

CAHIER DE RECHERCHE #2503E
Département de science économique
Faculté des sciences sociales
Université d'Ottawa

WORKING PAPER #2503E
Department of Economics
Faculty of Social Sciences
University of Ottawa

Measuring the contribution of racial stratification and social class at birth to inequality of opportunity

Paul Makdissi* and Myra Yazbeck†

May 2025

* Department of Economics, University of Ottawa, 120 University Private, Ottawa, Ontario, Canada, K1N 6N5; e-mail: paul.makdissi@uottawa.ca

† Department of Economics, University of Ottawa, 120 University Private, Ottawa, Ontario, Canada, K1N 6N5; e-mail: myazbeck@uottawa.ca

Abstract

This paper develops a framework for decomposing inequality of opportunity into racial stratification and social class components. We derive novel dominance conditions that enable robust rankings of joint distributions of income and birth circumstances, and develop additional dominance criteria for restricted classes of indices reflecting either pro-poor or meritocratic perspectives. Our framework includes an estimation approach and statistical tests for these stochastic dominance conditions, ensuring practical application with survey data. Using Health and Retirement Study data, we analyze inequality of opportunity in earnings among aging U.S. populations between 2010-2020. While social class-based inequality decreased for certain classes of indices, the racial stratification component increased, driving overall rising inequality of opportunity.

Key words: *Inequality of opportunity, stochastic dominance, stratification.*

JEL Classification: I31, I32, and Z13.

1 Introduction

This paper is motivated by the recent development in stratification economics (see Chelwa, Hamilton, and Stewart, 2022; Darity, 2022), the increased public interest in racial inequalities, and the importance of stratification and identity in analyzing inequality of opportunity. Its objective is to understand how racial stratification and social class at birth can contribute to the inequality of opportunity. In doing so, this paper links the stratification economics literature to the literature on inequality of opportunity (see Roemer, 1998; Bjerk, 2008; Ferreira and Gignoux, 2014; Lee and Seshadri, 2018). It presents two contributions to the literature on the inequality of opportunity. First, to link the equality of opportunity approach with the economics of stratification approach, it proposes a decomposition of the inequality of opportunity into a racial stratification component and a social class at birth component. Second, it develops new graphical tools and dominance conditions that allow comparing distributions using the proposed equality of opportunity approach. The dominance conditions we derive focus on three types of ethical principles. The first type of dominance conditions establishes an ethical framework that links Temkin’s (1986) complaint approach with Roemer’s (1998) inequality of opportunity definition, providing the foundational principles that all inequality indices in our analysis must satisfy. The second and third types of dominance conditions are based on two ethical principles we introduce in the paper: the pro-poor and the meritocratic ethical principles. All these three dominance conditions apply to the racial stratification component and the social class at birth component.

In our approach, inequalities of opportunity arising from racial stratification and social class at birth are weighted equally, as both represent circumstances beyond an individual’s control. While their sources differ, their impact on opportunity is treated symmetrically in our framework. This decomposition, however, serves an important purpose for policy

planning, as it helps identify the relative contribution of each source to overall inequality of opportunity and can guide targeted interventions. Moreover, understanding the role of stratification is particularly crucial as it may have broader societal implications beyond economic outcomes, potentially affecting social cohesion and the functioning of democratic institutions.

Interest in issues related to stratification, such as disparities and the extent of discrimination, has long been present in economics. A rich literature on intergroup inequalities focuses on choices and human capital investments to explain observed disparities. This research has been particularly important in the United States, where one of the most striking features is the persistence of large racial economic disparities (Bayer and Charles 2018; Chetty et al. 2020; Derenoncourt and Montialoux, 2021). However, the emergence of stratification economics as a distinct and rapidly growing field is more recent (e.g., Chelwa, Hamilton, and Stewart, 2022; Darity, 2022). The foundational principles of stratification economics were articulated in Darity’s (2005) seminal work, where he broadened the scope of inequality studies. Darity (2005) proposed an approach that moves beyond conventional perspectives, emphasizing the need to understand the sources of persistent inequalities between racial groups. Stratification economics, therefore, assesses the global institutional factors that may perpetuate disparities, focusing on measuring, understanding, and explaining racial inequalities from this broader perspective (see also Chelwa, Hamilton, and Stewart, 2022; Darity, 2022). Indeed, Darity, Hamilton, and Stewart (2015) argue that a racial group’s political influence and social inclusion are crucial factors contributing to these disparities and warrant careful scrutiny.

In explaining the persistence of racial disparities, Darity (2005) implicitly differentiates between decisions for which individuals should be held accountable and those that result from institutional contexts. The underlying assumption is that racial inequalities

are inherently unfair unless one believes that members of marginalized groups consistently make choices that leave them worse off than those in the dominant group. Consequently, a fairness-based approach to these racial inequalities is both natural and necessary. In this context, racial justice can be conceptualized as a scenario where all individuals, regardless of racial identity, face equal probabilities of attaining any outcome across all dimensions of human wellbeing.

This concept of racial justice aligns closely with Roemer’s (1998) equality of opportunity model. Specifically, the idea of equal probabilities of achieving any outcome across all dimensions of human wellbeing parallels Roemer’s (1998) notion of *accountable effort*, which is rooted in fairness. Roemer’s concept suggests that an individual’s social background and institutional constraints influence the capacity to exert the necessary *effort* to achieve desired outcomes. He argues that the *effort level* for which a person is responsible should be free from the effects of initial conditions and social constraints. Roemer (1998) further demonstrates that if residual luck is randomly distributed, equality of opportunity requires that the quantile function of outcomes, conditional on initial circumstances, be identical across all combinations of initial circumstances. Conceptually, if racial identity is considered one of these initial circumstances, Roemer’s (1998) equality of opportunity model and the concept of racial justice become equivalent. This idea can also be extended to other identity markers, such as ethnicity, language, religion, gender, sexual orientation, or any other characteristic, depending on the institutional context. By distinguishing between outcome differences due to individual responsibility and those arising from circumstances beyond one’s control (circumstances at birth), Roemer’s (1998) model offers a powerful framework for analyzing identity-based intergroup inequalities. This paper adopts this framework.

Given that we decompose the inequality of opportunity into a racial stratification component and a social class at birth component, we need to adapt the inequality of opportunity

framework. Thus, we propose a measurement framework that adjusts Temkin’s (1986) idea of inequality as an aggregation of individuals’ complaints when an individual is compared with *equally deserving* counterparts. Then, we adopt this notion of complaints and incorporate it into Roemer’s (1998) framework by comparing outcomes at the same level of *accountable effort*.¹ Roemer’s framework is well-suited for analyzing racial inequalities of opportunity because its underlying political philosophy closely aligns with the principles of stratification economics. By extending and adapting the inequality of opportunity approach to a stratification-based perspective, we can better capture the complexities of a stratified economy, where circumstances at birth, including race and other identity markers, determine whether individuals belong to dominant or marginalized groups. Our analysis, therefore, distinguishes between the portions of inequality of opportunity attributable to social class at birth and those arising from identity-based stratification.

Since Roemer’s (1998) inequality of opportunity condition relies on comparisons of quantile functions conditional on initial circumstances, our estimation approach naturally employs quantile regressions for estimating these conditional quantile functions. Our method, which involves estimating models of the conditional quantiles, is related to Pistoletti’s (2009) approach but differs in two key aspects. First, Pistoletti (2009) employs a Cox proportional hazard model to estimate the conditional cumulative distribution of outcomes. Second, we formulate an inequality of opportunity measurement framework derived from the conditional quantiles of earnings, as opposed to the standard earnings inequality indices used by Pistoletti (2009).

Our estimation approach also parallels the work of Brunori, Palmisano, and Peragine (2019) and Brunori, Ferreira, and Peragine (2021), who focus on identifying the circumstances that explain inequalities of opportunity. However, their estimation method differs

¹It is important to note that the framework we are proposing does not focus on individual outcomes but rather on the conditional distribution of outcomes linked with an individual’s initial circumstances, including her identity.

from ours. They compare conditional cumulative distributions of outcomes using random forest classification models, which are particularly suitable for contexts with binary circumstance variables. In contrast, our quantile regression modeling approach allows us to leverage data that includes continuous and count circumstances variables, such as parent’s education, parental earnings during childhood, or the individual’s age.

Our empirical application examines changes in inequality of opportunity in earnings among aging populations in the United States, focusing on the influence of social class at birth and racial stratification during the years 2010 and 2020. Francis and Weller (2021) highlight critical factors such as employment discrimination, occupational segregation, and wealth disparities that continue to drive economic inequalities as individuals approach retirement. Similarly, Viceisza, Calhoun, and Lee (2023) show that while retirement wealth has improved for Whites and individuals of other races between 1989 and 2016, these gains have not been mirrored among Blacks and Hispanics. Building on these insights, our study defines the marginalized group as Blacks and Hispanics and the dominant group as non-Hispanic Whites and non-Hispanic individuals of other races. Our findings reveal an overall increase in inequality of opportunity in earnings among aging individuals between 2010 and 2020. When we decompose this change, we find that inequality of opportunity due to social class at birth decreases for certain indices, while inequality arising from racial stratification consistently increases over the period. By distinguishing the contributions of social class at birth and racial stratification, our study provides a more nuanced understanding of the sources of inequality of opportunity.

The remainder of the paper runs as follows. Section 2 presents the measurement framework. Section 3 offers the dominance conditions for identifying robust orderings of distributions in regards to inequality of opportunity, inequality of opportunity due to racial stratification, and inequality of opportunity due to social class at birth. To apply these

dominance conditions to survey data, Section 4 presents the estimation and statistical inference approach. Section 5 offers an application of the measurement approach using US data. Finally, Section 6 presents a brief conclusion and some directions for future research.

2 Measurement framework

The inequality of opportunity approach splits inequalities into two broad categories: inequalities due to individual’s responsible decisions (i.e., *accountable effort*) and inequalities due to the birth lottery (i.e., initial circumstances at birth). Roemer (1998) shows that if one assumes that the distribution of residual luck² is independent of initial circumstances, then equality of opportunity translates into a condition on the quantile function. More specifically, Roemer’s condition stipulates that the quantile function of an outcome conditional on a given vector of initial circumstances should be the same for all potential values for this vector of initial circumstances. Also, Roemer (1998) explains that under this assumption on residual luck, we can reduce these inequalities of opportunity by focusing on interventions that shift the lower contour of the set of conditional quantile functions.

In this paper, we adopt Roemer’s view of inequality of opportunity and propose a new measurement framework based on Temkin’s (1986) definition of inequality as the sum of complaints and the concept of *equally deserving individuals* (Cowell and Ebert, 2004). We think Temkin’s concept of *equally deserving individuals* fits nicely in an equality of opportunity framework if one is willing to consider that individuals with the same level of *accountable effort* are *equally deserving*. From this perspective, we define the complaint as resulting from differences in the reward associated with each level of *accountable effort*.

As in Temkin (1986), when assessing the extent of a complaint, we specify a reference point. There are three possible reference points: (1) the average member, (2) the best-off member, and (3) all other people who are better off. This paper uses the idea underlying

²The residual luck is the portion of luck not determined by the birth lottery.

the *best-off equally deserving individual* as a reference point because it is consistent with Roemer’s view of reducing inequalities of opportunity.

