Sovereign Wealth and Risk Management

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Abstract

This paper sets out a new approach to sovereign wealth and risk management, based on the theory of contingent claim analysis (CCA). To manage sovereign risk, it is essential to analyse the sovereign's balance sheet. The state has to solve an asset-liability management (ALM) problem between its sources of income and its expenditure. The analytical framework for this approach covers all public entities, not only the state budget, and includes implicit guarantees to the private sector. It has a number of essential applications for sovereign wealth management, particularly with respect to sovereign wealth funds (SWFs) and foreign exchange reserves. We present the conceptual framework, tools and data needed to carry out this type of analysis. We then focus on Chile to provide a practical example of sovereign balance sheet estimation and sovereign ALM.

1. Introduction

The subprime mortgage crisis and its economic impact prompted governments to significantly expand both their balance sheets and their risk exposure. The fact that some developed countries suddenly found themselves on the brink of default led to growing awareness of the importance of sovereign risks and the need for satisfactory monitoring tools. In addition, a number of natural resource-dependent countries have realised that those resources were not inexhaustible and should be managed and transformed into a lasting source of national income, especially since new risks are looming, notably demographic risk, which involves thorny problems of pension management. Against this backdrop, the issue of how countries manage their resources and wealth has come under the spotlight.

Increasingly, institutions other than budgetary government are engaging in large-scale financial operations. Central banks' prerogatives in managing foreign currency reserves are well known. But since the subprime crisis, central banks have considerably expanded both their role and their balance sheets. In addition, a large number of sovereign wealth funds (SWFs) have been set up over the past twenty years to collect and manage the tax revenues that states receive either from natural resources (Norway, Chile, Middle Eastern countries, etc.) or from exports (China, Singapore, etc.). Finally, a third type of institution managing sovereign money is the public pension fund, often the result of an excess of contributions over benefits during a demographic transition. The newfound importance of these institutions, which are outgrowths of sovereign

governments, has blurred the boundaries of the state's sphere of influence, especially as the government may decide to commandeer the fund's assets to meet short-term obligations in times of crisis. In 2010, for example, in the wake of the subprime crisis, Russia, Ireland, Kazakhstan and Qatar used SWFs or public pension fund assets to invest in banks or shore up equity markets. Segregating the various items on government balance according to the institutions that manage them is a delusion because, when serious problems arise, all assets are fungible.

Until now, the academic work on optimal allocation of sovereign wealth has viewed the funds as independent entities, each with its own objective. Bernardell et al. (2004) and Beck and Rahbari (2008) determined the optimal allocation of central banks' foreign exchange reserves with the dual objective of avoiding domestic currency appreciation and building up a cushion against a liquidity crisis. Independently, others have addressed the optimal allocation for an SWF (Scherer (2009), Brown et al. (2010), Martellini and Milhau (2010)) by examining exogenous liabilities of the SWF set by the government and proxied by an inflation-linked investment benchmark. The example of the recent crisis clearly shows that other sovereign liabilities have to be taken into account (debt, contingent liabilities, etc.). When a government is short of liquidity to meet its debt payments, the SWF's assets are automatically available to substitute for the funds initially earmarked for this purpose.

In this paper we revisit the issue of sovereign wealth management, considering the "sovereign" concept in the broad sense, including all the related institutions (budgetary

government, central bank, SWFs, pension funds and public entities placed under the sovereign's authority). In essence, managing a sovereign seems little different to managing an individual's assets. Each is endowed with capital (human, financial and/or natural resource-related) and must meet expenditure needs that vary over time and are linked to economic objectives. A sovereign may manage its income and save wisely using appropriate vehicles so that it can meet its spending requirements (on education, ageing populations, etc.) and cope with the risk of financial crises and the like that it will eventually have to face. It has to manage the mismatch between its revenue sources and its expenditure, thus solving a classic asset-liability management problem. The good news is that financial tools exist to deal with questions of this type. They were initially developed to manage personal wealth (Merton (1969), Bodie et al. (1992), Bodie et al. (2008)), and are used routinely for managing pension funds (Bodie et al. (2009), Bagliano et al. (2009)). But they are only starting to be used to manage sovereign risks. Gray et al. (2007) and Gray and Malone (2008) laid the groundwork in this area by adapting the contingent claim analysis (CCA) model to sovereign balance sheet with the objective to measure sovereign credit risk.

Financial management of government resources and expenditures raises difficult questions, in which economic policy and financial management objectives are closely entwined. Standard macroeconomic tools are ill-suited to this task. Most of the macroeconomic variables monitored at present describe flows, not stocks, and are unsuitable for evaluating intangible assets such as human and natural capital (Aglietta (2010)). Moreover, our traditional macroeconomic data lack a significant

dimension, namely risk (Gray et al. (2007)). In the case of sovereign balance sheets, risks are linked on the one hand to market price fluctuations (for commodities, export goods, wage costs, etc.) that cause the government's income and expenditures to fluctuate, and on the other hand to inventory changes (natural resource depletion, population growth, etc.) But in a financial economy, what really needs to be measured is the actual nature of financial risks, with their non-linear features (contingent liabilities modelled as options, etc.), and the accumulation phenomena that lead to systemic risks.

This paper proposes an analytical framework to conceive the optimal asset allocation of sovereign wealth, based on the analysis of sovereign balance sheet. To do so, it extends the theory and practice of modern CCA to sovereign wealth and risk management. The sovereign balance sheet is not only analysed to evaluate the sovereign assets and default risk as in Gray et al. (2007) but also to measure the risk exposures of the various items composing assets and liabilities of the sovereign. This allows to define the optimal sovereign asset allocation in an integrated ALM framework. A real data application is provided on a simplified case study based on the example of Chile. Section 2 presents the conceptual framework. Section 3 assesses the variables currently available for estimating a sovereign balance sheet and focuses on the need to create new economic measurement tools that bridge the gap between macroeconomics and finance. Section 4 presents Chile as a practical example of sovereign balance sheet estimation and sovereign ALM. Section 5 concludes.

2. The Conceptual Framework

What tools and analytical framework are necessary to manage a country's wealth in a way that is appropriate to its economic needs and the risks it encounters? First, it is important to consider the decision-making entity (which we shall call the "sovereign") in its entirety, with its objective function precisely identified. Second, the sovereign's balance sheet and risks need to be measured in detail. The difficult question of managing its wealth in accordance with its economic policy objectives may thus be framed as an exercise in asset-liability management.

2.1 Definition of the sovereign

We consider here the concept of "sovereign" in the broad sense, including not just the state's budgetary institutions and monetary authorities (central bank), but also the other institutions related to it, such as pension funds, SWFs and state-owned enterprises. Distinctions among various state entities are less and less meaningful, as recent crises have shown. In 2010 several countries turned to public institutions for assistance in coping with the crisis-linked credit crunch. Some countries used the assets of SWFs or national pension funds to invest in bank deposits (Russia and Kazakhstan) or to support equity market liquidity (Kuwait). Others used the resources to directly recapitalise ailing banks (Ireland, Kazakhstan and Qatar). For this purpose, states modified their funds' investment rules on a discretionary basis, exposing them to new risks. In some countries with greater borrowing capacities, the state did not commandeer the funds' assets but

tweaked their regulations to allow them to buy a larger share of the sovereign debt. These recent examples clearly show that a state facing a crisis can elicit contributions from the "off-budget" entities that it holds or controls (such as deposit insurance agencies, SWFs and pension funds) to meet its short-term obligations without unduly worsening the fiscal deficit.

