Tournemaine (2013) formalize a model where pollution has a direct effect on human capital accumulation and find that a tighter environmental policy always reduces income inequality, as lower-skilled are assumed to be more affected by pollution, and it can also improve growth, if the tax is not too high. On the other hand, in a framework where the health effects of pollution and human capital go through child mortality, Schaefer (2015) obtains that pollution imposes an increase in inequalities and a decline in growth, through the drop in the willingness of parents to invest in education. Here, we depart from these contributions by looking at a different health mechanism, i.e. adult mortality, and by endogenizing disparities in the health effect of pollution such that the vulnerability of an agent to pollution depends on her level of human capital and hence can evolve with it, in accordance with the empirical evidence mentioned above.

More precisely, we formalize an overlapping generations model, where agents can live up to three periods depending on their survival probability when old. Their longevity is endogenously determined by their human capital and the level of pollution. Pollution is represented as a flow due to aggregate production, while human capital is the source of endogenous growth but also of the heterogeneity among households.

We obtain that there may exist multiple balanced growth paths. There is always a long-term equilibrium without inequality, while one or several long-term equilibria with inequalities may also occur. Through a numerical illustration where the model is calibrated to match the United States features, which is particularly affected by health inequalities, we draw two cases. First, the

[5] Other contributions have also considered the morbidity aspect by considering the negative effect of pollution on productivity or on human capital accumulation (van Ewijk and van Wijnbergen , 1995 ; Aloi and Tournemaine , 2011 or Raffin , 2012).

balanced growth path without inequality is the only one but is a saddle point, so that it delimits a huge inequality trap where disparities worsen across time. Second, this long-term state without inequality is stable and coexists with a long-term equilibrium with inequalities delimiting also an inequality trap but of a smaller size. Thus, there always exists a trap where inequalities persistently grow but its size varies, especially according to the weight of intergenerational transmission in human capital accumulation which favors human capital divergence across generations. We highlight that the pollution intensity is critical for the long-term behaviors of the economy. The underlying mechanism goes through the fact that longevity influences the individuals' preferences for the future and hence their return on investment in education. When the pollution intensity is too large, the economy will be stuck in the inequality trap even if initial disparities are low.

Our analysis also emphasizes the cost that inequalities entail in the long run for the economy, notably in terms of growth and health. The importance of pollution in the risk to experience widening inequalities hence raises questions about the possible redistributive power of an environmental policy and about its effect on growth. We emphasize that a tighter tax on pollution associated with an investment in pollution abatement reduces the size of the inequality trap and thus can allow the economy to escape from the trap. It comes from the fact that the improvement in environmental quality increases more the return on investment in education of lower-skilled parents who are more vulnerable to the negative health effects of pollution than higher-skilled parents. However, a tighter tax on pollution may be insufficient to get out of the trap, especially when inequalities are too wide, because the health status of the more disadvantaged is so deteriorated that the necessary improvement in environment requires a very high tax rate which would prevent individuals to consume and hence damage their welfare. Consequently, such environmental policy is an efficient tool to address health inequalities but the government should implement it as soon as possible, before the gap among agents becomes too wide. Finally, we show that a tighter environmental policy enhances the long-term growth rate of the economy, through the positive effect of the decrease in pollution on life expectancy and the resulting incentive to invest more in education, which fosters human capital accumulation.

The paper is organized as follows. In Section 2, we set up the theoretical model. Section 3 focuses on the long-term equilibria of the economy. The implications of the environmental policy on the dynamics and growth are examined in Section 4. In Sections 3 and 4, we provide analytical results followed by a numerical illustration. Finally, Section 5 concludes and technical details are relegated to an Appendix.

2 The model

Consider an overlapping generations economy, with discrete time indexed by $t = 0, 1, 2, ..., +\infty$. Households may live three periods, childhood, adulthood, and old age depending on a longevity index. At each date t, a new generation of N heterogeneous agents is born. We assume no population growth, so we normalize the size of the population (N) to unity. Individuals are indexed by $i = u, s$, corresponding to the two groups of workers in the economy, lower-skilled (u) and higher-skilled (s), of size ξ and $1-\xi$ respectively.[6] The two groups of agents are characterized by an inequality in initial endowment in terms of human capital, so that agents born in $t-1$ differ only in the human capital level of their parents ($h^u_{t-1} < h^s_{t-1}$).

2.1 Consumer's behavior

Individual of type i born in $t-1$ cares abouts her adult consumption level c^i_t, her old-age consumption level d^i_{t+1} and about the future level of human capital of her child through paternalistic altruism h^i_{t+1}. Preferences are represented by the following utility function:

$$\ln(c^i_t) + \pi^i_t \left[\beta \ln(d^i_{t+1}) + \gamma \ln(h^i_{t+1})\right] \tag{1}$$

with γ and $\beta > 0$.

The weight π^i_t represents the agent's longevity or her survival probability in old age.[7] A higher life expectancy enhances the welfare obtained from consuming when old, but also from the future human capital of her child. Therefore, parents value more the future, for them and for their children, when they live longer. Reversely, parents with shorter life expectancy have shorter time horizons and hence put more weight on the present at the expense of investments whose return is future (including the future human capital of their children). To illustrate this point, Mariani et al. (2010) explain that parents will be relatively more affected by the success or the failure of their children if they live long enough to witness it, while Osang and Sarkar (2008) refer to the parent's "satisfaction from mental security in her old age of having an educated caregiver and companion", in line with the "companionship" argument of Ehrlich & Lui (1991).[8]

Longevity is an index of health status assumed to depend on individual's human capital h^i_t and pollution P_t, in accordance with empirical evidence mentioned in the Introduction. For the sake of simplicity, we assume a functional form for the life expectancy index, which is in line with the form adopted by Blackburn and Cipriani (2002), Chakraborty (2004), Castello-Climent and Domenech (2008) or Raffin and Seegmuller (2014):

Assumption 1

$$\pi^i_t = \pi\left(\frac{h^i_t}{P_t}\right) = \frac{\sigma h^i_t/P_t}{1 + h^i_t/P_t} \tag{2}$$

with $\sigma \in (0, 1]$*, the upper bound of longevity. Thus,* $\pi_t \in [0, 1]$*,* $\pi'(h^i_t/P_t) > 0$ *and* $\pi''(h^i_t/P_t) < 0$*.*

During childhood, individuals are reared by her parents and do not make any decisions. When

[6] The reason why we refer to lower-skilled individuals as u and to higher-skilled individuals as s is that the first category is relatively unskilled while the second is relatively skilled.

[7] Since individual i born in $t-1$ lives $2 + \pi^i_t$, we interchangeably use the terms "life expectancy", "longevity" and "survival probability" in the paper. We will also refer to it as health, although it is just one measure of health among others.

[8] Without such an assumption, we would have a counterfactual negative relationship between investments in education and longevity in the economy (see *e.g.* Jayachandran & Lleras-Muney , 2009 or Hansen , 2013).

adult, they supply inelastically one unit of labor remunerated at the wage w_t per unit of human capital. They allocate this income to consumption c_t^i, savings s_t^i and education of their children.

We assume that the education of a child requires that her parent invests $e_t^i \bar{h}_t$ units of human capital, such that schooling time e_t^i corresponds to an opportunity cost for the parent of $e_t^i \bar{h}_t w_t$ (as she does not use this efficient labor in production). Therefore, the total cost of education is the same for all types of agents and is relatively more expensive for poor parents.[9] When old, agents only consume. In line with Yaari (1965), Blanchard (1985) or Chakraborty (2004), we assume a perfect annuity market to abstract from the risk associated with uncertain lifetimes. Therefore, households deposit their savings to a mutual fund, which invests these amounts in physical capital. In return, the mutual fund provides them an actuarially fair annuity during retirement, corresponding to their savings increased by the gross return adjusted from their life expectancy R_{t+1}/π_t^i.

The two budget constraints for an adult born in $t-1$ are:

$$c_t^i + e_t^i \bar{h}_t w_t + s_t^i = w_t h_t^i \tag{3}$$

$$d_{t+1}^i = \frac{s_t^i R_{t+1}}{\pi_t^i} \tag{4}$$

Human capital of her child h_{t+1}^i depends on education e_t^i, human capital of the parents h_t^i and average human capital $\bar{h}_t$, representing the quality of the school system.

$$h_{t+1}^i = \epsilon (e_t^i)^{\mu} (h_t^i)^{\eta} (\bar{h}_t)^{1-\eta} \tag{5}$$

with $\epsilon > 0$, the efficiency of human capital accumulation. The parameters μ, η and their sum $\mu + \eta$ all $\in (0,1)$.[10] They are compatible with endogenous growth and capture respectively the efficiency of education and the intergenerational transmission of human capital within the family relatively to the transmission within the society.

The consumer program is summarized by:

$$\max_{e_t^i, s_t^i}, U(c_t^i, d_{t+1}^i, h_{t+1}^i) = \ln c_t^i + \pi_t^i \left[\beta \ln(d_{t+1}^i) + \gamma \ln(h_{t+1}^i) \right] \tag{6}$$

$$s.t \quad c_t^i + e_t^i w_t \bar{h}_t + s_t^i = w_t h_t^i$$

$$d_{t+1}^i = \frac{s_t^i R_{t+1}}{\pi_t^i}$$

$$h_{t+1}^i = \epsilon (e_t^i)^{\mu} (h_t^i)^{\eta} (\bar{h}_t)^{1-\eta}$$

[9] This assumption is perfectly equivalent to the one of de la Croix and Doepke (2003) or Schaefer (2015), where education is provided by teachers with a level of human capital equal to the average in the economy.

[10] We assume that $\mu + \eta < 1$ so that human capital convergence is possible. Education choice depends positively on h_t^i and negatively on $\bar{h}_t$ (representing the cost of education). Therefore, if $\mu + \eta > 1$, the return of h_t^i is always increasing and the return of $\bar{h}_t$ is negative, such that human capital convergence is impossible.

The maximization of this program (6) leads us to the following optimal choices in terms of education and savings.

$$e_t^i = \frac{\pi_t^i \gamma\mu}{1+\pi_t^i(\beta+\gamma\mu)}\frac{h_t^i}{\bar{h}_t} \tag{7}$$

$$s_t^i = \frac{\pi_t^i \beta}{1+\pi_t^i(\beta+\gamma\mu)} w_t h_t^i \tag{8}$$

Higher-skilled households invest more in savings and in children's education than lower-skilled households. The reason for this is twofold. First, there is a traditional income effect. The total wage of a worker depends on the wage rate w_t, equal for all agents, and on the level of human capital of this worker h_t^i. Therefore, higher-skilled parents benefit from a higher pay than lower-skilled one and can afford to spend more on education and savings. Second, longevity plays also an important role in the optimal choices for education and savings. A higher-skilled individual lives longer and hence gives more weight to the future, which enhance her savings and her investment in child's education.

One can notice that the optimal choice in terms of education is determined by the relative human capital of parents (with respect to the average level of human capital in the economy) rather than by the absolute level. The rational for this is that education implies an opportunity cost associated with the investment in human capital agents have to do to educate their children. This investment is the same for all and is equal to the average human capital in the economy, in order to represent a standardized educational system (where a unit of schooling time is equivalent for all types of agents). Therefore, the schooling time that parents choose for their children depends on their relative human capital and will be relatively more expensive for lower-skilled parents.

2.2 Production

Production of the consumption good is carried out by a single representative firm. The output is produced according to a constant returns to scale technology:

$$Y_t = AK_t^\alpha L_t^{1-\alpha} \tag{9}$$

where K_t is the aggregate stock of physical capital, L_t is the aggregate efficient labor supply to production, $A > 0$ measures the technology level, and $\alpha \in (0,1)$ is the share of physical capital in the production. Defining $y_t \equiv \frac{Y_t}{L_t}$ as the output per unit of labor and $k_t \equiv \frac{K_t}{L_t}$ as the capital labor ratio, we have the following production function per unit of labor:

$$y_t = Ak_t^\alpha$$

The government collect revenues through a tax rate $0 \leqslant \tau < 1$ on production, which is the source of pollution. The firm chooses inputs by maximizing profits $(1-\tau)Y_t - R_tK_t - w_tL_t$, such that:

$$w_t = A(1-\alpha)(1-\tau)k_t^{\alpha} \tag{10}$$

$$R_t = A\alpha(1-\tau)k_t^{\alpha-1} \tag{11}$$

2.3 Pollution

The index of pollution, we consider in this paper, embodies mainly air pollution which represents the world's largest single environmental health risk according to the World Health Organization. An interesting feature of such a pollution is that its direct harmful effect on human health are due to its level before absorption, deposition or dispersion in the atmosphere. Moreover, the air pollutants identified as the most significant health threats, i.e. particulate matter and ground-level ozone, remain only for short periods of time (from hours to weeks) in the atmosphere. Therefore, we choose to consider pollution as the flow currently emitted in the economy rather than as a stock.[11]

Environmental degradation is a by-product of the current production process ($Y_t = y_tL_t$). The government can use the revenue of the pollution tax (τ) to reduce pollution, by providing a public environmental maintenance $M_t > 0$. This maintenance, also called pollution abatement, represents a public investment in favor of the environment.[12] Besides, looking at air pollution, it is clear that it is not exclusively due to the production of goods but also to transportation, energy consumption *etc*. In order to consider the effect of the whole economic activity on pollution, we weight the pollution flow by the total labor force ($\bar{h}_t$) over the labor only used in the production of the consumption good (L_t). Thus, we define the law of motion of pollution as:

$$P_t = (ay_tL_t - bM_t)\,\frac{\bar{h}_t}{L_t} \tag{12}$$

where the parameters $a > 0$ and $b > 0$ correspond to the emission rate of production and the efficiency of environmental maintenance respectively.