A key feature of the inequality of opportunity framework is that inequality is assessed not based on observed individual outcomes but on the conditional quantile function of these outcomes given the individual’s initial conditions. Roemer (1998) demonstrates that a suitable measure of *accountable effort* is the quantile rank of this conditional quantile function. In this framework, the conditional quantile function maps each level of *accountable effort* to a corresponding level of outcome. Thus, the conditional quantile function effectively represents the opportunity set available to individuals with a specific combination of initial circumstances.

2.1 Notation and definitions

To model this framework mathematically, we consider a society with a distribution of types $t \in \mathcal{T} \in \mathfrak{R}^{J+1}$ linked to individual initial circumstances, i.e., the aspects of the individual’s environments that are beyond her control. These circumstances could include childhood and family environment. We assume that this society is also composed of two identity groups, a dominant group, D , and a marginalized group, M .³ The definition of the group identity, $g \in \mathcal{G} := \{D, M\}$, is often determined by the society’s specific institutional context and may be based on race, ethnicity, language, religion, gender, sexual orientation or any other identity markers. In addition to their identity group, individuals differ in social classes. We define social class as encompassing all other circumstances at birth, denoted by $x \in \mathcal{X} \subseteq \mathfrak{R}^J$. These circumstances may include the year of birth of the individual, her region of birth, and her parents’ characteristics during childhood (e.g., parent’s education, employment statuses, earnings during childhood, and occupation).

³In this paper, we keep the stratification of the economy into two groups for ease of exposition. All methods and results presented in the paper can be generalized to multiple groups and even account for the intersectionality of stratification.

For each individual, we observe a social outcome. In this paper, we will focus on earnings, y , nevertheless, the analyst can consider any other social outcome such as education (Ferreira and Gignoux, 2014) or health (Davillas and Jones, 2020). Following Roemer (1998), we assume that earnings, $y \in \mathcal{Y} \in \mathfrak{R}_+$, has a production function $\phi(\cdot)$. The production function, $\phi(e_R, x, g, \ell)$, depends on a measure of raw effort, e_R , the individual's social class at birth, x , her identity group, g , and the residual luck, $\ell \in \mathcal{L}$ (i.e., luck not determined by the birth lottery). We also assume that $\phi(\cdot)$ is strictly monotone increasing in the raw level of effort, e_R . This raw effort variable may include, among other things, years of schooling, type of training, labor supply, and savings and investment in private pension plans. Roemer (1998) explains that an individual's capacity to produce a given level of raw effort may also depend on birth circumstances. For instance, a parent with a higher education level can offer more support to her children in their learning activities. A wealthier parent can also pay for private tutoring, which poor parents cannot afford. For this reason, Roemer suggests that we should purge the impact of circumstances at birth from the level of raw effort. To do so, he defines the concept of *accountable effort*, e , as the individual's rank in the distribution of raw effort of her type. Suppose an individual is of type $t = (x, g)$. In that case, her *accountable effort* is $e = F_{E_R|X,G}(e_R|x, g)$, where $F_{E_R|X,G}(e_R|x, g)$ represents the cumulative distribution of raw effort conditional on $X = x$, and $G = g$. Given that we cannot fully observe *accountable effort*, it is difficult to account for it without additional assumptions. Nevertheless, suppose we assume that $\ell \perp\!\!\!\perp X, G$, i.e., residual luck is statistically independent of types. In that case, the strict increasing monotonicity of $\phi(\cdot)$ in e_R implies that an individual will have the same rank in $F_{E_R|X,G}(e_R|x, g)$ as in $F_{Y|X,G}(y|x, g)$, the conditional cumulative distribution of earnings conditional to type $t = (x, g)$. This strict monotonicity assumption imposed on $\phi(e_R, x, g, \ell)$ is a key identification assumption of our model. This assumption allows us to overcome the need for a

specific model of $F_{E_R|X,G}(e_R | x, g)$ and, consequently, the need to define a measure of raw effort because the ranks in the distribution of raw effort will be mapped to the distribution of earnings. Thus, the only object that needs to be estimated is $F_{Y|X,G}(y|x, g)$ and/or its inverse function $Q_{Y|X,G}(e|x, g)$, the conditional quantile function. It is essential to mention that while we are naturally inclined to consider an individual's level of *accountable effort* as the main focus in this framework, empirically, it is impossible to do so. Specifically, we cannot identify an individual's level of *accountable effort* separately because individual i 's outcome, y_i , is a function of two unobserved factors: the level of raw effort, e_{Ri} , and the unobserved realization of residual luck, ℓ_i . These two unobserved factors cannot be disentangled from each other because they are both unobserved. However, while we cannot identify the individual's level of *accountable effort* separately, we can estimate the mathematical objects $F_{Y|X,G}(y|x, g)$ and $Q_{Y|X,G}(e|x, g)$. This allows us to conduct an inequality of opportunity analysis since Roemer (1998) argues that perfect equality of opportunity requires the conditional quantile functions to be the same for all types at birth.

2.2 Inequality of opportunity as a complaint

Let us assume that the outcome of interest is earnings. Then in a canonical earnings inequality measurement framework, the individual's outcome (y_i) is a function of the effort level chosen by this individual (e_{Ri}), her social class at birth (x_i), her group identity (g_i) and residual luck (ℓ_i) befalling her. From an inequality of opportunity framework perspective, the analyst's main object of interest is the opportunities offered to individuals. These opportunities are reflected in the distribution of earnings conditional on initial conditions (x, g), for all possible realization of residual luck, ℓ . In other words, from an inequality of opportunity perspective, the mathematical objects of interest are

$$F_{Y|X,G}(y|x, g) = \int_{-\infty}^{\infty} F_{Y|X,G,L}(y|x, g, \ell) dF_L(\ell), \quad (1)$$

and

$$Q_{Y|X,G}(e|x,g) = \inf\{y \in \mathfrak{R}_+ | e \leq F_{Y|X,G}(y|x,g)\}. \quad (2)$$

For each possible level of *accountable effort*, the complaint should be defined for each combination of initial conditions (x, g) . This complaint is defined as the relative difference between the expected outcome at *accountable effort* level e conditional on the initial conditions (x, g) and the reference expected outcome associated with the same level of *accountable effort* e . In the context of this paper, the reference outcome function, $\rho(e)$, represents the expected outcome had the opportunities been at the same level as those generating the 95th percentile of expected outcomes at this level of *accountable effort*. This approach is rooted in Temkin's (1986) idea of using the *best-off equally deserving individual* as a reference point, which aligns with Roemer's (1998) view of reducing inequalities of opportunity. By using the 95th percentile as a practical implementation of this concept, we ensure that our analysis remains consistent with these theoretical foundations while being empirically feasible with survey data.

Having defined the reference outcome function, $\rho(e)$, we use it in Figure [1](#) to illustrate the concept of a complaint at a level of *accountable effort* e_1 . The higher curve represents the reference outcome function $\rho(e)$. The lower curve represents the quantile function conditional to (x, g) . The absolute loss at a level of *accountable effort* e_1 is $\rho(e_1) - Q_{Y|X,G}(e_1|x,g)$. This concept of absolute loss can be used to define $\kappa(e, x, g)$, the complaint of a person of type (x, g) at a level of *accountable effort* e , as the proportion of earnings lost due to inequality of opportunities:

$$\kappa(e, x, g) = \max\left(0, \frac{\rho(e) - Q_{Y|X,G}(e|x,g)}{\rho(e)}\right). \quad (3)$$

The overall complaint associated with the type (x, g) is a socially weighted sum of complaints at all effort levels:

$$\tilde{\kappa}(x, g) = \int_0^1 \omega(e) \kappa(e, x, g) de, \quad (4)$$

where $\omega(e)$ is a social weight function for complaints at a given *accountable effort* level e , we assume that $\omega(e) \geq 0$ for all $e \in [0, 1]$ and that $\int_0^1 \omega(e) de = 1$ (for $\tilde{\kappa}(x, g)$ to be socially weighted average). An example of such social weight function is the case in which $\omega(e) = 1$ for all $e \in [0, 1]$. In this case, $\tilde{\kappa}(x, g)$ is the average complaint of type (x, g) or the average proportion of earnings lost due to unequal opportunities for individuals of type (x, g) . In the empirical application in Section 5, the only mathematical objects that are estimated at the individual level are the expected complaint function $\kappa(e, x_i, g_i)$ (for all $e \in [0, 1]$) and the expected overall complaint $E[\tilde{\kappa}(x_i, g_i)]$.

Let us define the effort-dependent inequality of opportunity index as an average of these complaints over the distribution of types and effort

$$I(F_{Y,X,G}) = E[\tilde{\kappa}(x, g)] = \sum_{g \in \{D, M\}} \Pr[G = g] \int_{\mathcal{X}} \tilde{\kappa}(x, g) dF_{X|G}(x|g). \quad (5)$$

One interesting property of the effort-dependent inequality of opportunity indices is that they are subgroup-perfectly decomposable. This perfect decomposability means that if one defines

$$I_g(F_{Y,X,G}) = E[\tilde{\kappa}(x, g)|G = g] = \int_{\mathcal{X}} \tilde{\kappa}(x, g) dF_{X|G}(x|g), \quad (6)$$

as the effort-dependent inequality of opportunity index of group g , then the overall index is the sum of subgroup indices weighted by their population shares:

$$I(F_{Y,X,G}) = \sum_{g \in \{D, M\}} \Pr[G = g] I_g(F_{Y,X,G}). \quad (7)$$

Given that our main interest is to capture which part of the inequality of opportunity is attributable to stratification and which part of inequality is associated with social class at birth, we decompose the complaints of the marginalized group as follows:

$$\kappa(e, x, M) = \max \left(0, \frac{[\rho(e) - Q_{Y|X,G}(e|x, D)] + [Q_{Y|X,G}(e|x, D) - Q_{Y|X,G}(e|x, M)]}{\rho(e)} \right), \quad (8)$$

where $Q_{Y|X,G}(e|x, D)$ is the earnings of a person from the dominant group at the same level of *accountable effort* and the same initial conditions, except group identity. The overall

complaint is decomposed into a stratification component and another due to the social class at birth. Figure 2 illustrates this decomposition. The blue double-arrow segment indicates the absolute loss due to stratification at *accountable effort* level e_1 . We define the stratification complaint function for all effort levels $e \in [0, 1]$ as this loss in relative term, i.e.

$$\kappa^{Strat}(e, x, M) = \max\left(0, \frac{Q_{Y|X,G}(e|x, D) - Q_{Y|X,G}(e|x, M)}{\rho(e)}\right). \quad (9)$$

The green double-arrow segment in Figure 2 indicates the absolute loss due to social class at birth. The social class complaint function is defined for all effort levels as this loss in relative terms, i.e.

$$\kappa^{Class}(e, x, M) = \max\left(0, \min\left[\frac{\rho(e) - Q_{Y|X,G}(e|x, D)}{\rho(e)}, \frac{\rho(e) - Q_{Y|X,G}(e|x, G)}{\rho(e)}\right]\right). \quad (10)$$

In this context, we can define inequality of opportunity that is due to stratification as

$$I^{Strat}(F_{Y,X,G}) = \Pr[G = M] \int_{\mathcal{X}} \int_0^1 \omega(e) \kappa^{Strat}(e, x, M) dF_{X|G}(x|M). \quad (11)$$

Similarly, we can define the inequality of opportunity that is due to social class at birth as:

$$\begin{aligned} I^{Class}(F_{Y,X,G}) &= \Pr[G = D] \int_{\mathcal{X}} \int_0^1 \omega(e) \kappa(e, x, D) dF_{X|G}(x|D) \\ &\quad + \Pr[G = M] \int_{\mathcal{X}} \int_0^1 \omega(e) \kappa^{Class}(e, x, M) dF_{X|G}(x|M). \end{aligned} \quad (12)$$

The subgroup decomposability of $I(F_{Y,X,G})$ and the linearity of $\kappa(e, x, g)$ implies that

$$I(F_{Y,X,G}) = I^{Strat}(F_{Y,X,G}) + I^{Class}(F_{Y,X,G}). \quad (13)$$

In other words, total inequality of opportunity in a population is the sum of inequality of opportunity due to stratification and inequality of opportunity due to social class at birth.