The usual perception echoed in the recent literature on asset allocation by sovereign entities (Beck and Rahbari (2008), Scherer (2009), Brown et al. (2010), Martellini and Milhau (2010)), i.e. that every state-controlled body is a unique, separately managed entity, results in an incomplete and sub-optimal view. In a crisis, they become fungible, so it is more realistic to consider them from the outset in the aggregate. Notwithstanding this approach, some entities such as the central bank are legally independent, but their management at state level should be considered as part of a larger whole, including all sovereign assets.

2.2 The Sovereign Economic Balance Sheet

The sovereign has a multitude of objectives. Some are purely financial, such as debt repayment and setting aside foreign exchange reserves to cope with liquidity crises. Others are social, including pensions and financing of social services (infrastructures such as hospitals, schools and roads, education, military expenses, etc.). Yet others are economic, such as investment in key sectors or industries for future growth. These

objectives should be determined as precisely as possible to get a clearer idea of its upcoming expenditures.

To achieve its objectives the sovereign has a variety of resources, particularly future tax revenues, as well as income from other sources such as state-owned enterprises, fees, seigniorage, and possibly a stock of financial assets (foreign exchange reserves, SWF assets, public pension funds, etc.) Accordingly, the sovereign's financial wealth should be considered on a par with its other assets, which are managed to achieve one or more global objectives (its liabilities).

Definition of the Sovereign Economic Balance Sheet

The sovereign's global economic balance sheet is key to a full understanding of its situation and risks (Gray et al. (2007)). The idea is to estimate all the state's assets and liabilities at market price, and to measure the risks (volatility and sensitivity to economic shocks) linked to each balance sheet item. Just as a company's balance sheet is regularly used to assess the risk of bankruptcy (Merton (1974, 1977), KMV (1999, 2001, 2002)), the same analytical framework may be applied to a state. It is not only useful with regard to the state's debt repayment capacity (Gray et al. (2007), Gray and Malone (2008)), which is obviously a minimal objective, but more generally, as we shall see, to its ability to fulfil its long-term social and economic objectives.

How is a sovereign's economic balance sheet presented? Its assets break down as follows:

(1) International reserves: in general, foreign currency reserves held by the central bank, commodities (especially gold) and Special Drawing Rights with the International Monetary Fund. These reserves often have multiple objectives: to maintain currency stability or at least to avoid an excessive appreciation caused by export-linked inflows of currencies; to serve as a reserve asset in the event of a liquidity crisis.

(2) One or more SWFs, managed by the finance ministry or the central bank, also with multiple objectives which may include savings, macroeconomic stabilisation, and even political objectives.

(3) Pension fund assets

(4) Other public-sector assets (property, state-owned enterprises, etc.)

(5) Fiscal assets: taxes and revenues (fees, etc.) collected as tax receipts

A sovereign's liabilities include:

(1) The monetary base (currency in circulation, banks' reserves with the central bank)
(2) Local debt: debt denominated in the local currency of the monetary authorities, mainly held by domestic agents
(3) Foreign debt: debt denominated in foreign currency (frequently US dollars), mainly held by foreigners
(4) Pension fund liabilities

(5) Contingent liabilities, such as those in the banking sector (notably by too-big-to-fail institutions)

(6) Present value of expenses on economic and social development, security, government administration, benefits to other sectors

(7) Present value of target wealth to be left to future generations

Table 1 gives a simplified example of a sovereign balance sheet:

Table 1: Simplified Presentation of a Sovereign Balance Sheet

ASSETS	**LIABILITIES**
Foreign reserves, gold, special drawing rights Pension fund assets SWF Other public sector assets (state-owned companies, real estate) Present value of future taxes, fees, seigniorage	Base Money Local currency debt Foreign currency debt Pension fund liabilities Contingent claims: implicit guarantees (to banks etc.) Present value of expenses on economic and social development, security, government administration, benefits to other sectors Present value of target wealth to be left to future generations

Estimation of the Sovereign Economic Balance Sheet

An initial approach to measuring a sovereign's economic balance sheet is to estimate the market price and volatility of all its component assets and liabilities separately. But to do so, the present value of future income and expense flows has to be estimated, meaning that the sovereign must specify its economic objectives. An alternative method is to estimate the balance sheet on an integrated basis using market data only, as described by Merton (1974, 1977) and Gray et al. (2007). An implied value for the sovereign's assets can be estimated from the observed prices of liabilities and the balance sheet relationships between assets and liabilities. In this case, what is being measured is the market's valuation of the sovereign's balance sheet.

To do this, it is necessary to rearrange the balance sheet entries and adopt an integrated presentation, subtracting the present value of expenses from the present value of income, and subtracting the value of contingent liabilities from assets. This presentation avoids the need for a separate estimate of the balance sheet items related to the government's economic policy decisions: taxation levels, spending targets, and implicit liabilities (expected loss of the financial and corporate sector implicitly guaranteed by the government). These three items appear in aggregate form as a residual balance sheet item. Table 2 presents this aggregated form of the balance sheet.

Table 2: Aggregated Sovereign Balance Sheet

ASSETS	**LIABILITIES**
Foreign reserves, gold, special drawing rights Pension fund assets - liabilities SWF Other public sector assets (state-owned companies, real estate) *Present value of future taxes, fees, seigniorage – Present value of expenses on economic and social development, benefits to other sectors - Present value of target wealth to be left to future generations - Contingent Claims*	Base Money + Local currency debt Foreign currency debt

Following Merton (1974, 1979), Gray et al. (2007), the two liabilities can be valued as contingent claims on sovereign assets. The foreign currency debt is considered as a "senior claim" and the local currency debt plus base money as a "junior claim", which can be modelled as a call option on the total value of the sovereign's assets. Gray et al. (2007), KMV (1999, 2001, 2002) measure medium-term default risk, and the "distress barrier" is set as short-term foreign debt plus one-half of long-term foreign debt. Our objective here is different. We are not attempting to estimate the probability of a government's defaulting but to measure the best way for it to manage its wealth, given certain liabilities that must be considered in their entirety. In this context, the state's total short- and long-term debt must be taken into account.

Local currency liabilities expressed in foreign currencies $LCL_{\$}$ is the sum of the monetary base and local debt expressed in foreign currency:

$$LCL_{\$} = \frac{(M_{LC} e^{r_d T} + B_d) e^{-r_f T}}{X_F}$$

where r_d is the domestic interest rate, B_d the value of local debt, M_{LC} the monetary base, and X_F the forward exchange rate.

This liability in local currency can be seen as a call option on the value of sovereign assets $V_{\$Sov}$, also expressed in foreign currency, with a strike price equal to the default barrier B_f derived from payments promised in foreign currency until time *T*:

$$LCL_{\$} = V_{\$Sov} N(d_1) - B_f e^{-r_f T} N(d_2) \qquad (1)$$

with *N(.)* the cumulative standard normal distribution and r_f the foreign interest rate,

$$d_1 = \frac{\ln(\frac{V_{\$Sov}}{B_f}) + (\mu_{\$Sov} + \frac{\sigma^2_{\$Sov}}{2})}{\sigma_{\$Sov}\sqrt{T}}$$

$$d_2 = d_1 - \sigma_{\$Sov}\sqrt{T}$$

The real world asset drift is related to the risk-free rate according to the following relationship:

$$\mu_{\$Sov} = r_f + \lambda \sigma_{\$Sov}$$

Where λ is the market price of risk reflecting the risk aversion of the investor (here, the Sovereign entity).[1]

[1] For the empirical investigation, we follow KMV(2002) and fix $\lambda = 0.45$.