The government budget is balanced at each period such that the level of public environmental maintenance is equal to $M_t = \tau y_tL_t$. Thus, the pollution flow at a period t can be rewritten as:

$$P_t = (a - b\tau)y_t\bar{h}_t \tag{13}$$

In fine, pollution is composed of two elements. While production per unit of efficient labor y_t (which depends on capital intensity k_t) represents an index of pollution intensity, the aggregate human capital $\bar{h}_t$ corresponds to a "scale effect" because even if human capital is not polluting *per se*, it defines the size of economic activity and hence the scale of pollution emissions.

[11] The same choice has been theoretically done *e.g.* by Pautrel (2008) and Aloi and Tournemaine (2013).

[12] For example, it may embody clean air strategies implemented to reduce the use of fossil fuels through investment in renewable energy or green transportation subsidy.

In order to ensure that human activities lead to a positive pollution flow regardless the tax level, we assume:

Assumption 2 $a > b$

3 Equilibrium

The market clearing conditions for capital and labor are given by:

$$K_{t+1} = \xi s_t^u + (1-\xi) s_t^s \tag{14}$$

and

$$L_t = \xi[h_t^u - e_t^u \bar{h}_t] + (1-\xi)[h_t^s - e_t^s \bar{h}_t] \tag{15}$$

The presence of $e_t^i \bar{h}_t$ illustrates the investment of parents in terms of human capital, which does not enter the production of the consumption good. The values of e_t^i and s_t^i are given by the optimal choices of consumers (7) and (8) with the wage corresponding to (10). Thus, the market clearing conditions can be rewritten as:

$$K_{t+1} = A(1-\alpha)(1-\tau)k_t^\alpha \left[\xi h_t^u \frac{\pi_t^u \beta}{1+\pi_t^u(\beta+\gamma\mu)} + (1-\xi)h_t^s \frac{\pi_t^s \beta}{1+\pi_t^s(\beta+\gamma\mu)}\right] \tag{16}$$

and

$$L_t = \bar{h}_t \left[\xi x_t^u \frac{1+\pi_t^u \beta}{1+\pi_t^u(\beta+\gamma\mu)} + (1-\xi)x_t^s \frac{1+\pi_t^s \beta}{1+\pi_t^s(\beta+\gamma\mu)}\right] \tag{17}$$

The variable $x_t^i \equiv \frac{h_t^i}{\bar{h}_t}$ corresponds to the relative human capital of an individual i in period t. Using (5), the relative human capital of her child is described by:

$$x_{t+1}^i = \epsilon \left(\frac{\pi_t^i \gamma\mu x_t^i}{1+\pi_t^i(\beta+\gamma\mu)}\right)^\mu \frac{1}{g_t}(x_t^i)^\eta \tag{18}$$

with $g_t \equiv \frac{\bar{h}_{t+1}}{\bar{h}_t}$, the growth factor of average human capital. From the definition of the average human capital ($\bar{h}_t = \xi h_t^u + (1-\xi)h_t^s$), we can deduce the expression of the growth of human capital:

$$g_t = \epsilon(\gamma\mu)^\mu \left[\xi\left(\frac{\pi_t^u}{1+\pi_t^u(\beta+\gamma\mu)}\right)^\mu (x_t^u)^{\mu+\eta} + (1-\xi)\left(\frac{\pi_t^s}{1+\pi_t^s(\beta+\gamma\mu)}\right)^\mu (x_t^s)^{\mu+\eta}\right] \tag{19}$$

As the pollution flow corresponds to

$$P_t = (a - b\tau)Ak_t^\alpha \, \bar{h}_t \tag{20}$$

we can rewrite the longevity given in (2) in terms of the individual relative human capital and of the capital-labor ratio:

$$\pi_t^i = \pi\left(\frac{x_t^i}{P_t/\bar{h}_t}\right) = \frac{\sigma x_t^i}{(a-b\tau)Ak_t^\alpha + x_t^i} \tag{21}$$

From equations (16) to (21), we can define the dynamics of the economy as:

Definition 1. *Given the initial condition $K_0 \geqslant 0$, $h_0^u \geqslant 0$ and $h_0^s \geqslant 0$, the intertemporal equilibrium is the sequence $(k_t, x_t^u, x_t^s)_{t\in\mathbb{N}}$ such that the following dynamic system is satisfied for all $t \geqslant 0$:*

$$\left\{\begin{array}{lcl} k_{t+1} & = & \frac{A(1-\tau)(1-\alpha)k_t^\alpha}{g_t}\left[\xi x_t^u \frac{\pi_t^u\beta}{1+\pi_t^u(\beta+\gamma\mu)} + (1-\xi)x_t^s\frac{\pi_t^s\beta}{1+\pi_t^s(\beta+\gamma\mu)}\right] \\ & & \left[\xi x_{t+1}^u \frac{1+\pi_{t+1}^u\beta}{1+\pi_{t+1}^u(\beta+\gamma\mu)} + (1-\xi)x_{t+1}^s\frac{1+\pi_{t+1}^s\beta}{1+\pi_{t+1}^s(\beta+\gamma\mu)}\right]^{-1} \\ x_{t+1}^u & = & \epsilon\left(\frac{\pi_t^u\gamma\mu}{1+\pi_t^u(\beta+\gamma\mu)}\right)^\mu \frac{1}{g_t}(x_t^u)^{\mu+\eta} \\ x_{t+1}^s & = & \epsilon\left(\frac{\pi_t^s\gamma\mu}{1+\pi_t^s(\beta+\gamma\mu)}\right)^\mu \frac{1}{g_t}(x_t^s)^{\mu+\eta} \end{array}\right. \tag{22}$$

with g_t and π_t^i given by (19) and (21) respectively.

The evolution of the economy is summarized by the laws of motion of the physical to labor ratio (k) and of the relative human capital of lower-skilled agents (x^u) and higher-skilled agents (x^s). We can rewrite the dynamical system (22) by substituting the growth of the average human capital by its expression given in (19). Moreover, from the definition of average human capital, we can express the relative human capital of higher-skilled x_t^s as a function of the one of lower-skilled workers: $x_t^s = \frac{1-\xi x_t^u}{1-\xi}$. After some computations, it follows that the dynamical system given in Definition 1 can be simplified as a two dimensions system in terms of the capital-labor ratio in the production of the consumption good k and the relative human capital of lower-skilled agents x^u.

$$\left\{\begin{array}{lcl} k_{t+1} & = & \frac{A(1-\tau)(1-\alpha)k_t^\alpha}{\epsilon(\gamma\mu)^\mu}\left[\xi x_t^u \frac{\pi_t^u\beta}{1+\pi_t^u(\beta+\gamma\mu)} + (1-\xi x_t^u)\frac{\pi_t^s\beta}{1+\pi_t^s(\beta+\gamma\mu)}\right] \\ & & \left[\xi x_{t+1}^u \frac{1+\pi_{t+1}^u\beta}{1+\pi_{t+1}^u(\beta+\gamma\mu)} + (1-\xi x_{t+1}^u)\frac{1+\pi_{t+1}^s\beta}{1+\pi_{t+1}^s(\beta+\gamma\mu)}\right]^{-1} \\ & & \left[\xi\left(\frac{\pi_t^u}{1+\pi_t^u(\beta+\gamma\mu)}\right)^\mu (x_t^u)^{\mu+\eta} + (1-\xi)\left(\frac{\pi_t^s}{1+\pi_t^s(\beta+\gamma\mu)}\right)^\mu\left(\frac{1-\xi x_t^u}{1-\xi}\right)^{\mu+\eta}\right]^{-1} \\ x_{t+1}^u & = & (x_t^u)^{\mu+\eta}\left(\frac{1-\xi x_t^u}{1-\xi}\right)^{-\mu-\eta}\left(\frac{1-\xi x_{t+1}^u}{1-\xi}\right)\left(\frac{\pi_t^u}{1+\pi_t^u(\beta+\gamma\mu)}\right)^\mu\left(\frac{\pi_t^s}{1+\pi_t^s(\beta+\gamma\mu)}\right)^{-\mu} \end{array}\right. \tag{23}$$

with π_t^i given by (21).

The relative human capital is a useful variable as it connects the two groups. A decrease in the relative human capital of lower-skilled individuals x^u corresponds to a proportional increase in the higher-skilled relative human capital x^s. The lower x^u is, the lower is the level of human capital of lower-skilled workers respectively to higher-skilled workers, and hence the wider are disparities. Consequently, we use the relative human capital of lower-skilled individuals x^u to approximate the level of inequalities in the economy. When there is no inequality, all individuals have the same human capital, which means that all relative human capital levels are equal ($x^u = x^s = 1$), while reversely when inequalities are maximum x^u and x^s tend to 0 and $\frac{1}{1-\xi}$ respectively.

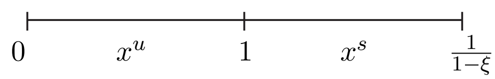


Figure 1: Representation of the relative human capital

In the rest of this section, we aim to analyze the long-run behavior of the economy. Thus, from Definition 1, we specify:

Definition 2. *A balanced growth path (BGP) is an equilibrium satisfying Definition 1 and where the stock of physical and human capital grow at the same and constant rate $(g-1)$.*

At a balanced growth path, the capital-labor ratio k_t, the growth of average human capital g_t, the relative human capital x_t^i and the flow of pollution P_t are constant.

3.1 Balanced growth paths with and without inequalities

From the dynamical system (23) and Definition 2, we obtain that:

Proposition 1 *Under Assumptions 1 and 2 and the condition $\alpha < \frac{1}{2}$:*

- ***[without inequality]:*** *There exists a balanced growth path without inequality $(k_E, 1, 1)$, whose growth rate is positive $(g_E - 1 > 0)$ for $\epsilon > \bar{\epsilon}$.*[13] *This BGP is locally stable when $\eta < \bar{\eta}(\tau)$ and corresponds to a saddle point otherwise. The thresholds $\bar{\epsilon}$ and $\bar{\eta}(\tau)$ correspond to:*

$$\bar{\epsilon} \equiv \left[\frac{(a-b\tau)Ak_E^{\alpha} + 1 + \sigma(\beta+\gamma\mu)}{\sigma\gamma\mu}\right]^{\mu} \quad \text{and} \quad \bar{\eta}(\tau) \equiv 1 - \mu\frac{2(a-b\tau)Ak_E^{\alpha} + (1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak_E^{\alpha} + (1+\sigma(\beta+\gamma\mu))}$$

- ***[With inequalities]:*** *There exists at least one BGP with inequalities when $2\mu + \eta > 1$ and $\eta < \tilde{\eta}(\tau)$, where the threshold $\tilde{\eta}(\tau)$ is the value of η such that:*[14]

$$\frac{A^{\frac{1}{\alpha}}(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}(a-b\tau)^{\frac{1-\alpha}{\alpha}}}{\epsilon\gamma^{\mu}(1+\sigma(\beta+\gamma\mu))^{\frac{1-\alpha(1+\mu)}{\alpha}}} = \frac{(1-\mu-\eta)^{\frac{1-\alpha}{\alpha}}\mu\left[(1+\beta\sigma)+\sigma\gamma(1-\mu-\eta)\right]}{(2\mu+\eta-1)^{\frac{1-\mu}{\alpha}}}$$

[13]Human capital accumulation has to be sufficiently efficient ($\epsilon > \bar{\epsilon}$) so that the growth rate is positive at the long-term state without inequality.

[14]The expression $\tilde{\eta}(\tau)$ is implicit but is deduced from Appendix 6.1.2. The effects of τ on $\bar{\eta}(\tau)$ and $\tilde{\eta}(\tau)$ are detailed in Section 4.1.