If one imposes a specific form on the social weight function, using these indices allows for a complete ordering of all joint distributions $F_{Y,X,G}$ in terms of inequality of opportunity, inequality of opportunity due to stratification, and inequality of opportunity due to social classes.

3 Identifying robust orderings in terms of inequality of opportunity

When using a specific index belonging to the class of effort-dependent inequality of opportunity indices, it is possible to have a complete ordering of distributions. However, this ordering will be contingent on the specific mathematical form of the index, i.e., the structure imposed on the social weight function $\omega(e)$. Nevertheless, it is always possible to test whether some rankings are robust to all potential functional forms the analyst may impose on the social weight function. This section aims to lay down the conditions for identifying such robust orderings.

To identify robust orderings of inequality of opportunity the analyst can use a dominance approach analogous to the one used in earnings inequality. Indeed, in the earnings inequality literature, the non intersection of the Lorenz curves can be used to identify robust orderings of earnings inequality. In such a case, the distribution with a Lorenz curve closer to the 45-degree line has the lowest inequality. This result holds for any inequality index.

In the inequality of opportunity framework of this paper, the object of interest is not the individual's earnings but the complaint function, $\kappa(e, x, g)$ associated with given initial circumstances. For this reason, in what follows we define new curves based on these complaint functions.

3.1 Inequality of opportunity orderings

First, we define the complaint incidence curve. For each level of *accountable effort*, this curve represents the expected complaint in the population. The formal mathematical definition of this curve is

$$CI(e, F_{Y,X,G}) = \sum_{g \in \{D,M\}} \Pr[G = g] \int_{\mathcal{X}} \kappa(e, x, g) dF_{X|G}(x|g). \quad (14)$$

Analogously, one can define the stratification and the social class complaint incidence curves as:

$$CI^{Strat}(e, F_{Y,X,G}) = \Pr[G = M] \int_{\mathcal{X}} \kappa^{Strat}(e, x, M) dF_{X|G}(x|M), \quad (15)$$

and

$$\begin{aligned} CI^{Class}(e, F_{Y,X,G}) &= \Pr[G = D] \int_{\mathcal{X}} \kappa(e, x, D) dF_{X|G}(x|D) \\ &\quad + \Pr[G = M] \int_{\mathcal{X}} \kappa^{Class}(e, x, M) dF_{X|G}(x|M). \end{aligned} \quad (16)$$

The curves described in [\[14\]](#), [\[15\]](#), and [\[16\]](#) provide a graphical representation of the distribution of expected complaint for all *accountable effort* levels. In addition, these curves can be used to identify robust rankings of inequality of opportunity.

Let us denote by Ω the set of all effort-dependent inequality of opportunity indices defined by equation [\(5\)](#). This class of indices is formally defined as

$$\Omega := \left\{ I(\cdot) \mid \omega(e) \geq 0 \ \forall e \in [0, 1], \text{ and } \int_0^1 \omega(e) de = 1 \right\}. \quad (17)$$

In our framework, robust rankings of inequality of opportunity are analogous to robust rankings of earnings inequality. For earnings inequality, it is well known that a robust ranking is identified when two Lorenz curves do not intersect. In our framework, we identify robust rankings of inequality of opportunity for all indices $I(\cdot) \in \Omega$ when complaint incidence curves do not intersect.

Let $\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) = I(F_{Y,X,G}^1) - I(F_{Y,X,G}^0)$ denotes the difference in inequality of opportunity between distribution $F_{Y,X,G}^0$ and $F_{Y,X,G}^1$.

Theorem 1. $\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$ for all indices $I(\cdot) \in \Omega$ if and only if

$$CI(e, F_{Y,X,G}^1) - CI(e, F_{Y,X,G}^0) \leq 0 \ \forall e \in [0, 1].$$

If we define analogously $\Delta I^{Strat}(F_{Y,X,G}^0, F_{Y,X,G}^1)$ and $\Delta I^{Class}(F_{Y,X,G}^0, F_{Y,X,G}^1)$ and use CI^{Strat} and CI^{Class} similar results hold. If the condition in Theorem [1](#) does not allow for a ranking

between two distributions, it is possible to consider subsets of indices. Thus an analyst can impose an additional structure on the ethical principles and check if a robust ordering exists for these subsets of the indices $I(\cdot) \in \Omega$. In this paper, we propose to consider two alternative normative views: a pro-poor normative view and a meritocratic normative view.

3.2 Pro-poor inequality of opportunity orderings

When an analyst has a pro-poor view of inequality of opportunity, she puts a higher weight on complaints associated with low levels of *accountable effort*. Thus, pro-poorness implies a non-increasing $\omega(e)$ function. A well-known example of such a function is $\omega(e) = \nu(1 - e)^{\nu-1}$, which represents the social weights of the rank-dependent social welfare function associated with the extended class of Gini indices. The parameter ν is an inequality aversion parameter in the Gini social welfare function. In our framework, since the ranking variable is *accountable effort*, such a parameter would be a parameter of pro-poor inclination. An infinity of other potential mathematical functions would satisfy a pro-poor view of the social weight function. Let us denote by Ω_P the set of all pro-poor effort-dependent inequality of opportunity indices. This class of indices is formally defined as follows:

$$\Omega_P := \left\{ I(\cdot) \in \Omega \mid \frac{d\omega(e)}{de} \leq 0 \ \forall e \in [0, 1], \text{ and } \omega(1) = 0 \right\}. \quad (18)$$

Drawing from the tools developed in the literature on progressive earnings taxation and the literature on socioeconomic health inequality, we use the absolute concentration curve (see Schechtman, Shelef, Yitzhaki, and Zitikis, 2008; Khaled, Makdissi, and Yazbeck, 2018)^[4] An absolute concentration curve is a graphical tool that can be used to identify robust orderings of inequality of opportunities for all pro-poor effort-dependent inequality of opportunity indices. In the context of this paper, we refer to these curves as the pro-poor complaint concentration curves. The formal mathematical definition of the pro-poor

⁴Khaled, Makdissi, and Yazbeck (2018) use the term generalized concentration curve instead of absolute concentration curve.

complaint concentration curves is as follows:

$$CC_p(e, F_{Y,X,G}) = \int_0^e CI(s, F_{Y,X,G})ds, \quad (19)$$

$$CC_p^{Strat}(e, F_{Y,X,G}) = \int_0^e CI^{Strat}(s, F_{Y,X,G})ds, \quad (20)$$

and

$$CC_p^{Class}(e, F_{Y,X,G}) = \int_0^e CI^{Class}(s, F_{Y,X,G})ds. \quad (21)$$

For a given level of *accountable effort*, e , the pro-poor complaint concentration curve represents the average complaint that would prevail in the population if only *accountable effort* levels less than or equal to e were to generate complaints. When $e = 1$, the pro-poor complaint concentration curve equals the average complaint level in the population.

The analyst can use these curves to identify robust rankings of distributions for all pro-poor inequality of opportunity indices belonging to Ω_P .

Theorem 2. $\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$ for all indices $I(\cdot) \in \Omega_P$ if and only if

$$CC_p(e, F_{Y,X,G}^1) - CC_p(e, F_{Y,X,G}^0) \leq 0 \quad \forall e \in [0, 1].$$

We can derive similar results for $\Delta I^{Strat}(F_{Y,X,G}^0, F_{Y,X,G}^1)$ and $\Delta I^{Class}(F_{Y,X,G}^0, F_{Y,X,G}^1)$ using CC_p^{Strat} and CC_p^{Class} .

3.3 Meritocratic inequality of opportunity orderings

When an analyst has a meritocratic view of inequality of opportunity, she puts a higher weight on complaints associated with high levels of *accountable effort*. The meritocratic view implies a non-decreasing $\omega(e)$ function. One example of such a function would be $\omega(e) = \alpha e^{\alpha-1}$, where the parameter α is a parameter of meritocratic inclination. Nevertheless, one may select another functional form as there is an infinity of functional forms

for $\omega(e)$ that would satisfy a meritocratic view. Let us denote by Ω_M the set of all meritocratic effort-dependent inequality of opportunity indices defined by equation (5). The mathematical definition of this class of indices is

$$\Omega_M := \left\{ I(\cdot) \in \Omega \mid \frac{d\omega(e)}{de} \geq 0 \ \forall e \in [0, 1], \text{ and } \omega(0) = 0 \right\}. \quad (22)$$

How can we reconcile a meritocratic normative view with the well-known aversion to inequality concept? To do so, one needs to remember that the complaint function, $\kappa(e, x, g)$, assigns a complaint with each possible level of *accountable effort*, $e \in [0, 1]$, a person could have exerted given the initial circumstances (x, g) . This complaint function captures inequality aversion. Thus, if one accepts the assumption that, from the individual's perspective, making an effort is costly and that the cost of effort increases with the effort level, it would be natural for some to put more weight on complaints associated with a higher level of effort. This ethical principle, combined with the stratification component, is also linked with an aversion to glass ceilings (see Bjerk, 2008).

Similarly to pro-poor complaint concentration curves, one can define the meritocratic complaint concentration curves as follows:

$$CC_m(e, F_{Y,X,G}) = \int_e^1 CI(s, F_{Y,X,G}) ds, \quad (23)$$

$$CC_m^{Strat}(e, F_{Y,X,G}) = \int_e^1 CI^{Strat}(s, F_{Y,X,G}) ds, \quad (24)$$

and

$$CC_m^{Class}(e, F_{Y,X,G}) = \int_e^1 CI^{Class}(s, F_{Y,X,G}) ds. \quad (25)$$

For a given level of *accountable effort*, e , the meritocratic complaint concentration curve represents the average complaint that would prevail in the population if only *accountable effort* levels greater than or equal to e were to generate complaints. When $e = 0$, the merito-

cratic complaint concentration curve equals the average complaint level in the distribution. In other terms, $CC_m(0, F_{Y,X,G}) = CC_p(1, F_{Y,X,G})$.

As in the case of the pro-poor complaint concentration curves, these curves can be used to identify rankings that are robust to all meritocratic inequality of opportunity indices belonging to Ω_M .

Theorem 3. $\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$ for all indices $I(\cdot) \in \Omega_M$ if and only if

$$CC_m(e, F_{Y,X,G}^1) - CC_m(e, F_{Y,X,G}^0) \leq 0 \quad \forall e \in [0, 1].$$

We can derive similar results for $\Delta I^{Strat}(F_{Y,X,G}^0, F_{Y,X,G}^1)$ and $\Delta I^{Class}(F_{Y,X,G}^0, F_{Y,X,G}^1)$ using CC_m^{Strat} and CC_m^{Class} .

