Rethinking the Edgeworth–Walras Convergence on Perfect Competition à la Debreu–Scarf

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Sept. 2010

1   Introduction

This paper is part of a larger research project on the evolution of the *Perfect Competition* concept in a historical perspective. We try to follow the changes this concept has gone through from the different alternative views during the so–called "Marginal Revolution" towards the consolidation of the price–taker hypothesis.

Some recent theoretical developments have underlined the importance of giving up the hypothesis of price taking agents.[1] These developments plead for a different conception of the theoretical functioning of the market. This implies to model a perfectly competitive market based on strategic behaviour rather than using the traditional Walrasian conception of agents. These works intend avoiding the common trend in economic theory where imperfect competition is increasingly taking the place of perfect competition as the general framework because of, as has been stated by Arrow (1959), if we accept the price–taker hypothesis as the equivalent of perfect competition we have no other alternative than to introduce the Walrasian auctioneer. This 'pessimistic view' on perfect competition pushes Arrow to postulate that in order to give a more realistic interpretation of economic reality (without the fiction of the centralizing auctioneer) we need to build imperfect competition models. The only difference with the basic Walrasian competitive model being the abandonment of the hypothesis of price–taker agents.

The Economics of Imperfect Competition has reached an important stage of development in terms of complexity without losing its capacity to raise and answer an important number of issues. This has been mainly built upon the abandonment of the so–called "walrasian perfect competition framework". One of the main arguments advanced since the 1970s against the walrasian framework is the absence of an explicit treatment of decentralized exchange and money. Both issues are

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[1] See Makowski & Ostroy (2001) and Gale (2000) for two essential references on the recent literature about the reestablishment of the perfect competition framework within a strategic behaviour approach as an alternative view regarding the traditional price–taker hypothesis.

evidently linked. The failures of the walrasian theory of money are often identified with the presence of a centralized exchange process which would be linked with a centralized price mechanism and the auctioneer. However, it seems generally accepted that the well known Debreu–Scarf theorem (Debreu & Scarf 1963) on the convergence of the core to the set of walrasian equilibria is the final proof of the strong relation between Edgeworth's and Walras's views on perfect competition. Following this interpretation, Edgeworth's theory of recontracting is a suitable way to make explicit what is only implicit within the walrasian framework: the very conditions under which a perfect competitive allocation emerges endogenously from agents' descentralized interactions. Edgerwoth's approach, still following this interpretation, would show the process of negotiation between agents leading to a solution which in the walrasian approach is left to the auctioneer fiction. This is a story of two players and two winners. From the walrasian point of view, the competitive price vector is seen as more than a spurious representation of an efficient allocation. It is the limit case of economic competition and it appears to be a very robust situation. While from an edgeworthian view, the Debreu–Scarf result has been seen as the demonstration of his genius conjecture, and a rigorous generalisation of Edgeworth's ideas. See Makowski & Ostroy (2001) and Gale (2000) for two essential references on the recent literature about the reestablishment of the perfect competition framework within a strategic behaviour approach as an alternative view regarding the traditional price–taker hypothesis.

Our paper evaluates the traditional interpretation of the Walras–Edgeworth convergence on perfect competition. Following modern literature on the Debreu–Scarf theorem, we show that this convergence can be questioned from two main points of view. First, a main challenge for the walrasian approach is revealed by the theoretical framework within which the notion of core and recontracting has been built by Debreu and Scarf. The Walrasian General Equilibrium model is based on hypothesis about individual behaviour and institutional structure of markets that avoid a lot of questions which are unavoidable once the edgeworthian recontracting process is considered.

General Equilibrium theory, being incapable of solving questions on stability, presents this convergence as an important result which can, at least partially, push this threat away. The Walrsian framework can be modified in order to incorporate an important solution concept from game theory, inspired from Edgeworth: the core. The convergence between Edgeworth and Walras is thus usually presented as the definitive proof of an ancient conjecture: perfect competition can be obtained as the limit situation of an economy with a great number of agents. Building upon the Debreu–Scarf theorem, economists also assert that efficient and economic competition converges, apparently giving an answer to those who conceived Game Theory and the Edgeworthian heritage as an alternative to Walras:

> *Consider "competition": the common sense meaning is one of struggle with others of fight, of attempting to get ahead or at least to hold one's place. It suffices to consult any dictionary of any language to find that it describes rivalry, fight, struggle, etc. Why this word should be used in economic theory in a way that contradicts ordinary language is difficult to see. No reasonable case can be made for this absurd usage which may confuse and must repel any intelligent novice.*

In current equilibrium theory there is nothing of this true kind of competition: there are only individuals firms or consumers, facing given prices fixed conditions, each firm or consumer for convince insignificantly small and having no influence whatsoever upon the existing conditions of the market (rather mysteriously formed by tâtonnement) and therefore solely concerned with maximizing sure utility or profit... (Morgenstern 1972, p.1164)