To solve the problem and find the values of the two unknowns $V_{\$Sov}$ and $\sigma_{\$Sov}$, we use a second equation, linking the volatility of the sovereign asset to that of the junior claim:

$$LCL_{\$}\sigma_{\$LCL} = V_{\$Sov}\sigma_{\$Sov}N(d_1) \qquad (2)$$

Thus, equations (1) and (2) can be solved to determine the value of the sovereign's assets $V_{\$Sov}$ and their volatility $\sigma_{\$Sov}$, as a function of the default barrier B_f.

2.3 Sovereign Wealth Management

Work on optimal allocation for SWFs or foreign exchange reserves has viewed these vehicles as independent entities, each with its own objective. Bernardell et al. (2004) and Beck and Rahbari (2008) considered the question of optimal management of a central bank's foreign exchange reserves. They took into account a dual objective: first, avoiding domestic currency appreciation when export income generates foreign currency inflows; second, creating a cushion against liquidity crises and withdrawal of foreign capital. Independently, other investigators have attempted to determine the optimal allocation for an SWF (Martellini and Milhau (2010)), e.g. for a commodity producing country (Scherer (2009), Brown et al. (2010)). These papers have developed an ALM strategy to take into account the fund's liabilities (e.g. inflation-linked in the case of a future-generations savings fund), which are assumed to be set by the government once and for all.

However, the crisis example illustrates that these liabilities are broader: they include sovereign debt and contingent guarantees to the private sector. When the government is short of liquidity to meet its debt payments, the SWF's assets are automatically available to substitute for the funds initially earmarked for this purpose. What therefore needs to be taken into account is the sovereign's "aggregate" liabilities, not only those that the state has assigned to the SWF (such as pension payments) but also those linked to other objectives, including some as crucial as short-term debt service. If problems arise, everything will be amalgamated. Assets, too, should be viewed in the aggregate – not just financial wealth but also the wealth derived from tax resources. The global allocation of available financial wealth may thus be defined in terms of the sovereign's overall assets and liabilities. If one or more sub-objectives are assigned to separate entities, there is a risk that they may have to be merged in the event of a crisis. Naturally, for practical or legal reasons, management of various investment "segments" may nevertheless be entrusted to different entities.

From a theoretical point of view, management of the sovereign wealth is not very different from that of an individual (Merton (1969), Bodie et al. (1992), Bodie et al. (2008)), a pension fund (Bodie et al. (2009)) or a foundation (Merton (1993)). The sovereign receives fiscal revenues each year. Part of this can be spent, and the residual can be saved in the SWF, central bank reserves or public pension fund. How much should be saved and how to invest is thus a classic ALM problem.

A first step for the sovereign is to measure as precisely as possible both its assets and their risks. These naturally include financial assets (held in reserve, sovereign wealth or public pension funds), but also "fiscal" assets (present value of future taxes), which may depend on various sources of risks, such as price fluctuations (commodities, exports, etc.) and inventory changes (human or natural capital). Estimating these risks properly can be a delicate step, as they may not depend only on the traded asset. The total wealth of the sovereign is the sum of the capitalised values of all the cash flow sources and the value of the investment funds.

The second step should be to define the sovereign's liabilities. Some are non contingent (such as repayment of existing debt); others may be determined by the state at its own discretion (public spending policy, private sector guarantees, etc.) and be subject to economic policy choices.[2] Contingent liabilities must also be evaluated at their market value. For each government-guaranteed institution, the probability of default and the expected loss due to default can be measured with Merton's model (this is the value of an implicit put option on the assets with a strike price equal to the default barrier, calculated as part of debt liabilities). Systemic CCA measures government liabilities by considering the co-dependence structure between institutions. The final estimate of the value of contingent liabilities also supposes that the government makes an economic policy choice, i.e. what fraction of the total expected loss is implicitly guaranteed.

[2] It will be up to policymakers to set achievable objectives (how much discrete expenditures), in view of constraints linked to the size and risks of total wealth.

The last step is to determine the state's optimal wealth allocation among traded assets in light of fiscal asset and unconditional liability items. The sovereign's objective is to maximise its expected utility, which is a function of its Global Sovereign Surplus (GSS), measured as assets minus liabilities. Basically, it will try to maximise the value of the target wealth to be left to future generations for a given amount of risk. In practice, it will choose the most appropriate risk measure for its situation (volatility, probability of a shortfall, expected shortfall, etc.). The optimal allocation and optimal expenditures of the sovereign will crucially depend on the nature and size of the fiscal asset and unconditional liabilities, and the sources of their uncertainty. A very similar problem has been solved analytically in a dynamic case by Merton (1993) for a university endowment fund. The optimal portfolio can be decomposed into speculative demand (the result of the standard mean variance optimal portfolio), and hedging demand components, intended to hedge the unanticipated changes in fiscal revenues and sources of expenses. Section 4.2 shows practically in a simplified framework how such an optimal asset allocation can be estimated, taking the example of Chile.

3. Public Finance Data and Their Limitations

Much of the data needed to construct a complete economic balance sheet of a sovereign is currently not available in a standardised form. Macroeconomic analysis usually considers flow data. When data for stocks are available, they are of very low frequency (generally annual) and thus cannot be used to evaluate the relevant risks

correctly. Furthermore, intangible assets and natural capital are never counted, except in the World Bank's 2006 initiative.

3.1 Traditional Public Finance Data

The usual practice in macroeconomic analysis consists in analysing a state via its flows. This is reflected in the macroeconomic data produced by the states themselves: income, expenditures, deficits, national saving, investing, exports, imports, funds flows, etc. Flow of funds statistics available in many countries provide balance sheet estimates of the different sectors (household, government, non-financial corporations, financial sector and rest of the world). They can be used to construct a sovereign balance sheet but must be approached with caution. The definition of the "government" entity differs by country[3] and may not exactly correspond to our broad definition of the sovereign. The IMF's GFS database, created in 2001, remedies these differences in scope from one state to another. It is a unified base of 153 countries' data on both flows ("Government Operations Table" and "Other Economic Flows") and stocks ("Government Balance Sheet" data). The scope of the sovereign entity under consideration is particularly broad, comprising not just the central government budgetary authority but also the central bank, SWFs, pension funds, deposit insurance fund, state-owned enterprises, subnational governments and other government agencies. Figure A1 in Appendix A presents the

[3] In the US, the "Flows of Funds" statistics consider state and local governments (excluding employee retirement funds), the federal government (including government-owned corporations and agencies that issue securities individually) and the monetary authority. In Europe, the European Central Bank and Eurostat "Euro Area Accounts" have a more restrictive definition. The general government sector comprises only central, state (regional) and local government and the social security or pension funds belonging to it. It does not include public enterprises, which are included in the corporate or financial sector and cannot be disentangled from it.

structure of the GFS analytical framework. Tables A2 to A4 present the available data in the GFS flows and stocks tables.