4 Estimation and inference

In previous sections, we introduced new inequality of opportunity measurement tools and dominance conditions for the robust ordering of inequality of opportunity and the impact of stratification and social class at birth on this inequality of opportunity. However, we assumed that the joint distributions, $F_{Y,X,G}$, were known to researchers, and we did not discuss uncertainty. This section will provide details regarding this paper's estimation and inference approach. It is important to note that while the measurement framework, the graphical tools, and the dominance conditions proposed in this paper are new, the estimation and the inference methodology we use are already available in the econometric literature.

Assume that we have two data sets of n_0 and n_1 observations with n_{0D} , n_{1D} , n_{0M} , and n_{1M} observations from the dominant and marginalized groups. All graphical tools and dominance conditions introduced in the previous two sections do not focus on the marginal cumulative distribution of earnings y but instead on the conditional quantile function that is associated with initial circumstances, $Q_{Y|X,G}(e|x, g)$. To estimate the graphical tools and conduct the dominance tests for conditions provided in Theorems [1](#), [2](#), and [3](#) the analyst

needs to estimate this conditional quantile function on a grid of *accountable effort* using canonical quantile regressions (Koenker and Bassett, 1978). Schechtman, Shelef, Yitzhaki, and Zitikis's (2008) offer a framework matching this paper's testing object of interest.

4.1 Estimation

In the first estimation step consists of estimating the quantile regressions for each group $g \in \{D, M\}$ on a grid of pre-specified *accountable effort* levels $\{e_1, e_2, \dots, e_L\}$. Note that if we chose a too small grid, we need to trim the two tails of the quantile function at some values $\varepsilon > 0$ and $1 - \varepsilon$. The trimming allows us to avoid issues associated with the estimation of tail quantiles (Koenker, 2005, p. 148).

In the second estimation step, we use the estimated models and predict the value of the conditional quantile function for each observation i , $\widehat{Q}_{Y|X,G}(e_\ell, x_i, g_i)$, based on the same grid of pre-specified *accountable effort* levels. For observations belonging to the marginalized group (i.e. if $g_i = M$), we also need a predicted value of the counterfactual conditional quantile function for each observation, $\widehat{Q}_{Y|X,G}(e_\ell, x_i, D)$.

In the third step, we estimate the reference outcome function on the grid of pre-specified *accountable effort* levels $\{e_1, e_2, \dots, e_L\}$. We use the 95th percentile of all the conditional quantile e_ℓ , conditional on the initial conditions, as the reference outcome. Formally,

$$\widehat{\rho}_i(e_\ell) = \inf \{y | \Pr [\widehat{Q}_{Y|X,G}(e_\ell, X, G) \leq y] \geq 0.95\}, \quad \forall e_\ell \in \{e_1, e_2, \dots, e_L\}. \quad (26)$$

Finally, the fourth step consists of predicting on the grid of pre-specified *accountable effort* levels $\{e_1, e_2, \dots, e_L\}$ a value of the complaint function for each observation i :

$$\widehat{\kappa}(e_\ell, x_i, g_i) = \frac{\widehat{\rho}(e_\ell) - \widehat{Q}_{Y|X,G}(e_\ell | x_i, g_i)}{\widehat{\rho}(e_\ell)}, \quad \forall e_\ell \in \{e_1, e_2, \dots, e_L\}. \quad (27)$$

For observations belonging to the marginalized group (i.e., if $g_i = M$), we also need to estimate the predicted values of the complaint due to stratification and the complaint due

to social class at birth:

$$\widehat{\kappa}^{Strat}(e_\ell, x_i, g_i) = \frac{\widehat{Q}_{Y|X,G}(e_\ell|x_i, D) - \widehat{Q}_{Y|X,G}(e_\ell|x_i, g_i)}{\widehat{\rho}(e_\ell)}, \quad \forall e_\ell \in \{e_1, e_2, \dots, e_L\}, \quad (28)$$

and

$$\widehat{\kappa}^{Class}(e_\ell, x_i, g_i) = \frac{\widehat{\rho}(e_\ell) - \widehat{Q}_{Y|X,G}(e_\ell|x_i, D)}{\widehat{\rho}(e_\ell)}, \quad \forall e_\ell \in \{e_1, e_2, \dots, e_L\}. \quad (29)$$

The set of complaint incidence curves can then be estimated on each point $e_\ell \in \{e_1, e_2, \dots, e_L\}$ using the values obtained in (27), (28), and (29):

$$\widehat{CI}(e, \widehat{F}_{Y,X,G}) = \frac{1}{N} \sum_{i=1}^N \widehat{\kappa}(e, x_i, g_i), \quad (30)$$

$$\widehat{CI}^{Strat}(e, \widehat{F}_{Y,X,G}) = \frac{1}{N} \sum_{i=1}^N \widehat{\kappa}^{Strat}(e, x_i, g_i) \mathbb{1}[g_i = M], \quad (31)$$

$$\begin{aligned} \widehat{CI}^{Class}(e, \widehat{F}_{Y,X,G}) &= \frac{1}{N} \sum_{i=1}^N \widehat{\kappa}(e, x_i, g_i) \mathbb{1}[g_i = D] \\ &+ \frac{1}{N} \sum_{i=1}^N \widehat{\kappa}^{Class}(e, x_i, g_i) \mathbb{1}[g_i = M], \end{aligned} \quad (32)$$

We estimate the pro-poor and meritocratic complaint concentration curves by integrating the expressions in equations (30), (31), and (32) using a Riemann sum approximation (see Appendix B for details).

4.2 Testing for dominance conditions

This section adopts a testing procedure that builds on Schechtman, Shelef, Yitzhaki, and Zitikis (2008) and Khaled, Makdissi, and Yazbeck (2018). This testing procedure uses a directional version of a statistic akin to the Kolmogorov-Smirnov statistic. Let us denote by $C(e)$ one of the curves defined in the preceding sections. $C(e)$ can be $CI(e)$, $CC_p(e)$, $CC_m(e)$, or one of their stratification or social class versions. Let $C_0(e)$ and $C_1(e)$ be the curve of two populations, 0 and 1. Assume that we have two i.i.d samples, S_0 and S_1 , of size n_0 and n_1 from these two populations. We are interested in testing one of the dominance

conditions in Theorems [1](#), [2](#), and [3](#). Formally, the null and alternative we are interested in are:

$$H_0 : C_1(e) - C_0(e) \leq 0, \forall e \in [0, 1]$$

$$H_1 : C_1(e) - C_0(e) > 0, \text{ for some } e \in [0, 1]$$

Note that if the chosen grid is too small, we should instead test for

$$H_0 : C_1(e) - C_0(e) \leq 0, \forall e \in [\varepsilon, 1 - \varepsilon]$$

$$H_1 : C_1(e) - C_0(e) > 0, \text{ for some } e \in [\varepsilon, 1 - \varepsilon]$$

It is important to mention that in the above test, we are not trying to establish dominance by imposing a null of non-dominance. Instead, we impose a null of dominance and test if this null can be rejected. There are two reasons why we adopt this testing approach which may seem counterintuitive. First, in a similar context, Davidson and Duclos (2013) have shown that testing a null hypothesis of non-dominance requires strong evidence against the null, which may be challenging to obtain. Second, since the conditions in Theorems [1](#), [2](#), and [3](#) only require weak dominance, we follow the usual practice from the empirical literature on stochastic dominance and test for $H_0^1 : C_1(e) - C_0(e) \leq 0, \forall e \in [0, 1]$ and for $H_0^2 : C_0(e) - C_1(e) \leq 0, \forall e \in [0, 1]$. Table [1](#) displays the decision rules for the dominance tests. For a level of significance α , we will consider that we have strong evidence in favor of dominance if the p -values of one of the aforementioned nulls are larger or equal to α while the p -values of the other are strictly lower.

Let $\tau = \sup_e [C_1(e) - C_0(e)]$, it is straightforward to construct a KS type directional test statistic $\hat{\tau}$ that is a non-parametric estimator of τ :

$$\hat{\tau} = \sqrt{\frac{n_0 n_1}{n_0 + n_1}} \sup_e [\hat{C}_1(e) - \hat{C}_0(e)] \quad (33)$$

The asymptotic distribution of $\hat{\tau}$ will be that of a functional of a two-dimensional Gaussian

process that is very complicated to compute.⁵ To overcome this computational issue, we build on Schechtman, Shelef, Yitzhaki, and Zitikis (2008) and Donald and Hsu (2014 and 2016) and use a selective recentering bootstrap approach. For a detailed description of the bootstrap procedure, please refer to Appendix C.

5 Empirical illustration: Inequality of opportunity among the aging population in the United States

To underscore the empirical applicability of our proposed measurement framework, we investigate changes in inequality of opportunity in earnings among aging populations in the United States, focusing on the years 2010 and 2020. Our analysis centers on the roles of social class at birth and racial stratification in shaping inequality of opportunity in earnings, with particular attention to the experiences of marginalized groups, defined here as Blacks and Hispanics, compared to the dominant group of non-Hispanic Whites and non-Hispanic individuals of other races.

The importance of this focus is underscored by existing literature. For instance, Francis and Weller (2021) highlight how employment discrimination, occupational segregation, and wealth disparities persist as significant drivers of economic inequalities as individuals approach and transition into retirement. Moreover, Viceisza, Calhoun, and Lee (2023) demonstrate that while retirement wealth has improved for Whites and individuals of other races between 1989 and 2016, these gains have not been mirrored among Blacks and Hispanics. This disparity points to persistent structural barriers that limit the accumulation of retirement wealth for marginalized groups, making it crucial to examine how these inequalities of opportunity evolve in later stages of life.

By focusing on individuals aged 50 and above, our empirical application captures a stage of life where most decisions related to *accountable effort* have already been made, allowing us

⁵Chernozhukov, Fernandez-Vál, and Melly (2013) refers to this issue as the Durbin problem.

to observe the enduring impact of initial circumstances on inequality of opportunity. This analysis not only tests the robustness of our methodological approach but also provides insights into the socio-economic dynamics affecting aging populations in the United States, particularly in the context of persistent racial and social class disparities.

5.1 Data and estimation strategy

The Health and Retirement Study (HRS) is a longitudinal panel study that surveys a representative sample of Americans over the age of 50. Initiated in 1992, the HRS aims to provide comprehensive data on the health, economic, and social factors influencing the lives of older adults in the United States. Conducted by the University of Michigan’s Institute for Social Research and funded by the National Institute on Aging (NIA) and the Social Security Administration (SSA), the HRS collects data biennially from its participants.

The HRS sample is nationally representative, employing a multi-stage area probability sampling method to ensure the inclusion of diverse subpopulations, including minority groups. The survey gathers detailed information on income, wealth, earnings, and other variables. Notably, the HRS also collects retrospective information on respondents’ early life circumstances, such as parental education, which is crucial for analyzing inequality of opportunity.

We utilize data from the years 2010, and 2020 included in the RAND HRS Longitudinal File 2020 (V2), a user-friendly version of the HRS data curated and maintained by the RAND Corporation. This file harmonizes the complex HRS data into a consistent and easy-to-use format. The 2020 version (V2) includes data from multiple waves of the HRS, up to and including the year 2020.

For our analysis, we use total household earnings as the outcome of interest. To adjust for the equivalent income of couples, we apply the OECD equivalence scale and divide by the square root of 2. To assess the initial conditions of each individual, we use race,

Hispanic ethnicity, year of birth, region of birth, father’s education, and mother’s education as circumstances at birth for our quantile regressions.