Proof. See Appendix 6.1. ■

From this proposition, we see that while there always exists a balanced growth path without inequality, one or several balanced growth path(s) characterized by inequalities among households may also occur. Therefore, the economy may converge in the long run to an equilibrium without inequality among households[15], but it is not always the case. It may also be stuck in situations where inequalities are persisting or increasing over time. In particular, when the long-term equilibrium without inequality E is a saddle point ($\eta > \bar{\eta}(\tau)$), human capital and longevity of individuals will most likely diverge within the population.[16]

The conditions such that we are in one situation or another depend on η and μ, i.e. the weights in human capital accumulation. The reason is that the persistence of inequalities or on the contrary the human capital convergence stem from the balance between several forces. Our model combines channels usually found in the literature on human capital and inequality (e.g. Tamura , 1991, Glomm and Ravikumar , 1992 or de la Croix and Doepke , 2003) and the uncommon longevity channel (see Castello-Climent and Domenech , 2008 for an exception).[17]

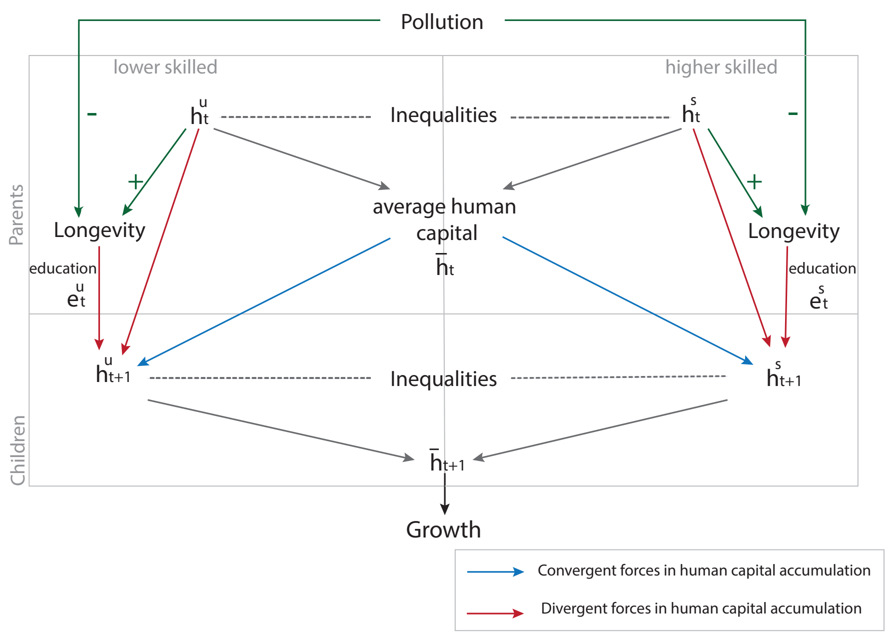


Figure 2: Structure of the model.

As represented in Figure 2, the human capital of a child is determined by the one of her parent (intergenerational transmission), by the education choice made by her parent and by average

[15]Such equilibrium is possible because agents differ only in the initial level of human capital. We do not assume in this model that poor individuals are less able to acquire skills.

[16]By definition $0 \leqslant x^u \leqslant 1$. Thus, the economy can still achieve the BGP where $x^u = 1$ for few initial conditions where x_0^u is very high and k_0 is very low. We analyze the extent of inequalities when $(k_E, 1, 1)$ is unstable in the numeriacal illustration Section 3.2.

[17]Castello-Climent and Domenech (2008) isolate the life expectancy channel by considering that human capital formation depends only on an investment in time. Thus, they do not consider the other aforementioned effects.

human capital representing the quality of the school system. The two first elements represent divergent forces in human capital accumulation, which perpetuate disparities among agents across generations, while the latter is a convergent force, which reduces them over time. Human capital disparities are transmitted to the next generation directly through the intergenerational spillover and more indirectly through education choices because human capital differences among parents correspond also to disparities in terms of income and life expectancy. Therefore, lower skilled agents can less afford the cost of educating their children and die sooner so that they give less value to the future and benefit less from their investment in education, whereas higher-skilled parents are more able and more willing to finance education. On the opposite, the presence of average human capital in the production of human capital represents a convergent force, which is crucial to ensure that human capital convergence is possible, as Tamura (1991) shows.

According to these effects, it emerges that the weights of the divergent forces, i.e. education and intergenerational transmission, in human capital accumulation has to be not too high so that it is possible for an economy to avoid inequalities in the long run. More precisely, we know that if the sum of these weights is at its maximum ($\mu + \eta \to 1$), there is only the long-term equilibrium without inequality and it is unstable, which means that inequalities would persistently grow for most of the initial conditions of the economy. Reversely, when it is at its minimum ($\mu + \eta \to 0$), the equilibrium without inequality is also the only one but is is stable so that inequalities vanish in the long run.[18]

Between these two extreme cases, the economy may or not exhibit long-term inequalities. In this paper, the diminishing return of the divergent forces in human capital accumulation (i.e. $\mu + \eta < 1$) is not sufficient to ensure convergence, contrary to what is usually obtained in the literature on human capital and inequality (see e.g. Tamura , 1991 or Glomm and Ravikumar , 1992).[19] This is due to the endogenous longevity. If longevity would be exogenous in our model, the growth of individual human capital would always be higher for lower-skilled so that the economy would always converge to the long-term equilibrium without inequality (under $\mu+\eta < 1$). Here, on the contrary, the determinants of individuals' health play a crucial role on the long-term behavior of the economy. Pollution and human capital affect the returns of investment in education through their impact on life expectancy. Moreover, this effect is not the same for all, as some agents have less human capital that makes them more susceptible to pollution. Therefore, when the levels of pollution and/or inequalities are too high, the growth of individual human capital can be lower or equal for lower-skilled.[20] In this case, the economy is stuck in an inequality trap, where disparities among households are persistent or widening across time.

[18] See Appendix 6.1.

[19] In Glomm and Ravikumar (1992), when education is public, human capital convergence always occurs, while when education is private, it is the case when the sum of the weights of education and of intergenerational transmission in human capital formation are lower than 1.

[20] The individual growth of human capital g^i is increasing and then decreasing in x^i. The maximum value of g^i is achieved in $x^i < 1$, while $g^i(0) = 0$ and $g^i(Max\{x^s\}) > 0$. Thus, there exists a level of x^u under (resp. above) which the individual growth of human capital is lower (resp. higher) for lower-skilled than higher-skilled agents $g^u < g^s$ (resp. $g^u > g^s$).

Analytically, we are not able to conclude on the dynamics of the long-term equilibrium(a) with inequalities. Therefore, we analyze numerically the model in the following section in order to get a more comprehensive overview of the different scenarios in which the economy may end up.

3.2 Numerical illustration

In this section, we provide a numerical analysis of the model in order to give more insight on the long-term behaviors of the economy. For that, we calibrate the model on the United States which is particularly affected by health inequalities and which officially intends to tacke this issue (U.S. Department of Health and Human Services , 2000). After motivating the choice of the parameters value, we study in details the characteristics of the different balanced growth paths.

3.2.1 Calibrations

To solve the model numerically, we give values to the parameters of technology and preferences so that they fit empirical observations and projections of the US economy. We assume that a period represents thirty years and the parameter in the longevity function σ is set to 0.9 to have realistic values of individuals life expectancy. Therefore, an individual starts working at 30, retires at 60 and may live for up to 87, according to her longevity.[21] In the real-business-cycle literature, the quarterly psychological discount factor is estimated to 0.99 (see Cooley , 1995 or de la Croix and Michel , 2002). Thus, β is set to $0.99^{4\times30} = 0.3$. Considering that the two groups of workers have the same size ($\xi = 0.5$)[22], we set the scale parameter ϵ to 6 and the preference for child human capital γ to 0.35 to match US data on the annual long-term growth rate (i.e. around 1.7%) and the US share of education expenditure in GDP at the balanced growth path (i.e. between 5 and 8%).[23]. Concerning the production technology, the share of physical capital in the production function α is set to 1/3 in accordance with empirical data and the total factor productivity A is a scale parameter set to 1. Concerning pollution, there is some flexibility. The weight of production and of environmental maintenance in the pollution flow are chosen to satisfy the condition $a > b$, ensuring that there is pollution emission in the presence of economic activities, i.e. $a = 0.6$ and $b = 0.4$.

The parameter μ represents the weight of education in human capital accumulation and corresponds more precisely in our model to the elasticity of human capital with respect to education. In the literature, the return to schooling in developed countries is estimated between 8% and 16% (see Ashenfelter and Krueger , 1994; Psacharopoulos , 1994 or Krueger and Lindahl , 2001). These figures correspond to Mincerian returns, which means that they include only an opportunity cost

[21] While the lower bound is equivalent to the US life expectancy at birth in the thirties (60), the upper bound is close the value expected by the US Census Bureau for 2060 (84 years).

[22] We provide a sensitivity analysis with respect to the repartition of the population ξ in Appendix 6.4.

[23] See the long-term projections for the US economy of OECD (2014a) on the growth rate and the Digest of Education Statistics 2012 of the US Department of Education for data on the education share.

(forgone earnings) and do not consider education expenditure. Following de la Croix and Doepke (2003), we assume that an additional year of schooling raises education expenditure by 20%. The resulting elasticity of schooling is from 0.4 to 0.8. Thus, we set μ to be 0.6. The weight of intergenerational transmission of human capital η being a key parameter in our analysis, we will consider all values allowed by the model, i.e. $\eta \in [0, 1-\mu)$. In the same way, we let the tax on pollution free by considering all tax rates $\tau \in [0, 1)$.[24]

3.2.2 Long-term behaviors of the economy

To get a clearer picture of the different scenarios, we analyze the existence and the dynamics of the balanced growth paths in the calibrated economy. For the set of parameters considered, we obtain the following result and represent the dynamics of the economy in Figure 3.[25]

Numerical result 1 (i) *When $\eta > \bar{\eta}(\tau)$, there exists a unique BGP, which is the one without inequality $(k_E, 1, 1)$. It is a saddle point, with a stable branch SS_E.*
(ii) *When $\eta < \bar{\eta}(\tau)$, there exist multiple balanced growth paths: the BGP without inequality $(k_E, 1, 1)$ and an additional BGP with inequalities (k_I, x_I^u, x_I^s). The first is stable, while the latter is a saddle point with a stable branch SS_I.*
Below SS_j, with $j = I, E$, the economy is stuck in an inequality trap, whereas above SS_j it converges to the BGP without inequality.

First, it is worth noticing that in the numerical analysis the threshold $\bar{\eta}(\tau)$, identified in Proposition 1, corresponds not only to the value under which the equilibrium without inequality is stable but also to the one under which a balanced growth path with inequalities appears, for all the levels of the tax.[26] It means that there always exists an inequality trap, where disparities are constantly widening, for realistic calibrations of the model on the US economy. Only the size of the trap varies. The higher is the weight of intergenerational transmission in human capital accumulation η, the heavier is the weight of divergent forces and therefore the larger is the size of the inequality trap.[27] In particular, when $\eta > \bar{\eta}(\tau)$ (i.e. case (i)), the economy is stuck in it for most of its initial conditions.

The underlying mechanism behind the coexistence of an inequality trap and a long-term equilibrium without inequality stems from the fact that the return on investment in education varies according to the levels of inequalities and pollution intensity in the economy, as we explained in Section 3.1. When the set of initial conditions is such that the economy is above the stable branch SS_j, inequalities are not too wide and the environmental quality is sufficiently high so that lower-skilled households have a higher return to education investment than higher-skilled

[24]More precisely, all the following numerical results are obtained by considering all η and τ with a pitch value of 0.000001 unit.

[25]The second dynamical equation in (23), represented by the blue curve in Figure 3, is discontinuous at $x^u = 1$.

[26] In Proposition 1, $\eta < \tilde{\eta}(\tau)$ such that at least one BGP with inequalities exists is a sufficient but not necessary condition.

[27]See the sensitivity analysis in Appendix 6.4 for more details on the effect of η on the results.

households, which allow them to narrow existing disparities over generations and to converge to an equal equilibrium in the long run.[28] On the contrary, when initial conditions are below the stable branch SS_j, disparities and/or pollution intensity are initially too large (low x_0^u and high k_0), thus health disparities are too wide for lower-skilled agents to be able to fill the gap (the return on the investment in education is larger for the higher skilled). Consequently, the economy is stuck in the trap where inequalities will steadily increase.