The IMF's GFS data nonetheless have significant limitations for estimating a sovereign's complete balance sheet. Contingent liabilities, such as too-big-to-fail guarantees to the financial sector and implicit guarantees to provide social benefits when various needs arise, are not estimated. They appear only when the event or the necessary condition for materialisation of the liabilities actually occurs. Furthermore, on the asset side of the balance sheet, there is no evaluation of the present value of future tax revenues. In commodity producing countries, these revenues depend on unmined mineral wealth or the capacity to exploit agricultural resources. This is significant insofar as flow data cannot be used to estimate available stocks, especially if these are exhaustible. Similarly, on the liabilities side of the balance sheet, there are no estimates of the expenditures the government intends to make in order to achieve its economic and social objectives, which depend crucially on future economic policy choices.

Finally, these balance sheet data, which are purely accounting-based and generally available on an annual basis, are not sufficient to measure the risks on each item. The value of these assets and liabilities fluctuates because of price movements and changes in financial inflows and outflows. This is particularly true for natural resources, which have highly volatile market prices and are exhaustible, so that the stock of available resources is bound to disappear over time. Accordingly, the volatilities and the sensitivities of each

balance sheet item to various economic and financial shocks and other factors should be measured by means of alternative scenarios, stress testing, or other methods.

3.2 World Bank's Wealth of Nations Estimation

In 2000 the World Bank took the unprecedented step of measuring the wealth of nations (World Bank (2006)). The total wealth of each nation is estimated as the present value of future flows of consumption. Consumption levels are based on past historical data but are adjusted to be "sustainable".[4] Total wealth is decomposed into: (1) produced capital (machinery and structures and urban land), (2) natural capital (energy resources, mineral resources, timber resources, non-timber forest resources, cropland, pastureland, protected areas) and (3) intangible capital (human, etc.). Intangible capital is calculated as a residual, the difference between total wealth and the sum of produced and natural capital.

These data are a very useful supplement to the existing figures because they provide an estimate of stocks[5] of natural resources and intangible assets. World Bank estimates of natural and human capital can be used to measure the present value of fiscal surplus (coming from work taxation or taxation of natural resource extraction for commodity exporting countries), by multiplying it by a certain percentage of desired

[4] For years when adjusted net savings are negative, the actual consumption rate is added to adjusted net savings.
[5] Flow variables are also available: depletion of natural resources, investment in education, domestic net investment.

taxation. Unfortunately, these data were estimated in 2000 for the World Bank's report and are not available as historical series.

4. From Theory to Practice: the Case of Chile

Chile is a particularly interesting case study. The central bank and finance ministry publish comprehensive balance sheet data. And the country has two extremely transparent SWFs, making it possible to produce relatively detailed estimates of the sovereign balance sheet.

4.1 Chile's Sovereign Economic Balance Sheet

Chile's central bank provides annual historical data on the stock of domestic and foreign debt issued by the government and on the amount of base money (M2) in circulation. This makes it possible to estimate the default barrier, the stock of domestic (junior) debt on the liabilities side, and, under the approach presented in Section 2, the sovereign entity's total assets and their volatility. Several assumptions are necessary. The volatility of domestic debt is considered identical to the historical volatility of Chile's domestic bond market[6] (JPMorgan Chile Local Government bond index, ELMI+) for the period 2000-2008.[7] Other financial data (exchange rates versus the US dollar, domestic

[6] An implied volatility of the debt market would have been more relevant, but futures markets are not available for government bonds in Chile.
[7] Data come from Datastream.

and foreign interest rates) are taken from the central bank website. The distress barrier for foreign currency denominated debt is taken as the whole stock of foreign debt. Resolving the system of equations (1) and (2) enables us to evaluate total sovereign assets in US dollars, as well as their volatility.[8] We use a long time horizon for this analysis, with T= 10 years.

Once the major aggregates (total assets, domestic and foreign debt) have been estimated, the balance sheet estimation can be refined by including asset sub-components. Chile has two SWFs. The first is a stabilisation fund, the Social Stabilization Fund (ESSF), launched in early 2007 and derived from the old Copper Stabilization Fund. Its goal is to stabilise fiscal spending. It aims at reducing the budget's dependency on global business cycles and the volatility of revenue derived from fluctuations of copper prices and other sources. Budget reductions originating in economic downturns or copper price declines can be financed in part with resources from the ESSF, reducing the need to issue debt. The fund received contributions in 2007 and 2008 but financed the budget deficit in 2009 and 2010. The second SWF is the Pension Reserve Fund (PRF), created at the end of 2006 in response to Chile's new demographic scenario. It serves as a supplementary source for the funding of future pension contingencies. Its objective is to support financing of government obligations arising from the government's guarantee to basic old-age and disability solidarity pensions. PRF has a fixed accumulation rule.[9] During its four years of existence, it received regular contributions from the budget.

[8] We use a Newton Raphson iteration technique, with 1% tolerance interval.
[9] Its capital increases each year by an amount equivalent to 0.2% of the previous year's gross domestic product (GDP). If the actual fiscal surplus exceeds 0.2% of GDP, the PRF receives a contribution equivalent to said surplus, up to a maximum of 0.5% of GDP.

Market values of both funds are available on their websites (at the end of 2008), as well as their detailed benchmark portfolio composition, which is the same for both funds. It is made up of money market investment and government bonds in USD, EUR and JPY exclusively. Table B1 in Appendix B reproduces the current benchmark portfolio composition[10]. In our balance sheet's estimation, the 2 fund's volatilities have been estimated by considering the historical volatility of their benchmark portfolio over the period 2000-2008. This approximation does not take into account the different speed of contributions and withdrawals inside the two funds which may be different by the very different nature of their role, and can potentially add volatility.

The central bank's reserves account for a substantial portion of assets. Their purpose is to guarantee secure and efficient access to international liquidity, and they are an instrument to safeguard currency stability (Chile has a floating exchange rate regime) and the normal functioning of domestic and external payment systems. They are invested in liquid foreign assets and are intended to permit intervention in the foreign exchange market in times of crisis. Information on the stock and composition of reserves at end-2006 is available on the central bank's website. These reserves are invested in assets under two main portfolios.[11] The Investment Portfolio (73.3% of the total reserves) is the largest, and includes short- and long-term foreign currency assets (with an average duration of 13 months) used to respond to unforeseen contingencies and long-term requirements. The Liquidity Portfolio is designed to cover requirements foreseeable in the short term. The structure for this portfolio corresponds to the currencies and

[10] Provided by the Chilean Ministry of Finance. We used exactly the same benchmarks and data sources.
[11] The reserves also contains 1.2% of other assets (detail not supplied).

maturities of programmed disbursements. Currency composition of both portfolios is provided in Table B2 in Appendix 2. In sum, the geographical breakdown of foreign reserves is very close to that of the two SWFs. The volatility of reserves is estimated as the volatility of a portfolio, with the proportions invested remaining constant. To conform to the reserves' composition in terms of issuers, data used are the Merrill Lynch BOFA Corporate and Government 1-3 years bond index in the US and EMU. Other foreign currencies have been approximated by JPY investment, and the Merrill Lynch BOFA 1-3Y Broad Index in Japan.