As is clear from this quotation, the echo of Edgeworth's criticism to Walras was related to scholars' inconformity concerning the walrasian price taker–hypothesis. The importance of this dissent has been ignored within the development of the cooperative game approach. On the other hand, historians of economic thought are well aware of the confrontation between these two important figures of the late XIXth century economics. However, this confrontation has been interpreted as a methodological misunderstanding.[2] Most scholars credit this misunderstanding to a difference in their conceptions of the dynamic process leading towards an equilibrium position. Making use of the authors' public and private opinions [3] (as in their correspondence), this debate has been well studied. The common point of view of historians of economics points mainly[4] to the idea of the possibility of an effective out of equilibrium exchange during the process of convergence to a general equilibrium situation. As Walker (1973) puts it abridging Edgeworth's position:[5]

... that is why Edgeworth rejected the tâtonnement *approach his alternative was an advance over the Walrasian conception, an early step in a new direction towards a line of investigation that is still being slowly worked out –a dynamic non–tâtonnement theory of exchange, production and consumption.* (Walker 1973, p.147)

From this interpretation of Edgeworth's analysis of the recontracting process it follows that the main difference between his and Walras's theory of perfect competition can only be observed if we take into account the out of equilibrium process. This interpretation finally joins the modern idea of convergence between both authors represented by the Debreu–Scarf theorem, as it is clearly recognized by Rebeyrol (1999):

Les économistes contemporains assimilent la concurrence parfaite à un comportement de "preneurs de prix" de la part des agents. Cette conception est associée à l'idée qu'en concurrence, les agents sont très nombreux et donc atomisés, sans pouvoir sur le rapport d'échange global qui prévaudra au marché. Elle est plus ou moins renforcée par la reconnaissance du théorème limite d'Edgeworth. (Rebeyrol 1999, p.99)

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[2] See Walker (n.d.), Rebeyrol (1999, p.78–100), Berta (2001) and Bridel & Huck (2002).

[3] Edgeworth (2003 [1889]*a*), (2003 [1889]*b*) y (2003 [1891]) and Walras (1992 [1888]) and (1965). It is also important to include the intervention of Bortkiewicz (1890) on "behalf"of Walras.

[4] With the important exception of Bridel & Huck (2002) that we follow very closely concerning the interpretation of this debate, though we go further in the analytical interpretation of the implications for modern economics.

[5] This idea is mainly developed in Negishi's interpretation of Edgeworth (Negishi 1985 and 1987). For an interesting analysis of this idea see Pignol (2000). We study different modern interpretations of Edgeworth's recontracting theory in another paper.

This paper tries to reconstruct Walras's and Edgeworth's conception of perfect competition to evaluate the well–founded of this convergence on the properties of the final situation (perfect competitive allocations) inspired by the Debreu–Scarf theorem. We will show that it is useful to reconstruct a historical debate using modern economic analysis. This allows not only shedding new light into the historians' debate but it also contributes to open new questions for modern economic theory.

In what follows, we first expose the main theoretical assumptions that Walras has to introduce in order to ensure the price–taker hypothesis (section 2). We show that, contrary to what modern economics does, it is not enough to assume price–taker agents without discussing the institutional framework within which this hypothesis can be sustainable. We show that Walras's entire institutional framework converges to a particular conception of individuals: non–strategic agents. In this sense, the Walrasian analysis is hard to be translated into a modern strategic game theoretical framework. This allows us to explain why the cooperative–game theoretical approach is more suitable to develop into a Walrasian tradition. It is exactly in this direction that the Debreu–Scarf theorem and the *Core* as an equilibrium solution tried to establish a convergence between Edgeworth and Walras. Next (section 3), we discuss this interpretation of Edgeworth as the forerunner of the cooperative–game tradition. We conclude that the main elements of the Debreu–Scarf theorem are extracted from a particular example given by Edgeworth that obscures the general idea this author wanted to convey. Following recent works on the analysis of the *Core* as an equilibrium solution Vind (1995) it is possible to recover the originality of Edgeworth's analysis and the interest to go further in a alternative way to the cooperative–game framework. We finally present some concluding remarks on the consequences of this analysis for modern "imperfect competition economics".

2   Walras's Theory: Perfect Competition and the Neutrality of Pure Economics

As stated by Jaffé (1980, p.530), Walras's pure economics is an integral part of a larger system of social philosophy. Walras tried to build a pure theory of economics as a neutral system with respect to the criteria of distributional justice. His pure theory of market, prices and money must satisfy commutative justice in order to describe production and exchange as free individual decisions isolated from distributional considerations. Agents must be free to exchange and produce within the limits of feasible allocations that respect individuals' budget constraints. These requirements lead Walras to impose a regime of free competition as the main (and sole) framework of his pure theory of economics.