We define the marginalized and dominant groups using two variables from the Health and Retirement Study (HRS). The first variable is race, with three categories: White, Black, and Other. The second variable is Hispanic ethnicity, with two categories: Hispanic and Not Hispanic. Based on these variables, we categorize observations into two groups: the marginalized group, which includes individuals who are either Black or Hispanic, and the dominant group, which includes non-Hispanic Whites and non-Hispanic individuals of other races. Regions of birth are grouped into five categories:

- The Northeast, which includes the New England and Mid-Atlantic regions.
- The Midwest, which includes the two northern central regions.
- The South, which includes the South Atlantic and the two southern central regions.
- The West, which includes the Mountain and Pacific regions.
- Other, which includes the US/NA division and individuals not born in the US.

We only keep individuals who are 50 and above in the datasets. After cleaning for missing values, we have 11,198 observations for 2010, and 7,083 observations for 2020. These covariates are used to estimate our quantile regressions. We use a grid size of 0.01 with tail trimming at the 0.05 and 0.95 quantiles. We use 999 bootstrap replications for our estimations and our dominance tests.

5.2 Average complaints for Black and Non-Black individuals

Figures [3](#) and [4](#) illustrate the average complaint incidences for marginalized and dominant older individuals for the years 2010 and 2020, respectively. These incidence curves depict the average complaint at various levels of *accountable effort*, representing the lost earnings

associated with conditions at birth for each *accountable effort* level an individual may choose to exert during their life. This highlights the constraints on opportunities.

The left panels of these figures present the average complaints within the marginalized group, which includes individuals who are either Black or Hispanic. The green area represents the loss linked to social class at birth, while the blue area represents the additional loss attributable to racial stratification. Conversely, the right panels depict the average complaints within the dominant group, which includes non-Hispanic Whites and non-Hispanic individuals of other races. As racial stratification does not impact this group, the overall complaint is illustrated solely by the green area, corresponding to the average loss of opportunities related to social class at birth for each level of *accountable effort*.

A preliminary examination of the figures indicates that visual assessment alone makes it challenging to discern trends in average complaints over time. Nevertheless, a noticeable trend emerges between 2010 and 2020 regarding the racial stratification component. Specifically, the portion of the average complaint associated with racial stratification appears to increase across almost the entire *accountable effort* spectrum, with the increase being more pronounced at the upper end. This trend is particularly concerning, as it indicates not only a general rise in inequality of opportunity due to racial stratification but also a heightened barrier at the upper levels of effort, suggesting an increase in the “glass ceiling” effect. To rigorously assess these temporal differences, we will formally test for variations between the years using the dominance conditions proposed in Section 3.

5.3 Inequality of opportunity

In this section, we analyze the evolution of overall inequality of opportunity among older individuals between the years 2010 and 2020. Figure 5 displays the pairwise comparisons of complaint incidence curves, pro-poor complaint concentration curves, and meritocratic complaint concentration curves for 2010 and 2020, while the first column in Table 2 provides

the corresponding p -values for the associated statistical tests.

The upper panel in the figure illustrates that the complaint incidence curve for 2010 consistently lies below that of 2020 at *accountable effort* levels where the confidence bands do not overlap. This visual observation is reinforced by the p -values, which indicate that inequality of opportunity in 2010 is significantly lower than in 2020 for any inequality of opportunity index within the general class Ω , with results significant at the 0.05 level. The left and right bottom panels illustrate respectively the pro-poor and meritocratic complaint concentration curves comparisons. Although not very clear from a quick visual inspection, both complaint concentration curves of 2010 consistently lie below that of 2020 in regions where the confidence bands do not overlap. The p -values associated with these dominance conditions reveal an increase of inequality of opportunity over this decade for indices within the general classes Ω_P and Ω_M , with these results being significant at the 0.01 level.

Overall, our analysis reveals a marked deterioration in equality of opportunity among older individuals during this period. To better understand the underlying factors driving these trends, the next section decomposes inequality into its social class and stratification components, allowing us to explore how these different dimensions contribute to the observed changes, particularly as they may differently affect marginalized and dominant groups.

5.4 Inequality of opportunity decomposed

In this section, we delve into the decomposition of inequality of opportunity among older individuals, focusing on the racial stratification and social class channels. Our analysis leverages pairwise comparisons of stratification and social class complaint incidence curves to shed light on the dynamics between the years 2010 and 2020.

Figure [6](#) presents the pairwise comparison of stratification complaint incidence curves for 2010 and 2020, showing that the curve for 2010 consistently lies below that of 2020. The corresponding p -values, detailed in the second column of Table [2](#) confirm this finding

at the 0.01 significance level, indicating a significant increase in inequality of opportunity due to racial stratification over the decade. Given the strength of this result at the 0.01 significance level, higher-order dominance tests are not required.

This trend is particularly concerning in light of the persistent structural barriers highlighted in the literature, such as employment discrimination, occupational segregation, and wealth disparities, which continue to exacerbate economic inequalities as individuals near retirement (Francis and Weller, 2021). The limited gains in retirement wealth among marginalized groups, as noted by Viceisza, Calhoun, and Lee (2023), likely contribute to these widening disparities.

Next, Figure 7 displays the pairwise comparisons of social class complaint incidence curves, pro-poor complaint concentration curves, and meritocratic complaint concentration curves for 2010 and 2020. The complaint incidence curves intersect, and both p -values are close to 0.05, indicating that caution is needed in interpretation. Since the statistical tests do not establish dominance by rejecting non-dominance, it is advisable to rely on higher-order dominance tests, in this context, the pro-poor and meritocratic tests, to gain clearer insights.

The pro-poor complaint concentration curves do not show a clear result, a finding confirmed by the associated statistical tests. However, although not obvious from visual inspection, the p -values associated with the meritocratic dominance condition indicate that the meritocratic complaint concentration curve for 2020 is below that of 2010 at the 0.05 level. This suggests an improvement in equality of opportunity related to social class from a meritocratic perspective.

Given that the dominant group is primarily affected by the social class component, these findings suggest that the impact of inequality of opportunity may be diminishing for this group. However, the marginalized group, which is impacted by both stratification and social

class components, presents a more complex picture. The opposing directions of movement in these components necessitate further testing for this group.

Unfortunately, the size of the dataset does not allow for significant results for the marginalized group when considered separately. However, our analysis reveals that overall inequality of opportunity has increased over the period, while the component due to social class at birth has decreased when focusing on meritocratic inequality of opportunity indices. Since the dominant group is primarily affected by the social class component, it is likely that the increase in inequality of opportunity is driven by the stratification component, which predominantly impacts the marginalized group. This robust rise in the stratification component is particularly concerning from an equity perspective, as it signals a growing divide in opportunities linked to racial stratification.

These results underscore the multifaceted nature of inequality of opportunity and highlight the distinct experiences of marginalized and dominant groups. While there may be signs of improvement in the social class component for both groups, the marginalized group may face increasing challenges due to the worsening stratification component. This highlights the need for targeted interventions to address these compounded inequalities and promote a more equitable distribution of opportunities across all racial groups.

6 Conclusion

In this paper, we developed a framework for measuring inequality of opportunity that combines Roemer’s (1998) definition with Temkin’s (1986) complaint-based approach. We demonstrated how to decompose these inequalities into social class at birth and identity-based stratification components, providing a robust measurement framework for stratification economics. Our theoretical contribution includes identifying dominance conditions for various indices, including pro-poor and meritocratic indices, and adapting existing econo-

metric methods to estimate and test these dominance conditions in our framework.

Applying this framework to the U.S. aging population between 2010 and 2020, we found increasing inequality of opportunity driven primarily by racial stratification. While meritocratic dominance tests suggest some improvement in class-based inequality, the intersection of complaint incidence curves indicates this improvement may not extend to non-meritocratic indices. Moreover, our analysis reveals divergent trends between dominant and marginalized groups: the former, affected primarily by social class, experienced modest improvements, while the latter, impacted by both class and racial stratification, faced mounting barriers to opportunity. These findings highlight the need for targeted policy interventions that address both class-based and identity-based barriers to opportunity.

Our measurement framework extends naturally to contexts with different identity markers and multiple intersecting identities. This flexibility, combined with its ability to separately identify class-based and identity-based sources of inequality, makes it a valuable tool for analyzing disparities in opportunity across diverse social and economic settings.

References

- [1] Bayer, P. and K.K. Charles (2018), Divergent Paths: Structural Change, Economic Rank, and the Evolution of Black-White Earnings Differences, 1940–2014, *Quarterly Journal of Economics*, 133, 1459-1501.
- [2] Bjerk, D. (2008), Glass Ceilings or Sticky Floors? Statistical Discrimination in a Dynamic Model of Hiring and Promotion, *The Economic Journal*, 118, 961-982.
- [3] Brunori, P., F.H.G. Ferreira, and V. Peragine (2021), Prioritarianism and Equality of Opportunity, IZA Discussion Papers Series, 14100.
- [4] Brunori, P., F. Palmisano, and V. Peragine (2019), Inequality of opportunity in sub-Saharan Africa, *Applied Economics*, 51, 6428-6458.
- [5] Chelwa, G., D. Hamilton, and J. Stewart (2022), Stratification Economics: Core Constructs and Policy Implications, *Journal of Economic Literature*, 60, 377-399.
- [6] Chernozhukov, V., I. Fernandez-Vál, and B. Melly (2013), Inference on counterfactual distributions, *Econometrica*, 78, 1093-1125.
- [7] Chetty, R., N. Hendren, M.R. Jones, and S.R. Porter (2020), Race and Economic Opportunity in the United States: an Intergenerational Perspective, *Quarterly Journal of Economics*, 135, 711- 783.
- [8] Cowell, F. and U. Ebert (2004), Complaints and inequality, *Social Choice and Welfare*, 23, 71-89.
- [9] Darity Jr., W.A. (2005), Stratification Economics: The Role of Intergroup Inequality, *Journal of Economics and Finance* 29, 144-153.

- [10] Darity Jr., W.A. (2022), Position and Possessions: Stratification Economics and Intergroup Inequality, *Journal of Economic Literature*, 60, 400-426.
- [11] Darity Jr., W.A., D. Hamilton, and J.B. Stewart (2015), A Tour de Force in Understanding Intergroup Inequality: An Introduction to Stratification Economics, *Review of Black Political Economy*, 42, 1-6.
- [12] Davidson, R. and J.-Y. Duclos (2013), Testing for Restricted Stochastic Dominance, *Econometric Reviews*, 32, 84-125.
- [13] Davillas, A. and A.M. Jones (2020), Ex ante inequality of opportunity in health, decomposition and distributional analysis of biomarkers, *Journal of Health Economics*, 69, 102251.
- [14] Derenoncourt, E. and C. Montialoux (2021), Minimum Wages and Racial Inequality, *Quarterly Journal of Economics*, 136, 169-228.
- [15] Donald, S.G. and Y.-C. Hsu (2014), Estimation and inference for distribution functions and quantile functions in treatment effect models, *Journal of Econometrics*, 178, 383-397.
- [16] Donald, S.G. and Y.-C. Hsu (2016), Improving the power of tests of stochastic dominance, *Econometric Reviews*, 35, 553-585.
- [17] Ferreira, F.H.G. and J. Gignoux (2011), The Measurement of Inequality of Opportunity: Theory and an Application to Latin America, *Review of Income and Wealth*, 57, 622-657.
- [18] Ferreira, F.H.G. and J. Gignoux (2014), The Measurement of Educational Inequality: Achievement and Opportunity, *World Bank Economic Review*, 28, 210-246.