Table B3 in Appendix B presents a simplified estimated economic balance sheet for the sovereign entity. The various assets and liabilities are shown, along with their volatilities. Note that senior debt could be evaluated either with bond market data (not necessarily compatible with the asset valuations made for the equity tranche) or derived from the assets' valuation. We chose the latter option. Subtracting financial wealth (SWF and reserve funds) from total assets, the residual balance sheet item (present value of fiscal surplus – contingent liabilities) is estimated. Note that its volatility could only be estimated in Merton's model by making assumptions about its correlations with other balance sheet items. In practice, the government can compare this estimate of the residual balance sheet item derived from market data with its own estimate of future income and expenditure flows based on its economic policy objectives.

4.2 Sovereign Wealth Management in Chile

In this section, we present a simplified case study of sovereign wealth management based on the example of Chile. To keep the framework simple, we consider that there are no contingent liabilities, that the fiscal surplus can be proxied by receipts indexed to Chilean inflation, copper prices and equities and that spending can be indexed to inflation.[12] What is the optimal allocation in this case? We consider a very simple objective function in which the government seeks to minimise the volatility of the Global Sovereign Surplus (GSS), measured as assets – liabilities, for a certain level of target return.

A first step is to estimate the present value of the fiscal surplus, written as the discounted sum of all the sovereign's revenue flows less the discounted sum of all its expenditures:

$$P_{fiscalsurplus} = \left[\sum_{i=1}^{\infty} \frac{R_i}{(1+r)^i}\right] - \left[\sum_{i=1}^{\infty} \frac{E_i}{(1+r)^i}\right]$$

Where R_i and E_i are respectively the revenues and expenditures expected for year *i*, *r* is the discount rate.[13]

The present value of government expenses can be broadly proxied by the price of an inflation linked bond, whose real coupon is equal to the expenditures (in real terms)

[12] In fact, the fiscal surplus is affected by the government's political decisions, e.g. choice of future expenditures, and tax policy. It ought to depend on intangible assets such as human capital, and take account of the fact that natural resources are finite. As we saw in Section 3, most of these data are not publicly available. We have taken a simplified case by way of illustration.

[13] The choice of the socially optimal discount rate may also be an economic policy choice (Gollier (2002)).

that the government expects to make in the future. This is a reasonable hypothesis as we find that the inflation sensitivity of the annual expenditure growth rate is very close to 1 over the period 1991-2009 (see Figure C1 and Table C3 in Appendix C).[14] The nominal "coupon" paid by the government at a future date *i* can thus be written, as in the case for inflation-linked bonds, as:

$$E_i = RR_i(1+\pi_i)$$

With RR_i the real coupon, and π_i the annual inflation rate until year *i*.[15]

The present value of revenues is more challenging to evaluate. Chile's tax revenues come largely from the sale of copper. According to Ffrench-Davis (2010), copper accounts for around 15% of fiscal income, including both taxes and all profits from CODELCO, the state-owned copper company, and taxation of private mining companies. We estimate the sensitivity of the expenditure growth rate to copper, Chile's equity index and inflation over the period 1991-2009 (see Figure C2 and Table C3 in Appendix C) and confirm Ffrench-Davis estimation. On our sample period, sensitivities to inflation, copper and equity prices are respectively 1.01, 0.17 and 0.10.[16] Following the same methodology as for expenses, the present value of fiscal revenues is estimated as a bond, whose real coupon is indexed on inflation, copper and equities. The "nominal coupon" received by the government at year *i* can thus be written as:

[14] This single factor explain more than 74% of the total volatility of government expenses.
[15] Note that the inflation indexation mechanism is done with a lag for inflation-linked bonds. Because our purpose is to approximate the present value of inflation-indexed expenses, we do not consider any delay in the indexation.
[16] The 3 factors explain more than 73% of the total volatility of government revenues.

$$R_i = RR_i(1 + \beta_{inflation} * \pi_i + \beta_{copper} * r_i^{copper} + \beta_{equity} * r_i^{equity})$$

With RR_i the real coupon, π_i the annual inflation rate until year *i*, r_i^{copper} and r_i^{equity} the copper and equity returns over year *i*, $\beta_{inflation}, \beta_{copper}$ and β_{equity} the sensitivities to copper and equity prices respectively (assumed constant over time).

In theory, we should consider the price of a perpetuity but in practice, the government may have low visibility on its expenditures and revenues in the distant future. Moreover, some of its resources, especially natural ones, may be exhaustible. For that reason, we consider a 20-year bond. Considering the history of real interest rates, inflation and copper prices,[17] we are able to reconstruct the historical evolution of the present value of the surplus since 1994.

The two liability items: local debt plus the monetary base, on the one hand, and foreign debt, on the other hand, have been proxied by the JP Morgan foreign and local Chile Bond Market indices (EMBI+ and ELMI+), respectively.[18] This approximation slightly overstates the risk of the liabilities because the indices do not include bonds maturing in less than a year. Also, for simplicity, we have not addressed the distinction between the monetary base and external debt.

The last stage is to determine the sovereign's optimal asset allocation (i.e. allocation of its currency reserves and SWF assets) that minimises the volatility of the

[17] Real rates quoted on Chile Inflation Adjusted Notes 20 years maturity, data available since 1994. Inflation is the Chile CPI headline index, equities is the Chile MSCI index. All data come from Datastream.
[18] Data come from Datastream.

GSS for a target rate of return. We therefore construct the corresponding efficient frontier. For the sovereign, the GSS is expressed as the difference between assets and liabilities. In our simplified case, this means:

GSS = Sovereign Assets + Fiscal Surplus – Foreign Debt – Domestic Debt

The optimisation program is written as:

$$Min_w \ \sigma_{GSS} \qquad (1)$$

$$r_{GSS} = \sum_{i=1}^{n} w_i r_i + r_{FS} - r_{FD} - r_{DD}$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0$$

Where σ_{GSS} and r_{GSS} are the volatility and return of the fiscal surplus, $r = (r_1, r_2, ..., r_n)$ is the annualised return of the n assets in portfolio over the investment horizon, $w = (w_1, w_2, ..., w_n)$ the fraction of capital invested in the asset i, r_{FS} r_{FD} and r_{DD} the fiscal surplus, foreign debt and domestic debt returns respectively.

We have allowed for a broader investment class than the one used today by Chile via its currency reserves and SWF, and we consider that the state can invest in eight diversified asset classes: emerging equities, emerging bonds, developed equities, developed bonds, world inflation-linked bonds, as well as three short term bond

investments in US Dollars, Euro and Yen.[19] We use monthly data for the period August 2000-December 2010.

Table C4 presents descriptive statistics for each balance sheet variable, as well as the asset classes taken into consideration for the sovereign allocation. Table C5 presents their correlations. Among the possible investments, emerging equities present the most attractive returns (15.41%) over the study period, but also the highest risk (25% volatility, with high extreme risks), followed by emerging bonds (10.33% return, 10.39% volatility). Unsurprisingly, USD short-term bonds are the least risky investment (1.65% volatility) as the balance sheet is expressed here in USD, but they also offer relatively low returns (4.40%) over the period compared to EUR and JPY short-term bonds (8.69% and 3.76% respectively). Assets currently in the allocation of Chile's SWF have very low returns compared to estimated liabilities (8.08% and 6.60% annualized returns for external and local debt respectively). This also motivates the use of a broad investment universe in our optimization exercise.