Since the first versions of his pure theory of prices, Walras intends to distinguish perfect competition from monetary exchange. In the following well known quotation from his 1873 lectures at the Académie de sciences morales et politiques de Paris, Walras presents money as a pure practical

economic device, excluded from the pure theory of competition:

> *Nous allons étudier le phénomène des prix se produisant dans ces conditions de concurrence supposées rigoureuses, en faisant abstraction des petites circonstances perturbatrices comme en physique, en mécanique, on fait abstraction tout d'abord, sauf à les introduire ensuite dans les formules, de la résistance des milieux, du frottement, etc. Seulement, nous écarterons aussi l'intervention de la monnaie. Pour avoir une idée exacte du mécanisme de la concurrence, il nous a bien fallu l'emprunter à l'un de ces marchés où se font des ventes et des achats de marchandises contre or et argent ; mais il est clair que l'intervention de la monnaie, qui est une simplification pratique, est une complication théorique qui doit être écartée.* Walras (1992 [1874], p.33)

The price taker behaviour and the "no–exchange–out–of–equilibrium" hypothesis are the main characteristics of Walras's perfect competition framework. This construction is founded on two elements: the numéraire and the fiction of a tâtonnement process. The former allows avoiding the treatment of strategic behaviours within a general equilibrium system and establishes a dichotomy between the exchange process and price formation. The latter prevents pure theory of prices from dealing with distributional effects of out–of–equilibrium exchanges.

2.1   The Numéraire and the Market Organisation

In the Eléments d'économie politique pure (henceforth EEPP[6]) Walras presents the foundations of his theory of prices and competition in a two commodities pure–exchange framework (Section II, 4th edition). Money is absent from this analysis (it shall remain absent until section VI in the 4th edition). Within this framework the two commodities are directly exchanged in a common market and consequently there exists a unique rate of exchange (relative price). Even in the simplest case of exchange, Walras introduces the hypothesis of price–taking. In order to fulfil the commutative justice requirements, agents exchange only at the equilibrium price of the market. As has been well noticed by Rebeyrol (1999, p.100): *"... l'absence de transaction en déséquilibre n'est une hypothèse exorbitante que si l'on admet l'absence de conscience du déséquilibre."* This hypothesis is the reduced form of the hypothesis of common knowledge of all exchange opportunities. Even if Walras does not explicitly describe the details of the organisation of this two–commodities–market, the reference to the stock–market structure leads to conclude that this is not a disorganised bilateral exchange process.

The generalisation of the price theory to the n–commodities case strengthens the requirements of a well organised market process. In section III of his EEPP 4th edition Walras attempts to demonstrate a "true theorem of general equilibrium". A simple extension of the two–commodities framework leads to a system of one independent market for every couple of commodities (i.e. $n(n-1)/2$ markets for n commodities). However, if nothing is said about the agents' level of information, this can result in an "incoherent" system of prices. A price system is incoherent if

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[6] Walras (1992 [1888]).

different exchange strategies lead to different final allocations for an agent. In other words, if some arbitrage gains of indirect exchange are still possible. The condition of a general equilibrium system of prices is resumed by Walras (1992 [1888]) in EEPP:

> *L'équilibre parfait ou général du marché n'a lieu que si le prix de deux marchandises quelconques l'une en l'autre est égal au rapport des prix de l'une et l'autre en une troisième quelconque.* (Walras 1992 [1888], p.161–163)

This condition is, yet again, a requirement of neutrality of the perfect competition solution. If there are un–exploited possibilities of arbitrage some agents loose exactly what other agents win by exchanging at those prices. The value of an individual's budget constraint is not independent of the order s/he follows in order to obtain her/his desired final allocation. And a generalised direct barter exchange within a system of isolated two–commodities markets may not necessarily result in a perfect general equilibrium situation (Walras 1992 [1888], p.163). However, if those possibilities of gains are common knowledge, the arbitrage strategies may be neutralised. Walras's solution to this problem is the introduction of a common unit of measure of individuals' budget constraints, namely the common numéraire. If prices are all quoted in terms of a common numéraire, agents can verify the coherence of the actual price system. Due to the common knowledge hypothesis on prices, arbitrages are neutralised. In consequence Walras concludes that *"... si on a crié des prix en numéraire, la condition d'équilibre général a été remplie ipso facto"* (Walras 1992 [1888], p.200).

Thus, the numéraire is more than a hypothesis aiming at simplifying the computation of a price system. The existence of a common measure of prices is accompanied of a particular conception of the organisation of markets. The numéraire is the necessary language for the common knowledge hypothesis. The consequence of these hypotheses is a market system where strategic behaviours are absent [7] (because they have already been neutralised!).

2.2 The Tâtonnement Process and Price–taking Behaviour

As Bridel (1997) and Bridel & Huck (2002) has clearly stated, the evolution throughout the different chapters and editions of the EEPP of Walras's theory of price formation explained by a tâtonnement process has important consequences on his monetary theory. Because of Walras's attachment to the internal coherence of his pure economics, the theory of stability and dynamics is subordinated to the notion of neutrality of the exchange process. This implies that the process of price formation in perfect competition is finally presented as an instantaneous adjustment of all markets towards equilibrium.