- [19] Francis, D.V. and C.E. Weller (2021), Race, ethnicity, and retirement security in the United States, Oxford Research Encyclopedia of Economics and Finance.
<https://doi.org/10.1093/acrefore/9780190625979.013.741>.
- [20] Khaled, M.A., P. Makdissi, and M. Yazbeck (2018), earnings-related health transfers principles and orderings of joint distributions of earnings and health, *Journal of Health Economics*, 57, 315-331.
- [21] Koenker, R., and G. Bassett, Jr. (1978), Regression quantiles, *Econometrica*, 46, 33-50.
- [22] Koenker, R. (2005), *Quantile Regression*, Econometric Society Monograph Series, Vol. 38., Cambridge University Press, Cambridge.
- [23] S.Y. Lee, A. Seshadri (2018) Economic Policy and Equality of Opportunity, *The Economic Journal*, 128, F114-F151.
- [24] Pistolesi, N. (2009), Inequality of opportunity in the land of opportunities, 1968–2001, *Journal of Economic Inequality*, 7, 411-433.
- [25] Roemer, J.E. (1998), *Equality of Opportunity*, Harvard University Press.
- [26] Schechtman, E., A. Shelef, A., S. Yitzhaki, and R. Zitikis (2008), Testing hypotheses absolute concentration curves and marginal conditional stochastic dominance, *Econometric Theory*, 24, 1044-1062.
- [27] Temkin, L.S. (1986), Inequality, *Philosophy & Public Affairs*, 15, 99-121.
- [28] Viceisza, A., A. Calhoun, and G. Lee (2023), Racial and Ethnic Disparities in Retirement Outcomes: Impacts of Outreach, *Review of Black Political Economy*.
<https://doi.org/10.1177/00346446231182343>

A Proof of theorems

Proof of Theorem 1. First note that equation (5) can be rewritten as

$$I(F_{Y,X,G}) = \int_0^1 \omega(e) \sum_{g \in \{D,M\}} \Pr[G = g] \int_{\mathcal{X}} \kappa(e, x, g) dF_{X|G}(x|g) de. \quad (34)$$

Combining equations (14) and (34) we get

$$I(F_{Y,X,G}) = \int_0^1 \omega(e) CI(e, F_{Y,X,G}) de. \quad (35)$$

Similarly, equations (11) and (12) can be rewritten as

$$I^{Strat}(F_{Y,X,G}) = \int_0^1 \omega(e) CI^{Strat}(e, F_{Y,X,G}) de. \quad (36)$$

$$I^{Class}(F_{Y,X,G}) = \int_0^1 \omega(e) CI^{Class}(e, F_{Y,X,G}) de. \quad (37)$$

Using equation (35), we can write

$$\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) = \int_0^1 \omega(e) [CI(e, F_{Y,X,G}^1) - CI(e, F_{Y,X,G}^0)] de. \quad (38)$$

Since $\omega(e) \geq 0$ for all $e \in [0, 1]$, $CI(e, F_{Y,X,G}^1) - CI(e, F_{Y,X,G}^0) \leq 0 \ \forall e \in [0, 1]$ implies that $\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$. Similar results hold for $\Delta I^{Strat}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$ and $\Delta I^{Class}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$. This proves the sufficiency of the condition.

Having provided a sufficiency condition let us now prove for the necessity of the condition. In order to prove necessity, consider the following social weight function:

$$\omega(e) = \begin{cases} 0 & 0 \leq e_c \\ 1/\varepsilon & e_c \leq e \leq e_c + \varepsilon \\ 0 & e \geq e_c + \varepsilon \end{cases} \quad (39)$$

The inequality of opportunity index $I_\varepsilon(\cdot)$ having the social weight function in (39) belongs to Ω . Now assume that for an arbitrary small $\varepsilon > 0$, we have $CI(e, F_{Y,X,G}^1) - CI(e, F_{Y,X,G}^0) \leq 0 \ \forall e \in [0, e_c] \cup [e_c + \varepsilon, 1]$ and $CI(e, F_{Y,X,G}^1) - CI(e, F_{Y,X,G}^0) > 0 \ \forall e \in [e_c, e_c + \varepsilon]$. In this case, $\Delta I_\varepsilon(F_{Y,X,G}^0, F_{Y,X,G}^1) > 0$. Similar results also hold for $\Delta I^{Strat}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$ and $\Delta I^{Class}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$. This proves the necessity of the condition. $\square$

Proof of Theorems 2 and 3. Integrating by parts equation (38) yields

$$\begin{aligned}\Delta I(F_{Y,X,G}^0, F_{Y,A,G}^1) &= \omega(e) [CC_p(e, F_{Y,X,G}^1) - CC_p(e, F_{Y,X,G}^0)] \Big|_0^1 \\ &\quad - \int_0^1 \frac{d\omega(e)}{de} [CC_p(e, F_{Y,X,G}^1) - CC_p(e, F_{Y,X,G}^0)] de. \quad (40)\end{aligned}$$

or

$$\begin{aligned}\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) &= -\omega(e) [CC_m(e, F_{Y,X,G}^1) - CC_m(e, F_{Y,X,G}^0)] \Big|_0^1 \\ &\quad + \int_0^1 \frac{d\omega(e)}{de} [CC_m(e, F_{Y,X,G}^1) - CC_m(e, F_{Y,X,G}^0)] de. \quad (41)\end{aligned}$$

Since $CC_p(0, F_{Y,X,G}) = 0$ and $\omega(1) = 0$ for all $I(\cdot) \in \Omega_P$, equation (40) can be rewritten as

$$\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) = - \int_0^1 \frac{d\omega(e)}{de} [CC_p(e, F_{Y,X,G}^1) - CC_p(e, F_{Y,X,G}^0)] de. \quad (42)$$

Similarly, since $CC_m(1, F_{Y,X,G}^1) = 0$ and $\omega(0) = 0$ for all $I(\cdot) \in \Omega_M$, equation (41) can be rewritten as

$$\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) = \int_0^1 \frac{d\omega(e)}{de} [CC_m(e, F_{Y,X,G}^1) - CC_m(e, F_{Y,X,G}^0)] de. \quad (43)$$

Let us start with equation (42). For indices $I(\cdot) \in \Omega_P$, $\frac{d\omega(e)}{de} \leq 0$ for all $e \in [0, 1]$. This implies that if $CC_p(e, F_{Y,X,G}^1) - CC_p(e, F_{Y,X,G}^0) \leq 0 \forall e \in [0, 1]$ then $\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$.

Let us now turn to equation (43). For indices $I(\cdot) \in \Omega_M$, $\frac{d\omega(e)}{de} \geq 0$ for all $e \in [0, 1]$. This implies that if $CC_p(e, F_{Y,X,G}^1) - CC_p(e, F_{Y,X,G}^0) \leq 0 \forall e \in [0, 1]$ then $\Delta I(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$.

Similar results hold for $\Delta I^{Strat}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$ and $\Delta I^{Class}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$. This proves the sufficiency of the conditions in Theorems 2 and 3.

Having provided a sufficiency condition let us now prove for the necessity of the condition. In order to prove necessity of the conditions in Theorem 2, consider the following social weight function:

$$\omega(e) = \begin{cases} \frac{2}{2e_c + \varepsilon} & 0 \leq e_c \\ \frac{2(e_c + \varepsilon - e)}{\varepsilon(2e_c + \varepsilon)} & e_c \leq e \leq e_c + \varepsilon \\ 0 & e \geq e_c + \varepsilon \end{cases} \quad (44)$$

The inequality of opportunity index $I_\varepsilon(\cdot)$ having the social weight function in (44) belongs to Ω_P . Now assume that for a arbitrary small $\varepsilon > 0$, we have $CC_p(e, F_{Y,X,G}^1) - CC_p(e, F_{Y,X,G}^0) \leq 0 \ \forall e \in [0, e_c] \cup [e_c + \varepsilon, 1]$ and $CC_p(e, F_{Y,X,G}^1) - CC_p(e, F_{Y,X,G}^0) > 0 \ \forall e \in [e_c, e_c + \varepsilon]$. In this case, $\Delta I_\varepsilon(F_{Y,X,G}^0, F_{Y,X,G}^1) > 0$. Similar results also hold for $\Delta I^{Strat}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$ and $\Delta I^{Class}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$. This proves the necessity of the condition for Theorem 2.

In order to prove necessity of the conditions in Theorem 3, consider the following social weight function:

$$\omega(e) = \begin{cases} 0 & 0 \leq e_c \\ \frac{2(e-e_c)}{\varepsilon(2e_c+\varepsilon)} & e_c \leq e \leq e_c + \varepsilon \\ \frac{2}{2-2e_c-\varepsilon} & e \geq e_c + \varepsilon \end{cases} \quad (45)$$

The inequality of opportunity index $I_\varepsilon(\cdot)$ having the social weight function in (45) belongs to Ω_P . Now assume that for a arbitrary small $\varepsilon > 0$, we have $CC_m(e, F_{Y,X,G}^1) - CC_m(e, F_{Y,X,G}^0) \leq 0 \ \forall e \in [0, e_c] \cup [e_c + \varepsilon, 1]$ and $CC_m(e, F_{Y,X,G}^1) - CC_m(e, F_{Y,X,G}^0) > 0 \ \forall e \in [e_c, e_c + \varepsilon]$. In this case, $\Delta I_\varepsilon(F_{Y,X,G}^0, F_{Y,X,G}^1) > 0$. Similar results also hold for $\Delta I^{Strat}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$ and $\Delta I^{Class}(F_{Y,X,G}^0, F_{Y,X,G}^1) \leq 0$. This proves the necessity of the condition for Theorem 3. $\square$

B Details on the Riemann sum approximations for the complaint concentration curves

The Riemann sum approximations for the complaint concentration curves are:

$$\widehat{CC}_p(e, \widehat{F}_{Y,X,G}) = \frac{1}{L} \sum_{\ell=1}^L \widehat{CI}(e_\ell, \widehat{F}_{Y,X,G}) \mathbb{1}[e_\ell \leq e], \quad (46)$$

$$\widehat{CC}_p^{Strat}(e, \widehat{F}_{Y,X,G}) = \frac{1}{L} \sum_{\ell=1}^L \widehat{CI}^{Strat}(e_\ell, \widehat{F}_{Y,X,G}) \mathbb{1}[e_\ell \leq e], \quad (47)$$

$$\widehat{CC}_p^{Class}(e, \widehat{F}_{Y,X,G}) = \frac{1}{L} \sum_{\ell=1}^L \widehat{CI}^{Class}(e_\ell, \widehat{F}_{Y,X,G}) \mathbb{1}[e_\ell \leq e], \quad (48)$$

$$\widehat{CC}_m(e, \widehat{F}_{Y,X,G}) = \frac{1}{L} \sum_{\ell=1}^L \widehat{CI}(e_\ell, \widehat{F}_{Y,X,G}) \mathbb{1}[e_\ell \geq e], \quad (49)$$

$$\widehat{CC}_m^{Strat}(e, \widehat{F}_{Y,X,G}) = \frac{1}{L} \sum_{\ell=1}^L \widehat{CI}^{Strat}(e_\ell, \widehat{F}_{Y,X,G}) \mathbb{1}[e_\ell \geq e], \quad (50)$$

and

$$\widehat{CC}_m^{Class}(e, \widehat{F}_{Y,X,G}) = \frac{1}{L} \sum_{\ell=1}^L \widehat{CI}^{Class}(e_\ell, \widehat{F}_{Y,X,G}) \mathbb{1}[e_\ell \geq e]. \quad (51)$$

If the grid size is small and tail trimming is required, we should adjust equation (46) for $e = \varepsilon$ using:

$$\widehat{CC}_p(\varepsilon, \widehat{F}_{Y,X,G}) = \varepsilon \times \widehat{CI}(\varepsilon, \widehat{F}_{Y,X,G}). \quad (52)$$

Equations (47) and (48) should be adjusted similarly. Additionally, equation (49) should be adjusted for $e = 1 - \varepsilon$ using:

$$\widehat{CC}_m(1 - \varepsilon, \widehat{F}_{Y,X,G}) = \varepsilon \times \widehat{CI}(1 - \varepsilon, \widehat{F}_{Y,X,G}). \quad (53)$$

Equations (50) and (51) should be adjusted in a similar manner.