The correlation matrix offers an interesting picture. EUR and JPY short-term bonds offer slightly negative correlations with the fiscal surplus (-2% and -3% respectively). Unsurprisingly (given the way we build it), the surplus is slightly correlated with emerging bonds and emerging equities (20% and 13% respectively). Local and

[19] Data used are MSCI emerging equity index, MSCI World index (developed countries), JP Morgan EMBI+ index (external debt), JP Morgan ELMI+ index (local debt), JP Morgan GBI Broad all-maturities index (developed countries government debt all maturities). The three short-term debt indices in US Dollars, Euro and Yen are the ones used for Chile's Central Bank reserve asset allocation: Merrill Lynch BOFA Corporate and Government 1-3Y bond index in the US and EMU, Merrill Lynch BOFA 1-3Y Broad Index in Japan. All data come from Datastream, except the Merrill Lynch indices coming from Bloomberg..

external debt have a relatively strong correlation with all debt markets. External debt has the highest correlation with world inflation-linked bonds (67%), whereas local debt presents the highest correlation with emerging government bonds (56%).

Chart C6 in Appendix C shows the efficient frontier obtained by solving programme (1). Table C7 presents the results of three optimal allocations: (1) the one that minimises the volatility of the GSS, (2) the one that minimises the volatility of the GSS for a 6% target rate of return, (3) and the one that minimises the volatility of the GSS for a 6.96% target rate of return (i.e. the maximum return achievable without shorting certain asset classes).

To minimise the volatility of the GSS, a substantial weight has to be given to equities, i.e. more than 50% of the allocation (developed markets 38%, emergings 24%), 27% needs to be invested in short-term bonds (in EUR, uncorrelated with the fiscal surplus and offering attractive returns over the period), while the remainder (11%) is allocated to emerging bonds, which are quite closely correlated with Chile's local and external debt and therefore provide relatively good protection for the liabilities. This allows to achieve a 4.83% return of the GSS. When the required return on the surplus is increased (6%), the proportion allocated to emerging equities also increases (to 48% for an expected return of 6%) whereas the portion allocated to developed equities declines (1%). A significant proportion is still allocated to EUR-denominated short-term bonds (25%). Lastly, the maximum return on the surplus is reached with a portfolio allocated 100% in emerging equities. Note that the volatility of the surplus is relatively insensitive

to changes in the allocation (rising from 13.24% to 13.79%). This is because only 31% of sovereign assets can be exposed to investment and we have not permitted short selling or derivatives, which would have made it possible to modify the overall exposure of the sovereign balance sheet more substantially.

5. Conclusion

The financial and economic crisis showed the importance of taking the risk on governments' balance sheets into consideration. Although the level of fiscal deficits has risen only slightly, fiscal risks have exploded as many countries have taken measures to recapitalise banks and extend loans and guarantees to the private sector.

This paper proposes an analytical framework for sovereign wealth and risk management, extending the theory of CCA. A complete approach to the sovereign balance sheet is necessary for fully understanding the country's risks and determining how it can best manage its wealth. This supposes the broadest possible definition of the sovereign entity, including, in particular, entities subordinated to the state, such as the central bank, SWFs, pension funds, government agencies and public enterprises. The reason is that all these entities become fungible if a crisis arises. This approach also assumes that all balance sheet items, both assets and liabilities, as well as their risks are measured precisely. To do this, it is necessary to measure not only the sovereign's financial wealth, but also its human and natural capital. Similarly, a relatively precise view of the government's economic objectives and an accurate estimate of contingent liabilities are also needed. A sovereign ALM strategy can thus be developed for managing asset risks in a way that is consistent with the sovereign entity's liabilities. One significant application of this analytical framework is the management of financial wealth under direct state control. Current practice consists in increasing the number of these entities – many countries now have more than two SWFs, foreign exchange reserves managed by the central bank, and in some cases a social security fund and a public

pension fund – and separating them according to discrete objectives[20] (Eaton and Ming (2010)). But, as the example of the recent crisis has shown, it is illusory to consider them as independent. An integrated sovereign ALM would necessitate important coordination between the various sovereign entities, both on the asset side (central bank, sovereign wealth fund) and on the liability side (debt management office, ministry of finance).

Two important and natural extensions of this work should be considered in the future. First, in our simplified framework, all items in the sovereign balance sheet have been treated as exogenous. But future GDP growth, tax revenue and liabilities are all endogenous. Governments have a large influence over their future path. Introducing some endogeneity in the processes would make it more realistic. Second, the issue of liabilities having a "default" boundary is also a strong assumption for sovereigns, subject to sovereign immunity. Repudiation, or financial repression (Reinhart and Sbrancia (2011)) are options for a sovereign unavailable to an individual or a corporate.

Practical application of this approach, which we present in an extremely simplified form in our Chilean example, still presents a number of difficulties. Conventional macroeconomic data are ill-suited to this type of analysis because they lack a significant dimension, namely risk. Moreover, intangible assets such as human and natural capital are difficult to take into account. The initiative taken in 2000 by World Bank to measure these components of national wealth calls for further development. We have taken a first step in this direction by offering a methodology for making a simplified

[20] In some cases, governments create rivalry between state investment bodies as a means to promote value-creation.

estimate of Chile's sovereign balance sheet, as well as a way of allocating assets that takes into account the risks on both the asset and the liability side.

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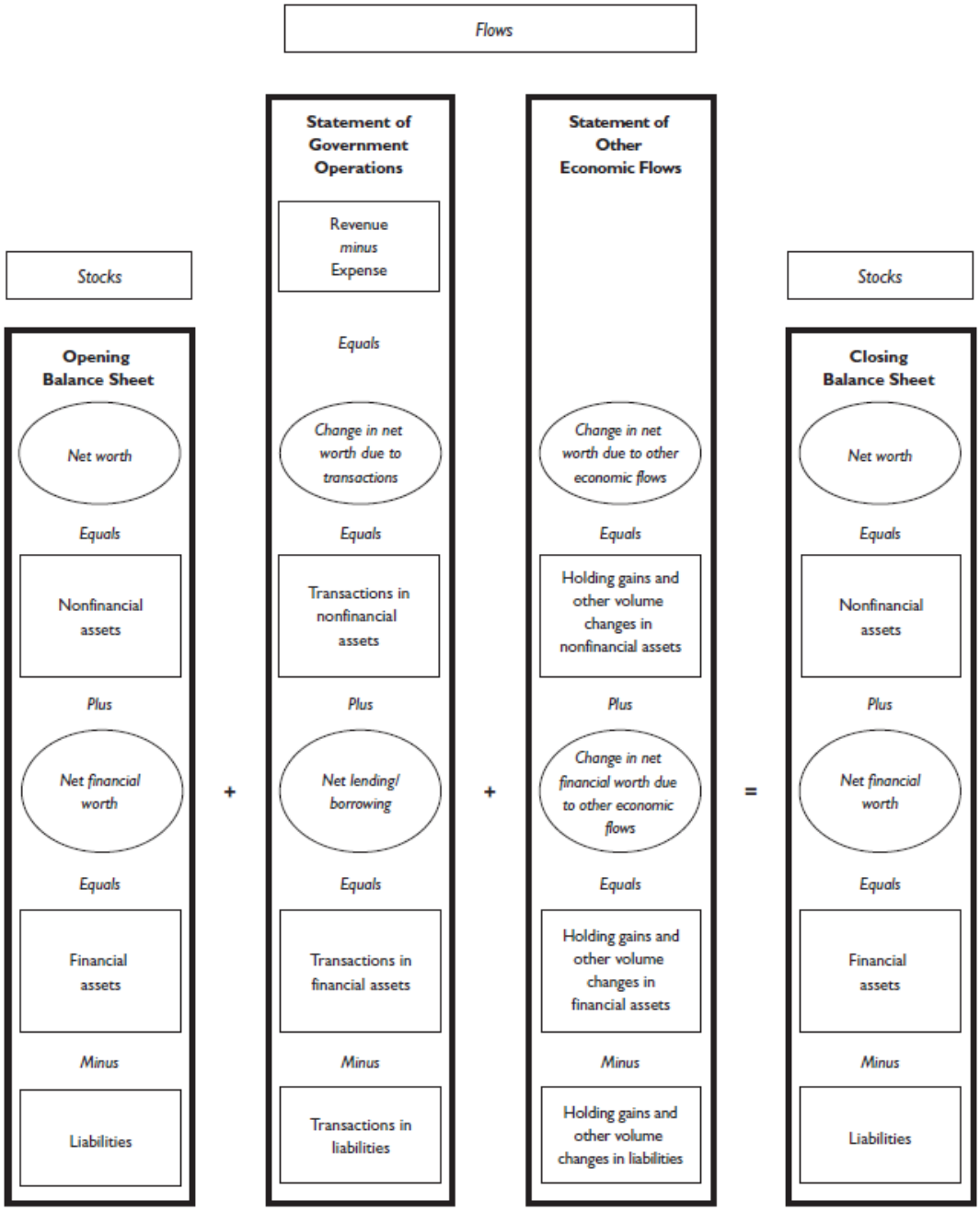
Appendix A