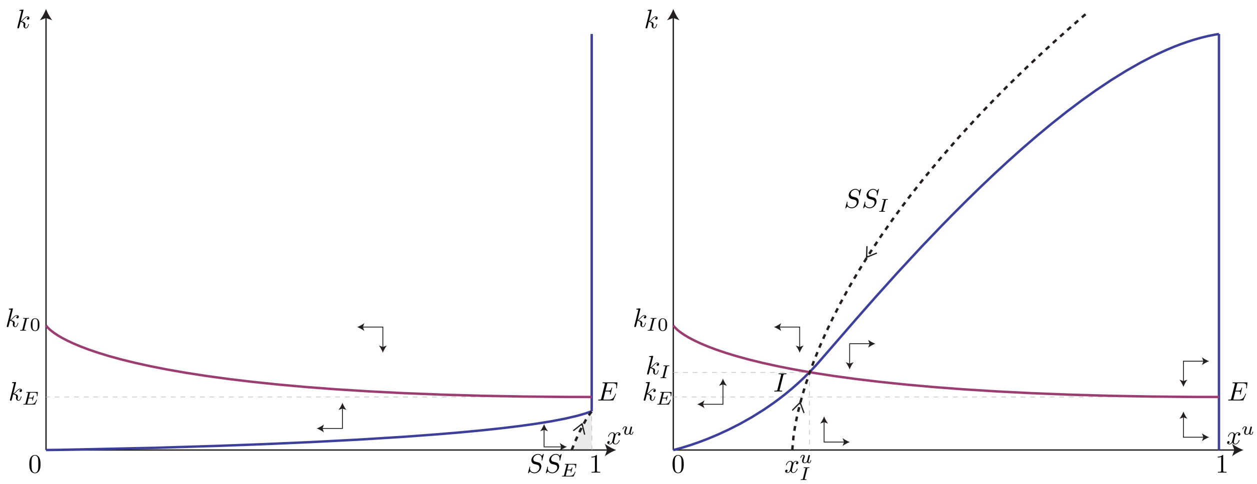


Figure 3: Representation of the dynamics of the economy in the cases (i) and (ii)

When the economy is in the inequality trap, it converges asymptotically to a state where the level of inequalities is maximum $(k_{I0}, x^u_{I0}, x^s_{I0})$. Such an extreme case is not achieved but it illustrates the constant worsening living conditions of the more disadvantaged households in such a trap. Indeed, at this state, the human capital of lower-skilled tends to zero ($x^u_{I0} \to 0$), as well as their income and their longevity π^u. Therefore, poor agents would die at the end of the second period of life (before retirement), and would not be able anymore of consuming, saving or educating their children so that the lower-skilled category would collapse. On the opposite, higher-skilled households tends to a state where they are richer, more educated and where they live longer than at the other long-term equilibria.

The numerical illustration gives us a clearer picture of how endogenous longevity favors the transmission of inequalities across generations. As Castello-Climent and Domenech (2008), it plays a crucial role. However, we consider additional forces in human capital accumulation and additional determinants of longevity that make our results contrast from theirs. While they obtain that the economy always converges to a long-term equilibrium with high but "sustainable" inequalities, we find that multiple long-term equilibria corresponding to a possible human capital convergence and an inequality trap where disparities among households are widening until a part of the population collapses. Moreover, we extent the mechanism of the transmission of inequality

[28]Note that this result holds even if the trap is delimited by E for initial conditions on the stable branch SS_E and above it (grey area in Figure 3) as the x^u is limited by definition to $[0, 1]$.

through longevity to the environmental dimension and emphasize that pollution is a critical determinant of the long-term behavior of the economy.

Concerning the characteristics of the two long-term equilibria E and I and of the end of the trap $I0$, the numerical illustration shows that:

Numerical result 2 *The wider inequalities in the long run ($x^s_{I0} > x^s_I > 1 > x^u_I > x^u_{I0}$)*

- *the larger are the physical capital to labor ratio and hence the pollution intensity ($k_{I0} > k_I > k_E$)*
- *the wider are disparities in terms of life expectancy ($\pi^s_{I0} > \pi^s_I > \pi_E > \pi^u_I > \pi^u_{I0}$) and the lower is the average life expectancy ($\pi_E > \bar{\pi}_I > \bar{\pi}_{I0}$)*
- *the lower is the long-term growth rate ($g_E > g_I > g_{I0}$).*

The numerical analysis of the model indicates that the long-term physical capital to labor ratio is higher at long-term states with inequalities. Given that the marginal propensity to save is larger for rich individuals, inequality consists in concentrating wealth in the hands of the one who saves more. It follows that aggregate savings is larger at such a state, which favors physical capital accumulation. The ratio capital labor being determinant for the pollution intensity, this latter is also larger at equilibria with inequalities.

The wider dispersion of human capital and the larger pollution intensity lead to wider inequalities in terms of health. While lower-skilled agents have a lower relative human capital and suffer from the higher pollution intensity, higher-skilled workers benefit from an increase in their relative human capital, which more than offsets the increase in the pollution intensity. The net outcome is that rich individuals live longer in the presence of inequality whereas poor individuals die sooner. Moreover, the loss of longevity for lower-skilled agents is larger than the benefit for higher-skilled so that the average life expectancy in the economy declines with inequality in the long run (even if population is equally distributed between the two categories).

In addition to this cost in terms of health, inequalities involve also a cost in terms of growth. As an illustration, for an intermediate weight of intergenerational transmission ($\eta = 0.2$) and without an environmental policy ($\tau = 0$), the estimated losses generated by inequalities are about 10 years of average life expectancy in the long run and about 0.4 percentage point of long-term economic growth per year, which represents a substantional damage on cumulative growth.[29] This latter result contributes to the wide literature on the implications of inequalities on growth, surveyed for example in Galor (2011). From these numerous analyses, it appears that inequalities have both positive and negative effects on economic performance and that the debate is still ongoing. Here, we conclude in favor of the theoretical and empirical studies that emphasize a net detrimental effect of disparities on economic growth through the channel of human capital. In this sense, OECD (2015) empirically confirms that inequalities reduce growth, by making

[29] The exact figures are $83,2$ and $72,4$ years for average life expectancy at E and $I0$ respectively and 1.68% and 1.31% for the compound annual economic growth rate at E and $I0$ respectively.

the poorer individuals less able to invest in education, whereas this phenomenon is explained theoretically through several mechanisms, as credit market imperfections (Galor and Zeira , 1993), fertility differentials (de la Croix and Doepke , 2003) or longevity differentials (Castello-Climent and Domenech , 2008). Our mechanism corresponds to the latter "longevity channel" where inequalities hamper growth by reducing the return on the investment in education.

In this section, we have emphasized that, for realistic calibrations of the model on the United States, there always exists a risk for the economy to be stuck in an inequality trap, where disparities are persistently widening, and that these inequalities are harmful for growth and health in the long run. Looking at the data, it appears that "inequality is today at its highest since data collection" in many OECD countries, including the US, and follows an upward trend.[30] Whether it is sufficient to imply that the United States is in such a trap or not, it certainly makes this risk a key issue. Moreover, we have pointed out the role played by pollution as a trigger or aggravating factor of this phenomenon. Even if inequalities are initially weak, the economy is most likely to be stuck in the trap when the pollution intensity is high. Therefore, it raises the question of the role that an environmental policy could play in breaking such vicious circle. This is the purpose of the following section.

4 Environmental policy implications

In this section, we assess the effect of a tighter pollution tax associated with an increase in public maintenance on the dynamics and on the growth of the economy. In particular, we want to know whether such an environmental policy can have a redistributive power, since we emphasized the role of pollution intensity in the persistence of inequalities, and whether it can allow to enhance the long-term economic growth, which is driven by human capital. For each point, we provide, first, an analytical analysis of the policy implications, while we illustrate them numerically in a second time.

4.1 Environmental policy implications on the balanced growth paths

From Proposition 1, we know that the conditions under the stability of the long-term equilibrium without inequality and under the existence of long-term equilibria with inequality both depend on the environmental tax τ. Examining how an increase in the pollution tax affects these thresholds, we make the following proposition.

Proposition 2 *Under Assumptions 1 and 2 and for $\alpha < 1/2$ and $2\mu + \eta > 1$, the thresholds $\bar{\eta}(\tau)$ and $\tilde{\eta}(\tau)$ depend positively on the tax rate τ. Moreover, there always exists a sufficient environmental tax rate τ such that:*

- *the long-term equilibrium without inequality is stable (i.e. $\eta < \bar{\eta}(\tau)$)*
- *there exists at least one BGP with inequalities (i.e. $\eta < \tilde{\eta}(\tau)$).*

[30] See OECD (2015) pages 5 and 25.

Proof. See Appendix 6.2. ■

When the tax on pollution is tighter, it allows the associated investment in environmental maintenance to increase, which reduces the pollution flow. Thus, the longevity of all agents increases, such that their preferences for the future and their investments in education increase too. It implies that the level of human capital is higher. However, although the individual growth of human capital increases for all, the decrease in pollution affects relatively more lower-skilled households, who are more susceptible to pollution.[31] Intuitively, even if all agents suffer from the same level of environmental damages, higher-skilled individuals are more able to protect themselves from the negative effect of pollution on health through knowledge, information or financial means.[32] All agents being proportionally taxed, it follows that an increase in the tax on pollution makes more likely the convergence toward the long-term equilibrium without inequality.

A tighter tax makes also more likely the existence of one or several balanced growth path(s) with inequalities. It could mean that the tax on pollution favors the persistence of inequality in long run but also that the tax restricts such inequality by stabilizing it (rather than let it worsen) or even by reducing the size of an inequality trap. To be able to conclude more precisely on the implications of the environmental tax on the long-term behavior of the economy, we use the numerical illustration started in Section 3.2.

For the parameters considered, we have found that the threshold $\bar{\eta}(\tau)$ represents the value under which both the long-term equilibrium E is stable and the long-term equilibrium with inequalities I exists, i.e. that the equilibrium without inequality E becomes stable when the one with inequalities I appears. It follows that:

Numerical result 3 *(i) When $\eta > \bar{\eta}(0)$, for low levels of the tax on pollution, there only exists the BGP without inequality E which is a saddle point and delimits the inequality trap. However, when the level of the tax becomes sufficiently high, η becomes lower than $\bar{\eta}(\tau)$, which implies that the long-term equilibrium without inequality E becomes stable, while the one with inequalities I appears and is a saddle point, delimiting the new trap.*[33]
(ii) When $\eta < \bar{\eta}(0)$, the condition such that the BGP without inequality E is stable and the BGP with inequalities I exists as a saddle point is satisfied for all levels of the tax on pollution.

Therefore, we deduce that, when $\eta > \bar{\eta}(0)$, a sufficient tax on pollution allows to reduce the size of the inequality trap, by making that the long-term equilibrium I rather than E delimits the trap. But beyond this case, we obtain that:

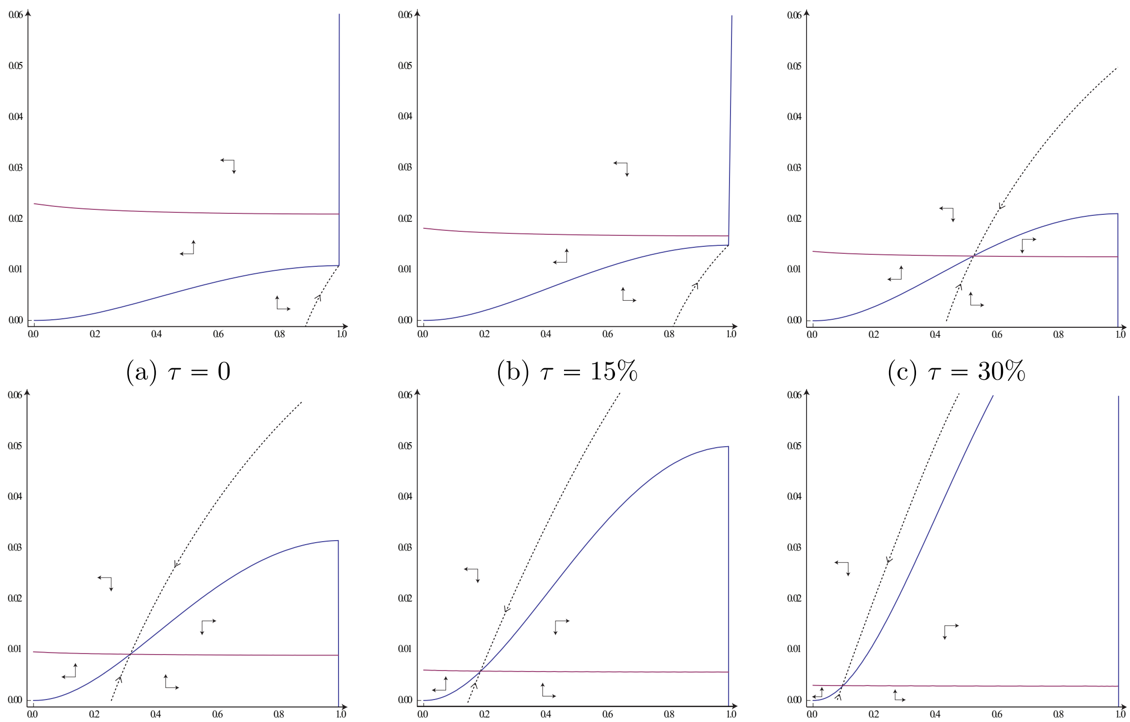
[31] More precisely, $\frac{\partial g^i}{\partial \tau} > 0$ but $\frac{\partial^2 g^i}{\partial \tau \partial x^i}$ is > 0 when x^i is small and < 0 when x^i is high, with a threshold equal to $\frac{(a-b\tau)Ak^{\alpha}(2\mu+\eta-1)}{(1+\sigma(\beta+\gamma\mu))(2-\mu-\eta)}$. Thus, the increase in the tax reduces this threshold, so that there are more levels of x^i such that lower-skilled are more affected by the decrease in pollution than higher-skilled.