C Bootstrap algorithm

The bootstrap algorithm is constructed as follows. Assume that we have an i.i.d. sample of size n_0 from the random variable corresponding to first theoretical curve L_0 and and and i.i.d. sample of size n_1 from the random variable corresponding to the second theoretical curve C_1 . Denote those samples by $\mathcal{S}_0$ and $\mathcal{S}_1$ respectively. Let $\widehat{C}_0$ and $\widehat{C}_1$ be the nonparametric estimators of C_0 and C_1 respectively, constructed from those two samples. Let

$$\widehat{\tau} = \sqrt{\frac{n_0 n_1}{n_0 + n_1}} \sup_e [\widehat{C}_1(e) - \widehat{C}_0(e)]$$

1. Repeat for $b = 1, \dots, B$

(a) Draw a sample of size n_0 from $\mathcal{S}$. Compute the nonparametric estimator $\widehat{C}_{0b}$.

(b) Draw a sample of size n_1 from $\mathcal{S}$. Compute the nonparametric estimator $\widehat{C}_{1b}$.

(c) Compute

$$\widehat{\tau}_b = \sqrt{\frac{n_0 n_1}{n_0 + n_1}} \sup_e [\widehat{C}_{1b}(e) - \widehat{C}_{0b}(e) - \widehat{C}_1(e) + \widehat{C}_0(e) + \widehat{\mu}_{1,2}(n)],$$

where $\widehat{\mu}_{1,2}(n) = [\widehat{C}_1(e) - \widehat{C}_0(e)] \mathbb{1}\{\widehat{C}_1(e) - \widehat{C}_0(e) < \alpha(n)\}$. Donald and Hsu (2014) explain that $\alpha(n)$ should be such that $\alpha(n) \rightarrow 0$ and $\sqrt{n}\alpha(n) \rightarrow \infty$.

We use $\alpha(n) = -\frac{\sqrt{\log(\log(n))}}{10\sqrt{n}}$

2. Using the sample $\widehat{\tau}_1, \dots, \widehat{\tau}_B$, compute the bootstrap p -value

$$\frac{1}{B} \sum_{b=1}^B \mathbb{1}(\widehat{\tau}_b > \widehat{\tau}).$$

Tables and Figures

Table 1: Interpretation of dominance tests for a level of significance α

p -values	Interpretation
$p_1 \geq \alpha$ and $p_2 \geq \alpha$	$C_1(e) = C_0(e)$
$p_1 < \alpha$ and $p_2 \geq \alpha$	$C_0(e) \leq C_1(e), \forall e \in [0, 1]$
$p_1 \geq \alpha$ and $p_2 < \alpha$	$C_1(e) \leq C_0(e), \forall e \in [0, 1]$
$p_1 < \alpha$ and $p_2 < \alpha$	$C_0(e)$ and $C_1(e)$ intersect

Table 2: Complaints dominance tests p -values

Year A Year B	Overall	Stratification	Social Class
$C_{2010}(e) \leq C_{2020}(e)$	$**$ 0.5375	$***$ 0.3363	0.0404
$C_{2020}(e) \leq C_{2010}(e)$	0.0100	0.0000	0.0606
$PC_{2010}(e) \leq PC_{2020}(e)$	$***$ 0.2883	-	0.2232
$PC_{2020}(e) \leq PC_{2010}(e)$	0.0000	-	0.3073
$MC_{2010}(e) \leq MC_{2020}(e)$	$***$ 0.4294	-	$**$ 0.0480
$MC_{2020}(e) \leq MC_{2010}(e)$	0.0000	-	0.6136
$**$: Dominance at the 0.05 level			
$***$: Dominance at the 0.01 level			

Figure 1: Complaint associated with an effort e_1 for type (x, g)

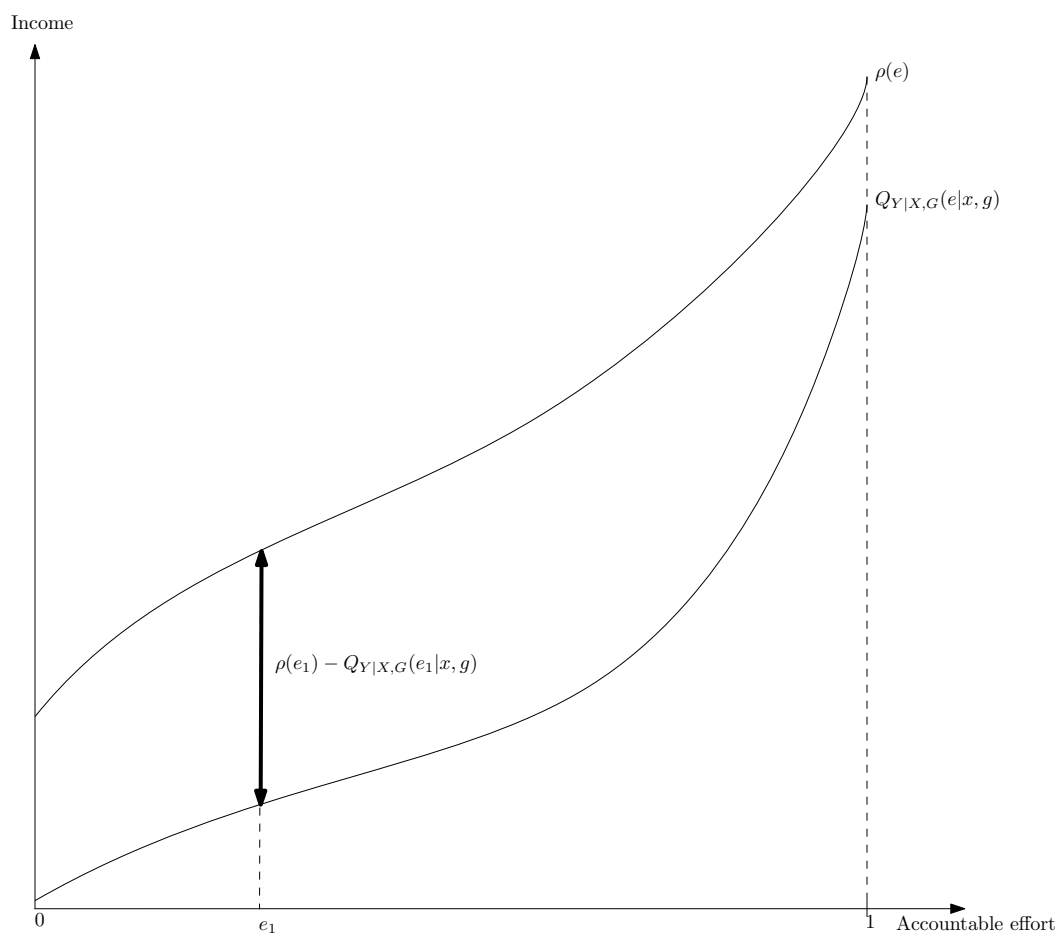


Figure 2: Decomposition of the complaint associated with an effort e_1 of type (a, x, M)