The well–known discussion of the walrassian tâtonnement appears in the first editions of the EEPP as a description of an adjustment process that actually takes place in the market. Here,

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[7] Costa (1988 and 2002) also analyses the problems Walras faces trying to avoid the arbitrage questions. He concludes that the analytical difficulties associated with those problems lead Walras to avoid an explicit treatment of competition and exchange. Following Rebeyrol (1999), we rather argue that it was not a matter of difficulty but a more profound attachment to a particular conception of pure economics.

Walras underlines the role that competition plays in this process. The literary description of the process, which will prevail until the last edition of the EEPP, sustains that during the formation of equilibrium prices agents have an active role and that competition is the result of individual decisions:

> *La valeur d'échange laissée à elle–même se produit naturellement sur le marché sous l'empire de la concurrence. Comme acheteurs, les échangeurs demandent à l'enchère, comme vendeurs, ils offrent au rabais, et leur concours amène ainsi une certaine valeur d'échange des marchandises tantôt ascendante, tantôt descendante et tantôt stationnaire. Selon que cette concurrence fonctionne plus ou moins bien, la valeur d'échange se produit d'une manière plus ou moins rigoureuse.* (Walras 1992 [1888], p.70)

One question arises after reading this passage: What does a more or less rigorous competition mean? Walras answers in the same paragraph:

> *Les marchés les mieux organisés sous le rapport de la concurrence sont ceux où les ventes et achats se font à la criée, par l'intermédiaire d'agents tels qu'agents de change, courtiers de commerce, crieurs, qui les centralisent, de telle sorte qu'aucun échange n'ait lieu sans que les conditions en soient annoncées et connues et sans que les vendeurs puissent aller au rabais et les acheteurs à l'enchère.* (Walras 1992 [1888], p.70)

That is, if the process of price formation is to take place under perfect competition, without interfering with distributive justice, agents cannot exchange at prices different from those of equilibrium. In order to guarantee this result, market intermediaries centralise supplies and demands thus avoiding exchanges out of equilibrium. These intermediaries are stock brokers. Therefore, in spite of the initial assertion according to which agents are active during the price formation process, Walras ends up reducing perfect competition to a situation where agents are price–takers because the active part of the market is left to crieurs and courtiers. However, these agents appear only as intermediaries during the price formation process but nothing is said about their role within the actual exchange process.

This description of tâtonnement, which seems to appeal to an empirical argument in the text presented above, is clearer in Walras's lecture before the Académie de sciences morales et politiques de Paris read in 1873 (Walras 1993). Here Walras explains that to avoid any possible ambiguity and to guarantee coherence with the aim of his pure theory[8] he assumes tâtonnement takes place through the use of an automatic calculator (a computer) which determines the quantities offered and demanded by each individual and can finally calculate general equilibrium prices

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[8]After the first edition of EEPP (1874), and probably as a consequence of Bertrand's and Edgeworth's criticisms, Walras makes more explicit his hypothesis of absence of exchange out of equilibrium. In spite of this textual evidence Walker (1990a and 1990b) considers, contrary to most interpretation, that it is wrong to assume that this hypothesis means Walras does not study the disequilibrium exchanges. Be this as it may, it is certain that as long as Walras's monetary theory is concerned, disequilibrium exchanges are absent. In lesson 29 of the EEPP 4th edition, "circulation"(exchange process) begins after equilibrium prices are quoted.

(Walras 1993). Walras thus avoids the problem of the consequences of disequilibrium on expectations (if the process takes place under an auctioneer) or on distributive effects (if exchanges take place in disequilibrium). As most of Walras's scholars since Jaffé's works (1976, 1977 and 1980) have remarked, the tâtonnement is a process during which no exchanges take place and can be interpreted (see Bridel 1997, chapter 4; 2002) as a method to solve a system of simultaneous equations whose roots are the equilibrium prices.

Finally, an important reason that completes and reinforces the requirements of commutative justice and explains why Walras focuses on the equilibrium situation is that it is optimal (i.e. Pareto optimal). If exchanges were to take place at disequilibrium prices there would be unsatisfied exchange possibilities that could induce an agent aware of this situation to propose a different price that would improve his condition and would not worsen that of the person willing to exchange with him. The normative[9] properties of general equilibrium are then imposed from the foundation of this theory as the main element economics would never give up. This is why the existence and welfare theorems can be dealt with independently from the dynamics. These theorems allow establishing a theoretical reference with regard to Walras's idea of commutative justice. Therefore, without demonstrating the stability of equilibrium Walras may assert, just as contemporary applied economics does, in his Studies of applied political economy [1898] that:

> *Les éléments du système économique sont des services qui, sous le régime de la libre concurrence, tendent naturellement à se combiner en produits de la nature et de la quantité propres à donner la plus grande satisfaction possible des besoins dans les limites de cette double condition que chaque service comme chaque produit n'ait qu'un seul prix sur le marché et que le prix de vente de chaque produit soit égal à son prix de revient en services.* (Walras 1992 [1988], p.77 our emphasis)

Walras considers that this situation should be the general frame for a pure economic theory. If there is to be a pure theory of money, it must be adapted to the frame of competitive general equilibrium. Besides from being static with passive agents as regards prices and only focusing on equilibrium situations, this framework leaves no place for a theory of the organisation of the exchange process without taking into account the perturbations of the equilibrium so induced. However, Walras attempted, through the different editions of his EEPP and other writings (in particular Walras (1992 [1988])), to introduce a monetary theory aiming at demonstrating the neutrality of the monetary exchange vis–à–vis the equilibrium real prices.