[32] As Blackburn and Cipriani (2002), we do not formalize health expenditures in this paper (neither private nor public one), but we consider that individual human capital includes the capacity of agents to spend in healthcare. For models with explicit healthcare spending, see e.g. Pautrel (2008), Varvarigos (2010) or Raffin and Seegmuller (2014).

[33] Note that when τ tends to 1, $\bar{\eta}(\tau)$ is always greater than η.

Numerical result 4 *An increase in the environmental tax always decreases the size of the inequality trap. Thus, a tighter pollution tax can allow an economy to escape the trap and to converge toward the BGP without inequality.*

The net effect of the environmental policy on inequality is hence negative. As we detailed previously, a tighter tax on pollution enables to reduce environmental damages, which improves the life expectancy of agents and hence their ability to look to the future. In this way, households pay more attention to the education of their children. This effect is even stronger for the more disadvantaged households, who are relatively more sensitive to environmental damages. Therefore, a tighter tax on pollution can allow unequal economies to escape the trap and to reduce disparities along the convergence to a long-term equilibrium without inequality.



(a) $\tau = 0$ (b) $\tau = 15\%$ (c) $\tau = 30\%$

(d) $\tau = 45\%$ (e) $\tau = 60\%$ (f) $\tau = 75\%$

Figure 4: Phase diagrams when $\eta = 0.35$, i.e. $\eta > \bar{\eta}(0)$, for different tax levels, with x^u on the X-axis and k on the Y-axis.

To illustrate the Numerical Results 3 and 4, we use the Figure 4 corresponding to the case (i) where $\eta > \bar{\eta}(0)$ since it includes all possible scenarios. In these phase diagrams, the inequality trap is bounded by the dotted line. When the economy is on the left of this line it is on the trap and inequality gets worse across generations, whereas when it is on the right of the line it converges to the long-term equilibrium without inequality. For all the level of the tax, we observe that as τ increases, the inequality trap moves to the left, which decreases its size, hence there is more conditions such that the economy converges to the long-term equilibrium without inequality. For low values of the tax on pollution, we have that $\eta > \bar{\eta}(\tau)$. Thus, the only long-term equilibrium is the one without inequality and it delimits a huge inequality trap (phase diagram (a)). When the tax on pollution increases, the inequality trap reduces even if it is still bounded by E (phase diagram (b)). When the tax becomes sufficiently high (phase diagram (c)), the condition $\eta < \bar{\eta}(\tau)$ is now satisfied such that the BGP with inequalities I appears and E becomes stable, which sharply diminishes its size. Finally, as the environmental tax continues to increase, the BGP I continues to move to the left and hence the trap further decreases (phase diagrams (d) to (f)).

It is important to notice that, unlike Aloi and Tournemaine (2013), an environmental policy does not always reduce inequalities. Indeed, this policy reduces the size of the trap but not the extent of inequalities directly, such that an economy may be still in the inequality trap after the increase in pollution taxation and hence continue to exhibit growing disparities. This is due to the fact that the difference in susceptibility to pollution, considered in this model, is endogenously determined according to the individual's level of human capital. When inequalities are initially too wide and/or pollution intensity is initially too large, the improvement in environmental quality may be insufficient to overcome the excessive initial disparities on the return on education investment.

Moreover, even if technically there always exists a level of the tax such that the economy can escape the trap, the more a society is unequal and the higher is the pollution intensity before the strengthening of environmental policy, the higher is the tax rate necessary to get out of the inequality trap. In extreme cases, the required tax can even be close to 100%. It entails, that for reasonable levels of pollution taxation, the environmental policy may be insufficient to reduce inequalities in the economy. The term reasonable refers to the fact that a tax whose level is too high prevents individuals to consume and hence harms their welfare.

Consequently, we conclude that an environmental policy, consisting in a public investment in environmental protection financed by a tax on pollution, is an efficient tool to reduce inequalities, through its positive effect on health. Nevertheless, to escape the inequality trap, the government should implement such a policy as soon as possible because the later it reacts, the larger is the required level of the tax on pollution.

When the policy is inefficient, other kinds of policy are necessary and can be combine with an environmental policy to favor both education of the less-advantaged agents and the environment, so that the economy can escape the trap. For that, an educational policy or public health spending oriented toward the lower-skilled agents would be much more efficient than an income transfer to them as the lack of education is due to a low preference for the future.

4.2 Environmental policy implications on growth

The growth factor of human and physical capital at the balanced growth path without inequality is given by:

$$g_E = \epsilon \left[\frac{\sigma\gamma\mu}{(a - b\tau)Ak_E^{\alpha} + 1 + \sigma(\beta + \gamma\mu)} \right]^{\mu} \tag{24}$$

The study of the effect of the environmental policy on this long-term growth rate reveals that:

Proposition 3 *Under Assumptions 1 and 2, a tighter tax on pollution improves the growth rate at the BGP without inequality ($g_E - 1$).*

Proof. See Appendix 6.3 ■

Several effects occur. On one hand, a tighter tax on pollution implies a negative income effect. Firms report the pollution tax on the wage rates (w_t) and the returns on savings of households (R_t). On the other hand, a higher tax rate leads to more maintenance activities, which decrease the level of pollution and hence improve health. Through this channel, individuals' longevity enhances, which leads to greater preferences for future motives in the utility function, i.e. savings and children's education. Concerning savings, the negative income effect outweighs the longevity effect, such that savings decrease with a tighter tax on pollution. However, concerning education, a third effect operates. It is important to keep in mind that education is an investment in terms of human capital done by parents and corresponds to an opportunity cost associated with the fact that they do not use this efficient labor to produce, and hence are not paid for that. Therefore, the negative income effect of the tax is neutralized by its positive effect through the reduction of the opportunity cost. Education is thus affected positively by the environmental policy through longevity and the stock of human capital improves with the tax. Human capital being the engine of growth in the economy, the long-term growth rate is also enhanced with a tighter environmental policy.

To provide a more complete picture of the effects of the environmental policy on economic growth, we analyze numerically what happens at the other long-term states of the economy, i.e. the balanced growth path I and the limit of the trap $(k_{I0}, 0, x_{I0}^s)$ where inequalities are maximum. We observe the following:

Numerical result 5 *A tighter tax $\tau \in [0, 1)$ decreases the long-term capital labor ratios (k_I and k_{I0}) and improves the long-term growth rates (g_I and g_{I0}).*

Thus, a tighter tax on pollution enables to improve the long-term growth of an economy even when it does not enable the economy to escape the inequality trap. In the same way that the environmental policy favors the economic growth at the balanced growth path without inequality E, it improves the life expectancy of all agents, which stimulates their investment in education. Therefore, the growth rate of average human capital increases. However in this case, the policy is not sufficient to make the return to education of lower-skilled parents larger than the one of higher-skilled parents, so that inequalities in human capital and life expectancy continue to get worse.

Finally, note that despite the fact that the environmental policy is found to be always growth-enhancing in this framework, it is important to remember that an excessive level of the tax is welfare-damaging through its negative effect on the ability of individuals to consume.

5 Conclusion

In this paper, we are interested in how pollution can exacerbate inequalities through health. In accordance with empirical evidence, we consider the role of pollution and of individual human capital on life expectancy. Developing a model that we study analytically and that we illustrate by calibrating it on the United States, we show that multiple long-term scenarios are possible. The economy may converge to a long-term equilibrium without inequality or be stuck in a trap with steadily increasing inequalities. Pollution plays a crucial role in determining the long-term behavior of the economy. Even if inequalities are initially low, the economy will be in the trap for a sufficient pollution intensity. The underlying mechanism stems from the negative effect of pollution on longevity, which discourages investments as education. Moreover, we show that inequalities are costly for the economy in the long-run in terms of economic growth and average life expectancy.

Therefore, we assess the implications of an environmental policy, consisting in a tax on pollution and a public investment in pollution abatement. We emphasize that such a policy can promote the long-term economic growth and enable the economy to escape the inequality trap. This is because a decrease in pollution enhances individuals' longevity thereby encouraging them to invest in education, but also because it favors more individuals who are more susceptible to pollution as long as disparities among agents are not too wide. We conclude that an environmental policy can be a useful tool to enhance growth and tackle inequalities in addition to improving the environment, but for that the government should implement it as soon as possible. Thus, our analysis provides new insight on the relationship between pollution and inequalities and on the role that an environmental policy can play on economic conditions through the channel of life expectancy.

6 Appendix

6.1 Proof of Proposition 1

For technical reasons, the study of the existence of balanced growth path equilibria is done in two parts: when there are no inequality among households (i.e. $x^u = x^s = 1$) and when inequalities exist among them (i.e. $x^u \neq x^s \neq 1$) at such long-term states.[34]

[34]When $x^u = x^s = 1$, rewrite the system (23) as two functions of k depending on x^u requires to divide by zero in the second dynamical equation.

6.1.1 BGP without inequality $x^u = x^s = 1$

Existence and uniqueness of a BGP without inequality

The dynamics of the economy described in (23) when there is no inequality ($x^u = x^s = 1$) reduces to:

$$k_{t+1} \frac{1 + \beta\pi_{t+1}}{1 + \pi_{t+1}(\beta + \gamma\mu)} = \frac{A(1-\tau)(1-\alpha)\beta k_t^\alpha}{\epsilon(\gamma\mu)^\mu} \left[\frac{\pi_t}{1 + \pi_t(\beta + \gamma\mu)} \right]^{1-\mu} \tag{25}$$

with $\pi_t = \frac{\sigma}{1+(a-b\tau)Ak_t^\alpha}$.
At this BGP, we have $k_{t+1} = k_t = k$. We rewrite equation (25) as $\Omega_1 = \Omega_2$ with:

$$\Omega_1 \equiv k \frac{(a-b\tau)Ak^\alpha + 1 + \beta\sigma}{(a-b\tau)Ak^\alpha + 1 + \sigma(\beta + \gamma\mu)}$$

and

$$\Omega_2 \equiv \frac{A(1-\tau)(1-\alpha)\beta k^\alpha}{\epsilon(\gamma\mu)^\mu} \left[\frac{\sigma}{(a-b\tau)Ak^\alpha + 1 + \sigma(\beta + \gamma\mu)} \right]^{1-\mu}$$

When $\alpha < \frac{1}{2}$ and under Assumptions 1 and 2, Ω_1 is increasing and convex in k and characterized by $\Omega_1(0) = 0$ and $\lim_{k \to +\infty} \Omega_1(k) = +\infty$, while Ω_2 is increasing and concave in k with $\Omega_2(0) = 0$ and $\lim_{k \to +\infty} \Omega_2(k) = +\infty$. Moreover, $\Omega_1'(0) < \Omega_2'(0)$. Thus, the two curves cross only once and there exists a unique positive BGP without inequality $(k_E, 1, 1)$.