Figure A1: Structure of the IMF GFS Analytic Framework



Source: GFS Manuel (IMF 2001)

Table A2: Structure of IMF GFS Government Balance Sheet data

ASSETS	LIABILITIES
Non financial assets - fixed assets - inventories - valuables - nonproduced assets	Domestic - currency and deposits - securities other than shares - loans - shares and other equity - insurance technical reserves - financial derivatives - other accounts payable Foreign - currency and deposits - securities other than shares - loans - shares and other equity (public corporations) - insurance technical reserves - financial derivatives - other accounts payable NET WORTH (balancing item)
Financial assets Domestic - currency and deposits - securities other than shares - loans - shares and other equity (public corporations) - insurance technical reserves - financial derivatives - other accounts receivable Foreign - currency and deposits - securities other than shares - loans - shares and other equity - insurance technical reserves - financial derivatives - other accounts receivable Monetary gold and SDR	

Table A3: Structure of IMF GFS Government Operations Table

REVENUES	EXPENSES
- Taxes - Social contributions - Grants - Other revenue	- Compensation of employees - Use of goods and services - Consumption of fixed capital - Interest - Subsidies - Grants - Social benefits - Other expense NET OPERATING BALANCE= Net acquisition of non financial and financial assets – Net incurrence of liabilities (balancing item)

Table A4: Structure of IMF GFS Other Economic Flows Table

REVENUES	EXPENSES
<u>Non financial assets</u> - Holding gains - Other volume changes <u>Financial assets</u> - Holding gains - Other volume changes	Liabilities - Holding gains - Other volume changes

Appendix B

Table B1: Benchmark composition of the ESSF and PRF

Benchmark Composition	Percentage of Total Portfolio
Money Market	**30.00%**
Merrill Lynch 6 Month Average	15.00%
Merrill Lynch Treasury Bills Index	15.00%
Nominal Sovereign Bonds	**66.50%**
Barclays Capital Global Treasury: U.S. Bond Index	31.50%
Barclays Capital Global Treasury: Germany Bond Index	28.00%
Barclays Capital Global Treasury: Japan Bond Index	7.00%
Inflation-Indexed Sovereign Bonds	**3.50%**
Barclays Capital Global Inflation-Linked: U.S. TIPS Index 1-10 years	3.50%

Data provided by the Chilean Ministry of Finance, 2010.

Table B2: Composition of Foreign Exchange Reserves by Portfolio and Currency

	Composition			
Type of Portfolio	USD	EUR	Others	Total
Investment Portfolio	46.8%	25.5%	1.0%	73.3%
Liquidity Portfolio	20.4%	0.0%	5.1%	25.5%
Other assets	-	-	-	1.2%

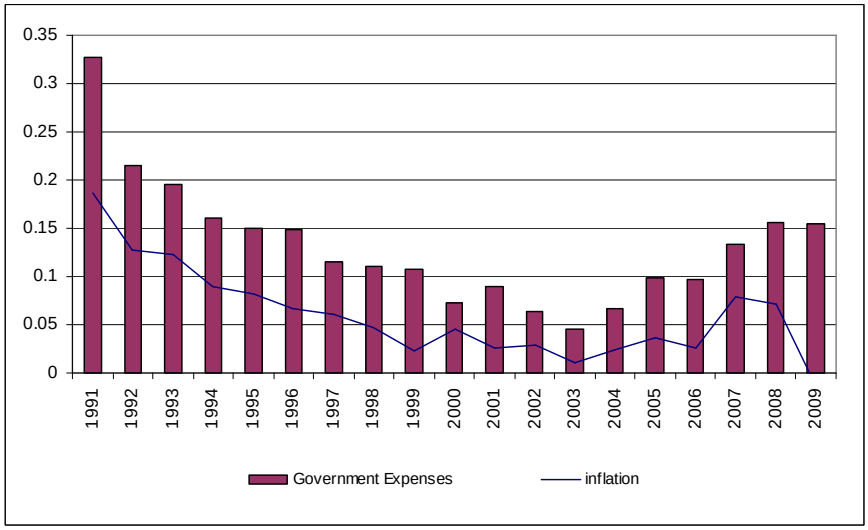
Data provided by the Central Bank of Chile, 2010.

Table B3: Estimation of Chile Balance Sheet, 2008

ASSETS (bn $)			**LIABILITIES (bn $)**		
Balances of:	Assets	Volatility	***Balances of:***	Liabilities	Volatility
INVESTMENTS	**22.7**	**4.8%**	**FOREIGN CURRENCY GVT DEBT**	**2.0**	**6.6%**
Stabilization Fund (ESSF)	20.2		(senior claim)		
Pension Reserve Fund (PRF)	2.5				
CURRENCY AND OTHER RESERVES	**23.2**	**3.8%**	**MONETARY BASE + LOCAL CURRENCY GVT DEBT** (junior claim)	**146.1**	**10.1%**
PRESENT VALUE OF FISCAL SURPLUS - GUARANTEES TO BANKS AND NON BANKS	**102.2**				
TOTAL ASSETS	**148.1**	**10.1%**	**TOTAL LIABILITIES**	**148.1**	**10.1%**