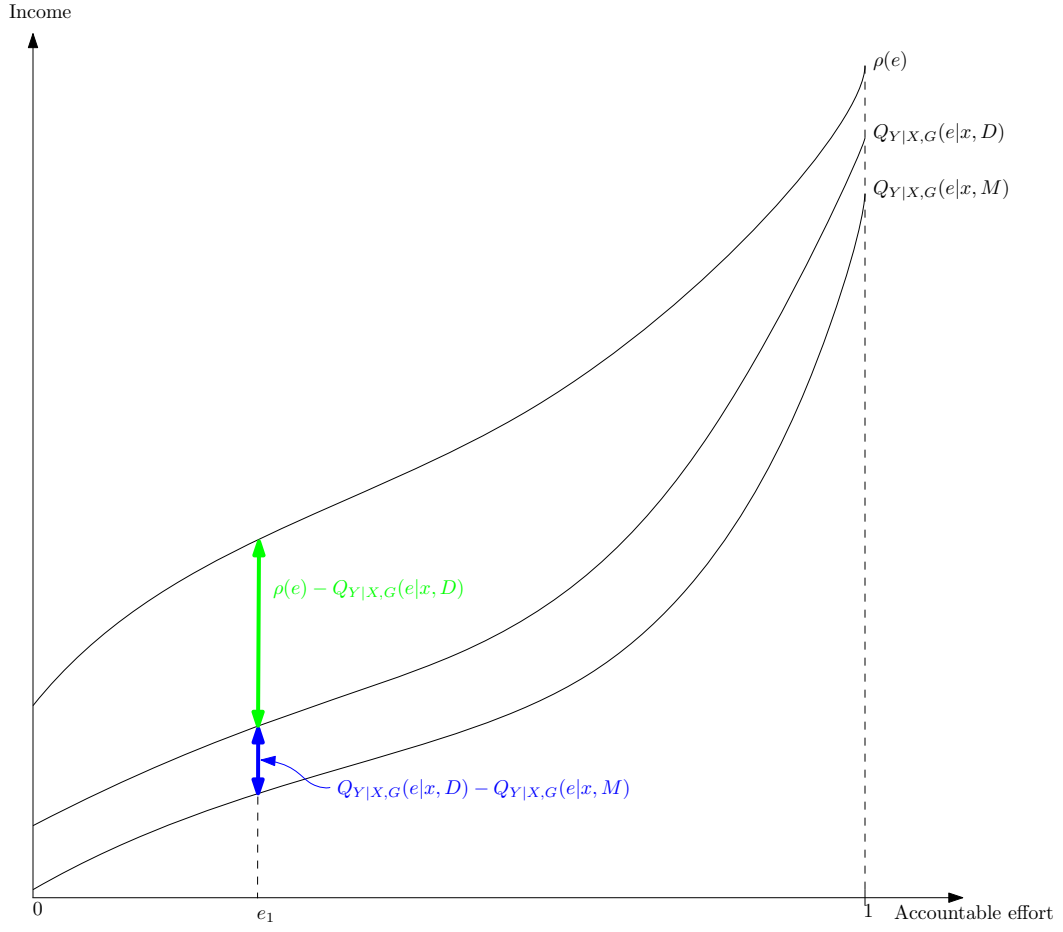


Figure 3: Complaint incidence by racial group, 2010

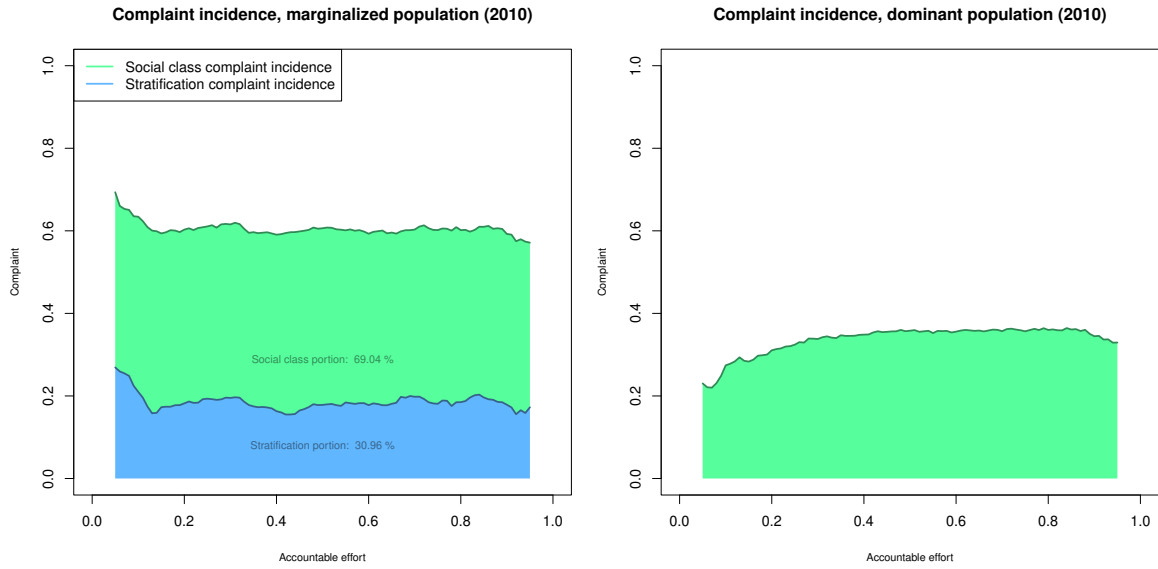


Figure 4: Complaint incidence by racial group, 2020

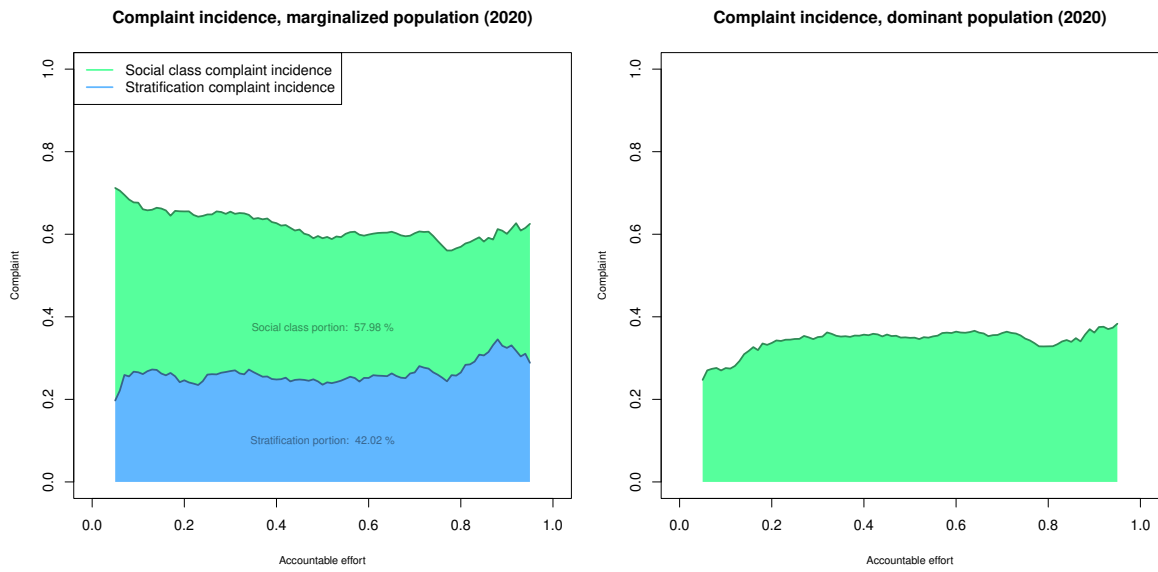


Figure 5: Complaint incidence curves

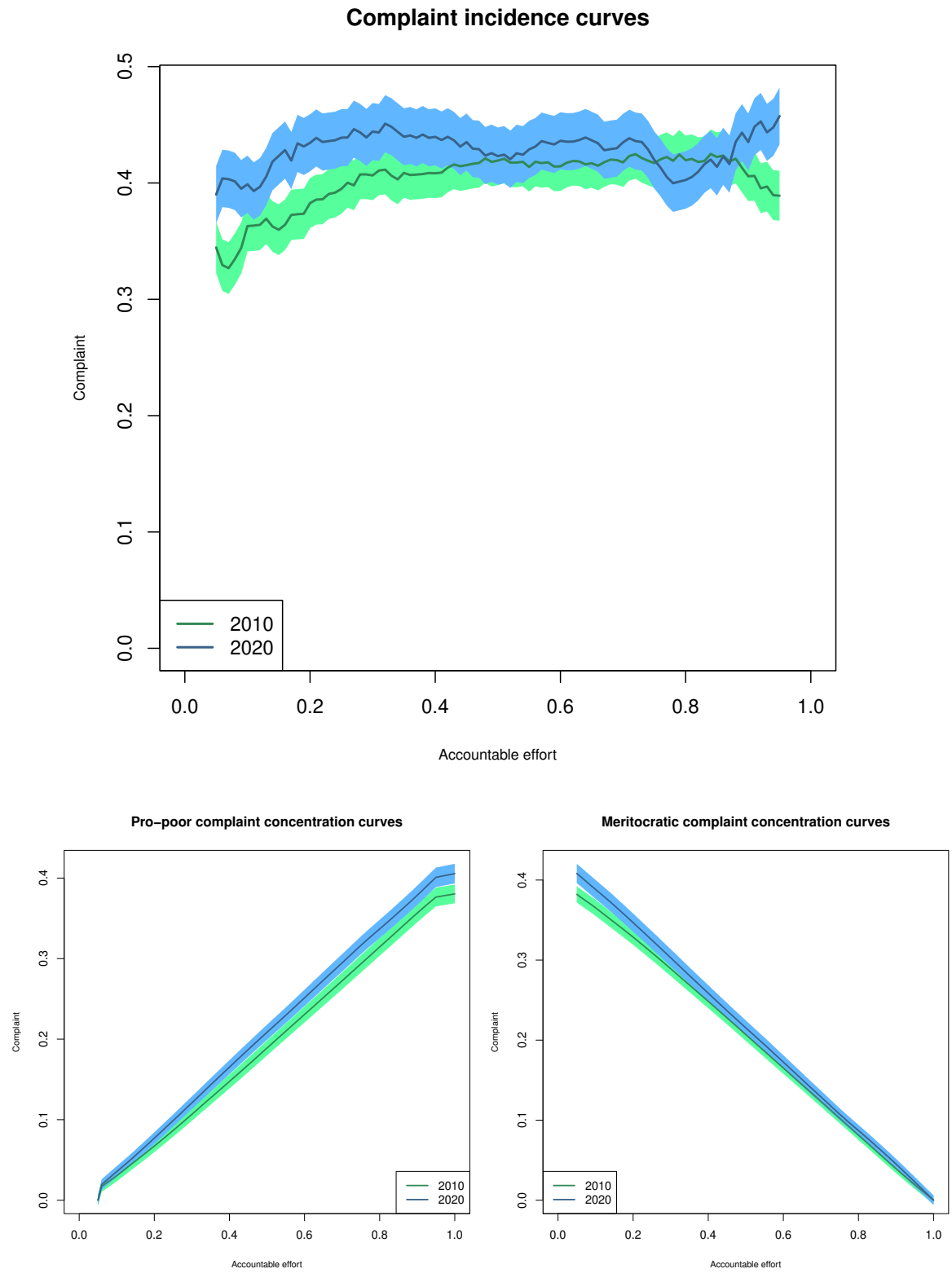


Figure 6: Stratification incidence curves

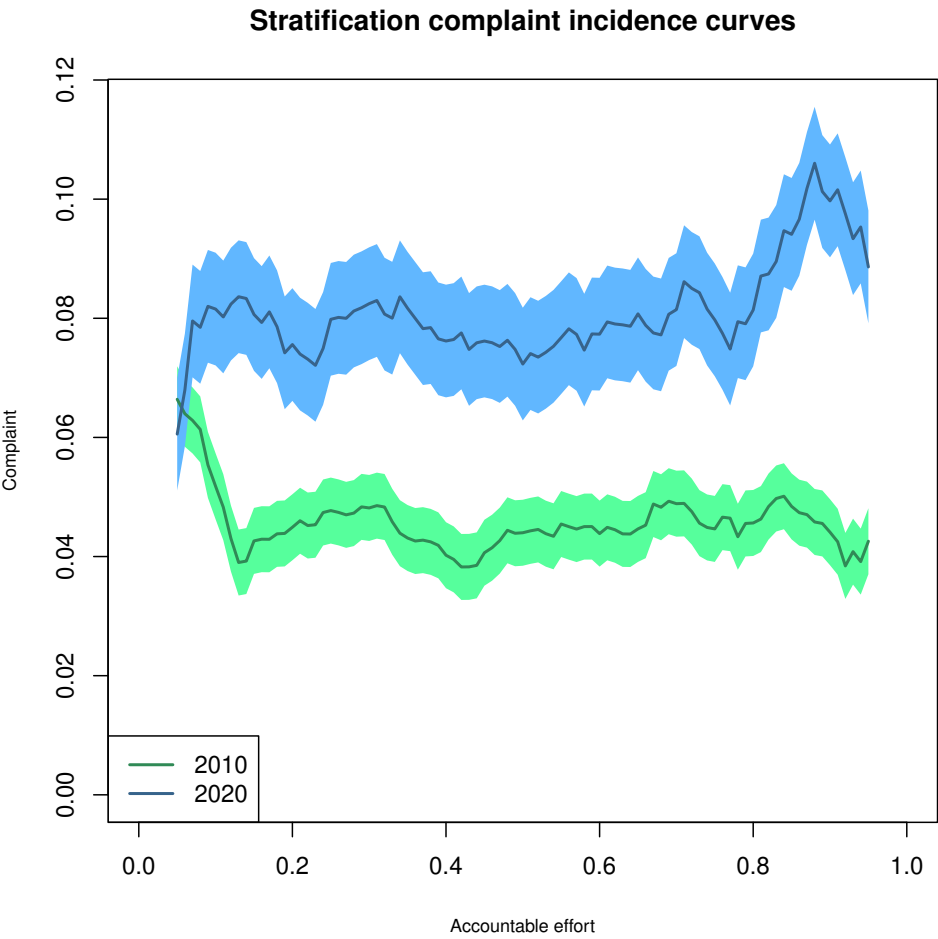


Figure 7: Social class complaint incidence curves