The growth factor at the BGP E corresponds to:

$$g_E = \epsilon \left[\frac{\sigma\gamma\mu}{(a-b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma\mu)} \right]^\mu \tag{26}$$

Thus, the growth rate is positive if $g_E > 1$, i.e.:

$$\epsilon > \left[\frac{(a-b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma\mu)}{\sigma\gamma\mu} \right]^\mu \equiv \bar{\epsilon} \tag{27}$$

Dynamics of the BGP without inequality

To analyze the stability of the BGP without inequality $(k_E, 1, 1)$, we compute the Jacobian matrix associated to the system (23) in E:

$$J(k_E, 1, 1) = \begin{pmatrix} \frac{\partial F_1}{\partial k_t}(k_E, 1, 1) & \frac{\partial F_1}{\partial x_t^u}(k_E, 1, 1) \\ \\ \frac{\partial F_2}{\partial k_t}(k_E, 1, 1) & \frac{\partial F_2}{\partial x_t^u}(k_E, 1, 1) \end{pmatrix} \tag{28}$$

where F_1 and F_2 are two implicit functions given by the dynamical system (23), such that: $k_{t+1} = F_1(k_t, x_t^u)$ and $x_{t+1}^u = F_2(k_t, x_t^u)$. Therefore, we use the implicit function theorem to obtain the elements of the Jacobian matrix.

The partial derivatives of the F_2 at a BGP (k, x^u) are given by:

$$\begin{gathered} \frac{\partial F_2}{\partial k_t}(k, x^u) = \mu \left(\frac{1-\xi x^u}{1-\xi} \right)^2 \left(\frac{(1-\xi)x^u}{1-\xi x^u} \right)^{2\mu+\eta} \left(\frac{(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu))} \right)^{\mu-1} \\ \left[\frac{(a-b\tau)A\alpha k^{\alpha-1}(1+\sigma(\beta+\gamma\mu))(x^u-1)}{((a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu)))^2} \right] \end{gathered} \tag{29}$$

$$
\begin{aligned}
&\frac{\partial F_2}{\partial x_t^u}(k,x^u) = \\
&\left(\frac{(1-\xi)x^u}{1-\xi x^u}\right)^{2\mu+\eta-1}\left(\frac{(a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu))}\right)^{\mu-1}\left(\frac{(1-\xi x^u)^2}{1-\xi}\right) \\
&\left[(2\mu+\eta)\frac{1-\xi}{(1-\xi x^u)^2}\left(\frac{(a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu))}\right)-\mu\,\frac{x^u}{1-\xi x^u}\,\frac{(1+\sigma(\beta+\gamma\mu))[(a-b\tau)Ak^\alpha+1+\sigma(\beta+\gamma\mu)]}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^2}\right]
\end{aligned} \tag{30}
$$

The partial derivatives of the F_1 at a BGP (k, x^u) are given by:

$$
\frac{\partial F_1}{\partial k_t}(k,x^u) = \frac{A(1-\tau)(1-\alpha)\beta}{\epsilon(\gamma\mu)^\mu(V_1V_2)^2}\;\frac{\alpha k^{\alpha-1}V_3V_1V_2+k^\alpha\left[V_3'V_1V_2-V_3\left(V_1V_2'+V_2\frac{\partial F_2}{\partial k_t}(k,x^u)W_1\right)\right]}{V_4} \tag{31}
$$

with

$$
V_1 \equiv \xi x^u\frac{(a-b\tau)Ak^\alpha+x^u(1+\sigma\beta)}{(a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu))}+(1-\xi x^u)\frac{(a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma\beta)}{(a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}
$$

$$
V_2 \equiv \xi\frac{\sigma^\mu(x^u)^{2\mu+\eta}}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^\mu}+(1-\xi)\frac{\sigma^\mu(\frac{1-\xi x^u}{1-\xi})^{2\mu+\eta}}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^\mu}
$$

$$
V_3 \equiv \xi\frac{\sigma(x^u)^2}{(a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu))}+\frac{\sigma\frac{(1-\xi x^u)^2}{1-\xi}}{(a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}
$$

$$
V_4 \equiv \frac{A(1-\tau)(1-\alpha)\beta k^\alpha}{\epsilon(\gamma\mu)^\mu}\frac{V_3}{V_2(V_1)^2}\left(\xi x^u\frac{(a-b\tau)A\alpha k^{\alpha-1}\sigma\gamma\mu x^u}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^2}+(1-\xi x^u)\frac{(a-b\tau)A\alpha k^{\alpha-1}\sigma\gamma\mu\frac{1-\xi x^u}{1-\xi}}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2}\right)+1
$$

$$
\begin{aligned}
W_1 = \xi\Bigg[&\frac{(a-b\tau)Ak^\alpha+x^u(1+\sigma\beta)}{(a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu))}-\frac{(a-b\tau)Ak^\alpha\sigma\gamma\mu x^u}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^2} \\
&-\frac{(a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma\beta)}{(a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}+\frac{(a-b\tau)Ak^\alpha\sigma\gamma\mu\frac{1-\xi x^u}{1-\xi}}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2}\Bigg]
\end{aligned}
$$

$$
V_2' = -\mu\sigma^\mu(a-b\tau)A\alpha k^{\alpha-1}\left[\frac{\xi(x^u)^{2\mu+\eta}}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^{1+\mu}}+\frac{(1-\xi)\left(\frac{1-\xi x^u}{1-\xi}\right)^{2\mu+\eta}}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^{1+\mu}}\right]
$$

$$
V_3' = -\sigma(a-b\tau)A\alpha k^{\alpha-1}\left[\frac{\xi(x^u)^2}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^2}+\frac{\frac{(1-\xi x^u)^2}{1-\xi}}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2}\right]
$$

and

$$
\frac{\partial F_1}{\partial x_t^u}(k,x^u) = \frac{\frac{A(1-\tau)(1-\alpha)k^\alpha\beta}{\epsilon(\gamma\mu)^\mu(V_1V_2)^2}\left[W_3'V_1V_2-V_3\left(V_1W_2'+V_2\frac{\partial F_2}{\partial x_t^u}(k,x^u)W_1\right)\right]}{V_4} \tag{32}
$$

with

$$
\begin{aligned}
W_2' = \xi\sigma^\mu\Bigg[&\frac{(2\mu+\eta)(x^u)^{2\mu+\eta-1}}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^\mu}-\frac{(x^u)^{2\mu+\eta}\mu(1+\sigma(\beta+\gamma\mu))}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^{1+\mu}} \\
&-\frac{(2\mu+\eta)(\frac{1-\xi x^u}{1-\xi})^{2\mu+\eta-1}}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^\mu}+\frac{\left(\frac{1-\xi x^u}{1-\xi}\right)^{2\mu+\eta}\mu(1+\sigma(\beta+\gamma\mu))}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^{1+\mu}}\Bigg]
\end{aligned}
$$

$$
W_3' = \xi\sigma\left[\frac{2x^u(a-b\tau)Ak^\alpha+(x^u)^2(1+\sigma(\beta+\gamma\mu))}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^2}-\frac{2\frac{1-\xi x^u}{1-\xi}(a-b\tau)Ak^\alpha+\left(\frac{1-\xi x^u}{1-\xi}\right)^2(1+\sigma(\beta+\gamma\mu))}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2}\right]
$$

At the BGP without inequality, $\frac{\partial F_2}{\partial k_t}(k_E,1) = \frac{\partial F_1}{\partial x_t^u}(k_E,1) = 0$, while $\frac{\partial F_1}{\partial k_t}(k_E,1)$ and $\frac{\partial F_2}{\partial x_t^u}(k_E,1)$ are greater than 0. Therefore, the two eigenvalues are given by: $\frac{\partial F_1}{\partial k_t}(k_E,1)$ and $\frac{\partial F_2}{\partial x_t^u}(k_E,1)$. Under Assumptions 1 and 2, under the condition $\alpha < 1/2$, and substituting the expression of k_E given in (25) at the BGP $(k_E,1,1)$, we have $0 < \frac{\partial F_1}{\partial k_t}(k_E,1) < 1$. Thus, the BGP E is stable iif $\frac{\partial F_2}{\partial x_t^u}(k_E,1) < 1$, which is equivalent to:

$$1-\left(2\mu+\eta-\frac{\mu(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak_E^{\alpha}+(1+\sigma(\beta+\gamma\mu))}\right)>0 \tag{33}$$

When the condition (33) is satisfied, the BGP without inequality is locally stable (a sink), otherwise it is a saddle point. This condition can be rewritten in terms of η as $\eta<\bar{\eta}(\tau)$ with

$$\bar{\eta}(\tau)\equiv 1-\mu\frac{2(a-b\tau)Ak_E^{\alpha}+(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak_E^{\alpha}+(1+\sigma(\beta+\gamma\mu))} \tag{34}$$

Thus, the BGP without inequality E is stable when $\eta<\bar{\eta}(\tau)$ and corresponds to a saddle point when $\eta>\bar{\eta}(\tau)$. Note that when $\mu+\eta\to 0$, the condition (33) is always satisfied, i.e. the BGP E is always stable, while when $\mu+\eta\to 1$, (33) is never satisfied, i.e. the BGP E is always a saddle.

□

6.1.2 BGP with inequalities $x^u \neq x^s$

Now, we study the existence and uniqueness of a BGP with inequalities ($x^u<1<x^s$). After computations, the dynamical system (23) at the BGP with inequalities, where $x^u_{t+1}=x^u_t=x^u\neq 1$ and $k_{t+1}=k_t=k$, corresponds to:

$$\begin{cases} k^{1-\alpha}\frac{\epsilon(\sigma\gamma\mu)^{\mu}}{A(1-\tau)(1-\alpha)\beta}\left[\frac{\left[(a-b\tau)Ak^{\alpha}+\frac{1-\xi x^u}{1-\xi}(1+\sigma\beta)\right][(a-b\tau)Ak^{\alpha}+x^u(1+\sigma(\beta+\gamma\mu))]+\xi x^u(a-b\tau)Ak^{\alpha}\sigma\gamma\mu\left(\frac{1-\xi x^u}{1-\xi}-x^u\right)}{(a-b\tau)Ak^{\alpha}\sigma\left(\frac{(1-\xi x^u)^2}{1-\xi}+\xi(x^u)^2\right)+\frac{1-\xi x^u}{1-\xi}\sigma x^u(1+\sigma(\beta+\gamma\mu))}\right] \\ \left[\frac{\xi(x^u)^{2\mu+\eta}}{[(a-b\tau)Ak^{\alpha}+x^u(1+\sigma(\beta+\gamma\mu))]^{\mu}}+\frac{(1-\xi)\left(\frac{1-\xi x^u}{1-\xi}\right)^{2\mu+\eta}}{\left[(a-b\tau)Ak^{\alpha}+\left(\frac{1-\xi x^u}{1-\xi}\right)(1+\sigma(\beta+\gamma\mu))\right]^{\mu}}\right]-1=0\equiv\mathcal{A}(k,x^u) \\ k=\left[\frac{1+\sigma(\beta+\gamma\mu)}{(1-\xi)A(a-b\tau)}\frac{(1-\xi)x^u(1-\xi x^u)^{\frac{2\mu+\eta-1}{\mu}}-((1-\xi)x^u)^{\frac{2\mu+\eta-1}{\mu}}(1-\xi x^u)}{((1-\xi)x^u)^{\frac{2\mu+\eta-1}{\mu}}-(1-\xi x^u)^{\frac{2\mu+\eta-1}{\mu}}}\right]^{\frac{1}{\alpha}}\equiv\Psi_2(x^u) \end{cases} \tag{35}$$

Properties of the function Ψ_2

The second equation of (35) defines $k=\Psi_2(x^u)$. Under Assumptions 1 and 2 and the conditions $2\mu+\eta>1$ and $\alpha<1/2$, the properties of this function are:

- $Sign(\Psi_2')=u'v-uv'$ with:

$$u=(1-\xi)x^u(1-\xi x^u)^{\frac{2\mu+\eta-1}{\mu}}-((1-\xi)x^u)^{\frac{2\mu+\eta-1}{\mu}}(1-\xi x^u)<0$$

$$v=((1-\xi)x^u)^{\frac{2\mu+\eta-1}{\mu}}-(1-\xi x^u)^{\frac{2\mu+\eta-1}{\mu}}<0$$

$$v'=\frac{2\mu+\eta-1}{\mu}\left[((1-\xi)x^u)^{\frac{\mu+\eta-1}{\mu}}(1-\xi)+(1-\xi x^u)^{\frac{\mu+\eta-1}{\mu}}\xi\right]>0$$

$$\begin{aligned} u' \quad = \quad & \xi((1-\xi)x^u)^{\frac{2\mu+\eta-1}{\mu}}+(1-\xi)(1-\xi x^u)^{\frac{2\mu+\eta-1}{\mu}} \\ & -\frac{2\mu+\eta-1}{\mu}\left[\xi((1-\xi)x^u)(1-\xi x^u)^{\frac{\mu+\eta-1}{\mu}}+(1-\xi)(1-\xi x^u)((1-\xi)x^u)^{\frac{\mu+\eta-1}{\mu}}\right] \end{aligned}$$

  We rewrite this last equation as $u'=\mathcal{I}(x^u)-\mathcal{J}(x^u)$, where $\mathcal{I}(x^u)$ corresponds to the first part (first line) of the equation and $\mathcal{J}(x^u)$ corresponds to the second one.