This reasoning may be easily understood from what is nowadays known as recontracting in a model à la Edgeworth. The famous debate between Walras and Edgeworth regarding the tâtonnement process leads to the conclusion that, although Walras does not accept the terms of Edgeworth's argument, he accepts that at least for the pure theory of prices the criteria of commutative

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[9]It is important to underline that here we are referring to the modern concept of "Pareto optimality" which has nothing to say in terms of distributive justice or even of first best results. Then we accept the critical point made by Jaffé (1977) against Hicks and Baumol concerning Walras's extremely optimistic view of a free market issue in terms of a social welfare criterion in terms of first best allocation or distributive justice.

justice and distributive neutrality imply that *"Il est parfaitement juste de proposer immédiatement le prix unique"* (Walras 1896: 184). We shall show in the following section how many theoretical differences between both authors are present behind this apparent methodological misunderstanding.[10]

3 Edgeworth's theory of perfect competitive field: the limit case of a decentralized exchange economy

3.1 Edgeworth's "final settlement" and his equilibrium solution:

Following the developments of General Equilibrium Theory during the 1970's it seems generally accepted that the concept of the "core"as an equilibrium solution is the mathematical representation of Edgeworth's "final settlement". As summarized by Hildenbrand:

> *Since an economy with a finite number of agents is not perfectly competitive, Edgeworth introduced a new concept of equilibrium; the 'final settlement' or in today's language, the core. The 'Limit Theorem' express that under 'perfect competition' these two concepts coincide* (Hildenbrand 1993).

In the present section we shall show how this, now common, interpretation of Edgeworth's theory is not only misleading but it also contributes to erroneously reinforce the idea of the convergence of the price–taking hypothesis and the recontracting analysis. It is necessary to revisit Edgeworth's texts[11] in order to show that the source of this confusion belongs to him using an example of, in modern terminology, a 'replica economy'. This was a simple illustration of Edgeworth's mechanism of recontracting. The convergence of an economy with a great number of agents to a perfect competitive situation has been developed in the XXth century economics as the general Edgeworthian framework. It is precisely within this framework that the idea of a Walras–Edgeworth convergence and the consequent disregarding of their difference and the founding of the price–taker hypothesis as the sole requirement for perfect competition takes its roots.

There are two main consequence derived from the traditional interpretation of Edgeworth as exposed in the above quotation from Hildenbrand. Both of them point out to the direction of the interpretation of Edgeworth as a theorist of the cooperative–games line and the forerunner of the notion of the core as a solution concept. We will discuss these notions from Edgeworth's texts. This allows us to plead in favour of Edgeworth's originality and of the need to rescue his theory of perfect competition.

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[10] For an in–depth discussion of this point see Rebeyrol (1999, p.90–100) and Bridel & Huck (2002). For an opposite point of view see Walker (n.d.).

[11] All references to and quotations from Edgeworth's texts are taken from the recent edition of his EconomicWorks by Newman (ed.) (2003). This contains also the translation of some articles originally published by Edgeworth in French.

3.2 Blocking Coalitions and Combinations

The first consequence of this now traditional interpretation of Edgeworth is that the notion of "final settlement" is necessarily linked to the formation of *blocking–coalitions* as it has been developed within the cooperative game theory. Put in a nutshell, the core is the set of contracts that cannot be improved, from the point of view of an individual, by an alternative arrangement with a different group of agents. This notion thus leads to a representation of Edgeworth's *recontracting* process as a permanent formation and dissolution of coalitions. In this way, the set of actions available to individuals are replaced by the actions of a coalition. Furthermore, the very notion of cooperative games implies that the coalition may be interpreted as a collective decision body. Within this collective body individual strategies are subject to the distributional rule imposed within the coalition. However, this interpretation does not really follow from Edgeworth's texts as we shall now show.

Two textual elements allow us to assert that the language of the cooperative game theory, and the subsequent modern theory of the core, do not correspond to Edgeworth's theory. (i) The modern notion of coalition seems to be an interpretation of Edgeworth's concept of combination and of cooperative associations. However, these two notions are deeply different when one examines them more carefully. Edgeworth clearly establishes a difference between a group of agents entering in a series of bilateral contract relations and a combination. The logic of the former is purely individual in the sense of an individual improvement accepting bilateral contracts within a new group rather than the bilateral contracts the individual has formerly agreed to within another group. In fact, within the new improving–group all contracts continue to be bilaterally established and, more important, these kinds of association do not necessary imply giving up all previous exchanges outside this group.