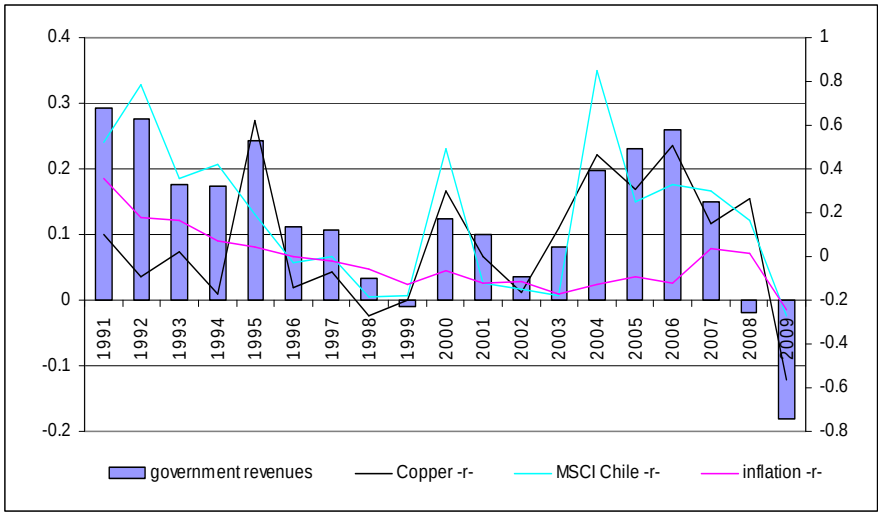
Appendix C

Figure C1: Annual Growth of Government Expenses and Annual Inflation in Chile, 1991-2009



Government Expenditures are provided by the Chilean Ministry of Finance, inflation is measured by the yearly change in headline CPI.

Figure C2: Annual Growth of Government Revenues and Annual Inflation, Copper and Equity returns in Chile, 1991-2009



Government revenues are provided by the Chilean Ministry of Finance, inflation is measured by the yearly change of headline CPI, the equity market is the MSCI Chile (total return including dividends).

Table C3: Results of Annual Regression of Expenses and Revenues Growth Rate on Inflation, Copper and Equity Returns, Chile, 1991-2009

	Expenses	Revenues
c	0.06*** (4.81)	0.03 (1.35)
$\beta_{inflation}$	1.18*** (7.04)	1.01 (1.50)
β_{copper}	-	0.17** (2.72)
β_{equity}	-	0.10 (1.51)
R^2	74.5%	73.2%
$AdjR^2$	73.0%	67.9%
SEE	0.034	0.068

***,**, * *significant respectively at the 1%, 5% and 10% level.*

Table C4 : Descriptive Statistics of Monthly Returns of Assets and Liabilities, Chile Balance Sheet, August 2000-December 2010

	USD	EUR	JPY	Emg Eqty	Dvp Eqty	Emg Bond	Dvp Bond	World IL Bond	Fiscal Surplus	External Debt	Local Debt
Ann Mean	4.40%	8.69%	3.76%	15.41%	3.63%	10.33%	7.44%	8.07%	12.78%	8.08%	6.60%
Median	0.36%	0.58%	0.02%	1.44%	0.94%	1.23%	0.46%	0.88%	1.30%	0.71%	0.59%
Maximum	1.72%	10.37%	8.47%	17.85%	11.91%	8.52%	7.33%	7.41%	13.82%	6.39%	7.79%
Minimum	-0.97%	-8.82%	-8.14%	-27.66%	-19.37%	-13.79%	-4.94%	-11.74%	-14.35%	-7.97%	-17.59%
Volatility	1.65%	11.20%	9.87%	25.00%	17.34%	10.39%	7.60%	8.58%	15.80%	6.64%	11.89%
Skewness	0.11	0.07	-0.26	-0.67	-0.71	-1.01	0.01	-0.96	-0.22	-0.59	-1.22
Kurtosis	3.41	3.82	3.40	4.29	4.25	7.32	3.20	7.43	4.51	6.84	8.04

Table C5 : Correlation Matrix between Assets and Liabilities, Chile Balance Sheet, August 2000-December 2010

	USD	EUR	JPY	Emg Eqty	Dvp Eqty	Emg Bond	Dvp Bond	World IL Bond	Fiscal Surplus	External Debt	Local Debt
USD	100%										
EUR	40%	100%									
JPY	32%	30%	100%								
Emg Eqty	-15%	35%	-9%	100%							
Dvp Eqty	-21%	38%	-5%	89%	100%						
Emg Bond	21%	38%	7%	62%	57%	100%					
Dvp Bond	60%	85%	65%	15%	16%	37%	100%				
World IL Bonds	52%	83%	32%	38%	36%	53%	83%	100%			
Fiscal Surplus	6%	-2%	-3%	13%	4%	20%	1%	10%	100%		
External Debt	57%	50%	6%	17%	12%	54%	57%	65%	9%	100%	
Local Debt	5%	35%	3%	55%	51%	56%	25%	39%	19%	31%	100%

Figure C6: Efficient Frontier, GSS Expected Return and Volatility tradeoff, August 2000-December 2010

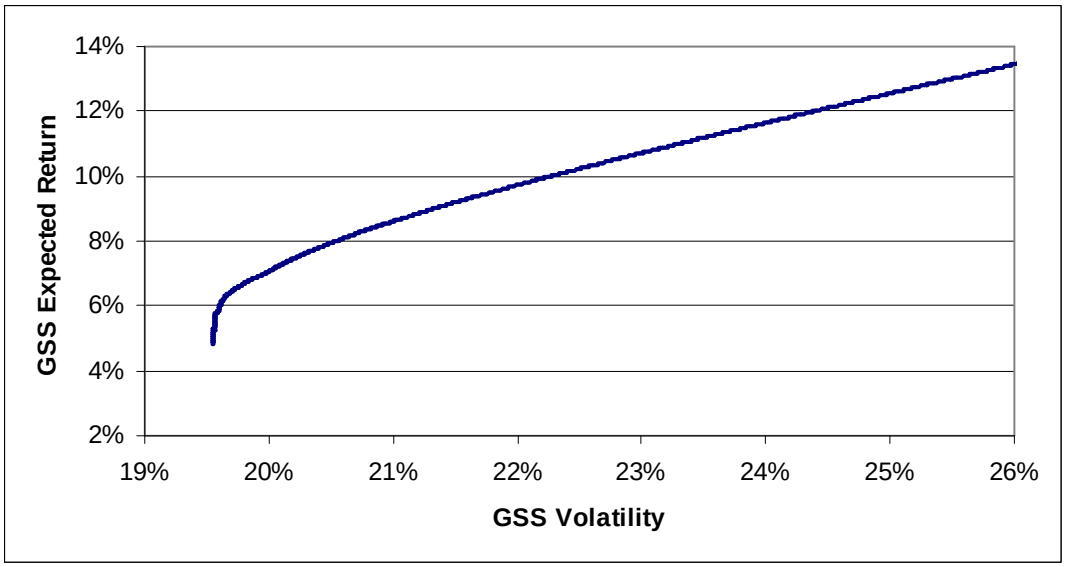


Table C7: Optimal portfolios for Chile, August 2000-December 2010

	Min Vol	GSS return=6%	GSS return=6.96%
Ann. Mean	4.83%	6.00%	6.96%
Median	0.51%	0.62%	0.44%
Maximum	12.14%	12.45%	12.74%
Minimum	-11.48%	-11.43%	-11.01%
Volatility	13.24%	13.31%	13.79%
Skewness	0.00	0.00	0.00
Kurtosis	3.92	3.98	3.83
		Weights	
USD	0%	0%	0%
EUR	27%	25%	0%
JPY	0%	0%	0%
Emg Eqty	24%	48%	100%
Dvp Eqty	38%	1%	0%
Emg Bond	11%	26%	0%
Dvp Bond	0%	0%	0%
World IL Bond	0%	0%	0%