  - $\mathcal{I}(0)=(1-\xi)$, $\mathcal{I}(1)=(1-\xi)^{\frac{2\mu+\eta-1}{\mu}}>\mathcal{I}(0)$ and $\mathcal{I}'(x^u)>0$.

- $\mathcal{J}(0) = +\infty$, $\mathcal{J}(1) = \frac{2\mu+\eta-1}{\mu}(1-\xi)^{\frac{2\mu+\eta-1}{\mu}} < \mathcal{I}(1)$ and

$$\begin{aligned}\mathcal{J}'(x^u) &= \tfrac{2\mu+\eta-1}{\mu}\Big[\xi(1-\xi)\left((1-\xi x^u)^{\frac{\mu+\eta-1}{\mu}} - ((1-\xi)x^u)^{\frac{\mu+\eta-1}{\mu}}\right) \\ &\quad + \tfrac{\mu+\eta-1}{\mu}\left((1-\xi)^2(1-\xi x^u)((1-\xi)x^u)^{\frac{\eta-1}{\mu}} - \xi^2((1-\xi)x^u)(1-\xi x^u)^{\frac{\eta-1}{\mu}}\right)\Big]\end{aligned} \tag{36}$$

  $\mathcal{J}'(x^u)$ is an increasing function of x^u ($\mathcal{J}''(x^u) > 0$) which is always negative in $x^u = 0$ but may become positive for high x^u when $\xi > 1/2$ ($\mathcal{J}'(1) > 0$ when $\xi > 1/2$).
- u' is negative as long as $\mathcal{J}(x^u) > \mathcal{I}(x^u)$. Thus, we can define a threshold $\hat{x}^u \in (0,1)$ under which u' is negative and above which u' is positive for high level of ξ.

- The condition $u' < 0$ is sufficient to ensure that $\Psi_2' > 0$. Thus, we show that there exists a threshold $\hat{x}^u \in (0,1)$ under which Ψ_2 is an increasing function of x^u and above which Ψ_2 may become decreasing (for high level of ξ).
- Moreover, $\Psi_2 \geqslant 0 \; \forall x^u$, $\Psi_2(0) = 0$ and

$$\lim_{x^u \to 1} \Psi_2(x^u) = \left[\frac{1+\sigma(\beta+\gamma\mu)}{A(a-b\tau)} \frac{1-\mu-\eta}{2\mu+\eta-1}\right]^{\frac{1}{\alpha}} > 0 \tag{37}$$

Properties of the function Ψ_1

The first equation of (35), $\mathcal{A}(k, x^u) = 0$, allows to define $k = \Psi_1(x^u)$, with $\Psi_1(x^u)$ an implicit function. Under Assumptions 1 and 2 and the conditions $2\mu + \eta > 1$ and $\alpha < 1/2$, we obtain that $\Psi_1(0)$ and $\Psi_1(1)$ are equal to two positive constants.

More precisely, in $x^u = 0$ we have:

$$\begin{aligned}&\mathcal{A}(k,0) = 0 \Leftrightarrow k^{1-\alpha} \tfrac{\epsilon(\sigma\gamma\mu)^\mu(1-\xi)^{2-2\mu-\eta}}{A(1-\tau)(1-\alpha)\beta\sigma} \tfrac{(a-b\tau)Ak^\alpha + \frac{1}{1-\xi}(1+\sigma\beta)}{\left[(a-b\tau)Ak^\alpha + \left(\frac{1}{1-\xi}\right)(1+\sigma(\beta+\gamma\mu))\right]^\mu} = 1 \\ &\Leftrightarrow k^{1-\alpha}\left[(a-b\tau)Ak^\alpha + \tfrac{1+\sigma\beta}{1-\xi}\right] = \tfrac{A(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu(1-\xi)^{2-2\mu-\eta}}\left[(a-b\tau)Ak^\alpha + \tfrac{1+\sigma(\beta+\gamma\mu)}{1-\xi}\right]^\mu\end{aligned} \tag{38}$$

We analyze the properties of $\Psi_1(0)$ by studying the last equation. For that, we name the function on the left side $f_0(k)$ and the function on the right side $g_0(k)$. Their properties are:

- f_0 is increasing and concave in k, $f_0(0) = 0$ and $\lim_{k\to\infty} f_0(k) = +\infty$.
- g_0 is increasing and concave in k, $g_0(0)$ is equal to a positive constant and $\lim_{k\to\infty} g_0(k) = +\infty$.
- In $k = 0$, $g_0 > f_0$. The two curves have not cross yet, thus $\Psi_1(0) > 0$.
- When $k \to \infty$, we have $\lim_{k\to\infty} f_0 > \lim_{k\to\infty} g_0$. Thus, the two curves cross only once and for a positive and finite value of k.

Therefore, $\Psi_1(0)$ is always a finite and positive constant.

In the same way, in $x^u = 1$ we have:

$$\begin{aligned}&\mathcal{A}(k,1) = 0 \Leftrightarrow k^{1-\alpha} \tfrac{\epsilon(\sigma\gamma\mu)^\mu}{A(1-\tau)(1-\alpha)\beta\sigma} \tfrac{(a-b\tau)Ak^\alpha + 1 + \sigma\beta}{[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)]^\mu} = 1 \\ &\Leftrightarrow k^{1-\alpha}\left[(a-b\tau)Ak^\alpha + 1 + \sigma\beta\right] = \tfrac{A(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^\mu\end{aligned} \tag{39}$$

As previously, we study the properties of $\Psi_1(1)$, by looking at the last equation. We name the function on the left side $f_1(k)$ and the function on the right side $g_1(k)$, whose properties are:

- f_1 is increasing and concave in k, $f_1(0) = 0$ and $\lim_{k\to\infty} f_1(k) = +\infty$.
- g_1 is increasing and concave in k, $g_1(0)$ is equal to a positive constant and $\lim_{k\to\infty} g_1(k) = +\infty$.
- In $k = 0$, $g_1 > f_1$, the two curves have not cross yet thus $\Psi_1(1) > 0$.
- When $k \to \infty$, we have $\lim_{k\to\infty} f_1 > \lim_{k\to\infty} g_1$. Thus, the two curves cross only once and for a positive and finite value of k.

Therefore, $\Psi_1(1)$ is equal to a finite and positive constant.

Comparison of Ψ_1 and Ψ_2

From the study of the properties of Ψ_1 and Ψ_2, we know that $\Psi_1(0) > 0$ and $\Psi_2(0) = 0$, it entails that $\Psi_1(0) > \Psi_2(0)$. It follows that if $\Psi_1(1) < \lim_{x^u\to 1} \Psi_2(x^u)$, there exists at least one BGP with inequalities.

From the study of Ψ_1, the condition $\Psi_1(1) < \lim_{x^u\to 1} \Psi_2(x^u)$ is equivalent to $f_1(k) > g_1(k)$ in $k = \lim_{x^u\to 1} \Psi_2(x^u)$ given in (37). We obtain that $\Psi_1(1) < \lim_{x^u\to 1} \Psi_2(x^u)$ if

$$\frac{A^{\frac{1}{\alpha}}(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}(a-b\tau)^{\frac{1-\alpha}{\alpha}}}{\epsilon\gamma^{\mu}(1+\sigma(\beta+\gamma\mu))^{\frac{1-\alpha(1+\mu)}{\alpha}}} < \frac{(1-\mu-\eta)^{\frac{1-\alpha}{\alpha}}\mu\left[(1+\beta\sigma)+\sigma\gamma(1-\mu-\eta)\right]}{(2\mu+\eta-1)^{\frac{1-\mu}{\alpha}}} \tag{40}$$

where the right side of the equation corresponds to a function $\mathcal{R}(\eta)$ which is decreasing in η. It follows that this condition can be rewritten as $\eta < \tilde{\eta}(\tau)$ where $\tilde{\eta}(\tau)$ is implicitly given by (40). Note that this condition is never satisfied when $\eta \to 1-\mu$, but can be otherwise.
Thus, under Assumptions 1 and 2 and the conditions $2\mu+\eta > 1$ and $\alpha < 1/2$, the condition $\eta < \tilde{\eta}(\tau)$ is sufficient so that there exists at least one BGP with inequalities.

Note that when $\mu+\eta \to 0$ or $\mu+\eta \to 1$, Ψ_2 corresponds to strictly negative values of k $\forall x^u$, so that there is no BGP with inequalities in these cases.

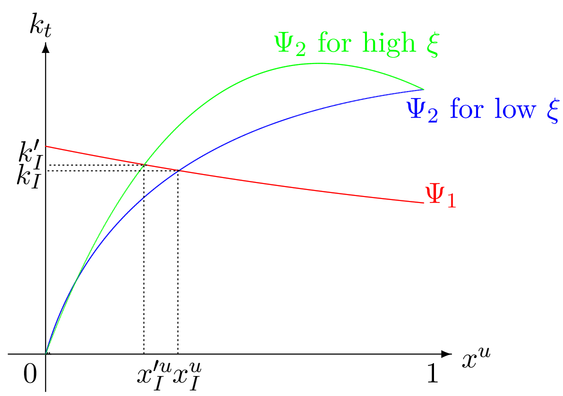


Figure 5: A representation of the dynamics when $x^u \neq 1$ (with Ψ_1 decreasing in x^u)

□

6.2 Proof of Proposition 2

Effect of τ on $\bar{\eta}(\tau)$: The threshold under which the BGP E is stable, i.e. $\bar{\eta}(\tau)$, is given by (34) in Appendix 6.1.1. To analyze the effect of τ on the dynamics of E, we compute $\frac{\partial\bar{\eta}(\tau)}{\partial\tau}$:

$$\frac{\partial\bar{\eta}(\tau)}{\partial\tau} = \frac{\mu(1+\sigma(\beta+\gamma\mu))Ak_E^{\alpha-1}}{\left[(a-b\tau)Ak_E^{\alpha}+1+\sigma(\beta+\gamma\mu)\right]^2}\left(bk_E-(a-b\tau)\alpha\frac{\partial k_E}{\partial\tau}\right) \tag{41}$$

The effect of the pollution tax on the dynamics at the BGP E depends on $\frac{\partial k_E}{\partial \tau}$. To compute this derivative, we use the dynamical equation (25) at the BGP:

$$\Phi(k,\tau) \equiv k\frac{(a-b\tau)Ak^\alpha + 1 + \beta\sigma}{(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)} - \frac{A(1-\tau)(1-\alpha)\beta k^\alpha}{\epsilon(\gamma\mu)^\mu}\left[\frac{\sigma}{(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)}\right]^{1-\mu} = 0$$

The effect of τ on k in E is given by the implicit function theorem:

$$\frac{\partial k}{\partial \tau}(k_E, 1) = -\frac{\frac{\partial \Phi}{\partial \tau}}{\frac{\partial \Phi}{\partial k_E}}$$

After computations, we obtain the two partial derivatives:

$$\begin{aligned} \tfrac{\partial \Phi}{\partial \tau} \quad = \quad & \Big(-bAk^{1+\alpha}\sigma\gamma\mu + \tfrac{A(1-\alpha)\beta k^\alpha \sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[Ak^\alpha(a-b(1-\mu(1-\tau))) + 1 + \sigma(\beta+\gamma\mu)\right] \\ & \left[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^\mu \Big) \left[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^{-2} \end{aligned} \tag{42}$$

$$\begin{aligned} \tfrac{\partial \Phi}{\partial k} \quad = \quad & \Big[((a-b\tau)Ak^\alpha)^2 + (a-b\tau)Ak^\alpha[2(1+\beta\sigma) + \sigma\gamma\mu(1+\alpha)] + (1+\sigma(\beta+\gamma\mu))(1+\sigma\beta) \\ & -\tfrac{A(1-\alpha)(1-\tau)\beta\alpha k^{\alpha-1}\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^\mu \left[Ak^\alpha\mu(a-b\tau) + 1 + \sigma(\beta+\gamma\mu)\right] \Big] \\ & \left[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^{-2} \end{aligned} \tag{43}$$

And, we have:

$$Sign\{\frac{\partial \bar{\eta}(\tau)}{\partial \tau}\} = Sign\{bk_E - (a-b\tau)\alpha\frac{\partial k_E}{\partial \tau}\}$$