The particular case, which is typically presented as the Edgeworthian archetype of the recontracting process (see Hildenbrand (1993)), where agents systematically renounce to their ancient contracts in order to recontract within a new group is only an example presented by Edgeworth. It is the very well known example of a *replica economy* where a great number of only two types of agents are gradually introduced in the market. Edgeworth intended to formulate a general concept, with multiple types of agents and unequal number of agents (Edgeworth 2003 [1881], p.43).

It is in fact, when Edgeworth actually discusses the extension of his *theorem of the perfect competitive field* that he presents the notion of a combination as a source of imperfect competition: "Combinations tend to introduce or increase indeterminateness; and the final settlements thereby added are more favourable to combiners than the previously existing ones "(Edgeworth 2003 [1881], p.44).

Furthermore, Edgeworth goes on to present a cooperative association as a particular form of combination as a source of imperfection. This type of collective arrangement is exactly what the cooperative game theory introduces as a coalition. Let's follow Edgeworth's text to clearly appre-

ciate it:

> *The fourth imperfection would seem likely to operate in the case of cooperative associations up to the time when the competitive field shall contain a practically infinite number of such bodies; that is, perhaps for a long time. To fix the ideas, suppose associations of capitalist–workmen, consisting each of 100 members, 50 contributing chiefly capital, and 50 chiefly labour. Let the field of competition consist of 1000 individuals. The point here indicated is that, notwithstanding the numerical size of the field, contract will not be more determinate than if the field consisted of 10 individuals.* (Edgeworth 2003 [1881], p.49)

From this, Edgeworth extracts a completely different conclusion than that implied by the coalitional formation presented in the cooperative game theoretical framework. The core of an economy, in this framework, is simply reduced by the presence of an increasing number of agents forming this kind of coalitions. From the Edgeworthian point of view, the number of agents does not per se guarantee increasing competition when this kind of cooperative arrangement is the rule.

3.3   The example of a replica–economy and the price–taker hypothesis

One last reason that confirms the distance between the cooperative game notion of coalition and the Edgeworthian pairwise arrangement within a subset of agents is that the former implies the presence of a unique–price rule within the coalition and the latter does not. In fact, when Edgeworth presents his example of a replica economy, gradually introducing the same number to both sides of a two–type of agents' economy, he clearly states that contracts continue to be bilateral and the implicit exchange–rates could be different. Actually, the settlements (contracts) prevailing within a group of, let's say two agents of type A and three agents of type B, are not the same. Exchange rates may be bilaterally defined and they could be different. However, when we consider a group composed by agents with the same characteristics and we suppose perfect fluidity of information it is impossible to establish this difference. However, the important question arising from this example is not whether the bilateral contracts among any A and any B could differ from another similar contract, but whether those contracts can be stable in the presence of imperfect information.[12] The real interest of the Edgeworthian framework is the possibility to discuss the presence of imperfections and not only to confine to a rule imposing a unique price within a coalition or for the whole economy.

In fact, Edgeworth compares this example, where a same price prevailes among all bilateral contracts within a subset of agents, with the result obtained by Jevons and Walras. This comparison is important because it allows Edgeworth to show the particularity of his example of a replica economy, compared with his broader notion of recontracting:

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[12] *"It is not necessary to resolve analytically the composite mechanism of a competitive field. It will suffice to proceed synthetically, observing in a simple typical case the effect of continually introducing into the field additional competitors."* Edgeworth (2003 [1881], p.34).

The dealing of an individual in an open market, in which there prevails what may be called the law of price, the relation between the individual's requirements and that quantity collectively–demanded–at–a–price, usually designated by the term Demand, between little d and big D in M. Walras's terminology, is elegantly exhibited by that author.

(...) Here it is attempted to proceed without postulating the phenomenon of uniformity of price by the longer route of contract–curve. When we suppose plurality of natures as well of persons, we have to suppose a plurality of contract–curves. (Edgeworth 2003 [1881], p.40)

Edgeworth constantly criticized the use Jevons and Walras make of the unique–price hypothesis. In fact, as we have forewarned in our introduction, the main point of contention between Edgeworth and Walras refers to the presence of this hypothesis. As Bridel & Huck (2002) have clearly stated:

Well beyond the static or dynamic nature of the tâtonnement, the opposition between the re-contracting principle and the notion of supply–and–demand at given prices, between 'fields of competition' and 'competitive markets' is at the heart of the controversy between Edgeworth and Walras. (Bridel & Huck 2002, p.531)

As is clear from the last quotation, the echo of Edgeworth's criticism to Walras was related to an inconformity by recent theoreticians with respect to the walrasian price–taker–hypothesis.[13]

The richness of Edgeworth's theoretical goal has been obscured in modern theory as a consequence of the enthusiasm generated by the Debreu–Scarf theorem and the reconciliation of those great fathers of modern economic analysis. Contrary to the hope of this line of reasoning, if there exists a convergence in terms of the result (namely the final allocations) of the perfect competitive situation between Edgeworth and Walras, there is no possible reconciliation in terms of the exchange mechanism that allow to establish those results. To put it shortly: Edgeworth's theory of recontracting cannot give content to the elusive notion of tâtonnement as an actual economic process. We think it is much more interesting for modern economic theory to take seriously the confrontation well stated by Edgeworth himself:

Equilibrium is attained when the existing contracts can neither be varied without recontract with the consent of the existing parties, nor by recontract within the field of competition. The advantage of this general method is that it is applicable to the particular cases of imperfect competition: where the conceptions of demand and supply at a price are no longer appropriate. (Edgeworth 2003 [1881], p.31)

In this sense, the interpretation given by Negishi on the convergence of the perfect competitive situation in both authors gives more credit to Edgeworth's claims but needs to be reinterpreted. Negishi, as well as Walker (1973), relies on the non–tâtonnement theory as the good interpretation

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[13] We refer here to the authors mentioned in the introduction of this paper: Gale, Makowski & Ostroy, etc...

of Edgeworth's message. However, he accepts that the convergence of the final outcome of both authors justifies the price–taker hypothesis:

> *The assumptions made in Cournot–Walras approach may not be realistic, unless is an auctioneer, as in the case of well–organized markets. Edgeworth's equivalent theorem justifies Walrasian assumption, however, since it is not the assumption the outcome that matters for a theory, and we can assume that the traders behave they were price takers, even though traders are not actually price takers, provided we have the same outcome as assured by Edgeworth.*
> (Negishi 1989, p.338)

We think, contrary to this view that the price–taker hypothesis has to be taken for what it is: an assumption. This implies not only a different conception of agent's behaviour, as we have shown in our interpretation of Walras's theory, but also a different institutional framework in order to neutralize all possible sources of strategic behavior. In other words, it is not possible to assume Walras's price–taker hypothesis as a consequence of a situation where there exists a huge number of individuals neutralizing all possible strategic (i.e. arbitrage) actions each other might take. The main result of our analysis of Walras's Pure Economics is that the auctioneer, the numéraire and the presence of brokers are institutions giving content to the price–taker hypothesis. The result of the tâtonnement (namely the general equilibrium price vector) and the non–trade–out–of–equilibrium notion are assured by this institutional framework.

In Edgeworth's theory, the institutional framework and individual behavior are completely different. However, the interpretation given in terms of the cooperative–game approach leaves no place for a different view. This framework contributes to reduce the individual behavior to a non–strategic one, as can be clearly seen from the opposition between the two main approaches on game theory. As Gabszewicz (1999) recognizes it:

> *... when a small number of sellers are conscious of the interactive strategic context created by their simultaneous selection of product supply, this selection can be viewed as a non–cooperative equilibrium when sellers do not coordinate their choices, and as cooperative equilibrium when sellers enter into collusion to select their aggregate supply and share the production of it among themselves.* (Gabszewicz 1999, p.26)

We come now to the second implication of the modern interpretation of Edgeworth as a fore-runner of the cooperative–game theory. This has to do with the conception of the theory of the Core, and thus Edgeworth's theory of *final settlement*, as a theoretical representation of the market with the simplest or even no institutional framework.

3.4 The Core as a free–of–institutions framework and Edgeworth's equilibrium exchange

The evolution of the cooperative–game theoretical approach leads economists, around the 1980´s, to believe that the *Core* as an equilibrium solution was very attractive because of its simplified Walrasian–like framework. The simplicity came from the fact that, as in the Walrasian–General–Equilibrium approach, the *Core* is a solution concept built with few data: tastes, technologies and endowments, without any institutional structure. As Mas-Colell (1982) states:

> *For the Cournot type analysis, one needs a specification of transaction institutions. This may be a cumbersome and non–trivial task. In contrast, the Core determines final allocations from the basic economic data: tastes, technologies, endowments. This is the feature of the Core which makes it so attractive, but the level of abstraction at which it moves has an unfortunate consequence. It is this: in the absence of an institutionally explicit model, the only sensible feasibility constraint for a blocking coalition is that the proposed reallocation affect can be attainable for all commodities simultaneously considered. This, of course, means that competition works by the setting of entirely separated subeconomies!* (Mas-Colell 1982, p.16)

As it arises from this quotation, the main characteristic of the *Core* approach, and thus of Edgeworth's, is that the economy seems to be completely described by those simple data. This is exactly the same conception of a Walrasian Economy as presented in the Arrow–Debreu model. This reveals how the Debreu–Scarf tradition constructs the Edgeworth–Walras convergence upon a common view on what is a complete description of an economy. Both representations are misleading. To interpret Walras's pure economics as a free–of–institutions framework is equal to ignoring that an auctioneer, the numéraire, non–out–of–equilibrium–exchange, etc... are all institutional hypothesis that are needed if one wants to ensure the price–taker hypothesis and thus the perfect competitive economy.

This misleading interpretation of Walras's and Edgeworth's theories prevailing in modern economic theory gives place to the mistreatment of the informational context and the exchange technology. It is a common place to say that a Walrasian economy explicitly assumes perfect information and a non–frictional exchange technology. Both ideas are far from being accurate. De Vroey (2003) has remarkably well shown that Walras's theory presumes less informational requirements than Marshallian, and we can say now, than Edgeworthian frameworks. The reason for this is simple, and has to do with the price formation and technology of exchange those authors assume. As we have shown above for Walras, the presence of the auctioneer implies that agents only need the price–vector information in order to make decisions. Furthermore, agents do not make conjectures on others' decisions, agents have no strategic behavior.

When strategic behavior is introduced and there is no centralized price–formation system (i.e. no auctioneer), and this is the case for Edgeworth, the informational flow becomes an important issue. Furthermore, the fluidity of information is the main issue in order to assure a perfect competitive field in terms of Edgeworth (1881 as in Newman (ed) 2003): There is free communication

throughout a normal competitive field. You may suppose the constituent individuals collected at a point, or connected by telephones. (p. 18)

Concerning the exchange technology and price–formation mechanism there is also a very different, but explicit, presence of institutions for both authors. Walras's has been stated in the second section of this paper. In the case of Edgeworth it has been mentioned that the recontracting process supposes a decentralized price pair–wise framework. The implicit exchange–rates of the recontracting process are fixed by a bilateral bargaining game. This is not a cooperative game as the strategies of all parties are (i) interdependent, and (ii) the context within which the decisions to accept or not a definitive exchange is explicitly modeled. In fact, the most interesting characteristic of Edgeworth's theory of perfect competition is that it is developed as a limit case of an imperfect competition framework. The notion of indeterminateness and the recurrent criticism addressed by Edgeworth to Cournot show how important the analysis of monopolies and all other forms of imperfect competition is for him. In fact, most of Mathematical Psychics is devoted to the analysis of "imperfect competition" and the policies that a scientific analysis can justify to solve the potential problems of imperfect competition.[14] Briefly, following Vind (1995):

> *Edgeworth's definition of competition and exchange equilibrium did not share the norms and goals of the later general equilibrium theorists. His notion of competition only makes sense in a society with institutions, and would not have regarded the core as an interesting solution concept even if it was instituion free.* (Vind 1995, p.1740)

4   Concluding remarks: Imperfect Competition ... on which notion of perfection do we stand?

It is obvious, and even uninteresting, to say that imperfect competition is a concept built upon a conception of what is a perfect competitive situation. However, this simple reasoning is not always adopted in economics. In fact, imperfect competition has rarely been constructed in relation to a clear–cut concept of perfect competition. It is usual to find a mere rhetorical argument explaining why the model violates the so–called Walrasian–competitive assumptiond by giving agents the power to establish prices. Once the price–taker hypothesis is left aside, Pandora's box is opened for theoreticians. It is the box of the institutional framework where the exchanges take place. However, the *walrasian allocations* are always kept as the benchmark. The way from the imperfect situation to the so–called competitive situation is however not explicit. This is in fact one of the reasons why the convergence between Edgeworth and Walras was a hopeful result for general equilibrium theoreticians. It seems to demonstrate that the central point may be the number of agents, without giving importance to other, more important, differences between those authors.

The cooperative–game approach has thus constructed an ideal framework in which it is possible to obtain the main results of the Arrow–Debreu model and, more importantly, to finally get

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[14] See pages 34 to 56 of Edgeworth (2003 [1881]).

rid of the annoying auctioneer. However, all the properties and shortcomings of the centralized Arrow–Debreu framework repapered within this new land of freedom. Even the wishes of the *founding–fathers* of Game Theory where betrayed by cooperative–game theory:

> *Let us look more closely at the type of economy which is represented by the "Robinson Crusoe" model, that is, an economy of an isolated single person or otherwise organized under a single will... Thus Crusoe faces an ordinary maximum problem the difficulties of which are of a purely technical... Consider now a participant in a social exchange economy. His problem has, of course, many elements in common with a maximum problem. But also it contains some very essential elements of an entirely different nature. He too tries to obtain an optimum result. But in order to achieve this he must enter into relations with others... Thus each person tries to maximize a function.., of which he does not control all variables. This certainly is no maximum problem but a peculiar and disconcerting mixture of several conflicting maximum problems.*
> (von Neumann & Morgenstern 1944, p.10)

This is exactly what the *Core* as a solution problem does not tackle. The centralized framework of Walras's theory translated into a game–theoretic approach produced a non–strategic behavior. At least for the sole case that interested the Debreu–Scarf limit theorem which is the perfect competitive allocation. There is a complete lack of a notion of competitiveness as common sense and economic rhetoric conceive it. Let's give the last word to an expert on the *Core* theory in order to express this point clearer:

> *... the present game theoretical treatment of General Equilibrium is also guilty of this lack of competitiveness. While the process leading to the establishment of the core is indeed a process of rivalry, the eventual solution is purely cooperative and can be approximated by a set of competitive prices. This is no surprise, in the sense that if we give two correct theories the same set of assumptions, they will arrive at the same conclusions.* (Schotter 1973, p.307)

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