Thus, $\frac{\partial \bar{\eta}(\tau)}{\partial \tau} > 0$ iif:

$$\begin{aligned} & bk_E\left[((a-b\tau)Ak_E^\alpha)^2 + (a-b\tau)Ak_E^\alpha[2(1+\beta\sigma) + \sigma\gamma\mu(1+\alpha)] + (1+\sigma(\beta+\gamma\mu))(1+\sigma\beta)\right] \\ & -\tfrac{A(1-\alpha)(1-\tau)\beta\alpha k_E^\alpha \sigma^{1-\mu} b}{\epsilon(\gamma\mu)^\mu}\left[(a-b\tau)Ak_E^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^\mu \left[Ak_E^\alpha\mu(a-b\tau) + 1 + \sigma(\beta+\gamma\mu)\right] \\ & +(a-b\tau)\alpha\Big(-bAk_E^{1+\alpha}\sigma\gamma\mu + \tfrac{A(1-\alpha)\beta k_E^\alpha\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[Ak_E^\alpha(a-b(1-\mu(1-\tau))) + 1 + \sigma(\beta+\gamma\mu)\right] \\ & \left[(a-b\tau)Ak_E^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^\mu\Big) > 0 \end{aligned} \tag{44}$$

It can be rewritten as:

$$\begin{aligned} & bk_E\left[((a-b\tau)Ak_E^\alpha)^2 + (a-b\tau)Ak_E^\alpha[2(1+\beta\sigma) + \sigma\gamma\mu] + (1+\sigma(\beta+\gamma\mu))(1+\sigma\beta)\right] \\ & +(a-b\tau)\alpha\Big(\tfrac{A(1-\alpha)\beta k_E^\alpha\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}Ak_E^\alpha(a-b)\left[(a-b\tau)Ak_E^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^\mu\Big) \\ & +(1+\sigma(\beta+\gamma\mu))\tfrac{A(1-\alpha)\beta\alpha k_E^\alpha\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[(a-b\tau)Ak_E^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^\mu(a-b) > 0 \end{aligned} \tag{45}$$

Under Assumption 2, the condition $bk_E - (a-b\tau)\alpha\frac{\partial k_E}{\partial \tau} > 0$ is always verified. Therefore, under Assumptions 1 and 2 and for $\alpha < 1/2$, the threshold $\bar{\eta}(\tau)$ depends positively on the tax rate τ. When $\tau \to 1$, $k_E \to 0$ and $\bar{\eta}(1) \to 1-\mu$ which is always true since we assume that $\mu + \eta < 1$. Thus, when τ tends to 1, the BGP without inequality is always stable.

Effect of τ on $\tilde{\eta}(\tau)$: Under Assumptions 1 and 2, for $2\mu + \eta > 1$ and $\alpha < 1/2$, we have obtained the condition (40) such that at least one BGP with inequalities exists. This condition can be rewritten as $\mathcal{L}(\tau) < \mathcal{R}$, where τ only intervenes in $\mathcal{L}$ and $\partial\mathcal{L}(\tau)/\partial\tau < 0$. Thus, an increase in the tax makes the condition (40) more easily satisfied. This condition corresponding also to $\eta < \tilde{\eta}(\tau)$, we deduced that the threshold $\tilde{\eta}(\tau)$ depends positively on τ. Moreover, (40) is always satisfied when τ tends to 1. Thus, a tighter tax on pollution increases the range of parameters for which there exists at least one BGP with inequalities.

□

6.3 Proof of Proposition 3

We analyze the effect of the tax rate on the growth factor at the BGP without inequality g_E, given by (24). Its derivative with respect to τ is:

$$\frac{\partial g_E}{\partial \tau} = \epsilon(\sigma\gamma\mu)^{\mu}\mu[(a-b\tau)Ak_E^{\alpha} + 1 + \sigma(\beta+\gamma\mu)]^{-\mu-1}k_E^{\alpha-1}\left\{bAk_E - (a-b\tau)A\alpha\frac{\partial k_E}{\partial \tau}\right\}$$

The effect of the pollution tax on the growth at the BGP E depends on $\frac{\partial k_E}{\partial \tau}$ and more precisely we have:

$$Sign\left\{\frac{\partial g_E}{\partial \tau}\right\} = Sign\left\{bk_E - (a-b\tau)\alpha\frac{\partial k_E}{\partial \tau}\right\}$$

From Appendix 6.2, we know that under Assumption 2, $bk_E - (a-b\tau)\alpha\frac{\partial k_E}{\partial \tau} > 0$. Therefore, under Assumption 2, we have that $\frac{\partial g_E}{\partial \tau} > 0$. The growth rate at the BGP without inequality g_E increases following an increase in the pollution tax.

6.4 Sensitivity Analysis

In this section, we analyze the robustness of our results with respect to two key parameters: the share of lower-skilled individuals in the economy ξ and the weight of intergenerational transmission in human capital accumulation η. Note that we have performed these analyses for a large number of tax rates, but results are similar. Thus, we have only reported here the results for $\tau = 0$ to save place.

Repartition of the two types of individuals in the population: ξ
The effect of ξ is illustrated in Figure 6 and Table 1. The share of lower-skilled individuals in the population does not affect the value of the BGP without inequality but modifies the BGP with inequalities. The higher the share of poor individuals in the population, the higher is the capital-labor ratio and the lower is the relative human capital of lower-skilled at the BGP with inequality. It entails that, at I, inequalities will be wider and that the growth will be lower. However, the dynamics of both BGP remains the same. Thus, the threshold in terms of initial inequalities under which the economy is in the inequality trap is lower. Other things being equal, a higher ξ implies that the relative disadvantage of lower-skilled agents with respect to the rest of the population is lower, which makes the human capital convergence easier.

Weight of intergenerational transmission in human capital accumulation: η
Figure 7 and Table 2 illustrate the evolution of the two BGP with respect to η. As for ξ, an increase in η has no effect on the values of the variables at the BGP without inequality E. On the contrary, at the BGP with inequalities, it reduces the capital labor ratio k_I and increases the relative human capital

of lower-skilled x_I^u. Thus, the growth rate at the BGP I is higher while the level of inequality is lower. However, looking at the dynamics, I is a saddle point and delimits the trap (when $\eta < \bar{\eta}(\tau)$). Therefore, up to the threshold $\bar{\eta}(\tau)$[35], an increase in η entails that the trap moves to the right and hence that the size of the inequality trap increases. At this threshold, I disappears and E becomes a saddle point and delimits the new trap of an even larger size. After $\bar{\eta}(\tau)$, a higher η continues to enlarge the trap by moving to the right the blue curve (representing the second dynamical equation of (23)). Therefore, other things being equal, a higher η favors the transmission of inequality and hence makes more likely that an economy is in the trap with widening inequalities.

Table 1: Sensitivity Analysis with respect to ξ when $\tau = 0$

ξ	k_I	x_I^u	g_I	$CAGR_I$	π_I^u	π_I^s	LE_I^u	LE_I^s	$AverageLE_I$
0.50	0.0241	0.0955	1.5413	1.452%	0.3197	0.8250	69.5919	84.7485	77.1702
0.70	0.0271	0.0738	1.4473	1.240%	0.2615	0.8515	67.8435	85.5435	73.1535
0.90	0.0355	0.0480	1.2439	0.730%	0.1762	0.8818	65.2848	86.4547	67.4018

ξ	k_E	g_E	$CAGR_E$	π_E	LE_E	$Eigenvalues_E$	$Eigenvalues_I$
0.50	0.0209	1.6506	1.684%	0.7724	83.1706	{0.911051; 0.318788}	{1.17236; 0.321711}
0.70	0.0209	1.6506	1.684%	0.7724	83.1706	{0.911051; 0.318788}	{1.20855; 0.324857}
0.90	0.0209	1.6506	1.684%	0.7724	83.1706	{0.911051; 0.318788}	{1.27981; 0.328051}

Notes: $CAGR_j$ represents the compound annual growth rate at the balanced growth path $j = E, I$, while LE_j^i corresponds to the life expectancy in years of the individual i at the BGP j.

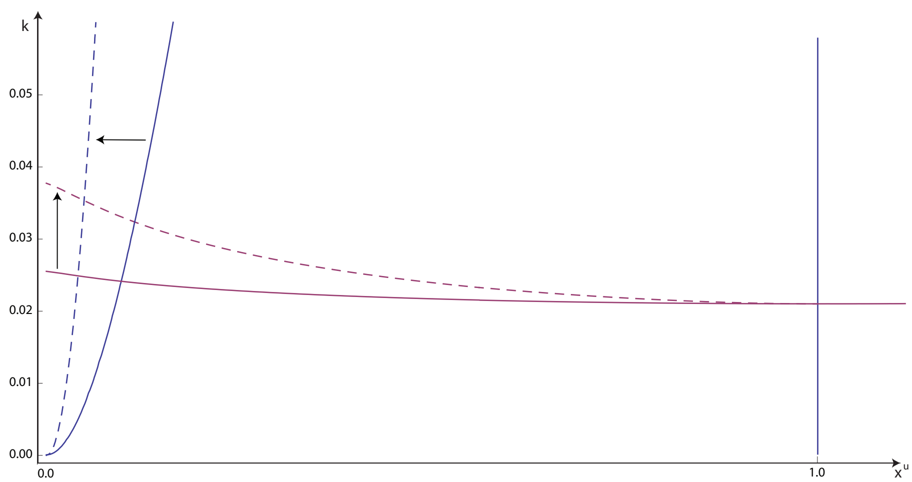


Figure 6: Sensitivity analysis with respect to ξ with $\tau = 0$, where the solid lines capture the cases $\xi = 0.5$, and the dashed lines capture the case $\xi = 0.9$.

[35] $\bar{\eta}(0) = 0.34$ and $\bar{\eta}(1) = 0.4$.

Table 2: Sensitivity Analysis with respect to η when $\tau = 0$

η	k_I	x_I^u	g_I	$CAGR_I$	π_I^u	π_I^s	LE_I^u	LE_I^s	$AverageLE_I$
0.20	0.0259	0.0444	1.4847	1.326%	0.1801	0.8251	65.4019	84.7530	75.0775
0.25	0.0241	0.0955	1.5413	1.452%	0.3197	0.8250	69.5919	84.7485	77.1702
0.30	0.0223	0.2365	1.6010	1.581%	0.5251	0.8213	75.7518	84.6403	80.1960
0.35	∅	∅							
0.39	∅	∅							

η	k_E	g_E	$CAGR_E$	π_E	LE_E	$Eigenvalues_E$	$Eigenvalues_I$
0.20	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.861051; 0.318788\}$	$\{1.2362; 0.32274\}$
0.25	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.911051; 0.318788\}$	$\{1.17236; 0.321711\}$
0.30	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.961051; 0.318788\}$	$\{1.08036; 0.320204\}$
0.35	0.0209	1.6506	1.684%	0.7724	83.1706	$\{1.01105; 0.318788\}$	
0.39	0.0209	1.6506	1.684%	0.7724	83.1706	$\{1.05105; 0.318788\}$	

Notes: $CAGR_j$ represents the compound annual growth rate at the balanced growth path $j = E, I$, while LE_j^i corresponds to the life expectancy in years of the individual i at the BGP j.

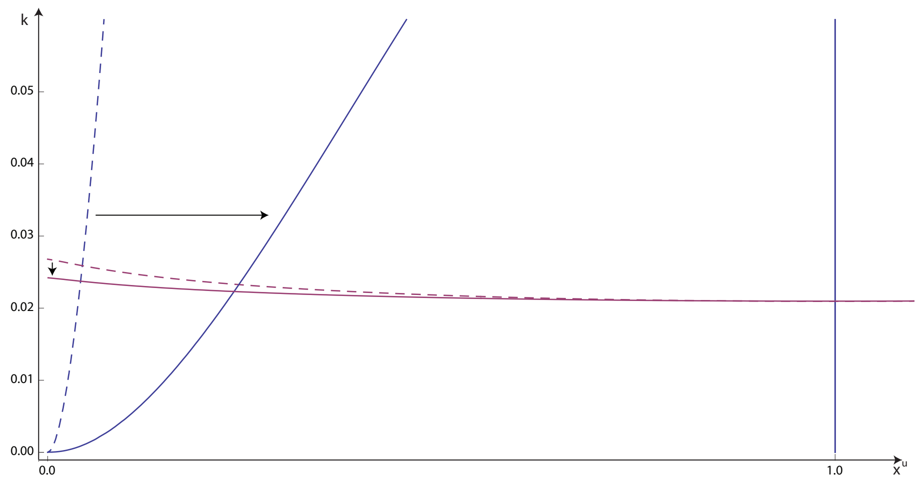


Figure 7: Sensitivity analysis with respect to η with $\tau = 0$, where the dashed lines capture the cases $\eta = 0.2$, and the solid lines refer to the case $\eta = 0.3$.

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